Coordination of pricing, inventory, and production reliability decisions in deteriorating product supply chains

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Abstract

In this article, we study a two level supply chain model for deteriorating items, in which the supplier's production system is unreliable and the retailer's demand is price sensitive. The supplier's production line may randomly shift from the in-control state to the out-of-control state. When the production line is in the out-of-control state, a proportion of the produced products will have bad quality. To mitigate the out-of-control risks, the supplier can improve the production line reliability by investing in high quality machines, highly skilled workers, or advanced maintenance technologies. We start with the study of pricing and inventory problems concerning endogenous reliability in the integrated and decentralized scenario. To better illustrate the proposed models, two applicable algorithms are designed to determine the optimal production reliability, ordering quantity and prices. Then, a cooperative reliability investment and revenue sharing contract is proposed to coordinate the supply chain. Numerical examples and sensitivity analysis of the equilibrium strategies and coordinating results on key system parameters (e.g. deterioration rate, production rate, etc.) are given to verify the effectiveness of the contract, and meanwhile get some managerial insights.

Keywords: Deteriorating items; Unreliable production; Reliability investment; Price; Inventory; Supply chain coordination

1. Introduction

Deterioration is defined as decay, change or spoilage through which the quality and/or the quantity of the items are decreasing (Ghare & Schrader, 1963). Such items include fresh fruits, fresh vegetables, blood, and fashion products, etc. Product deterioration takes place frequently in inventory systems and often causes great losses to inventory managers, especially in multi-level supply chains due to long distance transport or unmatched demand and supply. For example, about 15% of perishable foods are wasted in the food retailing sector (Ferguson & Ketzenberg, 2004). In China, more than 25% of fruits and vegetables are deteriorated during transportation, at wholesale markets and in shops[1]. In addition, in real industry, it is almost impossible for firms to have a perfect reliable production line in a long run production process. For example, the wear of machine parts reduces the reliability of production process, which affects the quality of the produced products (Atan & Snyder, 2014; Sana *et al.*, 2007).

Providing good quality products to customers will enhance customers' satisfactions, thus improve firms' competitiveness and economical benefits. Procurement managers are becoming more concerned about the quality of the products provided by the supplier to maintain high customer satisfaction in today's markets with fierce competition and fast changing customer preferences. Therefore, producers are incentivized to improve product quality and production reliability by investing in the installation of high quality machines, the employment of high-skill workers, or the adoption of advanced maintenance technologies (Huang *et al.*, 2017). The improved production reliability ensures high quality of the products selling through downstream buyers to customers. Therefore, the reliability investment not only benefits the producer itself, but also benefits the buyers.

In practice, to mitigate the negative impacts of producer's production risks, downstream buyers often adopting various risk preventing strategies including multi-sourcing, backup sourcing, and emergency purchases (see Dada *et al.*, 2011; Gurnani *et al.*, 2000; Tomlin, 2006). Nowadays, another increasingly popular approach is buyers providing subsidies on producers' production reliability investments to lower costs, increase products' quality, and improve reliability (see Liu *et al.*, 2010; Wang *et al.*, 2010). For example, it is a common practice in the automotive industry that companies like Honda, Toyota, BMW, and Hyundai are working with their suppliers to improve performance (Krause *et al.*, 2007).

[1] Roebuck M. (2015) "China must improve its cool supply chain to keep pace with demand for fresh food" The cool star. http://theloadstar.co.uk/coolstar/china-must-improve-its-cool-supply-chain-to-keep-pace-with-demand-for-fresh-food/ (Accessed on 2018/2/24)

As studied by Tang *et al.* (2014), with a well-designed subsidy strategy, the individual performance for both the buyer and producer can be improved. However, how product deterioration will affect the optimal process reliability, the associated pricing and inventory decisions are still unclear in literature. In this article, we study a two level supply chain consists of an unreliable supplier and a downstream retailer. The supplier produces and distributes deteriorating products to end customers through the downstream retailer. The supplier can also invest to improve the process reliability of its production system. First, we model the integrated and the decentralized supply chain for deteriorating items without cooperative investment, and study the determination of optimal process reliability, prices and inventories for both firms and total supply chain. Second, we designed a revenue sharing and cooperative investment contract to coordinate the supply chain. Through numerical simulations, we compared the optimal decision strategies in the integrated and the decentralized scenarios, and tested the efficiency of the coordinating contract. Some important managerial implications are presented as well.

The rest of the paper is organized as follows. In section 2, a comprehensive literature review is presented. In section 3, main assumptions and notations used in the paper are listed. In section 4, the model is formulated, which is followed by strategy analysis and coordination methods in section 5. Numerical analysis is presented in Section 6, along with some important managerial insights. Then, the paper is concluded in the last section.

2. Literature review

Our research is closely related to three streams: (a) supply chain management for deteriorating products, (b) inventory models considering production unreliability, and (c) gaming in deteriorating product supply chains, and (c) gaming in deteriorating product supply chains.

The first stream of literature refers to the inventory management problems for deteriorating products. According to Ghare and Schrader (1963) and Shah, Soni, and Patel (2013), deterioration is defined as decay, change, or spoilage so that the items are not in its initial conditions. There are two categories of deteriorating items. The first category refers to the items that become decayed, damaged, or expired with time, e.g. meat, vegetables, fruits, medicine, and so on. The second category is the items that lose part or total value with time, e.g. computer chips, mobile phones, fashion and seasonal products, and so on. Both kinds of items have short life cycles and after a period of existence in market, the items lose the original economical value due to the drop of consumer preference, product quality, and so on. Ghare and Schrader (1963) first proposed an exponentially decaying inventory model. Based on their work, people had done a lot on the economical ordering quantity (EOQ) problems for deterioration products. In this research area, different settings of critical factors, e.g. demand rate, deterioration rate, pricing strategies, and so on, have significant impacts on the formulation of the models, and the associated solutions and results. Firstly, for demand rate, it can either be a constant parameter (Mahata 2012) or be a time-dependent parameter (Dye, Chang, and Teng 2006; Giri, Jalan, and Chaudhuri 2003; Wang, Lin, and Jonas 2011). Also, demand can be backlogged (Dye, Chang, and Teng 2006), inventory level linked (Burwell et al. 1997), or price-sensitive (Dye and Hsieh 2012; Liang and Zhou 2011; Shah, Soni, and Patel 2013). Secondly, for deterioration rate, it can be a constant parameter (Liang and Zhou 2011; Sana, Goyal, and Chaudhuri 2004; Thangam and Uthayakumar 2009), a time-linked parameter (Skouri et al. 2009), preservation investment-linked parameter (Dye and Hsieh 2013; Hsu, Wee, and Teng 2010), or a stochastic parameter (Sarkar 2013, 2012; Sarkar and Sarkar 2013).

The above research only considers the single-stage inventory problems. Some people studied the problems in multi-level supply chains. Lee and Moon (2006) proposed a basic three-level producer-vendor-buyer model. Wang, Lin, and Jonas (2011) extended Lee and Moon (2006) by assuming that products suffer from time-linked deterioration rate. Besides, many researchers did a lot of work on integrated inventory and/or pricing decisions (Chen and Chen 2007; Lo, Wee, and Huang 2007; Noh, Kim, and Sarkar 2016; Sarkar et al. 2016), and cooperation strategies (Lin, Yu, and Wang 2009, 2010) in multi-level supply chains.

The second stream of literature is about inventory problems considering production unreliability. In the previous research, some papers consider product quality drop during production when the system is in the unreliable state (or out-of-control state). Rosenblatt and Lee (1986), Kim, Hong, and Chang (2001), Chung and Hou (2003), and Rahim and Al- Hajailan (2006) considered production process breakdown in an inventory model, in which, defective items are produced when the production process is disrupted. Then, Sana, Goyal, and Chaudhuri (2007), Sana (2010a), and Sana (2010b) developed an EPQ model or EMQ model to analyze the optimal production problem with unreliable production process. Following Sana (2010b), some more general models were developed. For example, Sarkar (2012) considered the production line is unreliable and demand is price linked. Sarkar and Saren (2016) took the inspection errors and warranty cost into consideration.

In addition to quality dropping problems, some papers also studied the production rate dropping problems when the system became out-of-control. For example, Glock (2013) considered the full breakdown problem. Abboud (1997), Abboud, Jaber, and Noueihed (2000), and Chakraborty, Giri, and Chaudhuri (2008) assumed that the machine unavailability 2 H. Huang et al. time or the repairing time length is random. Chung, Widyadana, and Wee (2011) extended the model of Chakraborty, Giri, and Chaudhuri (2008) by considering deteriorating items with shortage. Gharbi, Kenn, and Beit (2007) studied the preventive maintenance strategies in their inventory models with random machine breakdown. Wee and Widyadana (2012) extended the model of Gharbi, Kenn, and Beit (2007) by considering product deterioration. Giri, Yun, and Dohi (2005) and Jeang (2012) established an EPQ model when the production process was unreliable. In addition to the research on the 'full breakdown' models, some people study the 'partial breakdown' models, such as, Gavish and Graves (1981), Iravani and Duenyas (2002), Ben-Daya, Hariga, and Khursheed (2008), and Huang, He, and Li (2017), etc.

The third stream of literature is about the gaming problems with product deterioration. Zhang *et al.* (2015) studied a two-level price gaming problem between a supplier and a retailer when products are perishable. They proposed a revenue and preservation investment-sharing contract to coordinate the supply chain. Chen (2017) studied a non-cooperative decision problem between a manufacturer and a retailer with imperfect and deteriorating products. The impacts of the price scheme, the replenishment programme, imperfect quality, and the rework process are also analysed. Tiwari, Jaggi, and Gupta (2018) studied a manufacturer–retailer gaming problem for deteriorating products when the retailer has limited storage capacity. Huang, He, and Li (2018) modelled a three-level supply chain which consists of a retailer, a vendor, and a supplier, and studied a Stackelberg gaming problem considering supply uncertainty and preservation investment. He, Huang, and Li (2018) modelled a dual-channel supply chain with a manufacturer who sells through both its direct channel and a down- stream retailer. The impacts of product deterioration on both firms' pricing decisions are discussed.

In the aforementioned literature, no literature considers reliability investment, pricing, and supply chain coordination simultaneously in a two-level supply chain for deteriorating items. Our work extends the study of Huang, He, and Li (2017) to a two-echelon supply chain composed of a supplier and a retailer. Specifically, as the Stackelberg leader, the retailer sets retail price to maximise his own profit, while the following supplier pursues its maximum profit by determining the wholesale price as well as the reliability investment. Furthermore, we design a revenue-sharing and cooperative investment contract to coordinate the supply chain. The main contributions of this paper are threefold. First, this paper addresses the problem of reliability investment and pricing in a supply chain system, and fills the gap of supply chain coordination for deteriorating items. Second, effective algorithms are proposed to solve such a complex problem, providing an approach to deal with similar kind of issues. Third, numerical examples and sensitivity analysis are conducted, which contribute to the finding of some significant managerial implications for supply chain management of deteriorating items.

3. Assumptions and notations

We consider a two level supply chain consists of a supplier and a retailer, in which the supplier's production system is unreliable (see **Figure 1**). The production line may shift from the in-control state to the out-of-control state during the production run. In the in-control state, all the produced items are in good quality; however, in the out-of-control state, bad quality products will be produced, which further affects the product quality in the retail side. We study a case in which the supplier's production reliability is controllable by investment. In other words, the supplier can optimally set the system reliability with investment (Huang, He, and Li, 2017). The final goal is to determine not only the optimal reliability, but also the prices for both parties that maximize the profit of each party in an infinite time horizon.

Figure 1. Diagram of a two level supply chain with unreliable production

3.1. The endogenous production reliability

Following Ben-Daya, Hariga, and Khursheed (2008), in the in-control state, the supplier's production line is reliable. All the produced products in the in-control period are in good quality. However, in the out-of-control state, part of the produced products will be in bad quality. The in-control time length is a random variable, with probability density function $f(s)$ and cumulative distribution function $F(s)$. Assuming the expectation for s is $E[s] = \lambda$. Following the study of Huang, He, and Li (2017), when the expected in control time length $E[s] = \lambda$ is longer, the system is more reliable. On the contrary, when λ is smaller, out-of-control time of the production system becomes shorter and the system will be less reliable. In the following analysis, we use λ to denote the production reliability.

In real industry, companies can enhance the reliability λ of their production system by investing in the installation of high quality machines, the employment of high-skill workers, or the adoption of advanced maintenance technologies (Huang, He, and Li, 2017). In this paper, we assume that the distribution parameter λ is controllable by investing. The initial production system reliability is assumed to be λ_0. The distribution parameter can be increased to λ with a unit time investment $RIC(\lambda)$. The pattern of the investment cost function is assumed to be increasing and convex in λ, i.e. $RIC'(\lambda) > 0$ and $RIC''(\lambda) > 0$, which follows the law of diminishing marginal utility (DMU) and with condition $RIC(\lambda = \lambda_0) = 0$.

According to previous research, the in-control time length s may follow a uniform distribution, i.e. $s \sim U[0, 2\lambda]$ (Chakraborty, Giri, and Chaudhuri, 2008; Chung, Widyadana, and Wee, 2011) or an exponential distribution, i.e. $s \sim E(\lambda)$ (Jeang, 2012; Huang, He, and Li, 2017). In the model analysis, we first assume that s follows the uniform distribution, which helps us to get some analytical results. We present the numerical results for the model with an exponentially distributed s in Appendix A.1.

3.2. Notations

We present the notations in **Table 1**.

Table 1. Notations

Decision variables	
w	supplier's wholesale price
p	retail price
m	retailer's sales margin, $m = p - w$
Q	retailers ordering quantity
λ	expectation of in-control time length after reliability investment
γ	retailer's shared proportion of reliability investment
ϕ	supplier's revenue sharing rate to the retailer
Other parameters	
A_r, A_s	retailer's and supplier's fixed setup cost respectively
P	supplier's production rate, a constant
c_p	supplier's per unit item production cost
$D(p)$	customers' demand rate , a function of retail price $D(p) = b - ap$, in which b is the potential market size, and a is the price sensitive parameter of retailer price.
T_r	retailer's (supplier's) ordering cycle (production cycle)
t_s	supplier's production starting time in a cycle
T_s	supplier's production time length, $T_s = T_r - t_s$
θ	deterioration rate of the products
k	cost coefficient of reliability investment cost
$I_r(t), I_s(t)$	retailer's and supplier's inventory level at time t respectively
h	retailer's and supplier's per item per unit time inventory holding cost
s	elapsed time period during which the suppliers production line is in in-control state
$f_\lambda(s), F_\lambda(s)$	probability density function and distribution function of random parameter s. The subscript $(.)_\lambda$ denotes a distribution with mean value λ
λ_0	initial expectation of the in-control time length before investment
α	percentage of bad quality products produced when the production system is in the out-of-control state

r	retailer's penalty cost for selling bad quality products to customers in a cycle
M	supplier's restoration cost in transferring the production line from out-of-control state to in-control state
RIC	supplier's reliability investment cost per unit time
TP_i^j	unit time total profit for supply chain member i under channel structure j
$i = sc, s, r$	subscript, which denotes the whole supply chain, the supplier and the retailer respectively
$j = I, D, C$	superscript, which denotes the associated decisions of integrated, decentralized, coordinated supply chain respectively

3.3. Assumptions

We summarize the assumptions as follows:

(1) In each production cycle, the supplier's production system starts from an in-control state. After some time, the production line may shift to the out-of-control state. During the out-of-control period, a proportion α $(0 < \alpha < 1)$ of the produced products will have bad quality.

(2) At the end of a production cycle, when the out-of-control state appears, the supplier has to restore its production system with a fixed restoration cost M.

(3) The time takes to restore the production system is relatively shorter than the total production cycle length, which makes sure the supplier's production system is in-control at the start of a new production cycle.

(4) The initial reliability λ_0 can be increased to λ with a quadratic unit time investment cost $= k(\lambda - \lambda_0)^2/2$, where k captures the cost coefficient. (Huang, He, and Li, 2017)

(5) In the supply chain, the retailer is the stackelberg leader and the supplier is the follower. In real business, leading retailers refer to some large groceries including *Costco*, *Walmart*, and *Tesco*. These kinds of retailers have strong powers, and they act as leaders in the supply chains.

(6) The supplier adopts a lot-to-lot inventory policy (see Lee and Moon, 2006) to deliver the products to the retailer. Thus the supplier's production cycle length is equal to the retailer's ordering cycle.

(7) Production rate of the supplier is higher than the demand rate, which is to avoid shortages of the supplier.

4. Model formulation

The inventory patterns for both firms are presented in **Figure 2**. According to the assumptions and notations mentioned in section 2, we model the supplier, the retailer and the total supply chain profit respectively.

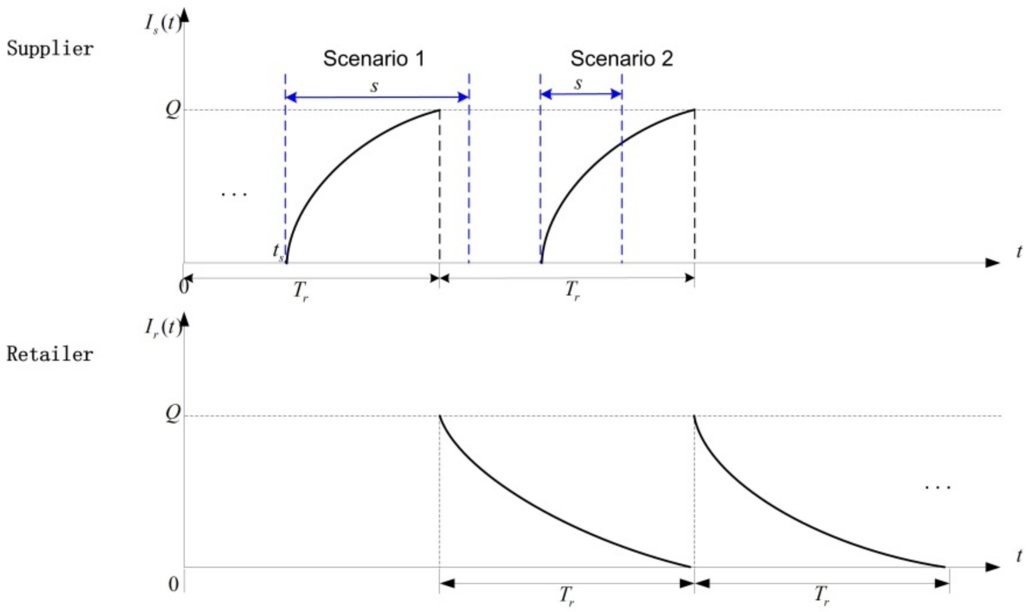

Figure 2. Inventory systems for the retailer and the supplier

4.1. The retailer's profit

Suppose the ordering cycle is T_r. At time, $t = 0$ the retailer places an order of Q units of products to the supplier, and the supplier will distribute the ordered products to the retailer at time $t = T_r$. During the time interval $[0, T_r]$, the retailer's inventory level depletes due to demand and deterioration, which drops to 0 at time $t = T_r$. According to the economical ordering quantity (EOQ) theories for deteriorating products, the inventory level satisfies the differential equation

$$\frac{\partial I_r(t)}{\partial t} = -D(p) - \theta I_r(t), t \in [0, T_r], \tag{1}$$

with boundary condition $I_r(T_r) = 0$.

Solve the differential equation, and we obtain the retailer's inventory level

$$I_r(t) = \frac{D(p)\left(e^{\theta(T_r - t)} - 1\right)}{\theta}, t \in [0, T_r]. \tag{2}$$

The ordering quantity can be presented as

$$Q = I_r(t = 0) = \frac{D(p)\left(e^{\theta T_r} - 1\right)}{\theta}. \tag{3}$$

During one cycle, the retailer's profit consists of sales revenue, replenishment cost, inventory holding cost, fixed starting cost and the penalty cost due to sale of bas quality products.

1) The sales revenue

The retailer's total revenue comes from the sales of the deteriorating products. The total sales revenue in a cycle can be calculated as $pD(p)T_r$.

2) The replenishment cost

The retailer has to pay an amount of money wQ to purchase the products from the supplier.

3) The inventory holding cost

The total accumulated inventory for the retailer in one cycle is $\int_0^{T_r} I_r(t)\,dt$. Therefore, the total inventory cost is $\int_0^{T_r} h\,I_r(t)dt$.

4) The fixed starting cost

The starting a new selling cycle, the retailer has to pay a lump sum cost of A_r.

5) The penalty cost

Customers who purchase and use the low quality products will be unsatisfied, and they may return the products or claim the refunds, which will hurt the retailer's reputation and economical benefit. Therefore, we normalize the retailer's losses caused by selling of bad quality products to a penalty cost r.

It is shown in Figure 1 that,

• (*Scenario 1*) when $s \geq T_s$, the supplier's production system is always in-control and the production line is perfectly reliable during the production run. In this situation, the produced products all have good quality and the penalty cost equals zero.

• (*Scenario 2*) when $s \leq T_s$, the production system will be shifted from the in-control state to the out-of-control state at time $t = s$. In other words, in the time interval $t \in [t_s + s, T_r]$, a proportion of the producedproducts may have quality risk. In this scenario, the penalty cost is incurred.

Combining the two scenarios, the probability that the bad quality products will be produced and sold to customers can be calculated as

$$\frac{Quantities\ of\ bad\ quality\ products}{Quantities\ of\ total\ production} = \frac{\int_0^{T_s} \alpha P(T_s - s) dF_\lambda(s)}{PT_s} = \frac{\alpha \int_0^{T_s} F_\lambda(s)ds}{T_s}.$$

Then, we further formulate the penalty cost in a cycle for the retailer as $\alpha \frac{\int_0^{T_s} F_\lambda(s)ds}{T_s}$.

Combining the above mentioned revenue and costs, the retailer's unit time total profit can be written as

$$TP_r(m) = \frac{1}{T_r}\left\{pD(p)T_r - wQ - \int_0^{T_r} h\,I_r(t)dt - A_r - r\alpha \frac{\int_0^{T_s} F_\lambda(s)ds}{T_s}\right\}$$

$$= (m+w)D(m+w) - \frac{wD(m+w)(e^{\theta T_r}-1)}{\theta T_r} - \frac{hD(m+w)(e^{\theta T_r} - \theta T_r - 1)}{\theta^2 T_r} - \frac{A_r}{T_r} - r\alpha \frac{\int_0^{T_s} F_\lambda(s)ds}{T_s T_r} \quad (4)$$

4.2. The supplier's profit

Based on the assumptions, the supplier adopts a lot-to-lot policy (Lee and Moon, 2006). During each cycle, the supplier starts to produce at time $t = t_s$, and the inventory level of the supplier rises due to the simultaneous influence of production and product deterioration, which follows the differential equation

$$\frac{\partial I_s(t)}{\partial t} = P - \theta I_s(t), t \in [t_s, T_r], \tag{5}$$

with boundary conditions $I_s(T_r - t_s) = 0$ and $I_s(T_r) = Q$.

Thus, the inventory level with respect to time can be derived as

$$I_s(t) = \frac{P}{\theta}\left(1 - e^{-\theta(t - T_r + T_s)}\right), t \in [t_s, T_r], \tag{6}$$

in which $T_s = T_r - t_s = -\frac{1}{\theta} ln\left(1 - \frac{D(p)(e^{\theta T_r} - 1)}{P}\right)$.

After the retailer sets its sales margin m, the supplier sets its wholesale price w and reliability parameter λ simultaneously to maximize its per unit time total profit. The supplier's profit consists of wholesale revenue, production cost, inventory holding cost, fixed starting cost, reliability investment cost, machine restoration cost.

1) The wholesale revenue

The supplier's total revenue comes from the wholesaling of the deteriorating products to the retailer. The total sales revenue can be calculated as wQ.

2) The production cost

The supplier's production time lasts for T_s, and the total cost can be calculated as $c_p P T_s$.

3) The inventory holding cost

During the production, the accumulated inventory holding quantity in a single cycle is $\int_{t_s}^{T_r} I_s(t)\, dt$. Thus the total inventory holding cost in a cycle is $\int_{t_s}^{T_r} h\, I_s(t) dt$.

4) The fixed starting cost.

When starting the production, the supplier needs to pay a lump sum fee A_s per cycle.

5) The reliability investment cost

To enhance the production reliability and reach a reliability level of λ, the supplier has to pay a quadratic unit time investment cost of $\frac{k(\lambda - \lambda_0)^2}{2}$. Therefore, the total cost in a production cycle is $\frac{k(\lambda - \lambda_0)^2 T_r}{2}$.

6) The expected restoration cost

When the out-of-control situation happens, the supplier has to restore its production system to the in-control state with a fixed cost M.

• (*Scenario 1*) when $s \geq T_s$, the supplier's production system is always in-control and there will is no restoration cost.

• (*Scenario 2*) when $s \leq T_s$, the production system will be shifted from the in-control state to the out-of-control state at time $t = s$. The restoration cost is incurred.

Therefore, the expectation of the total restoration cost can be calculated as $M\int_0^{T_s} f_\lambda(s)ds = MF_\lambda(T_s)$. Combining the above revenue and costs, the unit time total profit for the supplier can be presented as

$$TP_s(w,\lambda) = \frac{1}{T_r}\left\{wQ - c_pPT_s - \int_{T_r}^{t_s} h\,I_s(t)dt - A_s - MF_\lambda(T_s)\right\} - RIC(\lambda)$$

$$= \frac{wD(w+m)\left(e^{\theta T_r}-1\right)}{\theta T_r} - \frac{c_pPT_s}{T_r} - \frac{hP\left(e^{-\theta T_s}+\theta T_s-1\right)}{\theta^2 T_r} - \frac{A_s}{T_r} - \frac{MF_\lambda(T_s)}{T_r} - \frac{k(\lambda-\lambda_0)^2}{2} \quad (7)$$

in which $T_s = -\frac{1}{\theta}ln\left(1 - \frac{D(p)\left(e^{\theta T_r}-1\right)}{P}\right)$.

4.3. Supply chain's profit

The unit total profit for the whole supply chain is the sum of both firms' profit, which can be expressed as

$$TP_{sc}(p,\lambda) = TP_r(m) + TP_s(w,\lambda)$$

$$= pD(p) - \frac{c_pPT_s}{T_r} - \frac{hD(p)\left(e^{\theta T_r}-\theta T_r-1\right)}{\theta^2 T_r} - \frac{hP\left(e^{-\theta T_s}+\theta T_s-1\right)}{\theta^2 T_r}$$

$$-\frac{A_s}{T_r} - \frac{A_r}{T_r} - r\alpha\frac{\int_0^{T_s} F_\lambda(s)ds}{T_sT_r} - \frac{MF_\lambda(T_s)}{T_r} - RIC(\lambda)$$

$$= \left(p + \frac{h}{\theta}\right)D(p) - \frac{(c_pP+hP)T_s}{T_r} - \frac{A_s}{T_r} - \frac{A_r}{T_r} - r\alpha\frac{\int_0^{T_s} F_\lambda(s)ds}{T_sT_r} - \frac{MF_\lambda(T_s)}{T_r} - \frac{k(\lambda-\lambda_0)^2}{2} \quad (8)$$

5. Strategies analysis and supply chain coordination

In this section, we derive the optimal pricing and reliability investment under integrated and decentralized supply chain structures. In the integrated scenario, the two members jointly determine the retailer price and reliability investment. In the integrated supply chain, the retailer first decides the optimal sales margin, then the supplier determines the wholesale price and reliability investment, which aims to maximize the individual profit for each firm.

5.1. Integrated scenario

To simplify our analysis and to get some analytical results, we study the model when s uniformly distributed. The initial distribution is $U[0,2\lambda_0]$. After investment, the distribution turns to $U[0,2\lambda](\lambda \geq \lambda_0)$. We start with a benchmark case, in which the supplier and the retailer are vertically integrated. The profit function of the total supply chain can be simplified as

$$TP_{sc}(p,\lambda) = \left(p + \frac{h}{\theta}\right)D(p) - \left(c_pP + hP + \frac{r\alpha}{4\lambda} + \frac{M}{2\lambda}\right)\frac{T_s}{T_r} - \frac{k(\lambda-\lambda_0)^2}{2} - \frac{A_s}{T_r} - \frac{A_r}{T_r} \quad (9)$$

The target of the supply chain is to find the optimal selling price and an optimal reliability parameter λ. The corresponding optimization problem is formulated as

$$Max_{p,\lambda}: \{TP_{sc}(p,\lambda)\} \quad (10)$$

$$s.t.\quad p \le \frac{b}{a}, \lambda \ge \lambda_0.$$

For the optimization problem, we have the following proposition.

Proposition 1. In the integrated supply chain,

(1) for fixed p, the profit function is concave in λ. The optimal reliability parameter λ^I can be derived by solving the equation

$$(\lambda - \lambda_0)\lambda^2 = \frac{(r\alpha + 2M)T_s}{4kT_r} \tag{11}$$

When $\lambda^I > \lambda_0$; otherwise, $\lambda^I = \lambda_0$;

(2) for fixed λ, the profit function is concave in p. The optimal price p^I can be derived by solving the equation

$$b - 2ap - \frac{h}{\theta}a - \left(c_p P + hP + \frac{r\alpha}{4\lambda} + \frac{M}{2\lambda}\right)\frac{T_s'(p)}{T_r} = 0 \tag{12}$$

when $p < \frac{b}{a}$; otherwise, $p = \frac{b}{a}$; where $T_s = -\frac{1}{\theta} ln\left(1 - \frac{D(p)(e^{\theta T_r} - 1)}{P}\right)$, $T_s' = -\frac{a(e^{\theta T_r} - 1)}{\theta[P - D(p)(e^{\theta T_r} - 1)]}$.

Proof. See the proof of Proposition 1 in Appendix B.1. ■

Due to the complexity of the expressions in the Hessian matrix, it is impossible to prove the joint concavity of the profit function with respect to variables p and λ. With Proposition 1, we design an iteration algorithm (shown in **Table 2**) to derive the optimal solutions.

Table 2. Algorithm 1

Algorithm 1	
Step 1.	Start with $n = 0$ and initialize $\lambda_n = \lambda_0$.
Step 2.	Calculate the optimal retail price p_n from (12) for the given λ_n.
Step 3.	Calculate the optimal λ_{n+1} from (11) based on p_n.
Step 4.	If the difference between λ_{n+1} and λ_n is sufficiently small (i.e., e^{-4}), set $\lambda^I = \lambda_{n+1}$ and $p^I = p_n$, then output (λ^I, p^I) and stop. Otherwise, set $n = n + 1$ and go back to Step 2.

Substituting the equilibrium strategies p^I, λ^I into (8) yields the unit time total profits of the whole supply chain TP_{sc}. The concavity of the function is shown by numerical test, which is shown in **Figure 3**.

5.2. Decentralized scenario

In the decentralized supply chain, the retailer is a Stackelberg gaming leader and the supplier is the follower. As the gaming leader, the retailer first sets the sales margin. Then, based on the retailer's sales margin, the supplier sets its optimal wholesale price and reliability investment level to maximize its own profit. We use backward induction to solve the gaming problem. We start by solving the supplier's optimization problem to derive the response functions. Then, we study the optimal decisions of the retailer.

The supplier's optimization problem is

$$Max_{w,\lambda}:\{TP_s(w,\lambda)\} \tag{13}$$

$$s.t. \quad w+m \leq \frac{b}{a}, \lambda \geq \lambda_0.$$

Under receiving the retailer's sales margin m, some important results can be obtained when solving the supplier's problem.

Proposition 2. In the decentralized scenario,

(1) For constant w, the supplier's profit function is concave in λ. The optimal reliability parameter λ^D can be derived by solving the equation

$$(\lambda-\lambda_0)\lambda^2 = \frac{MT_s}{2kT_r}. \tag{14}$$

(2) For constant λ, the supplier's profit function is concave in w. The optimal reliability parameter w^D can be derived by solving the equation

$$b-2aw-am+c_p a+\frac{D(w+m)ha\left(e^{\theta T_r}-1\right)}{\theta P}+\frac{Ma}{2\lambda P}=0. \tag{15}$$

Proof. See the proof of Proposition 2 in Appendix B.2. ■

Corollary 1. For the same price retail ($p^I = w^D + m^D$), the supplier will invest more in reliability improvement under the centralized case, i.e., $\lambda^I \geq \lambda^D$.

Proof. See the proof of Corollary 1 in Appendix B.3. ■

From Equations (11) and (14), we find that the optimal reliability investment is closely linked to T_s, which is determined by selling price to end customers. Comparing the result in the decentralized and centralized model, we show in Corollary 1 that, firms will be more willing to invest in reliability improvement in the centralized model comparing to the decentralized model.

Due to the complexity of the expressions in the Hessian matrix, the concavity of the function is hard to prove by mathematical theory. But we can see the concavity in figure numerical test, which is shown in **Figure 4**. For the retailer, after knowing the best response function $w^D(m)$ and $\lambda^D(m)$, and substitute the functions to retailer's profit function, we have $TP_r\left(m, w^D(m), \lambda^D(m)\right)$. It is also hard to have an explicit expression of the profit function. However, we also shown in **Figure 5** that the retailer's profit function is concave in m. Thus, we design a two dimensional searching algorithm (shown in **Table 3**) to get the optimal decisions.

Table 3. Algorithm 2

Algorithm 2	
Step 1.	Initialize $TP_r = 0$.
Step 2.	Start with $k = 1$ and initialize $m_k = 0$. Given a small positive number $\delta = 10^{-3}$ (search step size).
Step 3.	Calculate the optimal wholesale price w_k and the optimal reliability level λ_k from (14) and (15) and then the profit of the supplier $TP_{s,k}$.
Step 4.	If $TP_r\|_{m_k} > TP_r\|_{m_{k-1}}$, set $m_{k+1} = m_k + \delta$ and go to Step 3. Otherwise, denote $m^D = m_k$, $w^D = w_k$ and $\lambda^D = \lambda_k$, then output (m^D, w^D, λ^D).

Substituting the equilibrium strategies m^D, w^D, λ^D into Equations (4) and (7) yields the unit time total profits of the retailer and the supplier TP_r and TP_s, respectively, and the profit of the whole supply chain TP_{sc}.

5.3. Supply chain coordination

In this subsection, we try to coordinate the supply chain with a cooperative reliability investment and revenue sharing contract. Assuming ϕ is the retailer's sharing rate of the supplier's sales revenue, and γ is the retailer's sharing rate of the supplier's reliability investment. The unit time total profits of the retailer and the supplier are respectively given by

$$TP_s(w,\lambda|\phi,\gamma) = \frac{(1-\phi)\left(wD(w+m)(e^{\theta T_r}-1)\right)}{\theta T_r} - \frac{c_p P T_s}{T_r} - \frac{hP(e^{-\theta T_s}+\theta T_s-1)}{\theta^2 T_r} - \frac{A_s}{T_r}$$
$$-\frac{(1-\gamma)(k(\lambda-\lambda_0)^2)}{2} - \frac{MT_s}{2\lambda T_r} \tag{16}$$

$$TP_r(m|\phi,\gamma) = (w+m)D(w+m) - \frac{(1-\phi)\left(wD(w+m)(e^{\theta T_r}-1)\right)}{\theta T_r}$$
$$-\frac{hD(w+m)(e^{\theta T_r}-\theta T_r-1)}{\theta^2 T_r} - \frac{A_r}{T_r} - \frac{r\alpha T_s}{4\lambda T_r} - \gamma\frac{k(\lambda-\lambda_0)^2}{2} \tag{17}$$

Proposition 3. Coordination can be achieved with a revenue sharing and cooperative investment contract only if the mechanism $(m^C, \gamma^C, \phi^C, w^C)$ satisfies

$$m^C(\phi^C) = 2p^I - \frac{b}{a} + \left\{c_p P + \frac{h_s P\left(1-e^{-\theta T_s(p^I)}\right)}{\theta} + \frac{M}{2\lambda^I}\right\}\frac{\theta T_s'(p^I)}{a(e^{\theta T_r}-1)(1-\phi^C)}, \tag{18}$$

$$\gamma^C = \frac{r\alpha}{r\alpha+2M} \in [0,1]. \tag{19}$$

$$w^C(\phi^C) = p^I - m^c(\phi^C). \tag{20}$$

Proof. See the proof of Proposition 3 in Appendix B.4. ■

According to the contract structure, four coordinating tools can be used by both players to establish an efficient solution, i.e., the wholesale price, the sales margin, the investment sharing rate and revenue sharing rate. As shown in Proposition 3, the supplier and the retailer can adjust their share rates of revenue to obtain a positive wholesale price and a proper subsidy proportion, i.e., $w^C > 0$, $m^C > 0$ and $0 < \gamma^C < 1$, and thus coordinate the whole supply chain. Failing this, the supply chain cannot be coordinated by means of the revenue sharing and cooperative investment contract.

Corollary 2. In the revenue sharing and cooperative investment contract, the wholesale price w^C is increasing in ϕ^C, while the retailer's sales margin m^C is decreasing in ϕ^C.

Proof. See the proof of Corollary 2 in Appendix B.5. ■

This corollary illustrates that to reach a coordination result, the supplier should provide a higher wholesale price and the retailer should set a lower sales margin when the revenue sharing rate is higher. This result is intuitive and realistic. Besides, both the supplier and the retailer are willing to adopt the mechanism $(w^C, m^C, \gamma^C, \phi^C)$, if and only if $TP_s^C \geq TP_s^D$ and $TP_r^C \geq TP_r^D$. Hence, the following proposition can be derived.

Proposition 4. Both the supplier and the retailer are willing to participate in the coordinating scenario rather than the decentralized setting only if $\underline{\phi} \leq \phi^C \leq \overline{\phi}$, where $\underline{\phi}$ and $\overline{\phi}$ are presented in Appendix B.6.

Proof. See the proof of Proposition 4 in Appendix B.6. ■

Proposition 4 reveals that the proposed contract is applicable only when the revenue sharing rate ϕ^C is moderate, otherwise, at least one of the two firms will not execute the contract. In the coordinating region $\phi \in \left[\underline{\phi}, \overline{\phi}\right]$, the supplier can retailer can adjust their profit by changing the value of ϕ^C, which also depends on the bargain power.

6. Numerical examples and sensitivity analysis

In this section, numerical examples and sensitivity analysis are presented to illustrate the proposed algorithms and the coordination results, which also help us to gain some managerial insights.

6.1. Numerical examples

We set the parameters for the numerical case in **Table 4**.

Table 4. Parameter settings

$b = 200$	$a = 10$	$k = 20$	$P = 200$
$h = 2$	$c_p = 2$	$A_r = 50$	$A_s = 80$
$T_r = 1$	$r = 100$	$\lambda_0 = 1.0$	$\theta = 0.2$
$\alpha = 0.5$	$M = 100$		

With the given data, results can be obtained by using Matlab software. As shown in **Figure 3**, the unit time total profit of the whole supply chain is concave in λ and p. Then, applying Algorithm 1, we obtain the optimal strategies in the integrated supply chain as $p^I = 12.38$, $\lambda^I = 1.564$, $Q^I = 84.41$, and the unit time total profit $TP_{sc}^I = 497.06$.

Similarly, **Figure 4** shows the concavity of TP_s in w and λ, and **Figure 5** indicates the concavity of TP_r in m for the given w and λ. Applying Algorithm 2, the equilibrium strategies of the manufacturer and the retailer are $m^D = 10.00$, $w^D = 6.35$, $\lambda^D = 1.303$, $Q^D = 40.38$, respectively. The corresponding unit time total profits $TP_r^D = 248.95$, $TP_s^D = 76.87$, and $TP_{sc}^D = 325.82$.

Comparing the strategies obtained from two supply chain structures, we find that the retail price (p^I) in the integrated scenario is less than that of the decentralized scenario ($w^D + m^D$) due to the double marginalization effect. Meanwhile, theoptimal reliability level and ordering quantity, as well as the unit time total profit under the integrated scenario are greater than those of the decentralized one.

As shown in Proposition 3, the supply chain can be coordinated by setting appropriate coordinating parameters. **Figure 6** shows the change of profit for the two parties before and after coordination. The supplier's profit drops while the retailer's profit rises in the revenue sharing rate ϕ. Based on Proposition 4, we also observe that there exists a feasible region of $\phi^C \in [0.562,0.828]$ in which the two parties will achieve a win-win situation.

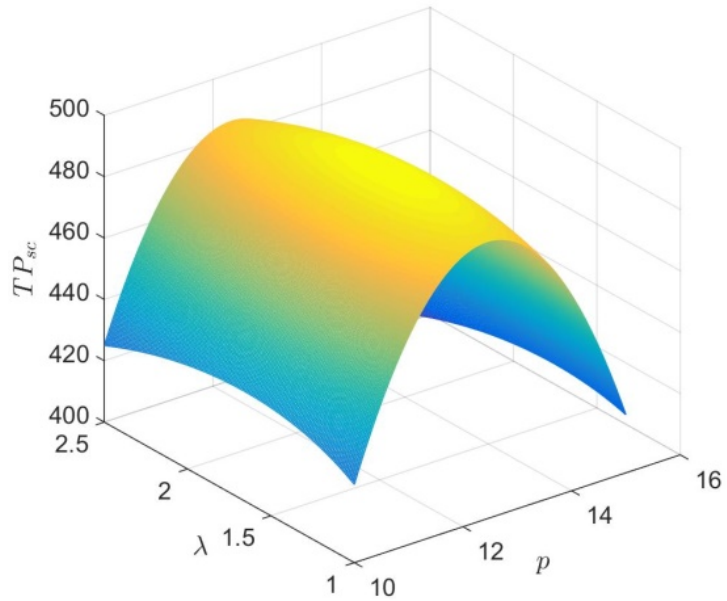

Figure 3. The supply chain's profit change with respect to λ and p in the centralized case.

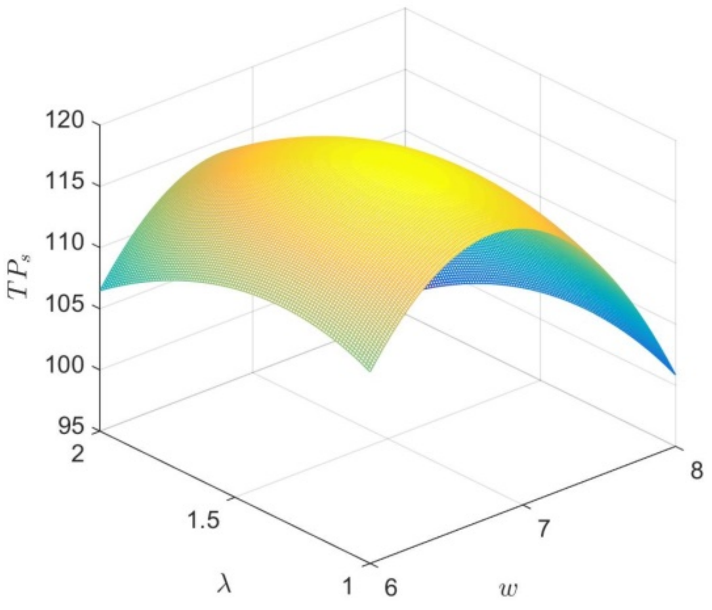

Figure 4. The supplier's profit change with respect to λ and w in the decentralized case when $m = 10$.

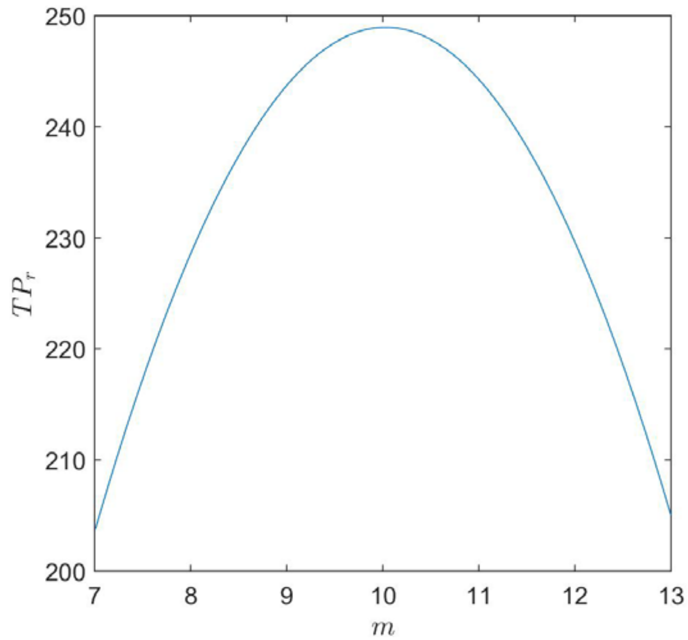

Figure 5. The retailer's profit change with respect to m in the decentralized case.

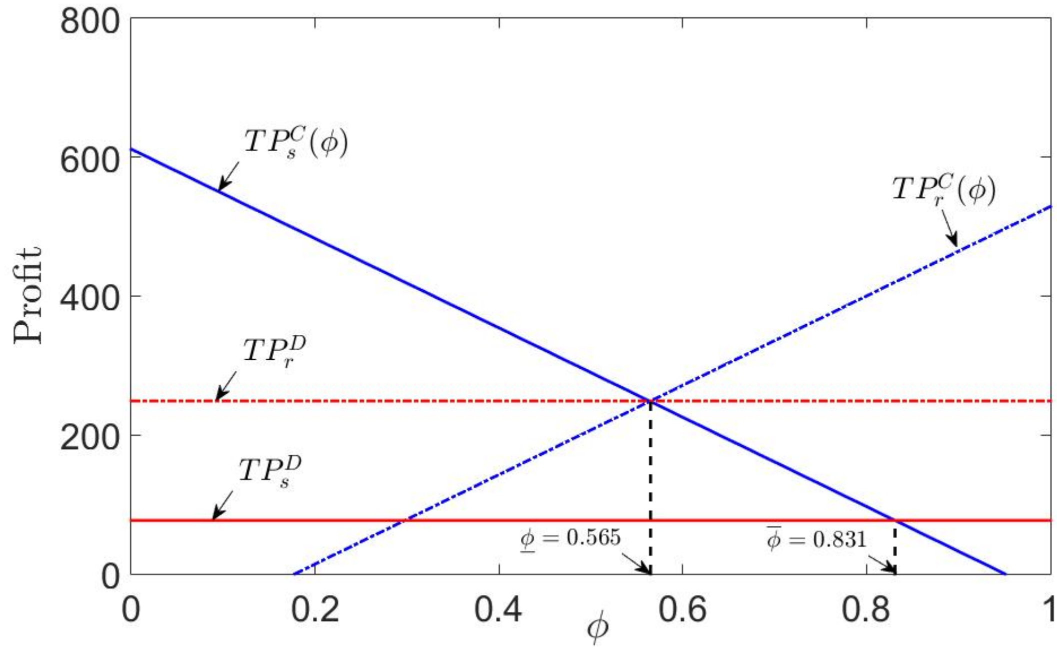

Figure 6. The retailer's and the supplier's profit change with respect to ϕ.

6.2. Sensitivity analysis on equilibrium strategies

In this subsection, we present the sensitivity analysis results on the equilibrium strategies with respect to key system parameters P, c_p, θ, α, M, λ_0 and h. We vary one parameter once and keep others fixed. **Table 5** summarizes the results for the centralized scenario and decentralized scenario, and provides some managerial insights for the supply chain management of deteriorating items.

Table 5. Sensitivity analysis with respect to system parameters P, c_p, h, θ, α and M.

Parameter values	Centralized Supply Chain				Decentralized supply chain						
	p^I	λ^I	Q^I	TP_{sc}^I	m^D	w^D	λ^D	Q^D	TP_r^D	TP_s^D	TP_{sc}^D
$P=180$	12.45	1.597	83.53	489.71	9.97	6.40	1.325	40.13	245.65	76.04	321.69
$P=190$	12.41	1.580	83.99	493.57	9.98	6.38	1.314	40.29	247.38	76.71	324.09
$P=200$	12.38	1.564	84.41	497.06	10.00	6.35	1.303	40.38	248.95	76.87	325.82
$P=210$	12.34	1.549	84.78	500.23	10.01	6.33	1.294	40.50	250.38	77.34	327.72
$P=220$	12.31	1.535	85.13	503.13	10.02	6.31	1.285	40.61	251.69	77.74	329.43
$c_p=1.6$	12.15	1.574	86.88	532.87	10.17	6.07	1.310	41.66	265.99	86.65	352.64
$c_p=1.8$	12.26	1.569	85.65	514.83	10.09	6.21	1.306	40.99	257.41	81.51	338.92
$c_p=2.0$	12.38	1.564	84.41	497.06	10.00	6.35	1.303	40.38	248.95	76.87	325.82
$c_p=2.2$	12.49	1.559	83.18	479.55	9.90	6.50	1.300	39.82	240.62	72.69	313.31
$c_p=2.4$	12.60	1.554	81.95	462.32	9.82	6.64	1.297	39.16	232.42	67.79	300.21
$\theta=0.16$	12.29	1.557	83.59	506.03	9.90	6.39	1.302	40.23	256.12	78.26	334.39
$\theta=0.18$	12.33	1.560	84.00	501.59	9.93	6.38	1.303	40.41	252.54	78.38	330.92
$\theta=0.20$	12.38	1.564	84.41	497.06	10.00	6.35	1.303	40.38	248.95	76.87	325.82

$\theta = 0.22$	12.42	1.567	84.80	492.44	10.02	6.35	1.305	40.61	245.34	75.37	320.71
$\theta = 0.24$	12.46	1.570	85.18	487.74	10.12	6.30	1.306	40.82	241.75	74.62	316.37
$\alpha = 0.1$	12.36	1.509	84.57	499.93	9.94	6.38	1.305	40.69	250.52	79.30	329.82
$\alpha = 0.3$	12.37	1.537	84.49	498.48	9.98	6.36	1.304	40.48	249.74	77.68	327.42
$\alpha = 0.5$	12.38	1.564	84.41	497.06	10.00	6.35	1.303	40.38	248.95	76.87	325.82
$\alpha = 0.7$	12.38	1.589	84.33	495.66	10.04	6.33	1.302	40.17	248.16	75.26	323.42
$\alpha = 0.9$	12.39	1.613	84.25	494.28	10.06	6.32	1.302	40.07	248.16	74.45	322.62
$M = 60$	12.35	1.448	84.73	502.92	10.03	6.30	1.212	40.59	251.66	78.94	330.59
$M = 80$	12.36	1.509	84.57	499.93	10.01	6.33	1.260	40.51	250.27	78.07	328.34
$M = 100$	12.38	1.564	84.41	497.06	10.00	6.35	1.303	40.38	248.95	76.87	325.82
$M = 120$	12.39	1.613	84.25	494.28	9.97	6.38	1.343	40.36	247.68	76.52	324.20
$M = 140$	12.40	1.659	84.11	491.60	9.95	6.41	1.379	40.30	246.46	75.81	322.28
$h = 1.0$	11.88	1.586	89.92	559.34	9.72	6.40	1.317	42.99	278.04	93.00	371.04
$h = 1.5$	12.14	1.575	87.12	527.54	9.87	6.37	1.310	41.61	263.20	84.36	347.56
$h = 2.0$	12.38	1.564	84.41	497.06	10.00	6.35	1.303	40.38	248.95	76.87	325.82
$h = 2.5$	12.61	1.553	81.79	467.83	10.13	6.33	1.297	39.18	235.27	69.66	304.93
$h = 3$	12.84	1.542	79.25	439.81	10.24	6.32	1.291	38.11	222.11	63.50	285.61

(1) Sensitivity analysis of production rate P

We find from **Table 5** that under the decentralized scenario, as production rate P increases, w^D and λ^D decrease, while m^D, Q^D, TP_r^D, TP_s^D and TP_{sc}^D increase. The selling price $w^D + m^D$ also decreases in P. This implies that a higher production rate P gives the supplier more initiatives to reduce the wholesale price and stimulate the retailer's demand. Accordingly, the ordering quantity increases with lower wholesale price, and the retail price decreases when the wholesale price is smaller. In addition, a larger P shortens supplier's production time, which reduces the probability of defective risk during one production cycle. To cut cost, the supplier decreases the reliability investment. Moreover, decreasing retail price stimulates sales, resulting in higher unit time total profits for the supplier, the retailer and the whole supply chain. Under the integrated scenario, as P increases, p^I and λ^I decrease, while Q^I and TP_{sc}^I increase. These results are in line with those in the decentralized case. Obviously, retail price in the integrated channel is significantly less than that of the decentralized one, i.e., $p^I < m^D + w^D$, and reliability investment is greater than that in the decentralized setting, i.e., $\lambda^I > \lambda^D$, which leads to higher profits for the supply chain.

(2) Sensitivity analysis of unit production cost c_p

Under the decentralized scenario, **Table 5** shows that when production cost c_p increases, w^D increases, while m^D, λ^D, Q^D, TP_r^D, TP_s^D and TP_{sc}^D decrease. From economic point of view, when the unit production cost rises, the supplier will raise the wholesale price to protect its sales margin, along with a higher retail price set by the retailer, which leads to a lower demand rate and the drop of ordering quantity.

This leads to reductions in profits of the supplier, the retailer and the whole supply chain. In the meantime, when the ordering quantity drops, the production cycle is shortened, which also reduces the defective probability. The maximum profit can be reached using a relatively low investment level. In the integrated scenario, when c_p increases, p^I increases, while λ^D, Q^D and TP_{sc}^D decrease, which share the same variation trends with those in the decentralized scenario. It is shown that the retail price in theintegrated channel is less than that of the decentralized one. Also, the reliability investment is greater than that of the decentralized channel, which contributes to higher profits.

(3) Sensitivity analysis of original deterioration rate θ

In the decentralized scenario, we show in **Table 5** that as the deterioration rate θ increases, m^D, Q^D and λ^D increase, whereas w^D, TP_r^D, TP_s^D and TP_{sc}^D decrease. This implies that, when deterioration rate θ is higher, more products will be ordered from the supplier. Therefore, the supplier's production cycle is extended, which may result in higher defective risk and lead to higher restoration cost. To reduce the cost, the supplier will invest more to enhance the reliability. Meanwhile, to reduce the deteriorating quantity, the retailer will set a higher sales margin, which aims to reduce demand rate and achieve lower inventory level. The high deterioration rate plays a negative role in the profits of the manufacturer, the retailer and the whole supply chain. The optimal strategies and profits in the integrated scenario have similar sensibilities against the deterioration rate with those in the decentralized scenario

(4) Sensitivity analysis of defective rate α

In the decentralized scenario, as the defective rate α increases, m^D increases, whereas w^D, Q^D, λ^D, TP_r^D, TP_s^D and TP_{sc}^D decrease. This implies that when defective rate α is higher, the retailer will try to maintain its profit by setting higher sales margin under a higher penalty cost. The raised sale margin leads to a lower demand rate and lower ordering quantity. To induce the retailer order more products, the supplier will set a lower wholesale price. The reduced ordering quantity shortens the production cycle, thus the defective probability drops and less investment is needed to achieve an appropriate reliability level. Since α incurs more cost, the supplier, the retailer and the supply chain profit all decrease. In the integrated scenario, the selling price p^I and reliability level λ^I increase, while the ordering quantity Q^I and total supply chain profit TP_{sc}^I decreases. It is interesting that λ^I increases in α in the decentralized scenario. This is because in the integrated scenario, the supplier only cares about its own profit and higher α do not affect its profit directly. However, in the integrated scenario, α affects the total profit directly. Investing more in reliability will reduce the total profit. So, in the integrated scenario, λ^I increases in α. Comparing the profits in the two scenarios, the total profit of the integrated scenario is significantly higher than that of the decentralized scenario.

(5) Sensitivity analysis of restoration cost M

In the decentralized scenario, with higher M, λ^D and w^D increases, whereas m^D, Q^D, λ^D, TP_r^D, TP_s^D and TP_{sc}^D decrease. This reveals that when restoration cost α is higher, the supplier will invest more in reducing the defective risk, thus compress the expected restoration cost. Meanwhile, the raised restoration cost pushes the supplier to set a higher wholesale price. When the supplier's wholesale price rises, the retailer also needs to set a higher selling cost ($w^D + m^D$), which will leads to the drop of ordering quantity. Although the selling price rises, the retailer's sales margin is cut down, its profit will be badly hurt. In addition, the supplier's profit and the supply chain profit drop as well due to the rise of supplier's restoration cost. The optimal strategies and profits in the integrated scenario have similar sensibilities against the deterioration rate with those in the decentralized scenario.

(6) Sensitivity analysis of unit inventory holding cost h

As seen from **Table 5**, in the decentralized scenario, when the unit inventory holding cost h rises, m^D increases, whereas w^D, λ^D, Q^D, TP_r^D, TP_s^D and TP_{sc}^D decrease. When the holding cost h is higher, the retailer inclined to avoid too much inventory by setting a higher sales margin and reducing the ordering quantity. To induce the retailer order more products, the supplier will reduce its wholesale price. Meanwhile, the reduction of ordering quantity shortens the production cycle and reduces the defective risk. To achieve the maximum of profit, the supplier is intended to set a lower reliability level. It conforms to intuition that higher holding cost will hurt both firms' profits and the supply chain profit. In the integrated scenario, when h increases, p^I increases, while λ^D, Q^D and TP_{sc}^D decrease, which have the same sensibilities against h with those in the decentralized scenario. It also shows that the supply chainprofit under integrated channel is significantly higher than that of the decentralized one.

We also presented two numerical examples with exponential distributed reliability in Appendix A.1 and cubic investment cost in Appendix A.2. Similar results can be obtained in the numerical results.

6.3 The impacts of unreliability ignorance

In the main model, we assume that both the supplier and retailer make pricing decisions under the consideration of unreliability costs. However, what if the supplier or/and the retailer ignore the costs? In this subsection, we study the impacts of unreliability ignorance to the firms' optimal pricing decisions and profit change in the Stackelberg game. We consider three cases, i.e., supplier ignorance, retailer ignorance, both firms ignorance, and compare the results with that in the main model.

In the supplier ignorance case, the supplier will make price decisions without considering the costs of selling bad quality products and the reliability investment; in the retailer ignorance case, the retailer make price decisions without considering the costs of selling bad quality products; in the both ignorance case, the supplier and retailer both ignore the costs of selling bad quality products and reliability investment. We use the data presented in **Table 4**. The numerical results are presented in **Table 6**.

Table 6. Comparative results when firms may ignore unreliability

	m^D	w^D	λ^D	Q^D	TP_r^D	TP_s^D
Both firms ignore	10.08	6.22	1	41.00	256.43	72.04
Supplier ignore	10.12	6.20	1	40.79	256.44	70.45
Retailer ignore	9.89	6.41	1.306	40.95	248.87	81.34
No firm ignore	10.00	6.35	1.303	40.38	248.95	76.87

In **Figure 7** and **8**, we testified the impacts of firms' unreliability ignorance to the price decisions and the corresponding profits. Firstly, we show that when the retailer ignores the unreliability costs when making price decisions, the retailer will set the lowest sales margin and the supplier will set the highest wholesale price. The distortion of the optimal prices for both firms when the retailer ignores the unreliability cost slightly hurts the retailer's benefit, however, it significantly contribute to the supplier's profit. Secondly, we also show that when the supplier itself ignores the unreliability costs when making price decisions, the retailer will set the highest sales margin, thus the supplier will set the lowest sales margin. As a result, the retailer's ignorance of unreliability cost hurts the retailer's profit, and benefit the retailer. Lastly, when both firms ignore the unreliability costs, the wholesale price will be lower and sales margin will be higher than that in the main model. The retailer's profit increases, while the supplier's profit dramatically decreases. In summary, ignoring the unreliability cost will cause uneconomical decisions and leads to profit drop.

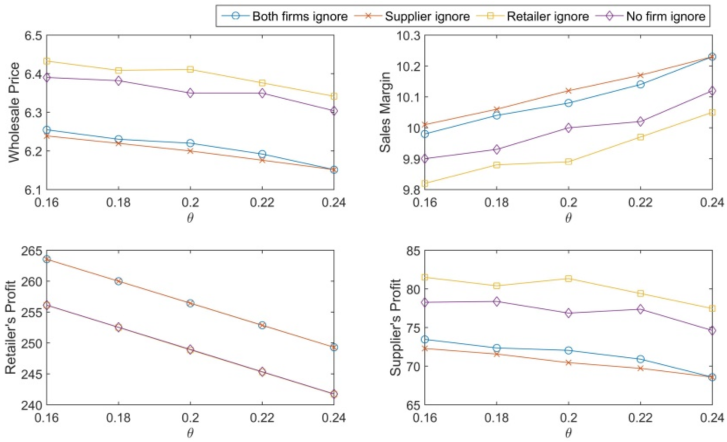

Figure 7. Impacts of reliability ignorance to price decisions and profits when θ changes

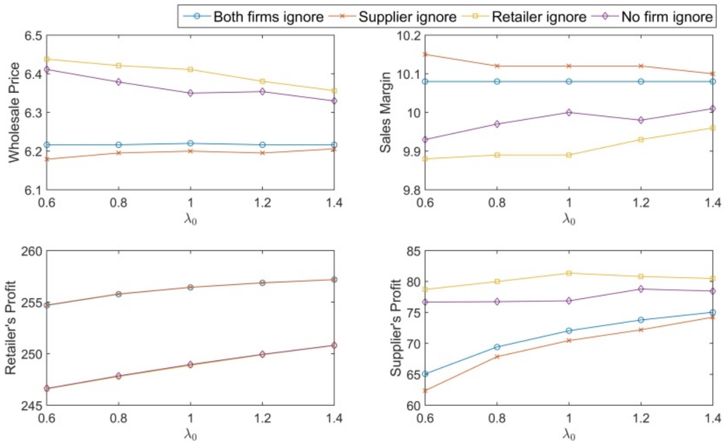

Figure 8. Impacts of reliability ignorance to price decisions and profits when λ_0 changes

6.4 Sensitivity analysis on coordinating results

The sensitivity analysis on feasible region of coordinating parameter ϕ with respect to system parameters P, c_p, θ, α, M and h is presented in **Table 7**. We show that both the upper bound ($\overline{\phi}$) and lower bound ($\underline{\phi}$) of the feasible region increases in c_p, θ, α and h, whereas decreases in P and M. It implies that for higher c_p, θ, α and h, to achieve the supply chain coordination, more supplier's wholesales revenue should be shared with the retailer. However, for higher P and M a relatively low sharing rate will help the two firms realize the coordination. We also tested the size of the feasible region, i.e., $\Delta\phi = \overline{\phi} - \underline{\phi}$, which is expanded for higher c_p, α and h, and lower M. It means that the supply chain will be more likely to be coordinated when the production cost, the defective rate and inventory cost is high, or the supplier's restoration cost is low.

Table 7. Coordination parameter ϕ with respect to system parameters P, c_p, θ, α, M and h.

P	180	190	200	210	220	Impact	c_p	1.6	1.8	2.0	2.2	2.4	Impact
$\overline{\phi}$	0.833	0.830	0.828	0.826	0.824	↓	$\overline{\phi}$	0.825	0.827	0.828	0.828	0.830	↑
$\underline{\phi}$	0.567	0.564	0.562	0.560	0.558	↓	$\underline{\phi}$	0.561	0.561	0.562	0.562	0.563	↑
$\Delta\phi$	0.266	0.266	0.266	0.266	0.266	–	$\Delta\phi$	0.264	0.266	0.2660	0.266	0.267	↑
θ	0.16	0.18	0.20	0.22	0.24	Impact	α	0.1	0.3	0.5	0.7	0.9	Impact
$\overline{\phi}$	0.820	0.823	0.828	0.830	0.837	↑	$\overline{\phi}$	0.825	0.827	0.828	0.830	0.831	↑
$\underline{\phi}$	0.554	0.559	0.562	0.568	0.569	↑	$\underline{\phi}$	0.562	0.562	0.562	0.562	0.563	↑
$\Delta\phi$	0.266	0.266	0.266	0.266	0.266	–	$\Delta\phi$	0.263	0.265	0.266	0.268	0.268	↑
M	60	80	100	120	140	Impact	h	1	1.5	2	2.5	3	Impact
$\overline{\phi}$	0.828	0.828	0.828	0.827	0.826	↓	$\overline{\phi}$	0.804	0.817	0.828	0.839	0.849	↑
$\underline{\phi}$	0.562	0.562	0.562	0.562	0.561	↓	$\underline{\phi}$	0.546	0.554	0.562	0.570	0.578	↑
$\Delta\phi$	0.266	0.266	0.266	0.265	0.265	↓	$\Delta\phi$	0.258	0.263	0.266	0.269	0.271	↑

Note: $\Delta\phi = \overline{\phi} - \underline{\phi}$, which denotes the size of the coordinating zone.

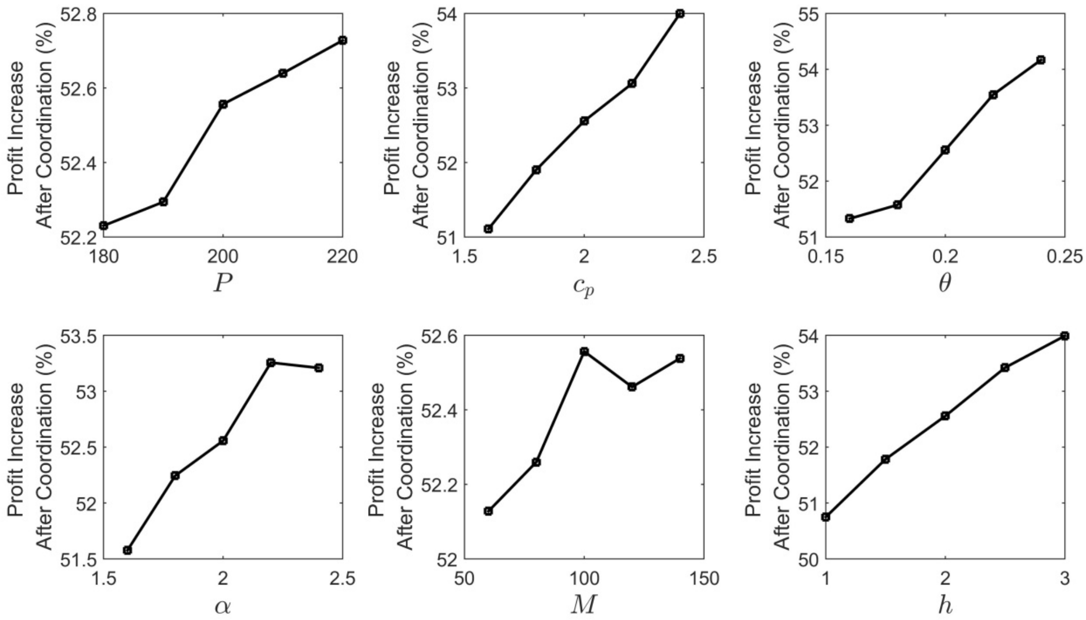

Figure 9. The retailer's and the supplier's profit change with respect to ϕ.

In **Figure 9**, we show the impacts of coordination contract implementation to the profit change of the total supply chain with respect to parameters $P,\ c_p,\ \theta,\ \alpha,\ M$ and h. It is shown that after coordination, the total supply chain profit increases over 50% in this numerical case. Firstly, we observe that profit increment after coordination increases with P and c_p, which indicates that the two firms will be more willing to cooperate when the supplier's production rate or production cost is high. Secondly, we find that when production deterioration rate or unit inventory holding cost is high, coordination will bring more profit to the supply chain. Thirdly, we can also see that profit increment is not monotonically increasing in α and M. When either α or M becomes high, the willingness to coordination may slightly drop. Overall, the cooperative investment and revenue sharing contract helps the supply chain to significantly increase the total profit.

7. Conclusions

In this paper, a two-echelon supply chain for deteriorating items involving a supplier and a retailer with endogenous reliability and price-dependent demand is studied. The supplier's production line is unreliable and defective items will be produced during the out-of-control period. Meanwhile, the supplier can invest to enhance its production line's reliability and reduce the expected defective rate. We first study the optimal decisions under the integrated and decentralized scenarios. To obtain the results, two algorithms are designed to searching for the optimal decisions. Comparing the optimal results in the two scenarios, we find that supply chain integration increases the performance of the total supply chain, and makes the production system more reliable. Then, we proposed a revenue sharing and cooperative investment contract to coordinate the supply chain, with which, the retailer shares part of the supplier's investment, in return, the supplier shares part of its revenue with the retailer. We use numerical simulations and sensitivity analysis to further illustrate the properties of the model, and gain some important managerial implications.

The model can be extended in three ways. Firstly, in this paper we studied a static pricing and investment problem. However, in real business, firms can dynamically change their pricing or investment decisions. Therefore, in the future, we can consider about the dynamic marketing and investing strategies. Secondly, in industry, there can be many firms in the same level in the supply chain. Therefore, studying how competition affects firms' pricing and reliability control decisions is more interesting and more practical, which is another direction of our future research. Thirdly, in this model, we have only studied a two level supply chain. In real industry, the supply chain can have multiple levels whose production may also be unreliable. How to control quality throughout the supply chain is also worth studying. Fourthly, in this paper, we only study the quality uncertain problems; however, quantity and quantity can both be affected, which can also be one of our future research directions. Lastly, in this paper, we assume that the firm can only produce one product. In the future, we can study firms' assortment planning with reliability control.

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Appendix A. Two numerical examples

Appendix A.1 Numerical results when s is exponentially distributed

We keep the parameters unchanged in the numerical example except for s, which follows a exponential distribution. In other words, $f_\lambda(s) = \frac{1}{\lambda} e^{-\frac{s}{\lambda}}$, $F_\lambda(s) = 1 - e^{-\frac{s}{\lambda}}$.

The optimal strategies in the integrated supply chain as $p^I = 13.86$, $\lambda^I = 1.614$, $Q^I = 150.90$, and the unit time total profit $TP_{sc}^I = 388.85$. The equilibrium strategies of the manufacturer and the retailer are $m^D = 11.12$, $w^D = 6.04$, $\lambda^D = 1.370$, $Q^D = 69.82$, respectively. The corresponding unit time total profits $TP_r^D = 183.13$, $TP_s^D = 72.74$, and $TP_{sc}^D = 255.87$. Sensitivity analysis with respect to system parameters P, c_p, θ, α, M and h are listed in **Table A.1**.

Appendix A.2 Numerical results when the investment cost follows a cubic form

We keep the parameters unchanged in the numerical example except for a cubic investment cost $= k(\lambda - \lambda_0)^3/2$

The optimal strategies in the integrated supply chain as $p^I = 12.37$, $\lambda^I = 1.599$, $Q^I = 84.43$, and the unit time total profit $TP_{sc}^I = 498.47$. The equilibrium strategies of the manufacturer and the retailer are $m^D = 9.99$, $w^D = 6.34$, $\lambda^D = 1.414$, $Q^D = 40.46$, respectively. The corresponding unit time total profits $TP_r^D = 249.73$, $TP_s^D = 77.69$, and $TP_{sc}^D = 327.43$. Sensitivity analysis with respect to system parameters P, c_p, θ, α, M and h are listed in **Table A.2**.

Table A.1. Sensitivity analysis with respect to system parameters $P,\ c_p,\ \theta,\ \alpha,\ M$ and h when s is exponentially distributed

Parameter values	Centralized supply chain				Decentralized supply chain						
	p^I	λ^I	Q^I	TP_{sc}^I	m^D	w^D	λ^D	Q^D	TP_r^D	TP_s^D	TP_{sc}^D
$P = 180$	14.01	1.632	147.31	377.23	11.13	6.11	1.387	68.65	178.39	70.53	248.92
$P = 190$	13.93	1.623	149.18	383.29	11.12	6.07	1.378	69.16	180.87	71.36	252.23
$P = 200$	13.86	1.614	150.89	388.84	11.12	6.04	1.370	69.82	183.13	72.74	255.87
$P = 210$	13.80	1.605	152.46	393.95	11.11	6.02	1.363	70.53	185.21	74.37	259.58
$P = 220$	13.74	1.597	153.89	398.65	11.10	5.99	1.355	70.86	187.13	74.85	261.97
$c_p = 1.6$	13.63	1.622	156.66	422.24	11.29	5.76	1.377	72.59	198.37	81.46	279.84
$c_p = 1.8$	13.75	1.618	153.77	405.37	11.22	5.89	1.373	71.04	190.68	76.52	267.20
$c_p = 2.0$	13.86	1.614	150.89	388.84	11.12	6.04	1.370	69.82	183.13	72.74	255.87
$c_p = 2.2$	13.98	1.610	148.03	372.66	11.01	6.20	1.366	68.70	175.72	69.37	245.10
$c_p = 2.4$	14.10	1.605	145.18	356.80	10.97	6.31	1.361	66.84	168.48	63.71	232.20
$\theta = 0.16$	13.63	1.609	150.24	411.21	10.85	6.15	1.370	70.54	197.02	78.62	275.64
$\theta = 0.18$	13.74	1.612	150.64	400.15	10.99	6.09	1.370	70.14	190.06	75.52	265.58
$\theta = 0.20$	13.86	1.614	150.89	388.84	11.12	6.04	1.370	69.82	183.13	72.74	255.87
$\theta = 0.22$	13.99	1.619	151.00	377.29	11.26	5.97	1.369	69.34	176.22	69.58	245.81
$\theta = 0.24$	14.12	1.618	150.94	365.50	11.36	5.94	1.369	69.27	169.33	67.79	237.13
$\alpha = 0.1$	13.84	1.539	151.46	393.25	11.12	6.04	1.370	69.81	185.55	72.74	258.29
$\alpha = 0.3$	13.85	1.578	151.17	391.02	11.12	6.04	1.370	69.81	184.34	72.74	257.08
$\alpha = 0.5$	13.86	1.614	150.89	388.84	11.12	6.04	1.370	69.82	183.13	72.74	255.87
$\alpha = 0.7$	13.88	1.648	150.62	386.71	11.17	6.01	1.368	69.28	181.92	71.00	252.93
$\alpha = 0.9$	13.89	1.681	150.35	384.62	11.17	6.01	1.368	69.28	180.72	71.00	251.72
$M = 60$	13.82	1.485	151.85	397.06	11.19	5.95	1.258	70.22	186.68	75.11	261.79
$M = 80$	13.84	1.552	151.36	392.88	11.17	5.98	1.317	69.84	184.87	73.35	258.23
$M = 100$	13.86	1.614	150.89	388.84	11.12	6.04	1.370	69.82	183.13	72.74	255.87
$M = 120$	13.88	1.671	150.43	384.93	11.10	6.07	1.417	69.48	181.44	71.15	252.60
$M = 140$	13.90	1.723	149.98	381.12	11.06	6.11	1.460	69.38	179.81	70.33	250.14
$h = 1.0$	12.88	1.646	175.05	501.99	11.09	5.58	1.400	81.69	259.24	113.26	372.51
$h = 1.5$	13.39	1.630	162.46	442.54	11.11	5.81	1.385	75.67	219.60	91.98	311.59
$h = 2.0$	13.86	1.614	150.89	388.84	11.12	6.04	1.370	69.82	183.13	72.74	255.87
$h = 2.5$	14.30	1.597	140.20	340.21	11.15	6.25	1.352	63.79	149.76	54.45	204.21
$h = 3.0$	14.70	1.580	130.27	296.09	11.19	6.46	1.334	57.73	119.44	37.57	157.01

Table A.2. Sensitivity analysis with respect to system parameters $P,\ c_p,\ \theta,\ \alpha,\ M$ and h when the investment cost follows a cubic form

Parameter values	Centralized supply chain				Decentralized supply chain						
	p^I	λ^I	Q^I	TP_{sc}^I	m^D	w^D	λ^D	Q^D	TP_r^D	TP_s^D	TP_{sc}^D
$P = 180$	12.45	1.621	83.55	491.16	9.97	6.39	1.431	40.16	246.45	76.53	322.98
$P = 190$	12.41	1.610	84.01	495.00	10.02	6.35	1.421	40.16	248.18	75.95	324.13
$P = 200$	12.37	1.599	84.43	498.47	9.99	6.34	1.414	40.46	249.73	77.69	327.43
$P = 210$	12.33	1.589	84.81	501.62	10.03	6.29	1.400	40.63	252.45	78.10	330.56
$P = 220$	12.30	1.579	85.15	504.50	10.01	6.28	1.387	41.01	254.73	80.29	335.02
$c_p = 1.6$	12.14	1.605	86.90	534.29	10.18	6.05	1.419	41.68	266.79	87.07	353.87
$c_p = 1.8$	12.26	1.602	85.66	516.25	10.06	6.21	1.417	41.22	258.18	83.58	341.77
$c_p = 2.0$	12.37	1.599	84.43	498.47	9.99	6.34	1.414	40.46	249.73	77.69	327.43
$c_p = 2.2$	12.48	1.596	83.20	480.96	9.88	6.50	1.412	39.95	241.39	73.91	315.31
$c_p = 2.4$	12.59	1.592	81.97	463.72	9.79	6.64	1.410	39.34	233.19	69.38	302.58
$\theta = 0.16$	12.28	1.595	83.61	507.44	9.90	6.37	1.413	40.25	256.90	78.68	335.59
$\theta = 0.18$	12.33	1.597	84.02	502.99	9.92	6.37	1.414	40.48	253.32	79.20	332.53
$\theta = 0.20$	12.37	1.599	84.43	498.47	9.99	6.34	1.414	40.46	249.73	77.69	327.43
$\theta = 0.22$	12.41	1.601	84.82	493.86	10.07	6.31	1.414	40.42	246.14	76.17	322.32
$\theta = 0.24$	12.46	1.603	85.21	489.16	10.14	6.28	1.415	40.39	242.54	74.64	317.18
$\alpha = 0.1$	12.35	1.562	84.60	501.26	9.99	6.34	1.414	40.46	251.19	77.69	328.89
$\alpha = 0.3$	12.36	1.581	84.51	499.86	9.99	6.34	1.414	40.46	250.46	77.69	328.16
$\alpha = 0.5$	12.37	1.599	84.43	498.47	9.99	6.34	1.414	40.46	249.73	77.69	327.43
$\alpha = 0.7$	12.38	1.616	84.35	497.10	9.99	6.34	1.414	40.46	249.00	77.69	326.70
$\alpha = 0.9$	12.38	1.631	84.27	495.74	9.99	6.34	1.414	40.46	248.27	77.69	325.97
$M = 60$	12.34	1.520	84.77	504.13	10.08	6.27	1.339	40.39	252.36	77.45	329.82
$M = 80$	12.35	1.562	84.60	501.26	10.05	6.30	1.379	40.37	251.03	77.15	328.18
$M = 100$	12.37	1.599	84.43	498.47	9.99	6.34	1.414	40.46	249.73	77.69	327.43
$M = 120$	12.38	1.631	84.27	495.74	9.99	6.36	1.444	40.29	248.48	76.25	324.73
$M = 140$	12.40	1.661	84.11	493.07	9.94	6.40	1.471	40.38	247.24	76.86	324.10
$h = 1.0$	11.87	1.613	89.94	560.77	9.75	6.36	1.424	42.85	278.84	92.15	370.99
$h = 1.5$	12.12	1.606	87.14	528.96	9.85	6.36	1.419	41.74	263.99	85.62	349.61
$h = 2.0$	12.37	1.599	84.43	498.47	9.99	6.34	1.414	40.46	249.73	77.69	327.43
$h = 2.5$	12.60	1.592	81.81	469.23	10.14	6.31	1.409	39.20	236.04	70.08	306.13
$h = 3.0$	12.83	1.585	79.28	441.20	10.28	6.28	1.404	37.99	222.88	62.76	285.65

Appendix B. Proofs

Appendix B.1. Proof of Proposition 1

Proof. (1) The second order derivative of the total profit function with respect to λ can be expressed as

$$\frac{\partial^2 TP_{sc}}{\partial \lambda^2} = -\frac{(r\alpha+2M)T_s}{4T_r}\frac{2}{\lambda^3} - k < 0. \tag{B.1}$$

So, the profit function is concave in λ. Thus, the optimal λ can be obtained by equating the first order derivative to zero, i.e., $\frac{\partial TP_{sc}}{\partial \lambda} = 0$. Simplify the function, and we obtain equation (11).

(2) The second order derivative of the total profit function with respect to p can be expressed as

$$\frac{\partial^2 TP_{sc}}{\partial p^2} = -2a - \frac{\left(c_p P + hP + \frac{r\alpha}{4\lambda} + \frac{M}{2\lambda}\right)}{T_r} T_s''(p) = -2a - \frac{\left(c_p P + hP + \frac{r\alpha}{4\lambda} + \frac{M}{2\lambda}\right)}{T_r} \frac{a^2\left(e^{\theta T_r} - 1\right)^2}{\theta\left[P - D(p)\left(e^{\theta T_r} - 1\right)\right]^2} < 0\,. \tag{B.2}$$

So, the profit function is concave in p. Thus, the optimal p can be obtained by equating the first order derivative to zero, i.e.,$\frac{\partial TP_{sc}}{\partial p} = 0$. Simplify the function, and we obtain equation (12).

This ends the proof of Proposition 1. ■

Appendix B.2. Proof of Proposition 2

Proof. (1) The second order derivative of the total profit function with respect to λ can be expressed as

$$\frac{\partial^2 TP_s}{\partial \lambda^2} = -\frac{MT_s}{T_r}\frac{1}{\lambda^3} - k < 0. \tag{B.3}$$

So, the profit function is concave in λ. Thus, the optimal λ can be obtained by equating the first order derivative to zero, i.e., $\frac{\partial TP_s}{\partial \lambda} = 0$. Simplify the function, and we obtain equation (14).

(2) According to the Taylor series theory, T_s can be approximated as $T_s \approx \frac{D(w+m)}{\theta P\left(e^{\theta T_r} - 1\right)}$. The second order derivative of the total profit function with respect to p can be expressed as

$$\frac{\partial^2 TP_s}{\partial p^2} = -\frac{2a\left(e^{\theta T_r} - 1\right)}{\theta T_r} - \frac{ha^2\left(e^{\theta T_r} - 1\right)^2}{\theta^2 T_r} < 0. \tag{B.4}$$

So, the profit function is concave in p. Thus, the optimal p can be obtained by equating the first order derivative to zero, i.e., $\frac{\partial TP_s}{\partial p} = 0$. Simplify the function, and we obtain equation (15).

This ends the proof of Proposition 2. ■

Appendix B.3. Proof of Corollary 1

Proof. From proposition 1 and 2, we have

$$(\lambda^I - \lambda_0)(\lambda^I)^2 = \frac{(r\alpha+2M)T_s}{4kT_r}. \tag{B.5}$$

$$(\lambda^D - \lambda_0)(\lambda^D)^2 = \frac{2MT_s}{4kT_r}. \tag{B.6}$$

Letting (B.5)-(B.6), we have

$$(\lambda^I - \lambda^D)(\lambda^{I2} + \lambda^I\lambda^D + \lambda^{D2} - \lambda_0\lambda^D - \lambda_0\lambda^I) = \frac{r\alpha T_s}{4kT_r} \geq 0 \tag{B.7}$$

Because $\lambda^I, \lambda^D \geq \lambda_0$, we have $\lambda^{I2} + \lambda^I\lambda^D + \lambda^{D2} - \lambda_0\lambda^D - \lambda_0\lambda^I \geq 0$. Thus $\lambda^I - \lambda^D \geq 0$ is satisfied.

This ends the proof of Corollary 1. ■

Appendix B.4. Proof of Proposition 3

Proof. To achieve the coordination, after setting a sales margin m and investment sharing proportion γ, the optimal decisions for the supplier is wholesale price $w = p^I - m$ and reliability parameter $\lambda = \lambda^I$. That is, the first order derivative of $TP_s(w, \lambda|\phi, \gamma)$ w.r.t. and should satisfy

$$\frac{\partial TP_s(w,\lambda|\phi,\gamma)}{\partial w}|_{w=p^I-m,\lambda=\lambda^I} = 0, \tag{B.8}$$

$$\frac{\partial TP_s(w,\lambda|\phi,\gamma)}{\partial \lambda}|_{w=p^I-m,\lambda=\lambda^I} = 0, \tag{B.9}$$

That is

$$\frac{(1-\phi)(b-2ap^I+am)(e^{\theta T_r}-1)}{\theta T_r} - \frac{c_p P T_s'(p^I)}{T_r} - \frac{h_s P\left(1-e^{\theta T_s(p^I)}\right)T_s'(p^I)}{\theta T_r} - \frac{MT_s'(p^I)}{2\lambda^I T_r} = 0. \tag{B.10}$$

$$\frac{MT_s(p^I)}{2\lambda^{I2}T_r} - (1-\gamma)k(\lambda^I - \lambda_0) = 0. \tag{B.11}$$

in which $T_s(p^I) = -\frac{1}{\theta}ln\left(1 - \frac{D(p^I)(e^{\theta T_r}-1)}{P}\right), T_s'(p^I) = -\frac{a(e^{\theta T_r}-1)}{\theta[P-D(p^I)(e^{\theta T_r}-1)]}$. Substituting equation (11) into (B.11), we finally have $\gamma^C = \frac{r\alpha}{r\alpha+2M}$. Solving equations (B.10) and substitute the results into $w = p^I - m$, we obtain equation (18) and (20).

This ends the proof of Proposition 3. ■

Appendix B.5. Proof of Corollary 2

Proof. The first order derivative of m^C and w^C with respect to ϕ^C is

$m^{C'} = \left\{c_p P + \frac{h_s P\left(1-e^{-\theta T_s(p^I)}\right)}{\theta} + \frac{M}{2\lambda^I}\right\}\frac{\theta T_s'(p^I)}{a(e^{\theta T_r}-1)(1-\phi^C)^2} < 0$, and $w^{C'} = -m^{C'} > 0$, respectively.

This means that m^C is decreasing, while w^C is increasing in ϕ^C.

This ends the proof of Corollary 2. ■

Appendix B.6. Proof of Proposition 4

Proof. The results can be obtained by solving equations

$$TP_s^C(\phi) \geq TP_s^D \text{ and } TP_r^C(\phi) \geq TP_r^D. \tag{B.12}$$

Solving the two inequalities, we have

$$\underline{\phi} = 1 - \frac{\left\{\begin{array}{c}\left(p^I D(p^I) - \frac{hD(p^I)\left(e^{\theta T_r} - \theta T_r - 1\right)}{\theta^2 T_r} - \frac{A_r}{T_r} - \frac{r\alpha T_s(p^I)}{4\lambda^I T_r} - \gamma^C \frac{k\left(\lambda^I - \lambda_0\right)^2}{2} - TP_r^D\right)\theta T_r + \\ \left(c_p P + \frac{h_s P\left(1 - e^{-\theta T_s(p^I)}\right)}{\theta} + \frac{M}{2\lambda^I}\right)\frac{\theta T_s'(p^I)}{a} D(p^I)\end{array}\right\}}{D(p^I)\left(e^{\theta T_r} - 1\right)\left(\frac{b}{a} - P^I\right)}, \tag{B.13}$$

$$\overline{\phi} = 1 - \frac{\left\{\begin{array}{c}\left(TP_s^D + \frac{c_p P T_s(p^I)}{T_r} + \frac{hP\left(e^{-\theta T_s(p^I)} + \theta T_s(p^I) - 1\right)}{\theta^2 T_r} + \frac{A_s}{T_r} + \frac{\left(1 - \gamma^C\right)\left(k\left(\lambda^I - \lambda_0\right)^2\right)}{2} + \frac{M T_s(p^I)}{2\lambda^I T_r}\right)\theta T_r + \\ \left(c_p P + \frac{h_s P\left(1 - e^{-\theta T_s(p^I)}\right)}{\theta} + \frac{M}{2\lambda^I}\right)\frac{\theta T_s'(p^I)}{a} D(p^I)\end{array}\right\}}{D(p^I)\left(e^{\theta T_r} - 1\right)\left(\frac{b}{a} - P^I\right)}. \tag{B.14}$$

Therefore, when $\underline{\phi} \leq \phi^C \leq \overline{\phi}$, the supply chain can be coordinated.

This ends the proof of Proposition 4. ■