Nuanced Robustness Analysis with Limited Information

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Abstract

This paper presents a nuanced robustness analysis for structures when only limited information is available. A new methodology based on fuzzy set theory is proposed to cope with scarce information as a major problem in the performance assessment of existing structures. The developed robustness measure provides information on the relationship between structural robustness and the magnitude of uncertainty in the damage of the structure. This feature is enabled through a nuanced consideration of imprecision in the damage assessment via alpha-level discretization. An entropy-based robustness measure is formulated as a function of imprecision in the damage state. On this basis different design solutions can be compared, in a one-swoop analysis, with respect to their robustness for different magnitudes of damage. This approach can, further, be used to assess effort for inspection versus gain in precision of the predicted structural performance. The development is of a general nature. Herein, it is elucidated in the context of a typical offshore en-

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gineering problem in order to demonstrate its application in practical cases. Fixed offshore platforms with different brace configurations are compared in view of robustness with respect to damage from corrosion.
Keywords: Robustness assessment; Imprecision; Entropy measure; Fuzziness; Alpha-level discretization; Marine corrosion; Fuzzy modeling

1. Introduction

A nuanced robustness assessment is developed to compare structural design solutions regarding their performance in dependence on the magnitude of uncertainy in the assessed damage of the structures. To address the particular relevance of the development to current industrial challenges, we focus on offshore structures under only vaguely known corrosion damage. In this context, robustness is a measure to assess a jacket structure's ability to sustain damage with a limited loss of ultimate capacity and, therefore, reliability [1]. A "robust" structure has inherent redundancies in terms of alternative load paths that allow the structure to withstand global damage caused by various events such as ship impact, extreme storms, explosions, etc. For less robust structures, however, a small damage event may significantly diminish the platform's global capacity resulting in a high-risk situation which requires immediate response such as platform de-manning, platform shutdown, or emergency repair. Robustness consideration in this context usually aims to mitigate the risk from disproportionate failure or progressive collapse due to damage caused by extreme loads or accidental loads. In the literature, robustness of fixed offshore platforms is usually evaluated through the ultimate strength analysis of structures in both intact and damaged states, which leads to a number of deterministic performance measures using the concept of reserve strength and residual strength, see [2]. The prescribed damage scenarios are frequently associated with removal of one critical member or several members in the intact state, see [3]. However, there are other sources of damage that, in contrast to damage suddenly provoked by accidental actions, arise gradually in time from aging of structures and may also involve disproportionate effects, including marine corrosion, see [4]. Besides the deterministic performance measures, the inevitable uncertainty in engineering practice has led to the development of probabilistic robustness measures based on reliability and risk analysis of structures, see [5, 6, 7]. A brief review of these measures is given in Section 2.

Robustness can also be understood as a structure's capacity to withstand the normal fluctuations of environmental conditions without noticeable effects on its serviceability. In this context, robustness denotes a high degree of independence between the uncertainty of structural parameters and the associated uncertainty in structural responses. Assessments of this type of robustness are devoted to obtaining global statements about the degree of structural response variation with respect to input fluctuations at once. Commonly, all uncertain parameters are described as random variables, which enables the application of probabilistic measures to assess structural robustness. As pertinent developments in robustness assessment in this context rely heavily on probabilistic models, a proper treatment of uncertainty is of vital importance for this point of view in understanding robustness. This includes the characterization of the deterioration of structural strength due to marine corrosion, which has adverse effects on the safety of offshore structures. The corrosion effects on the reliability of offshore structures has been studied in [8] where the probabilistic corrosion model from [9] for mild steel immersed in seawater was adopted to estimate the uncertainty in the corrosion depth for a relatively short period.

In engineering applications, the knowledge about the fluctuations of the structural parameters can be quite limited so that a clear probabilistic specification of uncertainty can be problematic. This is associated with rare and imprecise data. Examples are uncertain quantities for which mere bounds or linguistic expressions are known. For this type of information, alternative, non-probabilistic and mixed models provide reasonable properties [10]. Measures for the associated information content are available [11]. The usefulness and capabilities of these models and approaches, such as interval analysis, fuzzy set theory, evidence theory, imprecise probabilities and fuzzy random variables, have already been demonstrated in the solution of practical problems in civil and mechanical engineering [12, 8, 13, 14]. For the envisaged development the concept of fuzziness is selected to cater for the subjective character of the assessment of deterioration due to imprecise marine corrosion. This selection is motivated by the growing demand for quick structural performance assessments based on quite limited information from coarse inspections without detailed measurements as quantification basis. In such cases, the available information does not provide a proper basis for a probabilistic modeling but can still be sufficient to derive reasonable decisions when it is coarsely translated into effects on structural performance.

In this paper, we propose a nuanced robustness assessment based on fuzzy set theory and an assessment of fuzziness using an analog to SHANNON's entropy [15]. We tap on the robustness measure proposed in [16] and expand the concept from assessing robustness with a static number to the formulation of a robustness function depending on the magnitude of uncertainty in the structural conditions. This function enables the exploration of dependencies between structural robustness and the magnitude of uncertainty in the structural damage, which opens a new kind of insight into the structural problem and may facilitate a trade-off assessment for inspection effort versus gain in confidence for performance, safety and robustness predictions. The developments are elucidated by means of robustness asessment of aging offshore structures under marine corrosion with reference to the data provided in [17]. In the sequel, robustness measures from the literature are reviewed in Section 2. Section 3 is devoted to the development of the proposed nuanced robustness assessment. The usefulness of the proposed method is demonstrated in Section 4 by way of investigations of fixed offshore platforms with different brace configurations.

2. Review of Robustness Measures

2.1. Deterministic performance measures

Robustness is a measure to assess a platform's ability to sustain damage caused by extreme loads or accidental loads without disproportionate failure with respect to the causes of the damage itself. According to this understanding of structural robustness, deterministic performance measures are developed through comparing the structural performance in both intact and damaged states based on ultimate strength analysis. For the investigated frame structures, the ultimate strength depends on the nonlinear response of components of the frame and the nonlinear structural interaction between components through plastic deformation and load redistribution. The frames with different bracing configurations have different overall structural performance, usually described as "brittle" or "ductile" behavior. The concept of reserve strength and residual strength can be used to evaluate structural robustness associated with the ultimate conditions. The following three deterministic performance measures have been tested for a range of structural frames in [2].

Reserve strength can reflect the ability of an intact structure to sustain loads in excess of the design value. The Reserve Strength Ratio (RSR) is defined as

$$\mathrm{RSR} = \frac{\text{ultimate resistance of intact structure}}{\text{design environmental load}} . \tag{1}$$

Similarly, the Damage Strength Ratio (DSR) is defined to measure the ability of a damaged structure to sustain loads in excess of the design value,

$$\mathrm{DSR} = \frac{\text{ultimate resistance of damaged structure}}{\text{design environmental load}} . \tag{2}$$

The residual strength reflects the ability of having alternative load paths to carry loads shed from damaged members (i.e. redundancy). The Residual Resistance Factor (RRF) is defined as

$$\mathrm{RRF} = \frac{\text{ultimate resistance of damaged structure}}{\text{ultimate resistance of intact structure}} . \tag{3}$$

In addition, because the value of the residual strength corresponds to a particular displacement and different values may be achieved if the load is increased further, the following non-dimensional measure R_{twice} can be utilized when comparing structures with different brace configurations,

$$\mathrm{R}_{twice} = \frac{\text{environmental load at twice the ultimate deflection}}{\text{environmental load at ultimate deflection}}. \tag{4}$$

As previously pointed out that damage could also arise gradually in time from aging of structures, a general approach is presented in [18] to formulate a measure of time-variant structural robustness of concrete structures subjected to diffusive attacks from environmental aggressive agents based on the ultimate strength analysis. The amount of local damage is firstly obtained at the member level by means of a dimensionless damage index $0 \leq \delta \leq 1$ (for uniform corrosion c and original material thickness d, $\delta = \frac{c}{d}$) associated with the progressive deterioration of the material properties for steel bars $\delta_s(x,t)$ and concrete $\delta_c(x,t)$ at the spatial point x and time instant t. Then a global measure of damage $\Delta(t)$ at the cross-sectional level is evaluated by means of a weighted average of the local damage over the volume of the materials, as follows:

$$\Delta(t) = [1-\omega(t)]\Delta_c(t) + \omega(t)\Delta_s(t) \tag{5}$$

$$\Delta_c(t) = \frac{\int_{A_c} w_c(x,t)\delta_c(x,t)dx}{\int_{A_c} w_c(x,t)dx} \tag{6}$$

$$\Delta_s(t) = \frac{\sum_m w_{sm}(x,t)\delta_{sm}(x,t)A_{sm}}{\sum_m w_{sm}(x,t)A_{sm}} \tag{7}$$

where $\omega(t)$, $w_c(x,t)$, $w_{sm}(x,t)$ are weight functions (see [18]), A_c is the area of the concrete, and the A_{sm} is the area of the m^{th} steel bar. This cross-section formulation is finally extended at the structural level by an integration over all members of the system. By comparing the system performance in the intact state and in a damaged state, the time-variant measure of structural performance $\rho(t)$ is derived as,

$$\rho(t) = \frac{\lambda_c(t)}{\lambda_c(0)} \tag{8}$$

where the limit load multiplier $\lambda_c(t)$ corresponds to the ultimate capacity in a damaged state, and its initial value $\lambda_c(0)$ indicates the ultimate capacity in the intact state. Then, the structural robustness can be evaluated based on the relationship between $\rho(t)$ and the global damage $\Delta(t)$. In this approach damage can be defined in any way and gradually. The lambda reflects (quantifies) the degree of damage in the load carrying capacity through the structural analysis indirectly.

2.2. Probabilistic robustness measures

In order to take account of the unavoidable uncertainties in the environmental loading and structural resistance, probabilistic robustness measures have been developed based on either reliability analysis or risk assessment.

Based on system reliability analysis, the probabilistic measure of redundancy R_β is proposed in [5]

$$R_\beta = \frac{\beta_{\text{intact}}}{\beta_{\text{intact}} - \beta_{\text{damaged}}}\,, \tag{9}$$

where β_{damaged} is the reliability index of the damaged structural system and β_{intact} is the reliability index of the intact system. Similarly, a probabilistic measure called "damage factor" of a system was proposed in [7] as

$$R_{\text{d}} = \frac{P_{f,\text{intact}}}{P_{f,\text{damaged}}} \tag{10}$$

to assess its capacity to withstand damage without undesirable response. $P_{f,\text{damaged}}$ and $P_{f,\text{intact}}$ are the failure probabilities corresponding to damage and no damage in the system, respectively.

A framework of robustness assessment based on decision analysis theory has been proposed in [19], where the robustness is evaluated by computing both direct risk (R_{Dir}), which is associated with the direct consequences (C_{Dir}) of potential damages (D) to the system when an exposure (EX_{BD}) occurs, and indirect risk (R_{Ind}), which corresponds to indirect consequences (C_{Ind}) associated with subsequent system failure (F). A quantitative measure of robustness is then defined as,

$$\mathrm{I}_{Rob} = \frac{\mathrm{R}_{Dir}}{\mathrm{R}_{Dir} + \mathrm{R}_{Ind}}, \quad with \tag{11}$$

$$\mathrm{R}_{Dir} = \int_x \int_y \mathrm{C}_{Dir} f_{\mathrm{D}|\mathrm{EX}_{BD}}(y|x) f_{\mathrm{EX}_{BD}}(x) \mathrm{d}y \mathrm{d}x \tag{12}$$

$$\mathrm{R}_{Ind} = \int_x \int_y \mathrm{C}_{Ind} P(F|D = y) f_{\mathrm{D}|\mathrm{EX}_{BD}}(y|x) f_{\mathrm{EX}_{BD}}(x) \mathrm{d}y \mathrm{d}x \tag{13}$$

where $f_Z(z)$ is the probability density function of a random variable Z.

2.3. Entropy-based robustness measures

For uncertainty specified with the aid of fuzzy sets, as investigated in this study, an entropy-based robustness measure $R(\cdot)$ as proposed in [16] is useful. This is based on an analog to SHANNON's entropy. This analog to SHANNON's entropy provides features, which make uncertainties calculated for random variables and fuzzy variables somewhat related to one another.

In classical probability theory, SHANNON's entropy is a measure of the amount of uncertainty and the associated information, see [11]. Information comprises the elements x selected from a declared character set representing the fundamental set $\mathbf{X}$. The SHANNON's entropy H can be expressed by a probability distribution function $P(x)$ on a finite set using a functional of the form

$$H = -\sum_{x \in \mathbf{X}} P(x) \log_2 P(x) \ . \tag{14}$$

And for the case of an infinite set,

$$H = -\int_{-\infty}^{+\infty} f(x) \cdot \log_2 f(x) \mathrm{d}x \tag{15}$$

applies. In fuzzy set theory, for assessing the fuzziness of the fuzzy set $\widetilde{A}$ on $\mathbf{X}$, the functional values of the membership function $\mu(x)$ of $\widetilde{A}$ are applied as measure values of the elements. An entropy measure of fuzziness $H(\widetilde{A})$ analog to Shannon's entropy is introduced in [15] as

$$H(\widetilde{A}) = -k \cdot \int_{-\infty}^{+\infty} [\mu(x) \cdot \ln(\mu(x)) + (1 - \mu(x)) \cdot \ln(1 - \mu(x))] \mathrm{d}x \, . \tag{16}$$

The coefficient k is introduced when transforming the dyadic logarithm in the Shannon's entropy in Eq. (15) into the natural logarithm in Eq. (16). That is, $log_2(\mu(x)) = k \cdot \ln(\mu(x))$ and $k = \frac{1}{\ln(2)}$. Since entropies appear as ratios in our approach, k is cancelled out and does not have any influence. The entropy in Eq. (16) has the following properties:

- $H(\widetilde{A}) = 0$ if $\mu(x) = 0$ or $\mu(x) = 1.0$ for all x;
- $H(\widetilde{A})$ reaches maximum if $\mu(x) = 0.5$ for all x;
- If $\widetilde{A}_i$ is any sharpened version of $\widetilde{A}_j$ (that is, if $\mu_{A_j}(x) \leq 0.5$, then $\mu_{A_i}(x) \leq \mu_{A_j}(x)$, and if $\mu_{A_j}(x) \geq 0.5$, then $\mu_{A_i}(x) \geq \mu_{A_j}(x)$), then $H(\widetilde{A}_i) \leq H(\widetilde{A}_j)$;
- The symmetry property holds, i.e. $H(\widetilde{A}) = H(\widetilde{A}^c)$ where $\widetilde{A}^c$ is the complement of $\widetilde{A}$ and defined as $\widetilde{A}^c = \{(x, \mu_{A^c}(x)) | x \in \mathbf{X}; \mu_{\mathbf{A^c}}(\mathbf{x}) = \mathbf{1} - \mu_{\mathbf{A}}(\mathbf{x})\}$.

For example, $H(\tilde{z}_j)$ of the fuzzy sets $\tilde{z}_j$ shown in Fig. 3(b) are $H(\tilde{z}_1) = 0.72$, $H(\tilde{z}_2) = 0.60$, $H(\tilde{z}_3) = 0.60$, $H(\tilde{z}_4) = 0.30$ and $H(\tilde{z}_5) = 0.30$. These examples represent common shapes of membership functions in practical applications. Generally, this entropy measure evaluates the "steepness" of the membership function $\mu(x)$, which indicates $H = 0$ for a crisp set and H is maximum if $\mu(x) = 0.5$. In information theory the elements with $\mu_A(x) = 0.5$ represent the most interesting range of a fuzzy set $\widetilde{A}$ because $\mu(x) = 0.5$ characterizes the highest uncertainty in the decision to consider the associated element x either as belonging to $\widetilde{A}$ or as not belonging to $\widetilde{A}$.

The derivation of Eq. (16) from a probabilistic basis in information theory ensures reasonable compliance with probabilistic uncertainty measures. Let X be a random variable with normal distribution, and its uncertainty be measured in terms of the standard deviation σ_X. If the cumulative distribution function (CDF) $F_X(x)$ is substituted in Eq. (16) for the membership function $\mu(x)$, then a change of the standard deviation σ_X is associated with a proportional change of the entropy H. For example, let $X_i \sim (\mu_X, \sigma^2_{X_i})$ and $X_j \sim (\mu_X, \sigma^2_{X_j})$ be two normal random variables with $\sigma_{X_j} = 2\sigma_{X_i}$. If $\widetilde{A}_i$ and $\widetilde{A}_j$ are two fuzzy sets with their membership functions having values as the CDF of X_i and X_j, i.e., $\mu_{A_i}(x) = F_{X_i}(x)$ and $\mu_{A_j}(x) = F_{X_j}(x)$, then $H(\widetilde{A}_j) = 2H(\widetilde{A}_i)$.

According to [16], the robustness of a structural system $R(\cdot)$ can be defined as the ratio between the entropy of input parameters $\tilde{x}$ and the entropy of associated structural responses $\tilde{z}$ when the uncertainty of structural parameters is quantified as fuzziness,

$$R(\tilde{x}, \tilde{z}) = \frac{H(\tilde{x})}{H(\tilde{z})} \,. \tag{17}$$

And the following properties hold,

- $R(\cdot) \geq 0 \; \forall H(\tilde{x}),\; H(\tilde{z}) > 0\,;$
- $H(\tilde{z}_2) \leq H(\tilde{z}_1) \Rightarrow R_2(\cdot) \geq R_1(\cdot) \mid H(\tilde{x}_1) = H(\tilde{x}_2)\,;$
- $H(\tilde{x}) \to 0 \Rightarrow R(\cdot) \to 0 \mid H(\tilde{z}) > 0\,;$
- $H(\tilde{z}) \to 0 \Rightarrow R(\cdot) \to \infty \mid H(\tilde{x}) > 0\,.$

This robustness measure results in a global statement about the degree of variations in system output with respect to fluctuations in system input at once. The second property indicates that the smaller the uncertainty of the fuzzy outputs is obtained in relation to the uncertainty of the fuzzy inputs, the bigger the robustness of the structures is assessed. In practice, this means that moderate changes applied to structural parameters, for example as changes of design parameters or through quite common deviations from plans and moderate errors, affect the structural response (i.e. the structural performance) only marginally. Further, in the case that result uncertainty occurs even for crisp input, which represents instabilities, the robustness is zero. Robustness is not defined for the case that both the input uncertainty and the result uncertainty are zero. For further detailed explanations we refer to [16].

3. Nuanced Robustness Analysis

While deterministic performance measures and probabilistic robustness measures assess a structure under given conditions and uncertainties, the entropy-based robustness measure considered in Section 2.3 provides a potential to develop a nuanced robustness assessment in form of a robustness function depending on the magnitude of uncertainty in the structural conditions. Hence, a trade-off assessment for inspection effort versus gain in confidence for performance, safety and robustness predictions can be developed. We develop this nuanced robustness assessment based on the general idea of an entropy-based robustness measure.

3.1. Practical considerations

3.1.1. Applicability of entropy measure

One (first) problem that has been addressed by Section 2.3 is associated with the applicability of the entropy measure when considering the difference between an interval variable and a singleton. Mathematically, they have the same entropy values (i.e., $H = 0$), but it is counterintuitive as the interval possesses clearly a larger imprecision.

A similar (second) problem arises, as shown in Fig. 1, when the fuzzy output $\tilde{z}_1$ associated with the fuzzy input $\tilde{x}$ for system (1) and the fuzzy output $\tilde{z}_2$ associated with the same fuzzy input $\tilde{x}$ for system (2), have similar entropy values but quite different width of the system output at various membership levels with respect to same degrees of imprecision in the fuzzy input, i.e. $w(z_{1,\alpha_k}) > w(z_{2,\alpha_k})$. For example, mapping of the same fuzzy input $\tilde{x}$ in Fig. 3(a) through two systems (functions $f_2(x)$ and $f_3(x)$) gives two fuzzy outputs $\tilde{z}_2$ and $\tilde{z}_3$ in Fig. 3(b), with $H(\tilde{z}_2) = H(\tilde{z}_3) = 0.60$ but $w(z_{3,\alpha_k=0.5}) = 0.94 > 0.29 = w(z_{2,\alpha_k=0.5})$.

A third problem of the enropy measure, and hence of the robustness measure, concerns its dependence on scale and transformations. Since it is not invariant in this sense, an interpretation on the ratio scale is critical.

3.1.2. Suggested assumptions and extensions

The first problem is circumvented in this study; we assume that typical shapes of membership functions, such as triangular or quadratic, are adopted for the fuzzy inputs (as usually used in practical cases) and that the associated fuzzy outputs are obtained as fuzzy sets as well and not as precise numbers. A polygonal approximation of the shape of the fuzzy outputs using only a few membership levels is sufficient in most cases. If the analysis indicates strong nonlinearities in the membership functions, a more detailed alpha-discretizitation may be useful. Associated with this assumption, another restriction is made; the fuzzy variables $\tilde{x}$ considered herein possess one element $x \in \tilde{x}$ with $\mu(x) = 1$.

Next, the second problem raised above is considered. Robustness assessment based on the robustness measure in Eq. (17) leads to the same results for system (1) and system (2), $H(\tilde{z}_1) \approx H(\tilde{z}_2)$, i.e. to the conclusion that system (1) is as robust as system (2). However, this conclusion is only limited to a global view at the robustness of the two systems without reflection of the degree of independence between the imprecision of fuzzy inputs and the associated imprecision of fuzzy outputs at different membership levels. To implement this relationship between the α-level sets, the assessment from [16] is modified by utilizing alpha-level discretization as proposed in Section 3.2. This enables a consideration of a trade-off between additional information and an associated reduction of imprecision in the predicted structural response or reliability. Additional information and reduction of input imprecision can be understood as limitation of the analysis to the set of values

$\{x \in \mathbf{X} | \mu(x) \geq \alpha_k\}$ for the fuzzy input $\tilde{x}$ and the associated set of values $\{z | \mu_j(z) \geq \alpha_k\}$ for the fuzzy output $\tilde{z}_j$ in the assessment of robustness. Subsequently, the two systems may not exhibit similar robustness corresponding to the reduced imprecision in the fuzzy inputs.

The third problem concerning lack of invariance is circumvented by using this robustness measure on an ordinal scale rather than on a ratio scale. Although the meaning of the ratio value obtained for the robustness of an individual structure is not meaningful, it provides a useful basis for the comparison with a second structure, evaluated for the same problem with the same input and looking the same responses, in terms of robustness. Relating the robustness measures of two structures to one another in this manner translates the assessment to an ordinal scale, which is sufficient to decide which structure is more robust than the other one - without assessing numerically how big the difference in robustness is. Still, this difference in robustness between the structures related to the absolute robustness of one of the structures provides at least a rough sense about the magnitude of this difference. One can then see whether this difference is significant or not. Although this is a reasonable basis for deriving decisions in many practical cases, further research is needed to address this issue more rigorously.

3.2. Proposed approach

3.2.1. General description

The inconsistency explained in Section 3.1, see Fig. 1, can be resolved by computing the entropy-based robustness $R(\alpha_k)$ at various membership levels with respect to the degrees of imprecision in the fuzzy inputs and the associated imprecision of the fuzzy outputs.

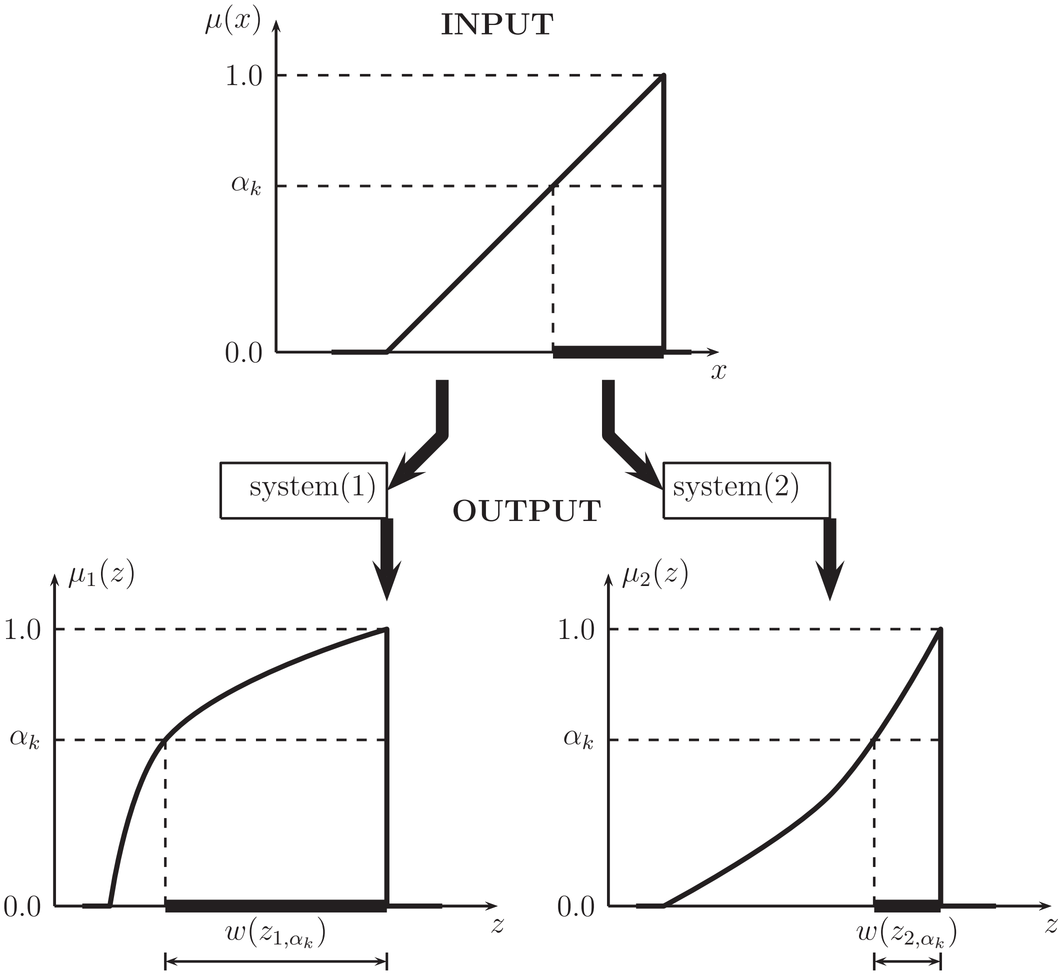


Figure 1: Illustration of problems with the existing robustness measure R

Given a fuzzy set $\tilde{A}$, at each alpha level $\alpha_k \in (0, 1]$, the crisp set A_{α_k}

$$A_{\alpha_k} = \{x \in \mathbf{X} | \mu_A(x) \geq \alpha_k\} \tag{18}$$

is called *α-level set*. Given another fuzzy set $\tilde{B}$, the *intersection* $\tilde{D}$ of the fuzzy sets $\tilde{A}$ and $\tilde{B}$ on $\mathbf{X}$ is obtained from

$$\tilde{D} = \tilde{A} \cap \tilde{B} = \{(x, \mu_D(x)) | x \in \mathbf{X}; \mu_D(x) = \min[\mu_A(x), \mu_B(x)]\}. \tag{19}$$

Since the fuzzy set theory, which permits the gradual assessment of the membership of elements in relation to a set, is a generalization of the classical set theory, the α-level set A_{α_k} can be viewed as a special fuzzy set. Thus, a new fuzzy set can be defined as the intersection of fuzzy set $\widetilde{A}$ and its α-level set A_{α_k}, denoted as $\widetilde{A}_{\alpha_k}$,

$$\widetilde{A}_{\alpha_k} = \widetilde{A} \cap A_{\alpha_k} \ , \tag{20}$$

as illustrated in Fig. 2. This concept is then applied to the fuzzy input $\tilde{x}$ and fuzzy output $\tilde{z}$ of the structural problem. The entropy-based robustness $R(\cdot)$ in Eq. (17) is calculated for each $\widetilde{A}_{\alpha_k}$ as the ratio between the entropy of $\tilde{x}_{\alpha_k} = \tilde{x} \cap x_{\alpha_k}$ of the fuzzy input $\tilde{x}$ and the entropy of $\tilde{z}_{\alpha_k} = \tilde{z} \cap z_{\alpha_k}$ of the fuzzy output $\tilde{z}$,

$$R(\alpha_k) = \frac{H(\tilde{x}_{\alpha_k})}{H(\tilde{z}_{\alpha_k})} \, . \tag{21}$$

The robustness $R(\cdot)$ in [16] is obtained as a special case of Eq. (21) for $\alpha_k = 0 + \varepsilon$ when $\varepsilon \to 0$. The robustness $R(\alpha_k)$ is not defined at $\alpha_k = 1$ because $H(\tilde{x}_{\alpha_k=1})$ and $H(\tilde{z}_{\alpha_k=1})$ are normally both equal to zero.

3.2.2. Illustrative example

The features of the modified robustness measure in Eq. (21) are demonstrated in the following illustrative example by means of analytical functions

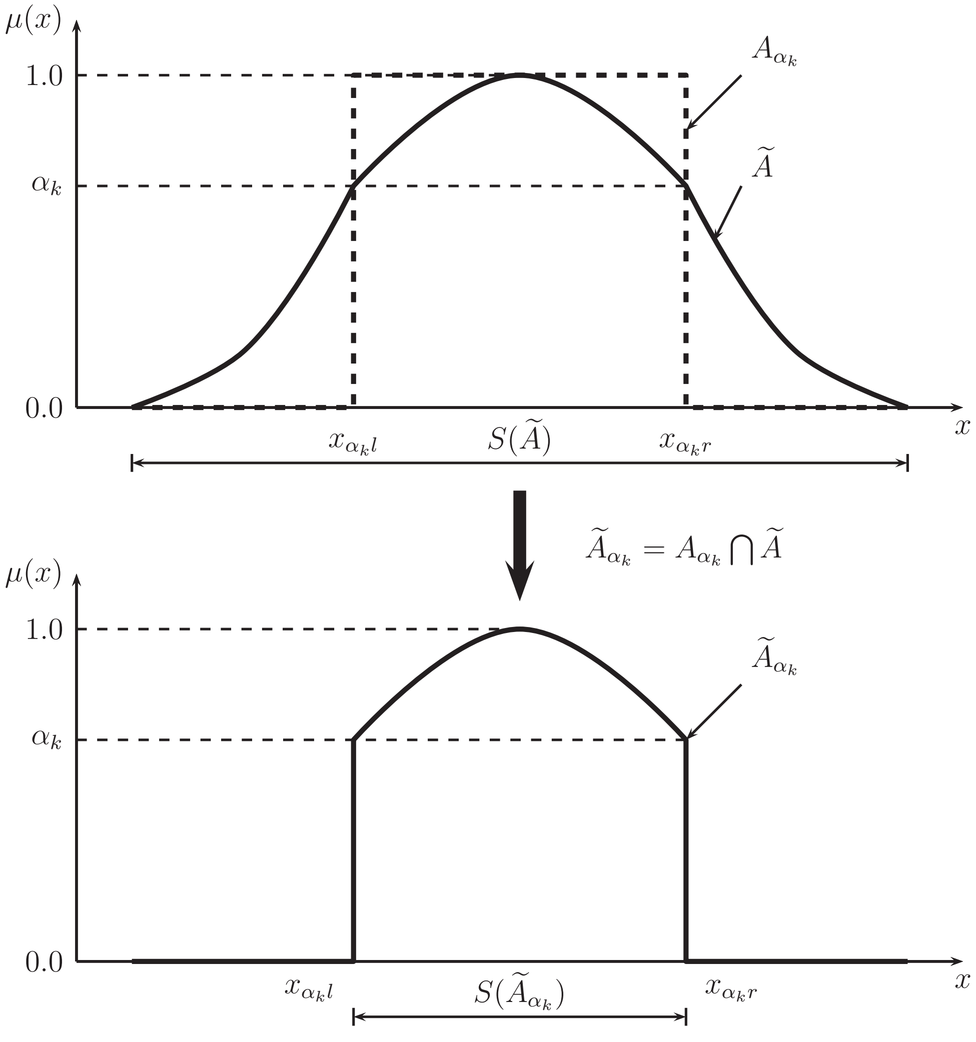


Figure 2: Intersection of the fuzzy set $\widetilde{A}$ with the α-level set A_{α_k}

specifically selected for this purpose. Consider the mapping of fuzzy input $\tilde{x}$ in Fig. 3(a) into the fundamental set $\mathbf{Z}$ with the aid of the following five mapping models $f_j(x)$:

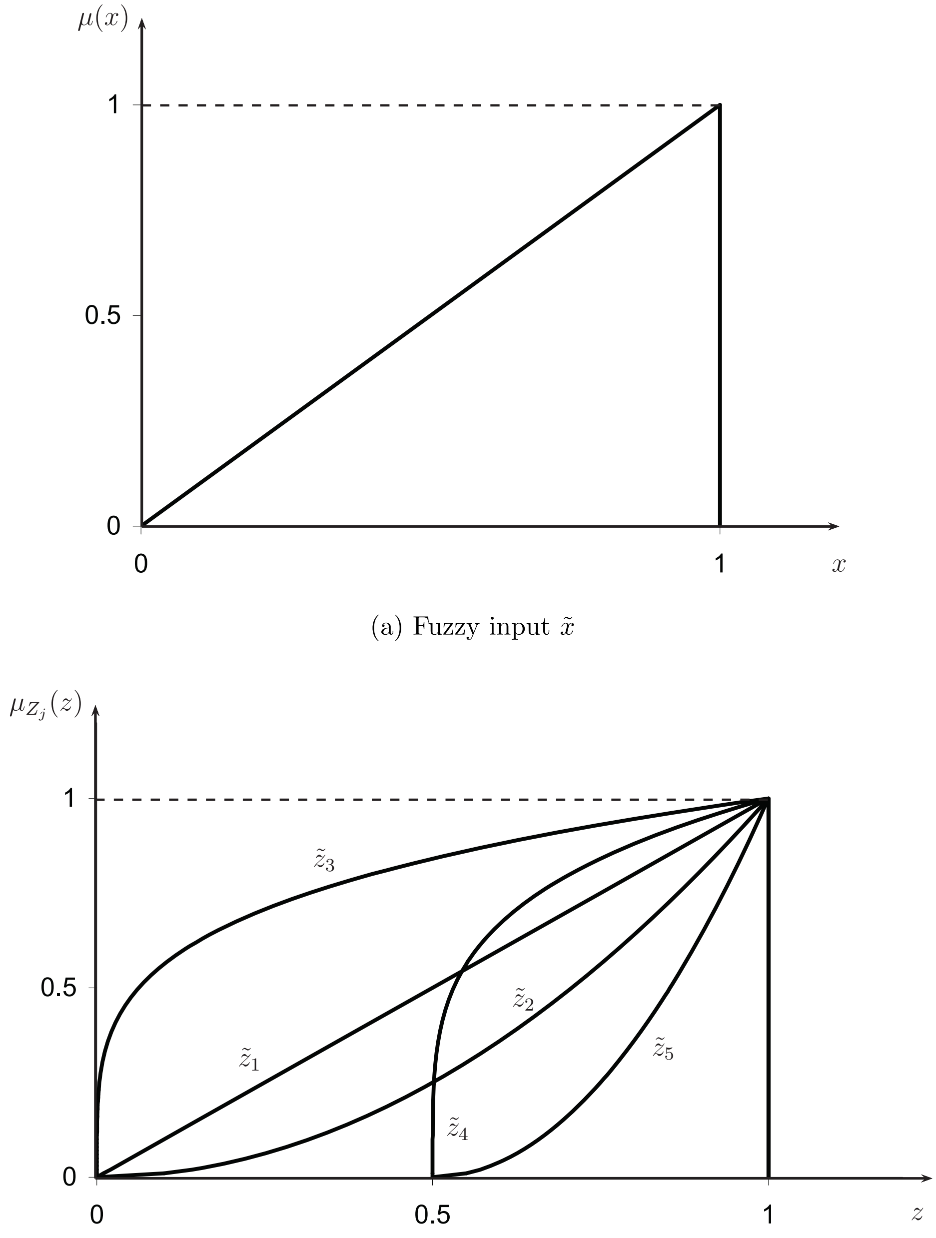
$$\begin{aligned} f_1(x) &= x \ , \\ f_2(x) &= x^{0.5} \ , \\ f_3(x) &= x^4 \ , \\ f_4(x) &= 0.5x^4 + 0.5 \ , \\ f_5(x) &= 0.5x^{0.5} + 0.5 \ . \end{aligned}$$

The membership functions for the fuzzy outputs $\tilde{z}_j$ can be obtained analytically,

$$\begin{aligned} \mu_1(z) &= z, \quad z \in [0,1] \\ \mu_2(z) &= z^2, \quad z \in [0,1] \\ \mu_3(z) &= z^{0.25}, \quad z \in [0,1] \\ \mu_4(z) &= (2z-1)^{0.25}, \quad z \in [0.5,1] \\ \mu_5(z) &= (2z-1)^2, \quad z \in [0.5,1] \end{aligned}$$

where $\mu_j(z) = 0$ for other values. The results are shown in Fig. 3(b).

The functions $f_i(x)$ have been chosen to illustrate the effects discussed below, which possess particular practical relevance and address the problem in Fig. 1. Specifically, $f_2(x)$ and $f_3(x)$ are selected to show that the associated fuzzy outputs $\tilde{z}_2$ and $\tilde{z}_3$ have similar entropy values but different shapes. The same applies to the selection of $f_4(x)$ and $f_5(x)$, but with a smaller uncertainty in the associated results $\tilde{z}_4$ and $\tilde{z}_5$ to work out discussion on this effect, as well. One could use further functions, as well, for this study.



(a) Fuzzy input $\tilde{x}$

(b) Fuzzy outputs $\tilde{z}_j$

Figure 3: Mapping $\tilde{x} \rightarrow \tilde{z}_j$: 3a fuzzy input $\tilde{x}$ and 3b fuzzy outputs $\tilde{z}_j$ associated with the mapping model $z_j = f_j(x)$

The entropy values associated with α_k of $\tilde{x}$ and $\tilde{z}_j$, normalized by $H(\tilde{x})$, are shown in Fig. 4. It clearly indicates a reduction of imprecision in the fuzzy input $\tilde{x}$ as α_k increases (i.e., collection of additional information) and the corresponding reduction of imprecision in the fuzzy outputs $\tilde{z}_j$. In an engineering context, it means that collection of additional information to reduce input imprecision has a trade-off in a reduction in imprecision of computational results, i.e., in predictions regarding structural behavior and reliability. However, for different mapping models, the reduction of imprecision in the outputs exhibits very different characteristics. For example, the imprecision in $\tilde{z}_{2,\alpha_k}$ and $\tilde{z}_{5,\alpha_k}$ decreases much faster than the imprecision in $\tilde{z}_{3,\alpha_k}$ and $\tilde{z}_{4,\alpha_k}$ for smaller values of α_k. It indicates that just a small reduction of imprecision in $\tilde{x}$ (i.e., low effort spent on collecting additional information) can result in a significant reduction in imprecision of $\tilde{z}_2$ and $\tilde{z}_5$. Thus, the mapping models f_2 and f_5 have more desirable properties than f_3 and f_4. They represent economical engineering design in the sense that only little effort in collecting input information has a significant trade-off in a substantial quality improvement of predictions regarding structural performance and reliability. This feature is reflected in the robustness measure in Eq. (21), which provides a quantitative assessment of the properties of the systems.

The entropy-based robustness $R(\alpha_k)$ is shown in Fig. 5 as a function of α_k. Several interesting conclusions can be drawn from the results:

- $R(\tilde{x}, \tilde{z}_2) = 1.22 \approx R(\tilde{x}, \tilde{z}_3) = 1.20 > R(\tilde{x}, \tilde{z}_1) = 1.00$ at $\alpha_k = 0 + \varepsilon$ when $\varepsilon \to 0$. This observation produces the robustness assessment in [16] as a special case. That is, if only the values of $R(\cdot)$ with respect to $\alpha_k = 0 + \varepsilon$ when $\varepsilon \to 0$ are considered to make a decision, it would

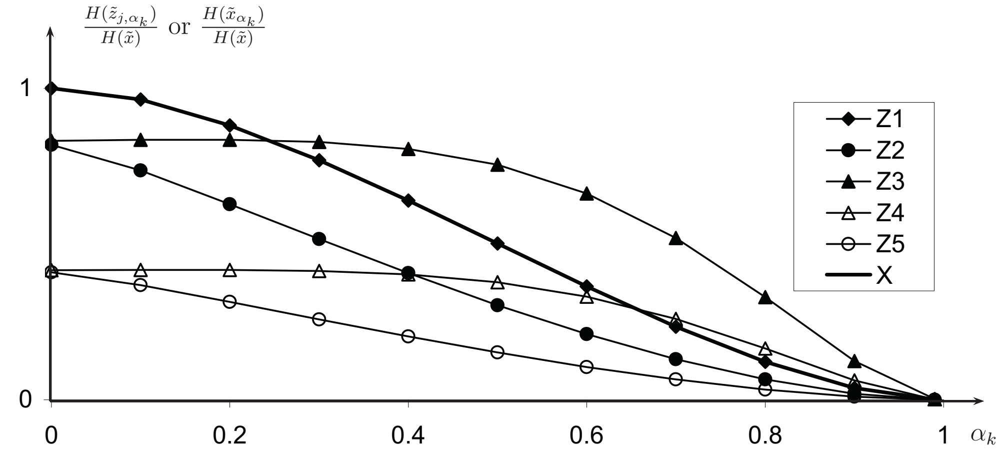


Figure 4: A reduction of imprecision in the fuzzy input $\tilde{x}$ as α_k increases and the corresponding reduction of imprecision in the fuzzy outputs $\tilde{z}_j$ (note: the curve for $\tilde{x}$ overlaps with that for $\tilde{z}_1$)

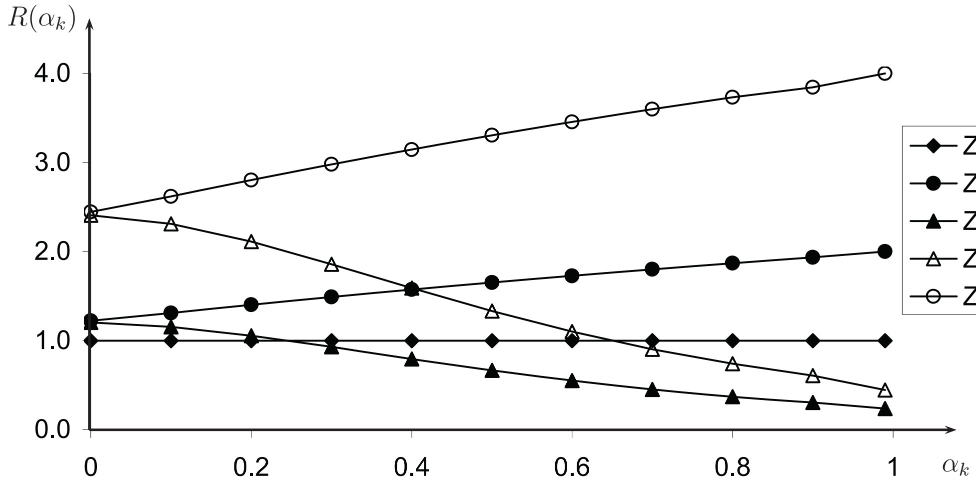


Figure 5: Robustness $R(\tilde{x}, \tilde{z}_j)$ associated with each mapping model $f_j(x)$ with alpha-level (α_k) discretization

be concluded that the mapping models $f_2(x)$ and $f_3(x)$ have a similar robustness and are both more robust than mapping model $f_1(x)$.

- Fig. 5 shows significant differences of the values $R(\cdot)$ corresponding to different values of α_k. As α_k is increased, $R(\tilde{x}, \tilde{z}_3)$ keeps decreasing while $R(\tilde{x}, \tilde{z}_2)$ keeps increasing. This indicates that the mapping models will exhibit different properties with respect to robustness when considering an increase of membership values $\alpha_k > 0$, i.e., a reduction of imprecision in the fuzzy input. The imprecision in the uncertain inputs can be reduced when more information is made available. This facilitates a trade-off analysis between collection of additional information and decision for a specific design variant. For example, at $\alpha_k = 0.4$, $R(\tilde{x}, \tilde{z}_2) = 1.57 > R(\tilde{x}, \tilde{z}_1) = 1.00 > R(\tilde{x}, \tilde{z}_3) = 0.79$. Thus, it would be concluded that mapping model $f_2(x)$ is the most robust system when input imprecision can be reduced to a degree corresponding to membership level 0.4. Furthermore, when considering all the values of $\alpha_k \in (0, 1]$, the mapping model $f_2(x)$ is more robust in overall than $f_3(x)$ although they have similar robustness values at $\alpha_k = 0 + \varepsilon$. The same situation appears when comparing the mapping models $f_4(x)$ and $f_5(x)$. Obviously, the mapping model $f_5(x)$ is a better choice.

- The mapping model $f_4(x)$ is more robust than $f_2(x)$ when $\alpha_k \leq 0.4$, especially, $R(\tilde{x}, \tilde{z}_4) = 2.41 \approx 2R(\tilde{x}, \tilde{z}_2) = 2.44$ at $\alpha_k = 0 + \varepsilon$. However, the values of $R(\cdot)$ associated with $\alpha_k \geq 0.4$ lead to the opposite conclusion that the mapping model $f_2(x)$ is more robust than $f_4(x)$. Again, the trade-off between collection of additional information and decision for a variant or improvement of the robustness assessment can be considered. If the collection of additional information is easy to facilitate, $f_2(x)$ would be the preferred model; otherwise $f_4(x)$. And the other may round, if $f_2(x)$ is selected, collection of additional information would be very useful and paid off; whilst for $f_4(x)$ collection of additional information does not lead to a benefit.

Hence, it is of vital importance to compute the structural robustness at various membership levels α_k. Robustness is not only a property of the structure, it is also dependent on the magnitude of imprecision/uncertainty in the input. It is a relative measure. Reduction of input imprecision can so lead to both increase and decrease in robustness depending on whether sensitivities are associated with the value ranges cut away in the reduction of imprecision or not. This consideration can substantially support design decisions in the context of availability of information and inspection cost.

It is noted that the membership functions of the result can be found in a closed form in the case of the simple illustrative example. In a practical structural analysis it is normally not possible to determine result membership functions in a closed form. However, they can be found in general via a numerical fuzzy analysis. A variety of intrusive and non-intrusive numerical approaches are available to perform this analysis, see [20, 21, 22, 23, 24].

4. Application to Offshore Structures

4.1. Structural models

Based on the environmental conditions and the information about the reference jackets provided in [3], two 2D frames are designed using software USFOS with some simplifications as well as some changes to the dimensions of members. USFOS (an acronym for Ultimate Strength for Frame Offshore Structure) is a numerical tool for nonlinear pushover analysis which helps to compute the reserve strength and residual strength of the frame structures before and after damage. The topologies for the X-bracing and K-bracing jacket structures are shown in Fig. 6. All structures are two-bay frames in water depth of $37m$. The environmental design loads are applied at the top two elevations of the frames with the values of 1334.5 kN and 667.2 kN, respectively. The diameter D and thickness t of all tubular members are listed in Table 1.

An assumption of fixed boundary conditions is made and all the tubular joints are assumed to be rigid. The structures are modelled with beam elements and material non-linearities are modelled by plastic hinges at element mid-span and element ends. Element formulation of this program also allows the considerations of large displacement effects and the coupling of lateral deflection and axial strain. This supports a realistic representation of the element behavior including column buckling. Nonlinear ultimate strength analysis can be carried out to determine the reserve and residual strength representing the degree of redundancy of the jacket structure. In the analysis, the load is applied incrementally until the ultimate resistance is reached and the load increment is automatically reversed when global instability is

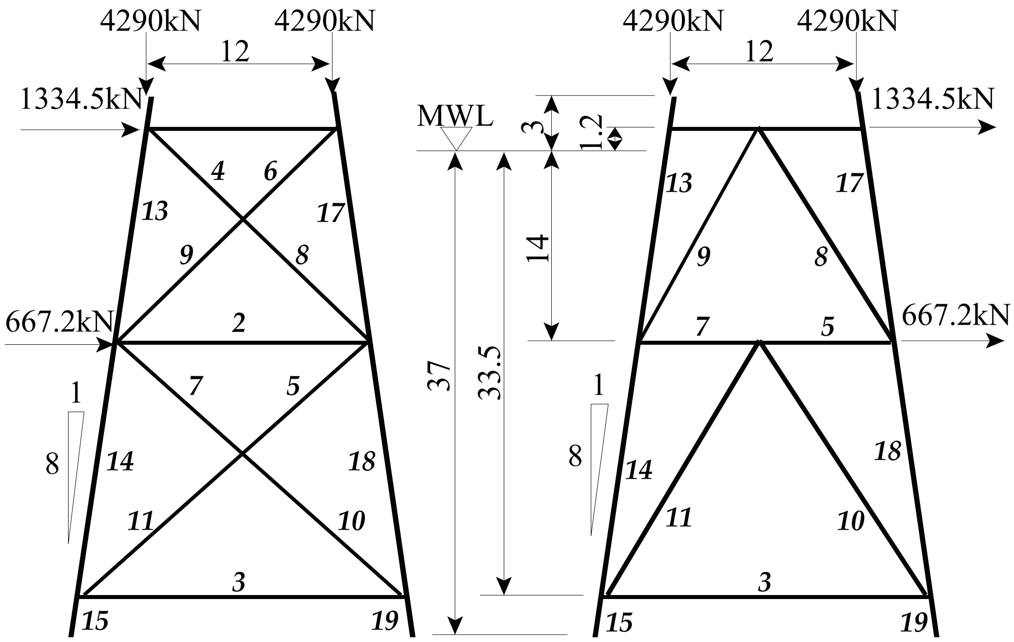


Figure 6: Structural models of the fixed offshore platforms (unit of length: m)

Table 1: Member sizes of the X-frame and K-frame

member	X-	frame	K-	frame
no.	D(m)	t(m)	D(m)	t(m)
2	0.32385	0.00953	-	-
3	0.3556	0.00953	0.508	0.0127
4	0.4572	0.0127	-	-
5	0.4572	0.0127	0.559	0.0127
6	0.4572	0.0127	-	-
7	0.4572	0.0127	0.559	0.0127
8	0.4572	0.0127	0.508	0.01588
9	0.4572	0.0127	0.508	0.01588
10	0.4572	0.0127	0.559	0.01715
11	0.4572	0.0127	0.559	0.01715
13	1.1684	0.0254	1.1684	0.0254
14	1.1684	0.03175	1.1684	0.03175
15	1.1684	0.03175	1.1684	0.03175
17	1.1684	0.0254	1.1684	0.0254
18	1.1684	0.03175	1.1684	0.03175
19	1.1684	0.03175	1.1684	0.03175

detected. The size of the increments may be varied along the deformation path, i.e. large steps in the linear range, and smaller steps with increasingly nonlinear behavior.

4.2. Damage modeling under imprecise marine corrosion

In this practical example, the fixed offshore platforms are assumed to be subjected to gradual deterioration caused by uniform corrosion. In order to investigate the corrosion effects with a longer period, the immersion corrosion data collected until 1994 provided in [17] are taken to specify the corrosion depth. With this approach we follow the general practise to consider only uniform corrosion when analyzing structural strength or structural capacity, see [25]. However, the proposed nuanced robustness analysis is not limited to this corrosion model. It can also be applied in association with non-uniform corrosion models. One may consider that corrosion tends to concentrate in the heat affected zone of the welds, and that stress-concentrations exists at the same spots. Hence, the damage maybe defined as an accumulation of those local damages in the connections.

Herein, we focus on uniform corrosion and define a fuzzy corrosion depth $\tilde{c}(t)$ associated with the exposure time t coarsely derived from the data in conjunction with a subjective assessment of deterioration, as shown in Fig. 7. The membership values $\mu(c)$ express the degree of subjective plausibility that particular values of $c(t)$ actually occur. That is, the membership values reflect a subjective assessment and the specification of the values are characterized by highly subjective factors. In this example, $\tilde{c}(t = 16)$ is considered and subjectively constructed according to the data points plotted in Fig. 7. A rational approach is to weigh the mean value 0.68 mm with a membership of $\mu(0.68) = 1.0$. The 5% and 95% bounds $[0.32, 1.40]$ mm at $t = 16$ provide a reasonable interval for the support of the fuzzy set $\tilde{c}(t = 16)$. However, it is observed that there are some outliers at $t = 15$, which are considered to be possible values but with lower degree of possibility, i.e., $\mu(c) \leq 0.1$ for $c \geq 1.40$ mm. Since the membership function is only a subjective assessment, complicated descriptions are often not necessary for practical purpose. It is appropriate to choose linear functional formulations for $\mu(c)$, as shown in Fig. 8.

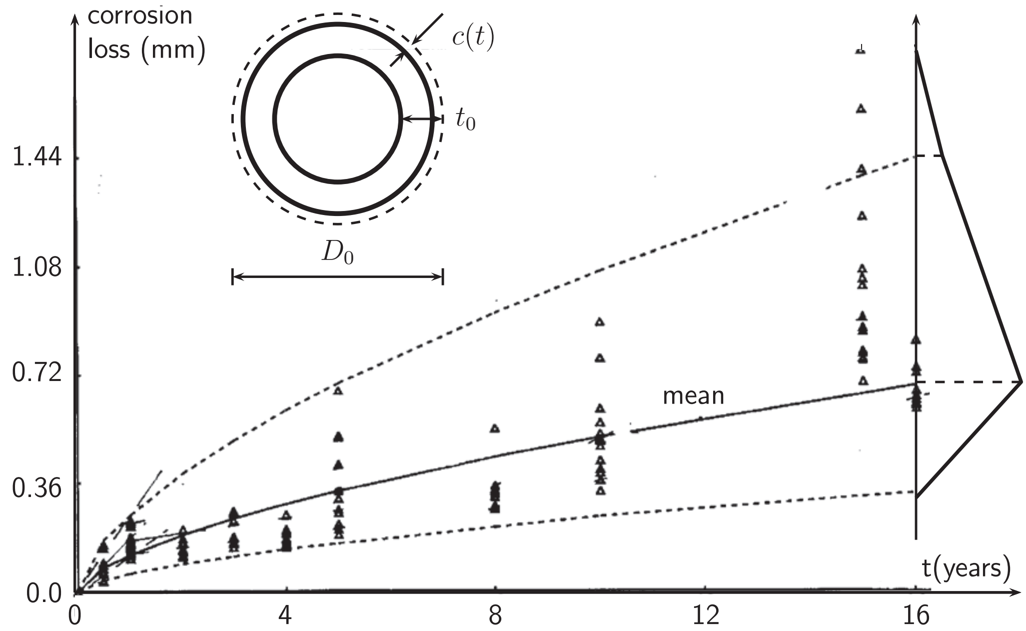


Figure 7: Immersion corrosion data for mild-steel coupons pooled from all available sources until 1994 subjected to an approximate temperature correction in [17] with 5 and 95 percentile bands

The concept of structural damage modeling from [4] is utilized herein to specify the amount of damage at the member level for a circular cross-section.

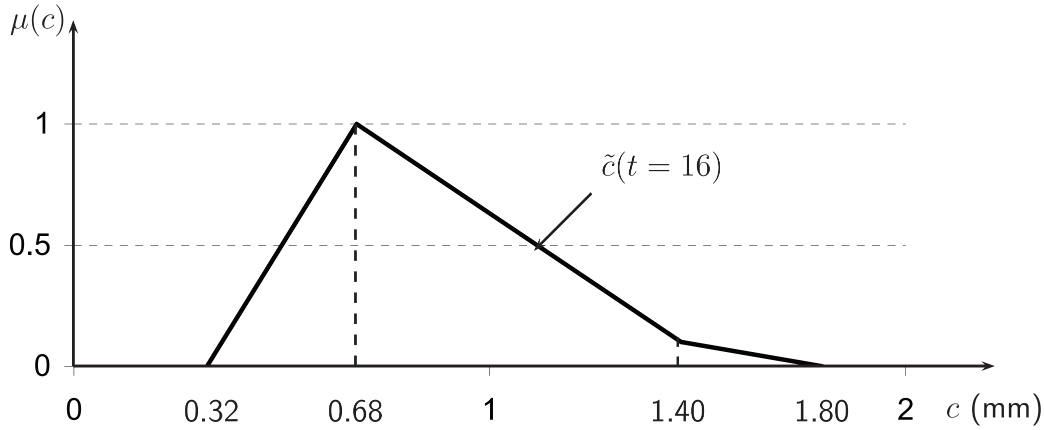


Figure 8: Fuzzy corrosion depth $\tilde{c}$ at $t = 16$ years according to the immersion corrosion data in Fig. 7

For the hollow steel tubes which are typically used in building offshore platforms, the damage at cross-sectional level can be represented by a ratio of the corroded area A_c and the original area A_0,

$$\beta = \frac{A_c}{A_0} = \frac{D_0}{(D_0 - t_0)t_0}c - \frac{1}{(D_0 - t_0)t_0}c^2 \tag{22}$$

where D_0 and t_0 are the diameter and wall-thickness, respectively, before deterioration. Finally, the formulation at the cross-sectional level is extended to obtain the total damage at the structural level by integration over all structural members, which is calculated as β_{total},

$$\beta_{\text{total}} = \frac{\sum \beta_i A_i L_i}{\sum A_i L_i} = \sum \omega_i \beta_i \tag{23}$$

where $\omega_i = A_i L_i / \sum A_i L_i$, and A_i is the cross-sectional area of a structural member with length L_i before deterioration.

As the corrosion depth is modeled as fuzzy variable $\tilde{c}(t = 16)$ as shown in Fig. 8, the total damage due to the marine corrosion is also a fuzzy variable and represented by $\tilde{\beta}_{\text{total}} = \sum \omega_i \tilde{\beta}_i$. Based on Eq. (22), $\tilde{\beta}_i$ can be calculated. It can be observed from the plot of $\beta = \beta(c)$ in Fig. 9 that there exists monotonic relationship between β and c when $0 \leq c \leq t_0$. Hence, the fuzzy result $\tilde{\beta}$ can be easily obtained by computing the alpha-level sets $[\beta_{\alpha_k l}, \beta_{\alpha_k r}]$ for $\alpha_k \in (0, 1]$, that is,

$$\beta_{\alpha_k l} = \frac{D_0}{(D_0 - t_0)t_0} c_{\alpha_k l} - \frac{1}{(D_0 - t_0)t_0} (c_{\alpha_k l})^2 \tag{24}$$

$$\beta_{\alpha_k r} = \frac{D_0}{(D_0 - t_0)t_0} c_{\alpha_k r} - \frac{1}{(D_0 - t_0)t_0} (c_{\alpha_k r})^2 \tag{25}$$

where $[c_{\alpha_k l}, c_{\alpha_k r}]$ is the alpha-level set at $\alpha_k \in (0, 1]$ of the fuzzy corrosion depth $\tilde{c}$. Based on Eq. (24) and Eq. (25), together with the linear function $\tilde{\beta}_{\text{total}} = \sum \omega_i \tilde{\beta}_i$, the total damage represented by $\tilde{\beta}_{\text{total}}$ can be obtained for the K-braced and X-braced frames, as shown in Fig. 10.

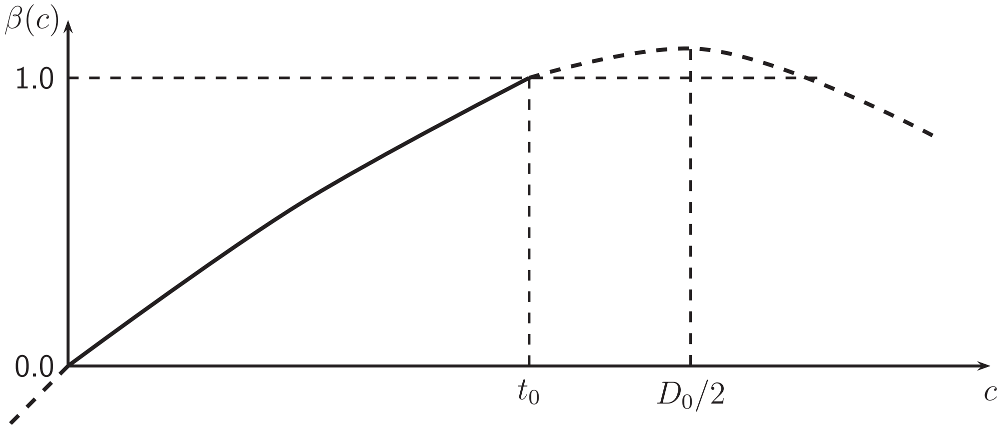


Figure 9: Plot of the damage represented by $\beta(c)$ for a hollow cross-section with diameter D_0 and thickness t_0. Note: $0 \leq c \leq t_0$

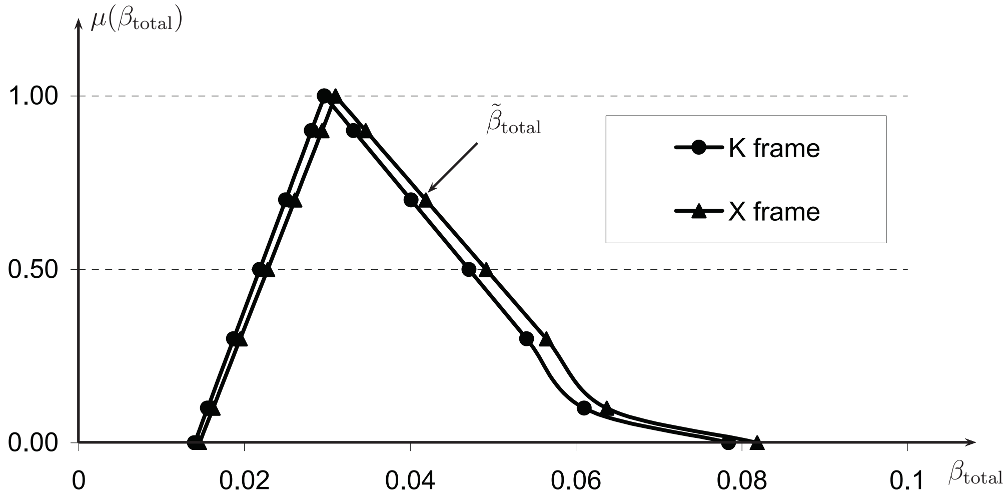


Figure 10: Total damage represented by $\tilde{\beta}_{\text{total}}$ for the K-braced and X-braced frames

4.3. Robustness assessment of fixed offshore platforms

The specified fuzzy variable $\tilde{c}(t = 16)$ for the corrosion depth is processed through the fuzzy structural analysis according to [26], which requires a repeated calculation of the fuzzy result values for varying corrosion depth. In this example, the non-dimensional measures based on ultimate strength analysis, RRF in Eq. (3) and R_{twice} in Eq. (4), are selected as fuzzy result values for each platform, respectively. For this purpose, the fuzzy structural analysis is coupled to the USFOS software and the fuzzy result values, $\widetilde{\text{RRF}}$ and $\widetilde{\text{R}}_{\text{twice}}$, are found by means of an optimization in the kernel of fuzzy structural analysis. The overall procedure includes two successive steps. First, a fuzzy structural analysis is performed with USFOS as a deterministic mapping model, as illustrated in Fig. 11. This deterministic mapping model provides the nonlinear ultimate strength analysis. And the fuzzy structural analysis delivers the fuzzy outputs to the second step. In the second step the entropy is calculated for different alpha-levels with the intersection of membership functions of both fuzzy inputs and outputs. Numerical sensitivities in the fuzzy structural analysis and in USFOS with respect to the fuzzy outputs can be minimized by appropriate selection of the algorithm parameters so that the corrosion effects can be well captured in the fuzzy outputs. The intersection of member functions, in the second step, is based on the mathematical operation of fuzzy sets and no additional effects will be introduced during this operation. Effects from entropy calculation by numerical integration of Eq. (16) are insignificant. Thus, the corrosion effects can be well reflected in the main results, which provide a sound basis to the application of the proposed approach.

$\widetilde{\mathrm{RRF}}$ reflects the imprecision of the ultimate capacity of the damaged platforms under corrosion at different membership levels, see Fig. 12. The entropy values associated with α_k of $\tilde{\beta}_{\mathrm{total},\alpha_k}$ and $\widetilde{\mathrm{RRF}}_{\alpha_k}$, normalized by $H(\tilde{\beta}_{\mathrm{total}})$, are shown in Fig. 13. It shows that the imprecision in $\widetilde{\mathrm{RRF}}_{\alpha_k}$ of the K frame decreases much faster than the imprecision in $\widetilde{\mathrm{RRF}}_{\alpha_k}$ of the X frame, especially for larger values of α_k. Thus, the K frame has advantageous properties over the X frame in view of the effects of imprecise marine corrosion on the ultimate capacity.

Based on the proposed approach for robustness assessment in Eq. (20) and Eq. (21), the entropy-based robustness $R(\alpha_k)$ at each alpha-level is calculated as the ratio between the entropy of $\tilde{\beta}_{\mathrm{total},\alpha_k} = \tilde{\beta}_{\mathrm{total}} \cap \beta_{\mathrm{total},\alpha_k}$ of the fuzzy input $\tilde{\beta}_{\mathrm{total}}$ and the entropy of $\widetilde{\mathrm{RRF}}_{\alpha_k} = \widetilde{\mathrm{RRF}} \cap \mathrm{RRF}_{\alpha_k}$ of the fuzzy output $\widetilde{\mathrm{RRF}}$. The result is shown in Fig. 14, which indicates that

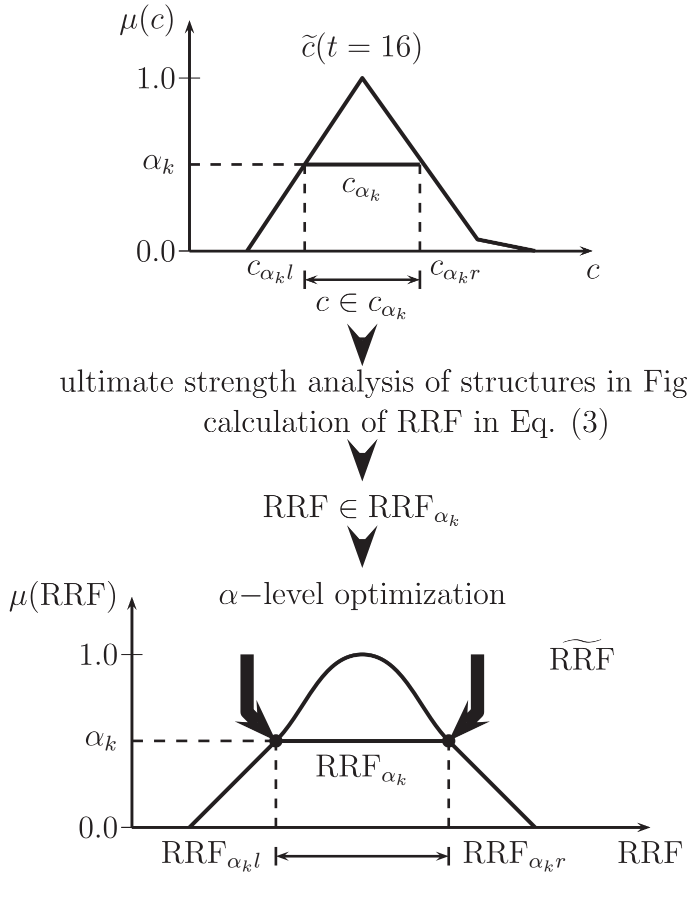


Figure 11: Fuzzy structural analysis with the nonlinear ultimate strength analysis as the deterministic mapping model

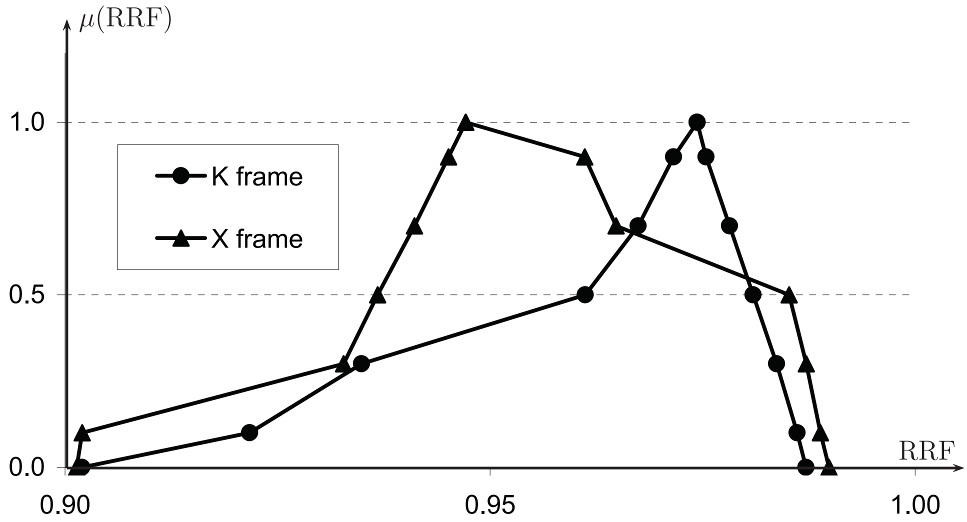


Figure 12: The membership functions of fuzzy RRF for the K-braced and X-braced frames

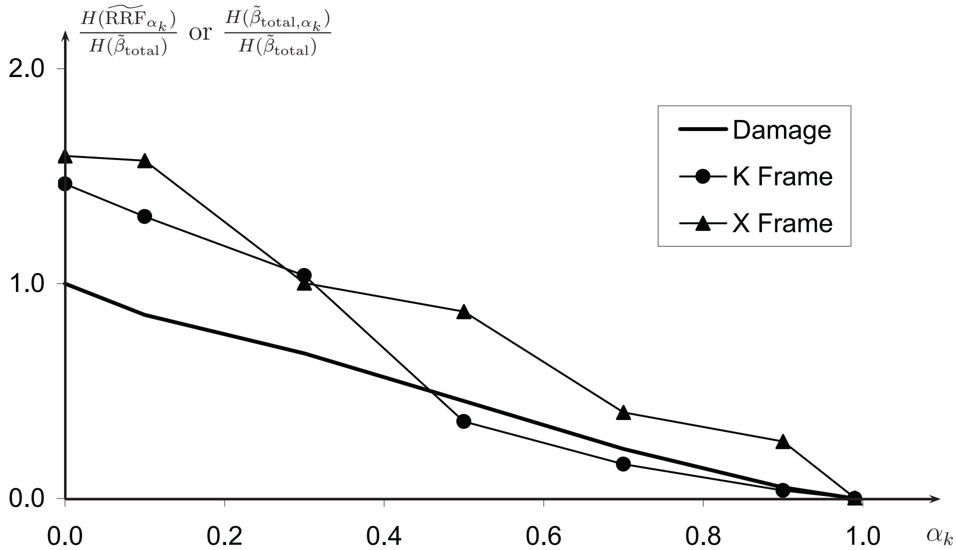


Figure 13: A reduction of imprecision in the fuzzy damage $\tilde{\beta}_{\mathrm{total}}$ as α_k increases and the corresponding reduction of imprecision in the fuzzy output $\widetilde{\mathrm{RRF}}$

the K-frame and the X-frame have a similar robust behavior with respect to imprecise corrosion effects when $\alpha_k \leq 0.3$. However, the K-frame shows a greater robustness than the X-frame when $\alpha_k > 0.3$. This result suggests that the robustness assessment for the K-frame can be significantly improved by collecting additional information about the corrosion, i.e. by reduction of input imprecision. However, collection of additional information regarding long time marine corrosion may be very difficult in offshore engineering practice. For the K-frame additional effort pays off, whereas for the X-frame, no clear benefit can be observed. This conclusion illustrates the potential of the proposed robustness measure for cost reduction and optimal resource allocation in inspection scheduling.

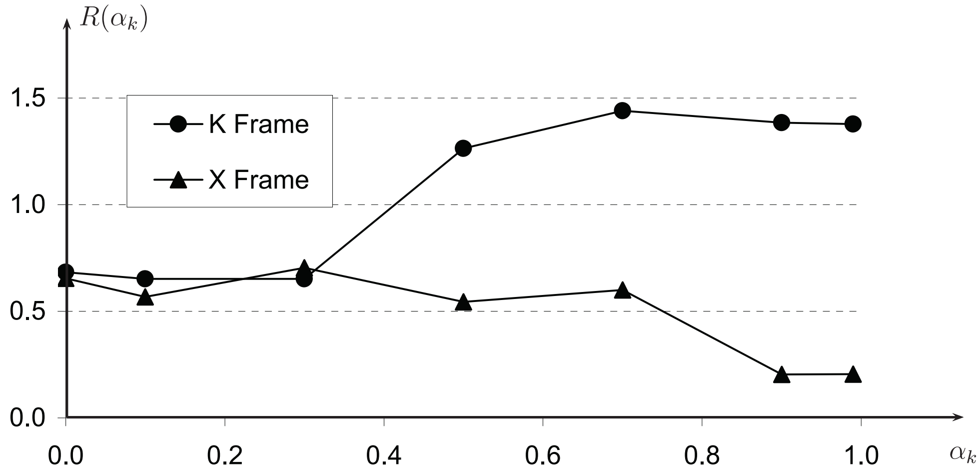


Figure 14: Robustness $\mathrm{R}(\tilde{\beta}_{\text{total}}, \widetilde{\mathrm{RRF}})$ associated with each frame with alpha-level (α_k) discretization

This observation that the K-frame shows a similar robust behavior as the X-frame when $\alpha_k \leq 0.3$ and a greater robustness when $\alpha_k > 0.3$ is quite different from the statement that the X-frame is more robust than the K-frame using the deterministic performance measures in [1, 2]. Furthermore, it is also known that the X-frame shows ductile behavior while the K-frame shows brittle behavior. However, these two statements are not conflicting with the new results as they refer to different aspects. While the deterministic investigation refers to ductility, the robustness assessment considered herein refers to the corrosion effects on the ultimate strength of the structure. A consideration of the residual load carrying capacity leads to an agreement in the conclusions. This can be observed in Fig. 15 by comparing the nominal distances of $\widetilde{\mathrm{R}}_{\mathrm{twice}}$ from the value 1.0, $\mathrm{d_X}(\widetilde{\mathrm{R}}_{\mathrm{twice}})$ and $\mathrm{d_K}(\widetilde{\mathrm{R}}_{\mathrm{twice}})$. In this result, $\widetilde{\mathrm{R}}_{\mathrm{twice}}$ reflects the imprecision of the residual strength of the damaged platforms under corrosion corresponding to twice the ultimate deflection at different membership levels. A smaller value of the distance indicates a smaller drop in the post ultimate strength, i.e., more ductility. This effect can be included in the robustness measures as constraint distance as proposed in [16]. Although the X-frame shows a better ductile behavior than the K-frame, as observed in Fig. 15, both frames show a similar robustness in view of the imprecise damage due to corrosion and the associated imprecision in $\mathrm{R}_{\mathrm{twice}}$, see Fig. 16. Further, it is indicated in Fig. 16 that $\mathrm{R}(\tilde{\beta}_{\mathrm{total}}, \widetilde{\mathrm{R}}_{\mathrm{twice}})$ keeps decreasing as α_k is increased. This indicates that the residual resistance $\widetilde{\mathrm{R}}_{\mathrm{twice}}$ is insensitive with respect to extreme values of the corrosion depth and rather shows sensitivities when the corrosion depth varies around the mean.

It is noted that the entropy results mainly reflect the sensitivities of the selected non-dimensional measures RRF and R_{twice} with respect to the uncertainty in corrosion depth. The interpretation of the results is focused on the trade-off between the effort for collection of additional information re-

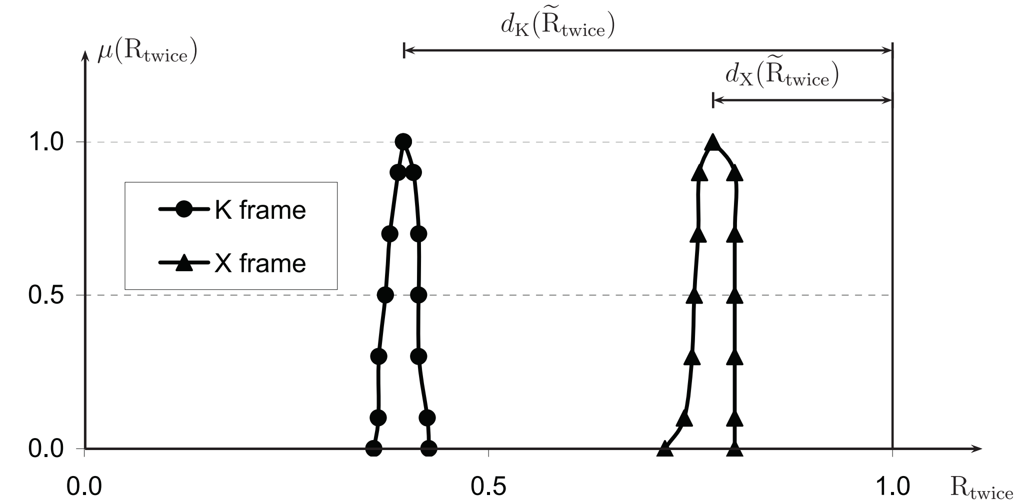


Figure 15: Membership function of $\widetilde{\mathrm{R}}_{\mathrm{twice}}$ for the K-braced and X-braced frames

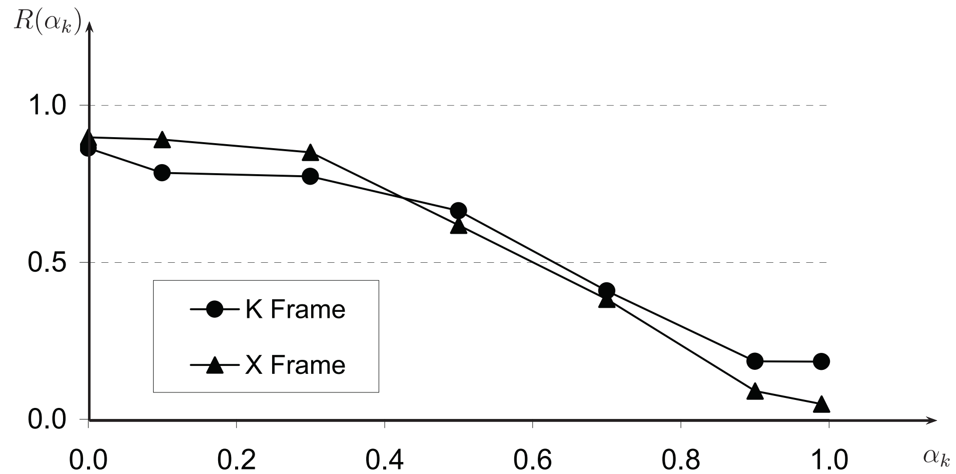


Figure 16: Robustness $\mathrm{R}(\tilde{\beta}_{\mathrm{total}}, \widetilde{\mathrm{R}}_{\mathrm{twice}})$ associated with each frame with alpha-level (α_k) discretization

garding the corrosion damage and the gain for the robustness assessment of the structures. However, the fuzzy outputs $\widetilde{\mathrm{RRF}}$ and $\widetilde{\mathrm{R}}_{\mathrm{twice}}$ are related to the frame behaviors, which becomes particularly clear in Fig. 15. The results show both the imprecision of the residual strength under corrosion damage and the property of structural redundancy.

The derived statements regarding to the effects of marine corrosion on the robustness of the two platforms designed in this numerical example may not be generalized to other gradual effects on the robustness or an alternative design. But the proposed approach provides a general basis for the robustness assessment of any newly designed or existing platforms with respect to imprecise effects of deterioration.

In summary, the different effects discussed and observed in Fig. 14$\sim$16 are not conflicting with each other but are complementary to formulate diverse views at the robustness of the X-frame and the K-frame. The influence of the framing configuration on the robustness of the fixed offshore platforms can be understood in a comprehensive way based on the proposed approach.

5. Conclusions

An improved methodology for nuanced robustness assessment of structures was proposed and demonstrated for aging offshore structures subjected to uncertain damage due to imprecise marine corrosion. Fuzzy variables were utilized to cater for the subjective character of the assessment of the corrosion effect. Structural robustness was evaluated at various membership levels to reflect various degrees of imprecision in the damage. It was shown that diverse views at the structural robustness can be formulated to provide more comprehensive understanding of the influence of the structural layout on the robustness. Engineering decisions for the design and re-analysis of structuras can so be generated on a broader basis. In the assessment of existing structures, an improved optimal resource allocation for inspection can be obtained. The proposed approach provides a general basis for the assessment of structural robustness under the consideration of fuzzy uncertainty in the structural parameters. It can also be applied to robust design. For a practical application one needs to implement a fuzzy structural analysis, as well, which can be numerically demanding for large structures. Further development on this side would benefit applications. Also, further development is needed to address the invariance issue of the entropy measure for fuzzy sets.

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