High-Robustness and Low-Complexity Joint Estimation of TOAs and CFOs for Multiuser SIMO OFDM Systems

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***Abstract*—We propose the first joint estimation scheme of time of arrivals (TOAs) and carrier frequency offsets (CFOs) for multiuser single-input multiple-output (SIMO) orthogonal frequency division multiplexing (OFDM) systems. By designing carefully preambles of K users and utilizing the properties of the channels and CFO vectors, a complex $2K$-dimensional ($2K$-D) joint multi-TOA and multi-CFO estimation problem is divided into $2K$ low complexity 1-D problems. The integer and fractional parts of each user's CFO are estimated in an integral and closed form. Simulation results show that the proposed scheme provides higher TOA and CFO estimation accuracy than the existing methods, while at much lower computational complexity. The achieved TOA estimation performance is robust against CFOs, and close to the idea case with no CFOs at receiver.**

***Index Terms*—Time of arrival (TOA), carrier frequency offset (CFO), single-input multiple-output (SIMO), orthogonal frequency division multiplexing (OFDM).**

I. Introduction

Time of arrival (TOA), defined as the propagation delay of the first arriving path, plays an important role in localization, synchronization and channel estimation of mobile networks, wireless sensor networks etc. [1]–[3]. A number of TOA estimators have been proposed for orthogonal frequency division multiplexing (OFDM) systems in the literature, such as multiple signal classification [1], estimation of signal parameters via rotational invariance technique (ESPRIT) [2], matrix pencil [3] etc. However, none of them proposed a scheme to combat carrier frequency offset (CFO), which is usually incurred by the mismatch between local oscillators at the transmitter and receiver or a Doppler frequency shift [4]–[7]. When the received signals are perturbed by CFO, the aforementioned TOA estimators will be biased. In [8], a TOA estimator robust against CFO was proposed for IEEE 802.15.4a Chirp Spread Spectrum (CSS) signals [8]. However, it is invalid for OFDM systems. Meanwhile, all aforementioned TOA estimators consider a single user only. To the best of our knowledge, the research of robust multi-TOA estimator against CFOs in OFDM systems is still an open problem.

Furthermore, accurate CFO estimation is essential for OFDM receiver design. CFO can be divided into an integer CFO (iCFO) and a fractional CFO (fCFO). Existing methods usually estimate CFO in two steps. The first step is to estimate the integer/fractional part of CFO whereas the second step is to estimate the rest fractional/integer part [4]–[7]. Nevertheless, they perform poorly if the residual fCFO or iCFO exists. Thus, it is crucial to develop a one-step CFO estimator so that iCFO and fCFO can be estimated as a whole rather than separately.

Moreover, the existing studies in OFDM systems usually estimated either TOAs [1]–[3] or CFOs [4]–[7], but not both together. A joint TOA and CFO estimation algorithm has been proposed for IEEE 802.15.4a CSS signals [9]. However, it has considered a single user only and is not applicable for OFDM systems. The joint estimation of TOAs and CFOs in multiuser OFDM systems is still an open issue. It is not straightforward to apply the above TOA and CFO estimators to multiuser OFDM systems as they will interfere with each other. Thus, they need to be performed sequentially. For example, multiple CFOs should be estimated first at base station (BS) and compensated at individual users via feedback. Only after CFOs compensation, can multiple TOAs be estimated. This requires separate training processes for CFOs and TOAs estimation, which is complex and spectrum inefficient.

In this correspondence, a joint multi-TOA and multi-CFO estimation (JMTMCE) scheme is proposed for a K-user single-input multiple-output (SIMO) OFDM system. Our work is different in that, this, to the best of our knowledge, is the first work to consider both TOAs and CFOs and their estimation together, for multiuser OFDM systems, without requiring separate training processes as by existing methods [1]–[7]. Through a careful preamble design for each user, a complex $2K$-dimensional ($2K$-D) joint multi-TOA and multi-CFO estimation problem is divided into $2K$ 1-D estimation problems. The estimation is performed in two steps: 1) TOAs and CFOs are separated into K groups corresponding to K users, by performing correlation between the received and transmitted preambles; 2) the TOA and CFO of each user are separated and estimated, by deriving a CFO-independent

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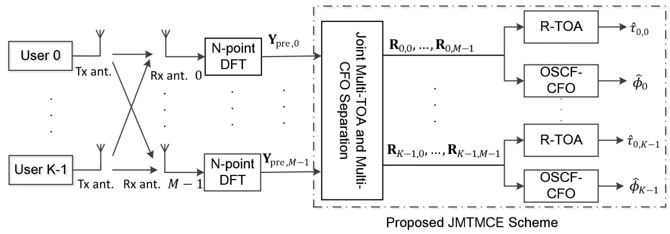

Fig. 1. Block diagram of the proposed JMTMCE scheme for a K-user SIMO OFDM system.

channel correlation matrix and a TOA-independent correlation matrix of the CFO vector. Also, the integer and fractional parts of each user's CFO are estimated in an integral and closed form, avoiding the error propagation due to sequential estimation of iCFO and fCFO in [4]–[7]. Simulation results show that the proposed scheme demonstrates higher TOA and CFO estimation accuracy than the existing methods [2], [4], [7], with a 30-fold complexity reduction. The achieved TOA estimation performance is robust against CFOs, and close to the ideal case with no CFOs at receiver.

Section II describes the system model. The preamble design and proposed JMTMCE scheme are presented in Sections III and IV. Complexity analysis and simulation results are given in Sections V and VI. Section VII draws conclusion.

Notations: Bold symbols represent vectors/matrices, and superscripts T, $*$, H and -1 denote the transpose, complex conjugate, complex conjugate transpose and pseudo inverse of a vector/matrix. diag$\{\mathbf{a}\}$ is a diagonal matrix with vector $\mathbf{a}$ on its diagonal. $\mathbf{I}_N$ and $\mathbf{0}_{M\times K}$ are the N-D identity matrix and a $M \times K$ zero matrix. $\otimes$ and $\mathbb{E}\{\}$ are the Kronecker product and expectation operator. $\mathbf{A}(r_1\colon r_d\colon r_2, c_1\colon c_d\colon c_2)$ is the submatrix of $\mathbf{A}$ with rows from r_1 to r_2 with step size r_d and columns from c_1 to c_2 with step size c_d. $\angle b$ is the angle of b. $\|\cdot\|_{\text{F}}^2$ is the Frobenius norm.

II. System Model

We consider an uplink K-user SIMO OFDM system, as depicted in Fig. 1. Each user and the BS are equipped with a single transmit antenna and M receive antennas respectively. A data frame consists of N_s OFDM blocks with N subcarriers each. Define $\mathbf{x}_k(i) = [x_k(0,i), \cdots, x_k(N-1,i)]^T$ as the signal vector of user k ($k = 0, \cdots, K-1$) in OFDM block i ($i = 0, \cdots, N_s - 1$), with $x_k(n,i)$ denoting the symbol on subcarrier n ($n = 0, \cdots, N-1$). Before transmission, each OFDM symbol block $\mathbf{x}_k(i)$ is processed by Inverse Discrete Fourier Transform (IDFT), and then a cyclic prefix (CP) of length L_{cp} is pre-pended.

The channel is assumed to remain constant for a frame's duration. Define L, $\alpha_{l,k,m}$ and $\tau_{l,k,m}$ as the number of resolvable propagation paths, the complex path gain and path delay of the l-th ($l = 0, \cdots, L-1$) path between the k-th user and the m-th receive antenna of the BS respectively. Since the scale of the transmit or receive antenna array is very small compared to the large signal transmission distance, the channels of different transmit-receive antenna pairs share very similar scatters [10]. Meanwhile, the path delay differences among the similar scatters are much smaller than the system sampling interval [10]. Hence, common time delays can be assumed for different transmit-receive antenna pairs [10], *i.e.*, $\tau_{l,k} = \tau_{l,k,m}$ for $m = 0, \cdots, M-1$. The time delay of the first arriving path $\tau_{0,k}$, either being line-of-sight (LoS) or Non-LoS, represents the TOA [1], [11], and requires to be estimated for the desired application. For example, the estimated TOA can be used by the relevant localization techniques in [11], either in LoS or NLoS scenarios, to determine the position of user k. The discrete channel impulse response (CIR) at the n-th ($n = 0, \cdots, N-1$) sampling point is given by

$$\hbar_{k,m}[n] = \sum\nolimits_{l=0}^{L-1} \alpha_{l,k,m}\delta(n/f_{\text{s}} - \tau_{l,k}) \tag{1}$$

where f_{s} is the sampling rate. Denote $\boldsymbol{\hbar}_{k,m} = [\hbar_{k,m}[0], \cdots, \hbar_{k,m}[N-1]]^T$ as the CIR vector. Its corresponding channel frequency response (CFR) vector is obtained as $\mathbf{h}_{k,m} = [h_{k,m}[0], \cdots, h_{k,m}[N-1]]^T$ with $h_{k,m}[n] = \sum_{l=0}^{L-1} \alpha_{l,k,m} e^{-j2\pi n \tau_{l,k} f_s/N}$ [12].

Define ϕ_k ($\phi_k = \phi_{\text{i},k} + \phi_{\text{f},k}$) as the CFO between the k-th user and the BS with $\phi_{\text{i},k}$ and $\phi_{\text{f},k}$ being the iCFO and fCFO, assuming all antennas at the BS share one local oscillator [7]. After removing the CP, the received signal at the m-th receive antenna in the frequency domain can be written as

$$\mathbf{Y}_m = \sum_{k=0}^{K-1} \mathbf{C}_k \mathbf{H}_{k,m} \mathbf{X}_k + \mathbf{W}_m \tag{2}$$

where $\mathbf{Y}_m = [\mathbf{y}_m(0), \cdots, \mathbf{y}_m(N_s-1)], \mathbf{y}_m(i) = [\mathrm{y}_m(0,i), \cdots, \mathrm{y}_m(N-1,i)]^T$ with $\mathrm{y}_m(n,i)$ as the received symbol on subcarrier n in OFDM block i; $\mathbf{C}_k = \mathbf{F}\mathbf{E}_k\mathbf{F}^H$ is the inter-carrier interference (ICI) matrix of the k-th user, in which $\mathbf{F}$ is the $N \times N$ DFT matrix with $\mathbf{F}(a,b) = 1/\sqrt{N} e^{-j2\pi ab/N}$, ($a, b = 0, \cdots, N-1$), and $\mathbf{E}_k = \text{diag}\{\mathbf{e}_k\}$ is the diagonal CFO matrix with $\mathbf{e}_k = [1, \cdots, e^{j2\pi(N-1)\phi_k/N}]^T$ being the CFO vector; $\mathbf{H}_{k,m} = \text{diag}\{\mathbf{h}_{k,m}\}$ is the diagonal CFR matrix; $\mathbf{X}_k = [\mathbf{x}_k(0), \cdots, \mathbf{x}_k(N_s-1)]$; $\mathbf{W}_m$ is the additive white Gaussian noise matrix.

III. Preamble Design

The preambles are designed for the joint estimation of multiple TOAs and CFOs. A preamble of P OFDM blocks $\mathbf{X}_{\text{pre},k} = [\mathbf{x}_k(0), \cdots, \mathbf{x}_k(P-1)]$ is designed for each user from two aspects. First, the preambles of different users should be orthogonal to each other, allowing the joint separation of multiple TOAs and multiple CFOs in Subsection IV-A. Second, for each user, the training symbols on the occupied subcarriers should be orthogonal to enable the independent estimation of TOA and CFO in Subsections IV-B and IV-C.

A Hadamard matrix $\mathbf{M}_P$ of size $P \times P$ can be applied to achieve these, in which any two different rows are orthogonal to each other [6]. Hence, every occupied subcarrier of each user should be assigned a unique row of $\mathbf{M}_P$. For each user, Q ($Q < N$) subcarriers are used for the joint estimation. Define $T = \frac{N}{Q}$ as the subcarrier spacing. Each user uses a unique initial subcarrier index, *e.g.*, j_k ($j_k = 0, \cdots, T-1$) for the k-th user. The subcarrier index of the q-th ($q = 0, \cdots, Q-1$)

pilot tone for the k-th user is $I_k^q = j_k + qT$. Denote $\mathbf{I}_k = [I_k^0, \cdots, I_k^{Q-1}]^T$. Then Q different rows of $\mathbf{M}_P$ ($P \leq KQ$) are randomly chosen and placed on those subcarriers with index $\mathbf{I}_k$. In contrast, the remaining $(N-Q)$ subcarriers are allocated with nulls. Define $\mathbf{R}_{kg} = \frac{1}{P}\mathbf{X}_{\text{pre},k}\mathbf{X}_{\text{pre},g}^H$ as the preamble correlation matrix averaged over P blocks between the k-th and the g-th ($g = 0, \cdots, K-1$) user. With this preamble structure, the correlation matrix $\mathbf{R}_{kg}$ becomes

$$\mathbf{R}_{kg} = \begin{cases} \mathbf{I}_Q \otimes \mathbf{Z}_k, & k = g \\ \mathbf{0}_{N\times N}, & k \neq g \end{cases} \tag{3}$$

where $\mathbf{Z}_k$ is a $T \times T$ single-entry matrix with $\mathbf{Z}_k(j_k, j_k) = 1$.

It is noteworthy that an orthogonal training sequence with the help of a Hadamard matrix was also designed in [6]. However, it allows only the estimation of multiple CFOs not the joint estimation of multiple TOAs and multiple CFOs. As P is lower bounded by KQ, the choice of Q is essential. The minimum value of Q for the TOA and CFO estimation is $2L$, as suggested in [12]. The shortest preamble length utilized $P = KQ$ is considered in this paper.

IV. Joint Multi-TOA and Multi-CFO Estimation

By performing correlation between the received and transmitted preambles, the TOAs and CFOs of K users are first separated jointly by user, dividing a $2K$-D joint multi-TOA and multi-CFO estimation problem into K 2-D joint TOA and CFO estimation problems. Then, for each of the K users, the 2-D estimation problem can be divided into two 1-D problems to allow independent estimations of TOA and CFO, by deriving a CFO-independent channel correlation matrix and a TOA-independent correlation matrix of the CFO vector. The integer and fractional parts of each CFO are estimated in an integral and closed form. The proposed JMTMCE scheme is illustrated in Fig. 1.

A. Joint Multi-TOA Separation and Multi-CFO Separation

As the preamble design is orthogonal in the user domain, the preamble of each user is used as a projection matrix, to separate K TOAs and K CFOs jointly. Let $\mathbf{Y}_{\text{pre},m} = [\mathbf{y}_m(0), \cdots, \mathbf{y}_m(P-1)]$ denote the received preamble matrix at the receive antenna m. Define $\mathbf{R}_{k,m} = \frac{1}{P}\mathbf{Y}_{\text{pre},m}\mathbf{X}_{\text{pre},k}^H$ as the correlation matrix between the received mixture of preambles from K users at the receive antenna m and the transmit preamble of user k averaged over P blocks. By using (2) and (3), $\mathbf{R}_{k,m}$ can be written as

$$\mathbf{R}_{k,m} = \mathbf{C}_k\tilde{\mathbf{H}}_{k,m} + \tilde{\mathbf{W}}_{\text{pre},m} \tag{4}$$

where $\tilde{\mathbf{H}}_{k,m} = \mathbf{H}_{k,m}(\mathbf{I}_Q \otimes \mathbf{Z}_k)$ and $\tilde{\mathbf{W}}_{\text{pre},m} = \frac{1}{P}\mathbf{W}_{\text{pre},m}\mathbf{X}_{\text{pre},k}^H$ is the noise matrix averaged over P blocks, with $\mathbf{W}_{\text{pre},m} = [\mathbf{W}_m(0), \cdots, \mathbf{W}_m(P-1)]$.

Hence, K $\mathbf{R}_{k,m}$ matrices at the m-th receive antenna have been separated with no interference from other users. Since the TOA and CFO of the k-th user are embedded in $\mathbf{R}_{k,m}$, the K TOAs and K CFOs have been separated jointly by user.

B. Multi-TOA Estimation

We propose a robust TOA (R-TOA) estimator, where the TOA can be recovered from $\mathbf{R}_{k,m}$ directly, without requiring any explicit procedures for CFO estimation and compensation in advance. The core idea is to obtain a CFO-independent channel correlation matrix from $\mathbf{R}_{k,0}, \cdots, \mathbf{R}_{k,M-1}$ firstly, and then to apply it with the ESPRIT algorithm to extract the TOA. The extraction of the CFO-independent channel correlation matrix can be summarized in three steps.

Step 1: N CFO-dependent channel correlation matrices are calculated from the diagonal vectors of $\mathbf{R}_{k,m}$ and the shift versions of $\mathbf{R}_{k,m}$ respectively. Let $\mathbf{d}_{k,m}^n = \text{diag}\{\mathbf{R}_{k,m}^n(j_k\colon T\colon N, j_k\colon T\colon N)\}$ denote the truncated diagonal vector of $\mathbf{R}_{k,m}^n$, where $\mathbf{R}_{k,m}^n$ indicates that $\mathbf{R}_{k,m}$ is up-shifted by n rows ($n = 0, \cdots, N-1$). $n = 0$ means there is no shift. Define $[c_k^0, c_k^1, \cdots, c_k^{N-1}]^T$ as the first column vector of $\mathbf{C}_k$. Then, the n-th CFO-dependent channel correlation matrix $\mathbf{R}_{k,m}^{\mathbf{dd},n}$ is computed by $\mathbf{R}_{k,m}^{\mathbf{dd},n} = \mathbf{d}_{k,m}^n(\mathbf{d}_{k,m}^n)^H$. Due to the Toeplitz property of $\mathbf{C}_k$ and the diagonal feature of $\tilde{\mathbf{H}}_{k,m}$, we can express $\mathbf{R}_{k,m}^{\mathbf{dd},n}$ as

$$\mathbf{R}_{k,m}^{\mathbf{dd},n} = |c_k^n|^2\bar{\mathbf{h}}_{k,m}\bar{\mathbf{h}}_{k,m}^H + \bar{\mathbf{W}}_{\text{pre},m}^n \tag{5}$$

where $\bar{\mathbf{h}}_{k,m} = \mathbf{h}_{k,m}(j_k\colon T\colon N, 1)$ is the truncated CFR vector and $\bar{\mathbf{W}}_{\text{pre},m}^n$ is the noise correlation matrix. Since $|c_k^n|^2$ varies with the CFO, $\mathbf{R}_{k,m}^{\mathbf{dd},n}$ is susceptible to the CFO.

Step 2: To achieve the TOA estimation independently of the CFO, we sum all of the N CFO-dependent channel correlation matrices. Denote $\mathbf{R}_{k,m}^{\mathbf{dd}} = \sum_{n=0}^{N-1}\mathbf{R}_{k,m}^{\mathbf{dd},n}$ as the summed correlation matrix. By considering $\mathbf{C}_k^H\mathbf{C}_k = \mathbf{F}\mathbf{E}_k^H\mathbf{F}^H\mathbf{F}\mathbf{E}_k\mathbf{F}^H = \mathbf{I}_N$, we can show $\sum_{n=0}^{N-1}|c_k^n|^2 = 1$, which is a constant. Thus, $\mathbf{R}_{k,m}^{\mathbf{dd}}$ can be given by

$$\mathbf{R}_{k,m}^{\mathbf{dd}} = \bar{\mathbf{h}}_{k,m}\bar{\mathbf{h}}_{k,m}^H + \bar{\mathbf{W}}_{\text{pre},m} \tag{6}$$

where $\bar{\mathbf{W}}_{\text{pre},m} = \sum_{n=0}^{N-1}\bar{\mathbf{W}}_{\text{pre},m}^n$. Hence, the impact of the CFO has been removed.

Step 3: Thanks to the assumption of equal delays among antennas, M receive antennas can provide space diversity to enhance the TOA estimate. The spatial averaged CFO-independent channel correlation matrix $\mathbf{R}_k^{\mathbf{dd}}$ is calculated by $\mathbf{R}_k^{\mathbf{dd}} = \frac{1}{M}\sum_{m=0}^{M-1}\mathbf{R}_{k,m}^{\mathbf{dd}}$. Then, it is improved by the forward-backward (FB) averaging technique, obtaining $\mathbf{R}_{\text{FB},k}^{\mathbf{dd}} = \frac{1}{2}(\mathbf{R}_k^{\mathbf{dd}} + \mathbf{J}(\mathbf{R}_k^{\mathbf{dd}})^*\mathbf{J})$ where $\mathbf{J}$ is the $Q\times Q$ matrix whose components are zero except for ones on the anti-diagonal.

Last, $\mathbf{R}_{\text{FB},k}^{\mathbf{dd}}$ is applied to the Step 4 of the ESPRIT algorithm [2] to obtain the TOA estimate of user k as $\hat{\tau}_{0,k}$.

C. Multi-CFO Estimation

We propose a one-step closed-form CFO (OSCF-CFO) estimator, where the CFO is extracted independently of the TOA and its integer and fractional parts are estimated as a whole. The core idea of the CFO estimation is similar to that of the TOA estimation. Specifically, a TOA-independent correlation matrix of the CFO vector is computed firstly, which is then applied with the ESPRIT algorithm to extract the unknown CFO. The computation of the TOA-independent correlation matrix of the CFO vector involves four steps.

TABLE I
ANALYTICAL COMPUTATIONAL COMPLEXITY (N: SIZE OF AN OFDM SYMBOL, K: NUMBER OF USERS, Q: NUMBER OF SUBCARRIERS FOR PREAMBLE DESIGN, M: NUMBER OF RECEIVE ANTENNAS, P_1 AND Q_1 ARE THE CORRESPONDING P AND Q IN [4], P_2 AND P_3: NUMBER OF SYMBOL BLOCKS OF [7] AND [2]. SEP.: SEPARATION, EST.: ESTIMATION.)

Item		JMTMCE	ICFO [4]+FCFO [7]+ESPRIT [2]
Multi-CFO and Multi-TOA sep.		$O(2N^2KQM)$	$O(Q^2K^2M)$
Multi-CFO est.	Multi-iCFO est.	$O(K(4N^2Q+5Q^3+2Q^2NM))$	$O(KQ_1(4N^2P_1+5M^2NP_1))$
	Multi-fCFO est.		$O(\frac{3}{4}KM^3P_2^3N\log_2 N)$
Multi-TOA est.		$O(K(9Q^3+2Q^2NM))$	$O(2Q^2MP_3K+5Q^3K)$

Step 1: A channel-perturbed CFO matrix $\tilde{\mathbf{E}}_{k,m}$ is calculated by performing IDFT on $\mathbf{R}_{k,m}$, *i.e.*, $\tilde{\mathbf{E}}_{k,m}=\mathbf{F}^H\mathbf{R}_{k,m}\mathbf{F}$. Substituting (4) into it, $\tilde{\mathbf{E}}_{k,m}$ can be given by

$$\tilde{\mathbf{E}}_{k,m}=\mathbf{E}_k\mathbf{T}_{k,m}+\tilde{\mathbf{W}}_{\text{pre},m} \tag{7}$$

where $\mathbf{T}_{k,m}=\mathbf{F}^H\tilde{\mathbf{H}}_{k,m}\mathbf{F}$ is a kind of channel circulant matrix with its first column vector being $\hat{\boldsymbol{\hbar}}_{k,m}=[\hat{\hbar}^0_{k,m},\cdots,\hat{\hbar}^{N-1}_{k,m}]^T$ and $\tilde{\mathbf{W}}_{\text{pre},m}=\mathbf{F}^H\mathbf{W}_{\text{pre},m}\mathbf{F}$ is the noise matrix.

Step 2: N TOA-dependent correlation matrices of the CFO vector are extracted from the diagonal vectors of the original and shifted versions of $\tilde{\mathbf{E}}_{k,m}$. Denote $\tilde{\mathbf{e}}^n_{k,m}=\text{diag}\{\tilde{\mathbf{E}}^n_{k,m}(j_k\colon T\colon N, j_k\colon T\colon N)\}$ as the truncated diagonal vector of $\tilde{\mathbf{E}}^n_{k,m}$, where $\tilde{\mathbf{E}}^n_{k,m}$ means that $\tilde{\mathbf{E}}_{k,m}$ is right-shifted by n columns ($n=0,\cdots,N-1$). Then, the n-th TOA-dependent correlation matrix of the CFO vector $\mathbf{R}^{\mathbf{ee},n}_{k,m}$ is computed by $\mathbf{R}^{\mathbf{ee},n}_{k,m}=\tilde{\mathbf{e}}^n_{k,m}(\tilde{\mathbf{e}}^n_{k,m})^H$. Due to the diagonal feature of $\mathbf{E}_k$ and the circulant property of $\mathbf{T}_{k,m}$, $\mathbf{R}^{\mathbf{ee},n}_{k,m}$ can be represented as

$$\mathbf{R}^{\mathbf{ee},n}_{k,m}=|\hat{\hbar}^n_{k,m}|^2\hat{\mathbf{e}}_k(\hat{\mathbf{e}}_k)^H+\hat{\mathbf{W}}^n_{\text{pre},m} \tag{8}$$

where $\hat{\mathbf{e}}_k=\mathbf{e}_k(j_k\colon T\colon N,1)$ and $\hat{\mathbf{W}}^n_{\text{pre},m}$ is the corresponding noise correlation matrix. $|\hat{\hbar}^n_{k,m}|^2$ is the channel power of the n-th sampling point, whose value changes with the TOA.

Step 3: To enable the independency of the CFO estimation of the TOA, N TOA-dependent correlation matrices in (8) are added up. Denote $\mathbf{R}^{\mathbf{ee}}_{k,m}=\sum_{n=0}^{N-1}\mathbf{R}^{\mathbf{ee},n}_{k,m}$ as the summed correlation matrix, which can be written as

$$\mathbf{R}^{\mathbf{ee}}_{k,m}=\parallel\hat{\boldsymbol{\hbar}}_{k,m}\parallel^2_{\text{F}}\hat{\mathbf{e}}_k\hat{\mathbf{e}}_k^H+\hat{\mathbf{W}}_{\text{pre},m} \tag{9}$$

with $\parallel\hat{\boldsymbol{\hbar}}_{k,m}\parallel^2_{\text{F}}=\sum_{n=0}^{N-1}|\hat{\hbar}^n_{k,m}|^2$ and $\hat{\mathbf{W}}_{\text{pre},m}=\sum_{n=0}^{N-1}\hat{\mathbf{W}}^n_{\text{pre},m}$. Since $\parallel\hat{\boldsymbol{\hbar}}_{k,m}\parallel^2_{\text{F}}$ is the summed channel power and insusceptible to the TOA, the impact of the TOA has been mitigated.

Step 4: The TOA-independent correlation matrix is similarly enhanced by the spatial and FB averaging techniques, obtaining $\mathbf{R}^{\mathbf{ee}}_{\text{FB},k}=\frac{1}{2}(\mathbf{R}^{\mathbf{ee}}_k+\mathbf{J}(\mathbf{R}^{\mathbf{ee}}_k)^*\mathbf{J})$ with $\mathbf{R}^{\mathbf{ee}}_k=\frac{1}{M}\sum_{m=0}^{M-1}\mathbf{R}^{\mathbf{ee}}_{k,m}$.

The CFO of user k is obtained as $\hat{\phi}_k$, by introducing $\mathbf{R}^{\mathbf{ee}}_{\text{FB},k}$ to the ESPRIT algorithm. Let $\mathbf{s}_k$ denote the signal eigenvector corresponding to the largest eigenvalue of $\mathbf{R}^{\mathbf{ee}}_{\text{FB}}$. $\mathbf{s}_{k,1}$ and $\mathbf{s}_{k,2}$ are defined as the first and last $(Q-1)$ elements of $\mathbf{s}_k$ respectively. We can obtain $\hat{v}_k=(\mathbf{s}_{k,1})^{-1}\mathbf{s}_{k,2}$, and the iCFO and fCFO of user k are computed as a whole with $\hat{\phi}_k=\frac{N\angle\hat{v}_k}{2T\pi}$.

It is noteworthy that the proposed JMTMCE scheme is more suitable for long-range systems due to the assumption of equal time delays among different antennas. This assumption does not hold in short-range scenario, since the links between different transmit-receive antenna pairs are more likely to encounter different scatters. This can be solved by simply utilizing time diversity rather than space diversity for long-range case. Instead of utilizing M receive antennas, each user transmits M preambles, and the spatial averaged correlation matrices in Steps 3 and 4 of Subsections IV-B and IV-C are replaced by the temporal averaged correlation matrices. The other parts of the proposed TOA and CFO estimation scheme remain the same.

V. Complexity Analysis

In Table I, the computational complexity of the proposed JMTMCE scheme and the existing methods [2], [4], [7] are presented, in terms of the number of complex additions and multiplications. The integer CFO estimator in [4], the fractional CFO estimator in [7] and the ESPRIT-based TOA estimator [2], referred to as ICFO, FCFO and ESPRIT respectively, are selected for comparison. With $N=64, K=2, Q=16, M=4, Q_1=64, P_1=1, P_2=12$ and $P_3=6$, the numerical complexity of the proposed scheme and the existing methods [2], [4], [7] can be computed. The proposed scheme is found to be very computationally efficient, with complexity reduction around 30-fold than the existing methods [2], [4], [7].

VI. Simulation Results

A simulation study is carried out to demonstrate the performance of the proposed JMTMCE scheme, with $K=2$ users. System parameters are set as follows: each OFDM block contains $N=64$ subcarriers; the CP length is $L_{\text{cp}}=16$; the modulation scheme is quadrature phase shift keying; the number of subcarriers for preamble design is $Q=16$; the number of receive antennas at BS is $M=4$; the Long Term Evolution Extended Pedestrian A Model with the sampling rate $f_{\text{s}}=30.72$ MHz [13] is applied; the CFO values are randomly generated in $[-N/2,N/2)$, unless otherwise stated. The root mean square errors (RMSEs) of TOA and CFO estimations are defined as $\text{RMSE}_{\text{TOA}}=\sqrt{\mathbb{E}\{\frac{1}{K}\sum_{k=0}^{K-1}(\hat{\tau}_{0,k}-\tau_{0,k})^2\}}$ and $\text{RMSE}_{\text{CFO}}=\sqrt{\mathbb{E}\{\frac{1}{K}\sum_{k=0}^{K-1}(\hat{\phi}_k-\phi_k)^2\}}$ respectively.

Fig. 2 demonstrates the RMSE of TOA performances as a function of two CFOs for the proposed R-TOA algorithm and the existing ESPRIT algorithm [2] at SNR= 25 dB. The range of two CFOs is from -4 and 4. It can be seen that the RMSE of TOA performance of the ESPRIT algorithm [2] increases with CFOs until a large error floor is formed. However, the proposed R-TOA algorithm shows almost the same RMSE performances for different CFOs.

Fig. 3 shows the RMSE of TOA performances of the proposed R-TOA algorithm, in comparison to the existing methods [2], [4], [7]. The ESPRIT algorithm [2] always has a

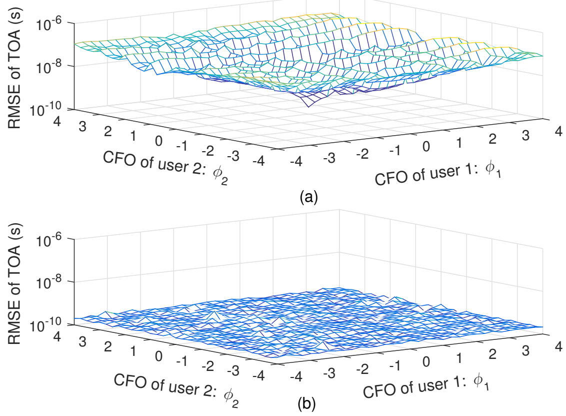

Fig. 2. RMSE of TOA performances as a function of two CFOs for: (a) the existing ESPRIT algorithm [2]; (b) the proposed R-TOA estimator.

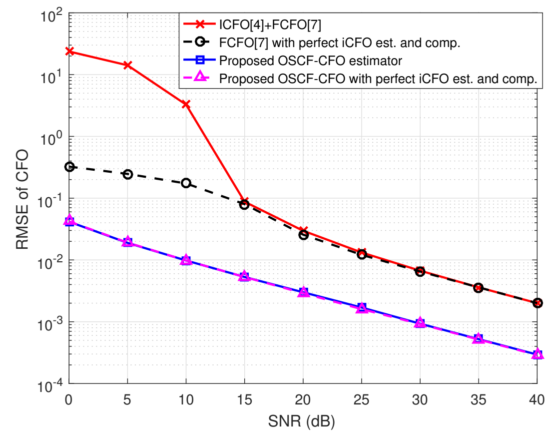

Fig. 4. RMSE of CFO performances of the proposed OSCF-CFO estimator and the existing CFO estimators [4], [7].

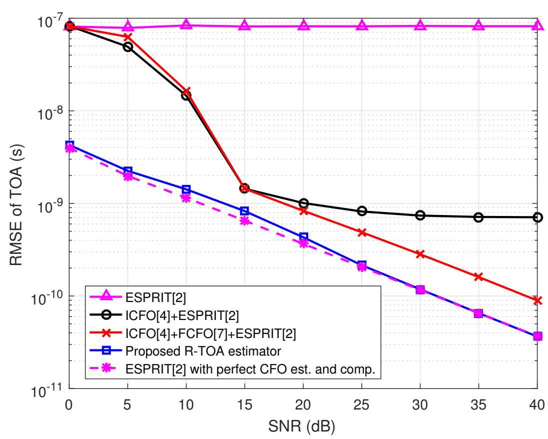

Fig. 3. RMSE of TOA performances of the proposed R-TOA estimator and the existing algorithms [2], [4], [7] (est.: estimation, comp.: compensation).

large error floor due to the presence of iCFOs and fCFOs. The ESPRIT algorithm [2] with iCFO estimation and compensation [4] is biased from SNR$=15$ dB to SNR$=40$ dB. This suggests that both the iCFO and fCFO are required to be removed for the ESPRIT algorithm [2]. Third, the proposed R-TOA algorithm outperforms the ESPRIT algorithm with both the iCFO and fCFO estimation and compensation especially at low SNRs, and also approaches the ESPRIT with perfect CFO estimation and compensation.

In Fig. 4, the RMSE of the proposed OSCF-CFO estimator and the existing CFO estimators [4], [7] are presented. Two groups of the existing CFO estimators are selected for comparison. In the first group ICFO [4] is used for iCFO estimation whereas in the second group a perfect iCFO estimation and compensation is assumed. FCFO [7] is applied to estimate the residual fCFOs for both two groups. It is easily observed that the two groups have a big performance gap at low SNRs. This is because the first group suffers from iCFO estimation error at low SNRs and the FCFO performs poorly in the presence of residual iCFOs. However, the proposed OSCF-CFO estimator is regardless of perfect iCFO estimation and compensation.

VII. Conclusion

A joint multi-TOA and multi-CFO estimation scheme has been proposed for multiuser SIMO OFDM systems, by decomposing a $2K$-D problem into $2K$ 1-D problems, thanks to the well-designed preambles. The proposed scheme provides higher TOA and CFO estimation accuracy than the existing methods [2], [4], [7]. Its TOA RMSE performance is close to the ideal case with perfect CFO estimation and compensation. The TOA and CFO estimation are shown to be more robust against CFOs, and can also reduce the overall complexity up to 30-fold than the existing methods [2], [4], [7].

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