**Robust Multi-Damage Localisation Using** **Common Eigenvector Analysis and Covariance Matrix Changes**

Shancheng Cao and Huajiang Ouyang

*Centre for Engineering Dynamics, School of Engineering.*

*The University of Liverpool, Liverpool, L69 3GH, U.K.*

*E-mail of the corresponding author:* [*h.ouyang@liverpool.ac.uk*](mailto:h.ouyang@liverpool.ac.uk)

**Abstract:**

Damage-induced local singularities in structural characteristic deflection shapes (CDS’s) are widely used in non-model-based damage localisation. Despite substantial advances in this kind of methods, several issues must be addressed to boost their efficiency and practical applications. This study deals with two essential problems of CDS-based damage localisation: the noise robustness of CDS estimation and the criterion to properly weigh damage information of several CDS’s. On the first problem, it is well known that CDS estimation is vulnerably compromised by various uncertainties such as measurement noise and computational errors, which will decrease the accuracy and increase the difficulties for damage localisation. A modified common eigenvector analysis (CEA) is proposed based on a bank of digital filters and a joint approximate diagonalisation technique, which statistically estimates the CDS’s as the common eigenvectors of a set of covariance matrices. On the second problem, a new robust damage index (DI) is proposed, which is comprised of damage-caused local shape distortions of several CDS’s weighted by their participation factors in the covariance matrix at zero-time delay. The advantage of doing this is to include fair contributions from changes of all CDS’s concerned and the proposed DI provides a measure of damage-induced changes in covariance matrix. Then a numerical study is presented to demonstrate the noise robustness of the modified CEA method over proper orthogonal decomposition and second-order blind identification in CDS estimation. Moreover, a comparison of the proposed DI over some traditional damage localisation methods is conducted based on an experimental study. The results of numerical and experimental studies demonstrate that the proposed CDS estimation method is more robust to noise and the proposed DI is highly accurate for multi-damage localisation.

**Keywords**: multi-damage localisation, operational modal analysis, common eigenvector analysis, joint approximate diagonalisation

# 1. Introduction

Structural damage detection and assessment are essentially important in maintaining the safety and reliability of engineering structures such as aircraft, bridges and buildings, which help maintain healthy states and prevent catastrophic incidents due to structural failures [1, 2]. This is an inverse problem and damage is likely to be detected, localised and quantified through monitoring certain structural dynamic parameters. The basic principle behind this is that damage-induced changes in certain structural properties will alter structural dynamic responses, which can be reflected in modal parameters or other dynamic features such as frequency response function and transmissibility [3-5].

Vibration-based damage identification methods are numerous and can be categorised according to different criteria such as levels of damage identification, linear or nonlinear vibration responses and whether using physics-based models or not [6-8]. Here, damage identification is classified according to the practical availability of baseline information. Basically, structural damage identification can be considered a pattern recognition problem, which compares the extracted features of current states of the structures with the benchmark features to determine the damage state [9-11]. But, establishing the damage feature bank for pattern recognition is challenging due to various possible damage scenarios. An alternative solution is to build the physics-based model of structures such as finite element model. In this case, the damage identification is accomplished via model updating techniques [12-15]. However, a well correlated structural model and the accurate initial state of the structure are primarily required. Moreover, the large number of updating parameters and the non-uniqueness of updated models increase the difficulties of model updating based damage identification methods [16, 17].

In the absence of a damage feature bank and physics-based model of structures, damage can be detected by comparing the damage features of damaged structures with baseline data of heathy structures [18-20]. Even when the baseline data of healthy structures is not available, structural damage identification in the form of detection, localisation and relative severity quantification can still be achieved by measuring the deviations from some properties of healthy structures such as normal distribution of probability density function under random excitation and smoothness of mode shapes for geometrically uniform and material-isotropic structures [21-24].

The purpose of this paper is to propose a robust multi-damage localisation method using structural characteristic deflections shapes (CDS’s) without baseline data of healthy structures, which is desirable and promising in practical damage identification of engineering structures. Here, CDS’s are referred to as structural spatial deflection vectors such as mode shapes, proper orthogonal modes (POMs) and operational deflection shapes, which are evaluated based on output-only vibration data.

Structural characteristic deflection shapes have been studied for damage detection and localisation in the last decades, but the crucial issue is the noise robustness of CDS estimation, which often involves substantial inaccuracies [25, 26]. There are mainly four sources of uncertainties in CDS estimation: operational, environmental, measurement and computational [27]. Among various output-only CDS estimation approaches, proper orthogonal decomposition (POD), also known as the [Karhunen–Loève](https://en.wikipedia.org/wiki/Karhunen%E2%80%93Lo%C3%A8ve_theorem) decomposition (KLD), is a multivariate statistical approach aiming at using a linear combination of orthogonal functions/vectors to represent the stochastic data. When applied to finite dimensional cases and truncated after a few terms, the POD method is equivalent to principal component analysis (PCA) [28]. The proper orthogonal modes of POD are estimated as the eigenvectors of the covariance matrix from vibration output responses, which are commonly calculated by Eigen-decomposition or singular value decomposition (SVD).

Galvanetto et al. [29] investigated POMs in beam structures under sinusoidal excitation of several frequencies. The POM differences between healthy and damaged structures were used to identify the damage. Shane and Jha [30] employed POMs to filter out the environmental influences on the data and with the help of POMs of undamaged structures to calculate the residual errors in the time series to detect the damage. Thiene et al. [31] combined POM and a gapped smoothing method together to localise damage using only the vibration data of damaged structures.

However, just using a single covariance matrix of random vibration responses to estimate all the POMs is not robust and reliable due to various uncertainties. From a statistical point of view, it is desirable and promising, for the accuracy and robustness, to estimate CDS’s based on a group of datasets or covariance matrices. For this purpose, a modified common eigenvector analysis (CEA) is proposed and investigated based on common principal components [32, 33], which statistically provides a kind of ‘average Eigen-structure’ shared by a set of covariance matrices from a group of datasets or from one dataset. In this paper, joint approximate diagonalisation (JAD) algorithm is used to get the solutions for CEA in time domain to avoid estimation of complex eigenvectors. Traditionally, JAD approach is extensively applied in blind source separation due to its better performance in addressing noisy data [34]. Moreover, second order blind identification (SOBI) based on JAD approach has been studied for operational modal analysis by many authors [35-37]. But in SOBI, a pre-whitening procedure is required before JAD procedure, which introduces a bias or error to the final solution. To overcome this issue, the pre-whitening procedure of SOBI is to be replaced by digital filters in this study. By doing this, the simultaneous estimation of all CDS’s in SOBI method is converted to evaluating each CDS individually to enhance its accuracy and noise robustness. For digital filters, infinite impulse response (IIR) filter is chosen, since it has much better frequency response when compared with finite impulse response filter of the same order.

The CDS estimation by the proposed modified CEA has three steps: (1) identify the resonant frequencies; (2) design a set of IIR filters with different orders and bandwidths around each resonant frequency and compute the covariance matrices of the filtered data; (3) apply JAD approach to the covariance matrices in step 2 to estimate the CDS’s. When there is only one mode in the filtered data, the estimated CDS will coincide with the mode shape regardless of the mass distribution [28].

With the CDS’s of damaged structures, damage localisation can be accomplished by comparing with baseline CDS’s of intact structures. Nevertheless, the main drawback of this approach is that the CDS’s of damaged structures and healthy structures are difficult to be matched under various operational conditions. Fortunately, the principle that CDS’s of healthy structures without stiffness and mass discontinuities are smooth can be used for damage identification in beam- or plate-type structures [38]. Based on this, a new damage index (DI) is proposed without baseline data of healthy structures, which uses the squared Euclidean distance of local shape distortions of several CDS’s. Moreover, the damage information in different CDS’s is weighted by their participation coefficients in the covariance matrix at zero-time delay. By doing this, the proposed DI indicates the damage-induced changes in covariance.

The paper is organised as follows. In Section 2, the proposed modified CEA method using a bank of IIR filters and JAD technique is presented for CDS estimation based on one dataset. In Section 3, a robust damage localisation index is proposed and its advantages are discussed as well. In addition, this damage index has the ability to indicate the relative damage severity. In Section 4, a cantilever beam with 2 open cracks is studied to statistically demonstrate the noise robustness of the proposed modified CEA method over POD and SOBI methods. Moreover, the sensitivity of the proposed damage localisation index to damage severity and noise levels is also studied. Then, an experimental study is presented in Section 5 to illustrate the efficiency and accuracy of the proposed damage localisation method. Furthermore, a comparison with some traditional damage localisation methods is conducted as well. Finally, the conclusions are summarised in Section 6.

# 2. Common eigenvector analysis based on one dataset

Although the accuracy of damage localisation is highly dependent on the density of measurement points, for the demonstration of the proposed CDS estimation method and the new damage localisation index, a few measurement points are enough. The measured velocity vector using Scanning Laser Vibrometer is acquired at locations with samples and it can be expressed as

(1)

where is modal coordinate velocity responses with indicating the number of natural frequencies (or modes) of the structural system, represents thematrix of mode shape vectors at the measured degrees of freedom and denotes a vector of measurement noise. Before computing the covariance matrix of vibration responses, a zero mean procedure is required. Provided that  and are uncorrelated, covariance matrix of the response should be computed from covariance matrices of the modal coordinates and noise as

(2)

where denotes time-delay, which is an integer multiple of the sampling period.

### 2.1. A bank of IIR filters at each resonant frequency

The purpose of this section is to design a set of IIR filters of the interested frequency bands around each interested resonant frequency. An estimate of the number of resonant frequencies in the output measurements can be determined in a few ways, for instance, using the number of peaks in the trace plot of power spectral density (PSD) matrix or in the singular value spectrum plot obtained by SVD of power spectral density matrix. Here, the latter is adopted and a singular value spectrum of PSD matrices using the velocity responses of numerical study in Section 4 is depicted in figure 1, which shows the first five singular values of PSD matrix at each frequency.



Figure 1 Singular value spectrum plot of the numerical study in Section 4.

At a certain resonant frequency ( indicates the singular peak such as peaks 1, 2 and 3 in figure 1), two parameters, the filter order and the band-pass width, are adjusted to obtain a series of IIR filters. An IIR filter includes both feedback and feed-forward terms, which is expressed as

(3)

where denotes the feedback filter order, indicates the feed-forward filter order and is the output signal. Taking the -transform of equation (3), the IIR filter is converted to

(4)

And its frequency response form is

(5)

In this study, the filter order and the band-pass control parameter are used, thus a total number of 16 IIR filters at each resonant frequency is obtained. Figure 2 shows an example of an eighth-order IIR filter with band-pass of 31-36Hz.

|  |  |
| --- | --- |
| (a) | (b) |

Figure 2 The eighth-order IIR filter with band-pass of 31-36Hz (a) the whole frequency band (b) local magnification (the red dashed line indicates the designed filter in theory while the blue line denotes the implemented filter).

Now, the covariance matrix of a bank of IIR filters of a certain resonant frequency is

,; (6)

where indicates the number of covariance matrices or the IIR filters and denotes the number of interested resonant frequencies.

### 2.2. CDS estimation based on common eigenvector analysis

Provided that only one mode is present in the selected frequency band, all the covariance matrices will possess the same mode shape. In this case, the estimated dominant common eigenvector associated with the largest eigenvalue will coincide with regardless of the mass distribution. If more than one mode is present in the selected frequency band, the estimated mode shapes for closely spaced modes will be biased due to the orthogonal property of CEA and the bias mainly affects the weak mode whilst the dominant mode shape is still robust. And the bias depends on the gap between the first and second eigenvalues: the bigger the gap, the smaller the error. In order to keep the robustness of the estimated CDS’s, only the largest dominant common eigenvector in the selected frequency band is estimated and used for multi-damage localisation.

Define an orthogonal basis with and are orthogonal complementary basis of the subspace spanned by. Assuming that the experimental data is not contaminated by noise, the spatial orthogonal basis of each covariance matrix in equation (6) can be calculated by SVD as

(7)

In equation (7), = is the common eigenvector while is not guaranteed to be identical for different covariance matrices. Moreover, is a diagonal matrix with nonnegative singular values in a decreasing order: . According to the hierarchy of similarities among several covariance matrices, this is a partial common eigenvector problem. The hierarchy of similarities between number of covariance matrices of dimensions is presented in table 1 [33].

Table 1 Hierarchy of similarities between covariance matrices of dimensions

|  |  |  |
| --- | --- | --- |
| Level | Similarities | Number of parameters |
| 1 | Equality |  |
| 2 | Proportionality |  |
| 3 | Common eigenvectors |  |
| 4 | Partial common eigenvectors of number |  |
| 5 | Arbitrary covariance matrices |  |

For practical applications, the experimental data and its computed covariance matrices are vulnerably compromised by noise but SVD method does not consider the noise effects on the covariance matrix during eigenvector estimation. In order to take into account the noise effects in covariance matrices as shown in equation (6), common eigenvector analysis is conducted here. For this partial common eigenvector problem in equation (7), it is reasonable to assume that the covariance matrices share all common eigenvectors, because the mode shape plays a dominant role in each covariance matrix while the contribution of is small. Now, the equation of applying the common eigenvector approach to obtain the close solution of partial common eigenvector problem is formulated as

, (8)

where is the orthogonal common eigenvector matrix (), is a diagonal matrix of real nonnegative values and denotes noise matrix which corresponds to in equation (6). For any, the problem is over determined and an exact common eigenvector matrix is not available with. A natural criterion to address this simultaneous diagonalisation problem is the least-squares criterion and equation (8) is converted to finding and that minimise the cost function

(9)

The advantage of applying the least squares criterion is that the large diagonal elements have a dominant influence on the solution, thereby it is guaranteed to obtain an accurate and robust dominant common eigenvector. The above procedure for common eigenvector analysis using least-squares criterion is also known as joint approximate diagonalisation approach. In this study, JAD method is implemented through Givens rotations without pre-whitening procedure. Moreover, the number of covariance matrices is adopted, as it has been found that this number of covariance matrices is good enough to obtain a robust and accurate CDS estimation.

# 3. Damage localisation index

With the estimated CDS’s of damaged structures, damage identification is conventionally accomplished by comparing them with the CDS’s of healthy structures. Normally, the damage-induced local shape distortions have a big variation of magnitudes in different CDS’s. In order to prevent a certain CDS from unfairly dominating the DI, the difference or distance between and at measurement point is normally measured according to the weighted squared Euclidean distance as

(10)

where denotes the weighting coefficient of the -th CDS at measurement point . In the case that the CDS matrix of healthy structures is not available, is obtained by polynomial smoothing approach of based on the assumption that the CDS’s of healthy structures are smooth. Here, gapped smoothing method (GSM) is used to estimate the CDS matrix oftheundamaged structure [24]. GSM is sensitive to the local peaks and valleys of CDS’s that are associated with mode shapes of higher frequencies. However, the higher modes are normally more sensitive to local damage. Hence, for different CDS’s, a compromise between damage identification accuracy and sensitivity should be made when using GSM. The proposed damage index (DI) is computed using only the CDS’s of damaged structures as

(11)

where denotes a damage index vector and the right-hand side term is a vector formed by the diagonal elements of. In addition, the weighting matrix is calculated as

(12)

In equation (12), representsCDS participation of covariance matrix of measured responses at zero-time delay. Moreover, according to equation (2), isclosely related tothecovariance matrix of modal coordinate responses. By doing this, the proposed damage index provides a fair measure of damage induced changes in the diagonal elements of auto-covariance matrix. In the following numerical and experimental studies, the proposed DI is employed by incorporating damage information of the first three CDS’s.

# 4. Numerical studies

A cantilever beam with two open cracks (indicated by red lines in figure 3) is simulated to demonstrate the noise robustness of the proposed CDS estimation method and DI. This damaged beam is modelled according to Euler-Bernoulli beam theory with Rayleigh damping, ( and), using 40 elements in MATLAB. Its geometrical and material properties are tabulated in table 2. In addition, the configurations of the cracks are presented in table 3 and the cracks are modelled according to fracture mechanics approach [39, 40]. Random excitation is applied at point 20 and velocity time series are ‘measured’ at the prescribed 20 points along the beam with an equal distance of 0.035m as shown in figure 3. In addition, the excitation force has a normal distribution with mean value being 0 and standard deviation being 100 N.



Figure 3 Cantilever beam with two open cracks.

Table 2 Geometrical and material properties of steel beam.

|  |  |
| --- | --- |
| Property | Value |
| Length (m) | 0.7 |
| Cross-section () | 0.020.02 |
| Young’s modulus (GPa) | 210 |
| Mass density () | 7850 |
| Poisson ratio | 0.33 |

Table 3 Crack information of numerical study.

|  |  |  |  |
| --- | --- | --- | --- |
| Cracks | Location | Measurement points | Depth percentage |
| Crack 1 | 0.199 |  | 20% |
| Crack 2 | 0.399 |  | 20% |

The element stiffness matrix for elements away from the cracked elements can be regarded as unchanged within a certain limitation of element size. The stiffness matrix of the cracked element is expressed as

(13)

where transformation matrix and flexibility matrix of a cracked element are

(14)

where denotes the element length, is Poisson’s ratio, is the second moment of the cross-sectional area, indicates the beam width, is the beam depth and is the depth of the crack. is the approximation factor for stress intensity factors and expressed as

(15)

In order to compare the noise robustness of estimated CDS’s of CEA method with those of POD and SOBI methods, Gaussian white noise is generated to contaminate the velocity responses in the form of

(16)

where **d** is a vector containing normally distributed random values with a zero mean and variance being 1, is the noise level range of [0 1] and represents standard deviation of vibration responses at -th point. Noise is added independently to 1000 times of the same level of 5%. With each noise realisation,the first three CDS’s ranked by their contributions to the vibration responses are calculated by POD method, SOBI method and CEA method, respectively. The normalised mean CDS’s and coefficients of variation (CV) of CDS’s over 1000 noise realisations are given in figure 4. The coefficient of variation is defined as the ratio of the standard deviation to the absolute mean value.



|  |  |
| --- | --- |
| (a) Normalised first CDS | (b) CV of firs CDS |

|  |  |
| --- | --- |
| (c) Normalised second CDS | (d) CV of second CDS |



|  |  |
| --- | --- |
| (e) Normalised third CDS | (f) CV of third CDS |

Figure 4 Normalised mean CDS’s and CV of CDS’s over 1000 noise realisations of numerical study with two cracks of 20% of the beam depth and 5% noise.

It is obvious in figures 4 (b), (d), (f) that the identified CDS’s of CEA method are much more noise robust than CDS’s estimated by POD and SOBI methods due to a smaller CV. As the estimated CDS’s of CEA method are the most robust to noise, the associated damage index values tend to be more robust to noise than the other two methods as well, which is demonstrated in figure 5(b). Figure 5(a) presents the mean damage index values by the three methods over 1000 noise realisations, which indicates that all the three methods are able to localise both cracks accurately. The advantage of DI by CEA method in figure 5 (a) is not very obvious because the damage in this particular case is severe.



|  |  |
| --- | --- |
| (a) | (b) |

Figure 5 (a) Mean DI values and (b) CV of DI values over 1000 noise realisations of numerical study with two cracks of 20% of the beam depth and 5% noise.

Moreover, in order to illustrate the damage identification capacity of individual CDS and CDS curvature (CDSC), the damage-induced shape changes in the first three CDS’s and CDS curvatures by CEA method are presented in figure 6, which are evaluated by using gapped smoothing method without requiring the baseline CDS’s or CDS curvatures of the health state. From figures 6 (a), (c) and (e), it is clear that the damage-induced shape differences have a big variation in magnitude for different CDS’s. Another conclusion is that each CDS is sensitive to damage at some positions while less sensitive to damage at other locations. Obviously, if the position of a crack coincides with a nodal point of the CDS, damage cannot be detected based on that CDS. For example, the second crack is near a node of the third CDS depicted in figure 4(e) and consequently it will not lend itself to easy detection, as shown in figure 6 (e). Thus, in order to prevent undue influences of a certain CDS from dominating the DI and fairly use the damage information of all CDS’s, a normalisation procedure or a proper weighting matrix should be utilised in multi-damage identification. In addition, the peaks of CDS differences in figures 6 (a), (c) and (e) can clearly indicate either crack 1 or crack 2 while there are several adjacent peaks in the differences of CDS curvatures in figures 6 (b), (d) and (f). This is due to the computational effects when estimating CDS curvatures from CDS’s by using the second-order central difference approach. It can be seen that the noise effects on CDS curvatures in figure 6 (b), (d) and (f) are not obvious, which confirms that the proposed modified CEA is good at reducing the effects of the added 5% Gaussian white noise.

|  |  |
| --- | --- |
| (a) Damage-caused changes of first CDS | (b) Damage-caused changes of first CDSC |

|  |  |
| --- | --- |
| (c) Damage-caused changes of second CDS | (d) Damage-caused changes of second CDSC |

|  |  |
| --- | --- |
| (e) Damage-caused changes of third CDS | (f) Damage-caused changes of third CDSC |

Figure 6 Damage-induced changes of CDS’s and CDS curvatures in numerical study with two cracks 20% of the beam depth and 5% noise.

Finally, in order to validate the proposed DI in detecting smaller damage, three other damage levels are studied. They are 5%, 10% and 15% of the beam depth for both cracks specified in table 3. The damage localisation results of different damage and noise levels are presented in figure 7. Figures 7 (a)-(c) illustrate the damage identification results of the three damage levels without noise effects, respectively. It is found that by increasing the severity of the two cracks, the damage localisation results become more accurate, as the two cracks of 15% depth can be both correctly localised, whilst crack 2 of 5% depth is impossible to be detected. Moreover, the damage localisation results of SOBI and CEA methods are better than those of POD method. Figures 7 (d)-(f) and (g)-(i) present the damage localisation results of the three damage levels with noise levels of 3% and 5%, respectively. It can be observed that by increasing the noise level, the damage identification results of POD and SOBI become much worse whilst the DI values of CEA method remain almost the same by comparing figures 7 (d), (g) with (a) or figures 7 (e), (h) with (b). Therefore, the damage identification results of CEA method are the most robust to noise. In addition, the damage identification results of SOBI and CEA are equivalent in the case of no noise (figures 7 (a), (b) and (c)) or small noise level for severe damage (figure 7 (f)). Apart from damage localisation information in figure 7, by comparing DI magnitudes of different damage levels, it can be concluded that the DI magnitude effectively indicates the relative damage severity of cracks of the same positions.



|  |  |  |
| --- | --- | --- |
| (a) 5% depth without noise | (b) 10% depth without noise | (c) 15% depth without noise |



|  |  |  |
| --- | --- | --- |
| (d) 5% depth with 3% noise | (e) 10% depth with 3% noise | (f) 15% depth with 3% noise |

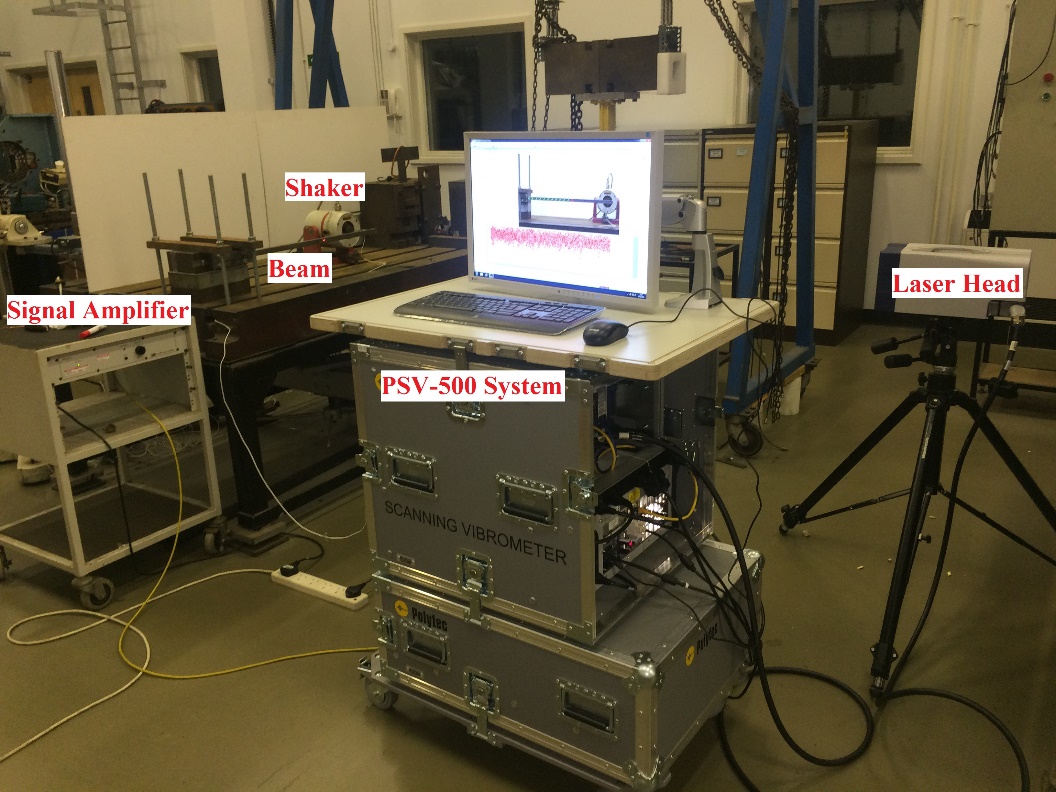


|  |  |  |
| --- | --- | --- |
| (g) 5% depth with 5% noise | (h) 10% depth with 5% noise | (i) 15% depth with 5% noise |

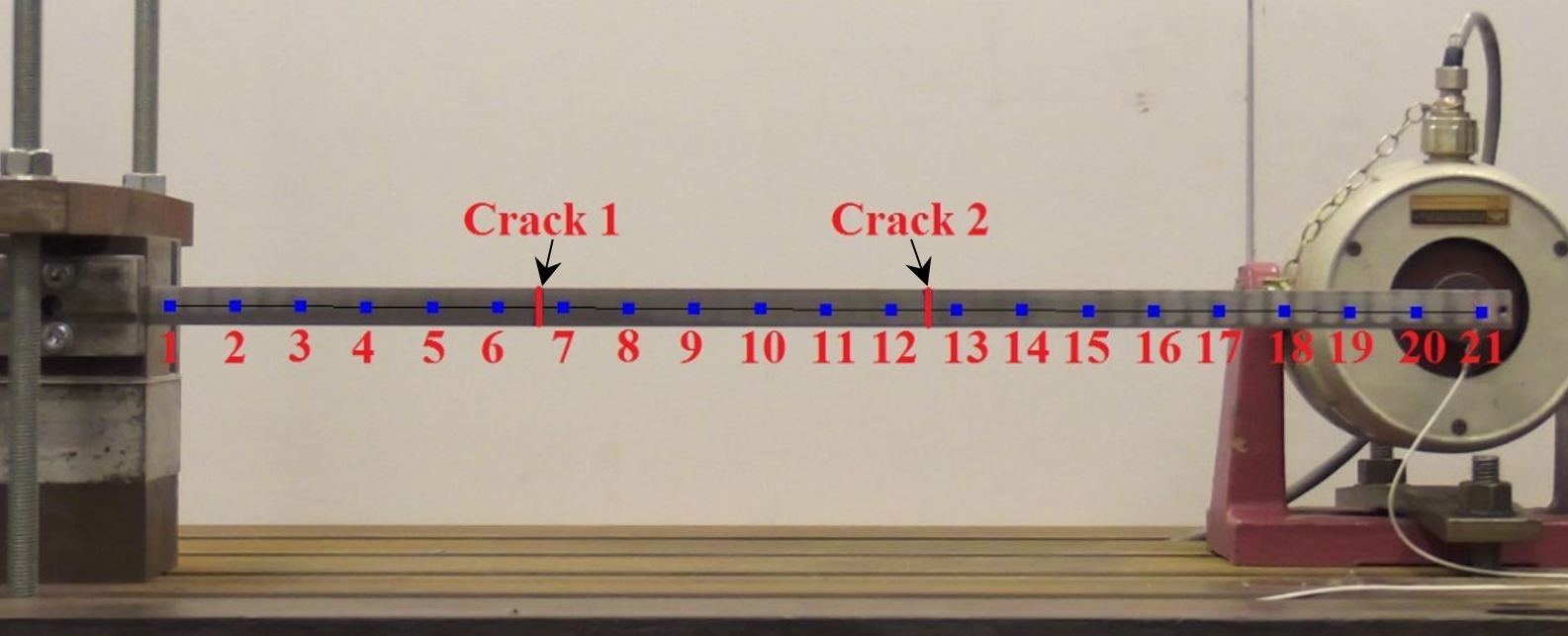
Figure 7 Damage localisation results of different damage and noise levels.

# 5. Experimental studies

The purposes of this section are twofold. First, the CDS’s estimated by POD method, SOBI method and the proposed modified CEA method will be compared experimentally. Secondly, the proposed DI will be validated and a comparison with some traditional damage indexes is presented as well. Two steel beams having dimensions of with two open cracks are tested. Experimental set-up is shown in figure 8(a). A PSV-500 Scanning Laser Vibrometer is used to acquire the velocity responses of prescribed 21 points as marked in blue points in figure 8(b). Pseudo-random excitation in frequency range of 0-800Hz is generated by the PSV-500 system and applied to the cantilever beam through a shaker (LDS V406). In addition, the driving force has a normal distribution with mean value being 0 and standard deviation being 13.2 N. Damage is cut as small slots at different locations and depths. The damage information of the two experimental examples is listed in table 4. In addition, the cracks are on the rear surface and marked as the red lines in the front view in figure 8(b).



(a)



(b)

Figure 8 (a) Experimental set-up (b) A cantilever steel beam with two cracks.

Table 4 – Configurations of cracks in two examples.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Examples | Cracks | Location | | Measurement points | Depth | Damage percentage | | Width |
| 1 | Crack 1 | | 0.2 m | 67 | 0.004 m | 20% | 0.001 m | |
| 1 | Crack 2 | | 0.4 m | 1213 | 0.004 m | 20% | 0.001 m | |
| 2 | Crack 1 | | 0.2 m | 67 | 0.006 m | 30% | 0.001 m | |
| 2 | Crack 2 | | 0.4 m | 1213 | 0.006 m | 30% | 0.001 m | |

Figure 9 presents the estimated normalised CDS’s and their curvatures for experimental example 1 by using output velocity responses. It is worth noticing that the CDS’s of SOBI method and CEA method are a good estimation of the mode shapes of the integrated system composed by the cantilever beam and the shaker. Another conclusion from figure 9 is that it is impossible to determine the positions of the two cracks by observing the irregular shape distortions in CDS’s or CDS curvatures. Hence, it is necessary to investigate further signal processing methods and use damage index for robust damage localisation.

|  |  |
| --- | --- |
| (a) Normalised first CDS | (b) First CDS curvature |

|  |  |
| --- | --- |
| (c) Normalised second CDS | (d) Second CDS curvature |

|  |  |
| --- | --- |
| (e) Normalised third CDS | (f) Third CDS curvature |

Figure 9 The estimated CDS’s and CDS curvatures in experimental example 1.

Based on gapped smoothing method, the damage-caused changes of each normalised CDS and their curvatures by CEA method in experiment 1 are shown in figure 10. The conclusion form figure 6, that different CDS’s have different levels of damage sensitivity, is also verified in figure 10. And in experiment 1, the second crack is also near a node of the third CDS, thus it cannot be well detected by using the third CDS, as demonstrated in figure 10 (e). However, different from damage-induced changes in CDS’s and CDS curvatures in figure 6, results in figure 10 tend to be more affected by the quality of measurement data. For instance, the damage-caused differences in the first CDS and CDS curvature in figures 6 (a) and (b) are able to provide useful information for the second crack but in figures 10 (a) and (b), the first CDS and CDS curvature only contribute to the detection of the first crack. Moreover, in figures 6 (d) and (f), the peaks appear only around the damaged areas whilst there are several false alarms at non-damage positions such as measurement point 11 in figure 10 (d) and measurement point 5 in figure 10 (f). The reason could be that the nature of measurement noise (random errors) is not well known and system errors are also present. Apart from these, it is hard to remove the effects of measurement noise completely and the second-order central difference approach for CDS curvature estimation may also amplify the errors of CDS’s. Although damage identification in reality (using experimental data) tends to be more difficult than in numerical studies, the proposed damage index based on CEA method is able to provide robust and accurate damage identification results, which will be demonstrated next.

|  |  |
| --- | --- |
| (a) Damage-caused changes of first CDS | (b) Damage-caused changes of first CDSC |

|  |  |
| --- | --- |
| (c) Damage-caused changes of second CDS | (d) Damage-caused changes of second CDSC |

|  |  |
| --- | --- |
| (e) Damage-caused changes of third CDS | (f) Damage-caused changes of third CDSC |

Figure 10 Damage-induced differences of CDS’s and CDS curvatures in experimental example 1.

To validate feasibility and effectiveness of the proposed DI in experiments, the damage localisation results of experimental examples 1 and 2 are given in figure 11. In figure 11 (a), both cracks are 20% of beam depth and it can be seen that the damage identification results of SOBI and CEA methods are much better than POD method when dealing with small damage. In figure 11 (b), both cracks are 30% of beam depth and it can be observed that all the three methods provide accurate localisation results. In addition to this, by comparing figure 11(a) with 11(b), it can be concluded that the proposed damage index is able to indicate the relative severity of damage at the same positions.

|  |  |
| --- | --- |
| (a) | (b) |

Figure 11 Damage index values: (a) experimental example 1 (b) experimental example 2.

In order to validate the proposed new weighing criterion and evaluate the performance of the proposed DI, a comparison with co-ordinate modal assurance criterion (COMAC) method, mode shape difference method (MSD), mode shape curvature difference method (MSCD) and damage index method (DIM), as tabulated in table 5, is presented in figure 12. Moreover, for easy comparison of the performance of different methods, all the damage indexes are normalised with the high values indicating the damage locations. For example, the quantity of COMAC is replaced by COMAC.

Table 5 A list of selected common damage identification methods

|  |  |
| --- | --- |
| Method | Damage index |
| Co-ordinate modal assurance criterion |  |
| Mode shape difference method |  |
| Mode shape curvature difference method |  |
| Damage index method | , |

It can be observed that all the damage identification methods show some peaks around the damage positions, but the proposed damage index is the most robust to noise and provides fewer false alarms than other selected methods. Besides, the two damage identification methods based on CDS curvatures in figures 12 (d) and (e) show more large DI values at non-damage locations. The two main reasons are as follows. First, the estimation of CDS curvatures from CDS’s amplifies the local details of CDS’s, which include not only the damage caused shape distortions but also the measured errors of CDS’s. Therefore, further signal processing such as wavelet transform and smoothing technique should be investigated to obtain better damage identification results when using CDS curvatures [41]. Secondly, an appropriate weighting strategy is not adopted in MSCD and DIM, which is important for effective multi-damage identification.

|  |  |
| --- | --- |
| (a) DI of the proposed method | (b) DI of 1-COMAC |

|  |  |
| --- | --- |
| (c) DI of MSD method | (d) DI of MSCD method |



|  |
| --- |
| (e) DI of DIM |

Figure12 A comparison with selected common damage identification methods based on experimental example 1

# 6. Conclusions

This paper presents experimental and theoretical studies of two vital problems for characteristic deflection shape (CDS) based non-destructive damage identification: robust CDS estimation and robust multi-damage localisation. For the CDS estimation problem, a review of proper orthogonal decomposition (POD), principle component analysis (PCA), singular value decomposition (SVD) and second order blind identification (SOBI) methods and their relationship is presented. Those approaches play a dominant role in covariance or correlation matrix based feature extraction. In this study, in order to enhance the CDS estimation accuracy, a modified common eigenvector analysis (CEA) is proposed and investigated, which deals with a set of covariance matrices for common spatial pattern feature extraction in structural dynamics. A numerical study of beams with two cracks contaminated by Gaussian white noise is carried out to prove the better noise robustness of CEA method in CDS estimation over POD and SOBI methods. Furthermore, damage index based on CDS’s of CEA method is found to be more accurate and noise robust than damage indexes using CDS’s of POD and SOBI methods. This study points out that the damage-induced shape distortions have a big magnitude variation in different CDS’s and it is hard to localise all the damage by one CDS. To overcome those problems, a new damage index is proposed to fairly use the damage information of several CDS’s. This damage index is validated numerically and experimentally and a comparison with some traditional damage localisation methods is made to show its advantages. In addition, the proposed damage index is demonstrated to have the capability of indicating the relative damage severity.

Apart from the contributions to non-destructive damage identification, the modified common eigenvector analysis is attractive and promising for robust operational modal analysis. Like the singular value decomposition method, CEA is a powerful eigenvector estimation technique and statistically has the advantage of providing more robust mode shape estimation.

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