High-Accuracy Joint Multi-CFO and Multi-TOA Estimation for Multiuser SIMO OFDM Systems

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***Abstract*—A joint multi-carrier frequency offset (CFO) and multi-time of arrival (TOA) estimation algorithm for multiuser single-input multiple-output (SIMO) orthogonal frequency division multiplexing (OFDM) systems is proposed. With carefully designed pilots, multiple CFOs and TOAs of K users are separated jointly, dividing a complex $2K$-dimensional estimation problem into $2K$ low-complexity mono-dimensional estimation problems. Two CFO estimation approaches, including a low-complexity closed-form solution and a high-accuracy null-subcarrier assisted accurate estimation approach, are proposed, where the integer and fractional parts of each CFO are estimated as a whole rather separately. Each TOA is estimated regardless of CFO by exploring the features of the inter-carrier interference matrix. The Cramér-Rao lower bounds (CRLBs) of multi-CFO and mutli-TOA estimation are derived for the first time for SIMO OFDM systems. Simulation results show that the proposed CFO and TOA estimators provide higher estimation accuracy than the existing approaches. They also achieve performances close to the CRLBs especially at high signal-to-noise-ratios (SNRs).**

I. Introduction

Multiple-input multiple-output (MIMO) and orthogonal frequency division multiplexing (OFDM) are key techniques for various wireless communications systems. However, OFDM is sensitive to carrier frequency offset (CFO) incurred by the mismatch between local oscillators at the transmitter and receiver or by a Doppler frequency shift [1]–[5]. CFO can be divided into an integer CFO (iCFO) and a fractional CFO (fCFO). iCFO leads to a cyclic shift of subcarriers, resulting in detection errors, while fCFO destroys the orthogonality among subcarriers and induces the inter-carrier interference (ICI). Moreover, location-awareness plays an important role in many commercial, social and military applications. Time of arrival (TOA), defined as the propagation delay of the radio signal arriving from the direct line-of-sight propagation path, is the most widely used solution in localization [6]–[11]. [11] presents a comprehensive review of different TOA-based localization algorithms. However, all of those algorithms are effective only when TOA measurements have been collected accurately. Hence, the estimations of both CFO and TOA are very essential. However, many studies in the literature separately estimate either CFO or TOA, but not both together.

Existing methods usually estimate CFO in two steps. The first step estimates the integer or fractional part of CFO whereas the second step estimates the rest fractional or integer part. Jiang et al. proposed two separate algorithms to sequentially estimate iCFO and fCFO for MIMO OFDM systems [1], however suffering from error propagations. [2] presented a low-complexity iCFO estimation structure, assuming a perfect fCFO estimation and compensation. [3], [4] and [5] demonstrated a pilot-aided, a precoding-aided and a blind-based fCFO estimation approaches respectively, under an assumption of perfect estimation and compensation of iCFO. Nevertheless, they perform poorly if residual fCFOs or iCFOs exist. Thus, it is crucial to develop a one-step CFO estimator so that iCFO and fCFO can be estimated as a whole rather separately.

High-resolution TOA estimation algorithms have been studied in the literature, such as multiple signal classification [6], estimation of signal parameters via rotational invariance technique (ESPRIT) [7], [12], matrix pencil [8], improved minimum variance distortionless response [10] etc. However, when the received signals are perturbed by CFOs, they lead to biased TOA estimates [9]. CFO estimation techniques [1]–[5] can be performed before these algorithms to mitigate the impact of CFOs. However, the TOA estimation performance is limited to CFO estimation performance. Oh, et al. [9] proposed a robust TOA estimator against CFO, but based on chirp signals instead of OFDM signals. To the best of our knowledge, the joint estimation of multiple CFOs and TOAs for multiuser single-input multiple-output (SIMO) OFDM systems is still an open area in the literature.

In this paper, a high-accuracy joint multi-CFO and multi-TOA estimation algorithm is proposed for multiuser SIMO OFDM systems. Our work is different in the following aspects. First, through careful pilot design, multiple CFOs and TOAs of K users are separated, diving a complex $2K$-dimensional ($2K$-D) CFO and TOA estimation problem into $2K$ low-complexity 1-D problems. In contrast, the existing methods [1]–[8], [10], [12] separate K CFOs and K TOAs sequentially, resulting in error propagations from CFO estimates to TOA estimates. Second, two CFO estimation approaches, including a computationally efficient closed-form solution and a null-subcarrier assisted accurate estimation approach, are proposed, where the integer and fractional parts of each CFO are estimated as a whole, while the existing CFO estimator approaches [1]–[5] estimate iCFO and fCFO separately through two sequential

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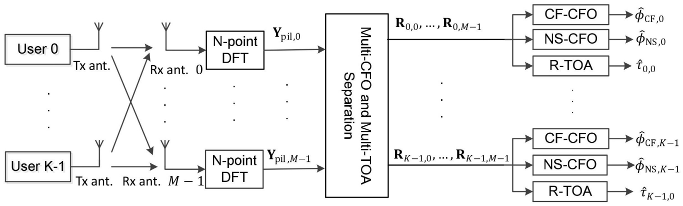

Fig. 1. Block diagram of the proposed joint multi-CFO and multi-TOA estimation algorithm for a K-user SIMO OFDM system.

algorithms, giving rise to error propagations. Third, each TOA is estimated regardless of CFO by applying the features of the ICI matrix. Nevertheless, the previous TOA estimators [6]–[8], [10], [12] require a CFO estimation and compensation procedure in advance. Fourth, to the best of our knowledge, this is the first work to derive the Cramér-Rao lower bounds (CRLBs) of CFO and TOA estimation for multiuser SIMO OFDM systems. Simulation results show that the proposed joint CFO and TOA estimation algorithm outperforms the existing methods [1], [5], [7] in terms of both the CFO and TOA estimation accuracy. Also, it provides CFO and TOA estimation performances close to the CRLBs especially at high signal-to-noise-ratios (SNRs).

System model and pilot design are described in Sections II and III respectively. Joint multi-CFO and multi-TOA estimation scheme is presented in Section IV. CRLB derivation and complexity analysis are given in Section V. Simulation results are presented in Section VI. Section VII draws conclusion.

Notations: Bold symbols represent vectors/matrices, and superscripts $*$, T, H, -1 denote the complex conjugate, transpose, complex conjugate transpose and inverse of vectors/matrixs. diag$\{\mathbf{a}\}$ is a diagonal matrix with vector $\mathbf{a}$ on its diagonal. $\mathbf{I}_N$, $\mathbf{1}_{M\times K}$ and $\mathbf{0}_{M\times K}$ are a $N\times N$ identity matrix, a $M\times K$ all-one matrix and a $M\times K$ zero matrix. $\otimes$, $\odot$ and $\mathbb{E}\{\}$ are the Kronecker product, Hadamard product and expectation operator. vec$\{\mathbf{A}\}$ is the vector-version of matrix $\mathbf{A}$. $\mathbf{A}(r_1\colon r_d\colon r_2, c_1\colon c_d\colon c_2)$ is the submatrix of $\mathbf{A}$ with rows from r_1 to r_2 with step size r_d and columns from c_1 to c_2 with step size c_d. $\angle b$ is the angle of b. $\Re\{\mathbf{A}\}$ and $\Im\{\mathbf{A}\}$ correspond to the real and imaginary parts of $\mathbf{A}$. $\|\cdot\|_{\text{F}}^2$ is Forbenius norm.

II. System Model

We consider an uplink multiuser SIMO OFDM system, where K users simultaneously transmit information to a base station (BS), as shown in Fig. 1. Each user and the BS are equipped with a single transmit antenna and M receive antennas respectively. A data frame consists of N_s OFDM blocks with N subcarriers each. Define $\mathbf{x}_k(i)=[x_k(0,i),\cdots,x_k(N-1,i)]^T$ as the signal vector of user k ($k=0,\cdots,K-1$) in OFDM block i ($i=0,\cdots,N_s-1$), with $x_k(n,i)$ denoting the symbol on subcarrier n ($n=0,\cdots,N-1$). Before transmission, each OFDM symbol block $\mathbf{x}_k(i)$ is processed by Inverse Discrete Fourier Transform (DFT), and then a cyclic prefix (CP) of length L_{cp} is pre-pended.

The channel is assumed to exhibit quasi-static block fading and the channel frequency response (CFR) remains constant for the duration of a data frame. For the k-th user, the CFRs of different transmit-receive antenna pairs share the common time delays [12]. Denote L as the number of propagation paths. The CFR $\mathbf{h}_{m,k}$ between the m-th antenna and k-th user is modelled as $\mathbf{h}_{m,k}=\mathbf{B}_k\mathbf{A}_{m,k}$, where $\mathbf{B}_k=[\mathbf{b}_k(0),\mathbf{b}_k(1),\cdots,\mathbf{b}_k(L-1)]$ with $\mathbf{b}_k(l)=[1,e^{-j2\pi\tau_{k,l}/(NT_{\text{s}})},\cdots,e^{-j2\pi(N-1)\tau_{k,l}/(NT_{\text{s}})}]^T$, $\mathbf{A}_k=[\mathbf{A}_{0,k}^T,\mathbf{A}_{1,k}^T,\cdots,\mathbf{A}_{M-1,k}^T]^T$ with $\mathbf{A}_{m,k}=[\alpha_{m,k,0},\alpha_{m,k,1},\cdots,\alpha_{m,k,L-1}]^T$. $\tau_{k,l}$ and $\alpha_{m,k,l}$ are the corresponding path delay and path gain of the l-th ($l=0,1,\cdots,L-1$) path, and T_{s} is the sampling time. Denote $\boldsymbol{\tau}_k=[\tau_{k,0},\tau_{k,1},\cdots,\tau_{k,L-1}]$. $\tau_{k,0}$ represents the TOA of the k-th user which is used for localization [6].

Define ϕ_k as the CFO between the k-th user and the BS, assuming that all antennas at the BS share one local oscillator. ϕ_k can be written as a sum of an iCFO $\phi_{\text{i},k}$ and a fCFO $\phi_{\text{f},k}$. Existing CFO estimators [1]–[5] estimate CFO ϕ_k through two steps, *i.e.*, iCFO ($\phi_{\text{i},k}$) estimation and fCFO ($\phi_{\text{f},k}$) estimation, while the proposed estimators estimate the CFO in one step where $\phi_{\text{i},k}$ and $\phi_{\text{f},k}$ are estimated as a whole rather separately.

After removing the CP at the m-th receive antenna, the received signal in the frequency domain can be written as

$$\mathbf{Y}_m=\sum_{k=0}^{K-1}\mathbf{C}(\phi_k)\mathbf{H}_{m,k}\mathbf{X}_k+\mathbf{z}_m \tag{1}$$

where $\mathbf{Y}_m=[\mathbf{y}_m(0),\cdots,\mathbf{y}_m(N_s-1)],\mathbf{y}_m(i)=[\mathrm{y}_m(0,i),\cdots,\mathrm{y}_m(N-1,i)]^T$ with $\mathrm{y}_m(n,i)$ as the received symbol on subcarrier n in OFDM block i; $\mathbf{C}(\phi_k)=\mathbf{F}\mathbf{E}(\phi_k)\mathbf{F}^H$ is the ICI matrix of the k-th user, in which $\mathbf{F}$ is the $N\times N$ DFT matrix with $\mathbf{F}(a,b)=1/\sqrt{N}e^{-j2\pi ab/N}$, ($a,b=0,\cdots,N-1$), and $\mathbf{E}(\phi_k)=\text{diag}\{\mathbf{e}_k\}$ is the CFO matrix with $\mathbf{e}_k=[1,e^{j2\pi\phi_k/N},\cdots,e^{j2\pi(N-1)\phi_k/N}]^T$ being the CFO vector; $\mathbf{H}_{m,k}=\text{diag}\{\mathbf{h}_{m,k}\}$ is the diagonal CFR matrix; $\mathbf{X}_k=[\mathbf{x}_k(0),\cdots,\mathbf{x}_k(N_s-1)]$; $\mathbf{z}_m$ is the additive white Gaussian noise matrix.

III. Pilot Design

The pilot structure for joint multi-CFO and multi-TOA estimation is introduced so that the complex $2K$-D CFO and TOA estimation is divided into $2K$ low-complexity 1-D CFO and TOA estimations. The system block diagram is illustrated in Fig. 1.

A pilot of P OFDM blocks is designed for each user, denoted as $\mathbf{X}_{\text{pil},k}=[\mathbf{x}_k(0),\cdots,\mathbf{x}_k(P-1)]$. Define $\mathbf{R}_{kg}=(1/P)\mathbf{X}_{\text{pil},k}\mathbf{X}_{\text{pil},g}^H$ as the pilot correlation matrix averaged over P blocks between the k-th and the g-th ($g=0,1,\cdots,K-1$) user. A good pilot design should meet $\mathbf{R}_{kg}=\mathbf{I}_N$ for $k=g$ and $\mathbf{0}_{N\times N}$ for $k\neq g$. This indicates all pilots for different users and different subcarriers should be orthogonal to each other. This can be achieved by a Hadamard matrix $\mathbf{M}_P$ of size $P\times P$, in which any two different rows are orthogonal to each other, and its auto-correlation matrix with $\mathbf{R}_{\mathbf{MM}}=(1/P)\mathbf{M}_P\mathbf{M}_P^H$ is an identity matrix $\mathbf{I}_P$ [3]. Hence, every subcarrier of each user should be assigned a unique row of $\mathbf{M}_P$ with $P\geq KN$. However, the required pilot length P is long if N is large.

To reduce the training overhead, estimations will only be performed on Q ($Q < N$) subcarriers, with $P \geq KQ$. Define $T = N/Q$ as the subcarrier spacing. Each user uses a unique initial subcarrier index, *e.g.*, j_k ($j_k = 0, \cdots, T-1$) for the k-th user. The subcarrier index of the q-th ($q = 0, 1, \cdots, Q-1$) pilot tone for the k-th user is $\mathbf{I}_k(q) = j_k + qT$. Then Q different rows of $\mathbf{M}_P$ are randomly chosen and placed on those subcarriers with index $\mathbf{I}_k$. By contrast, the remaining $(N-Q)$ subcarriers are allocated with null. With this pilot structure, the correlation matrix $\mathbf{R}_{kg}$ becomes

$$\mathbf{R}_{kg} = \begin{cases} \mathbf{I}_Q \otimes \mathbf{Z}_k, & k = g \\ \mathbf{0}_{N\times N}, & k \neq g \end{cases} \tag{2}$$

where $\mathbf{Z}_k$ is a $T \times T$ single-entry matrix with $\mathbf{Z}_k(j_k, j_k) = 1$.

It is noteworthy that an orthogonal training sequence with the help of a Hadamard matrix was also designed in [3]. However, it allows only the estimation of multiple CFOs not the joint estimation of multiple TOAs and multiple CFOs. As P is lower bounded by KQ, the choice of Q is essential. The minimum value of Q for multi-CFO and multi-TOA estimation is $2L$, as suggested in [12]. The required pilot length P of the proposed joint CFO and TOA estimation scheme needs to be at least $2KL$. The shortest pilot length utilized $P = KQ$ is considered in this paper.

IV. Joint Multi-CFO and Multi-TOA Estimation

The joint multi-CFO and multi-TOA estimation scheme is demonstrated in this section. First, the multi-CFO and multi-TOA separation scheme is presented. Second, a one-step closed-form and null-subcarrier based solutions are developed for each CFO whose integer and fractional parts are estimated as a whole rather separately. Then, two features, Toeplitiz matrix and unitary power in each row/column, of the ICI matrix are used to enable the TOA estimation robust against CFO.

A. Multi-CFO and Multi-TOA Separation

Let $\mathbf{Y}_{\text{pil},m} = [\mathbf{y}_m(0), \cdots, \mathbf{y}_m(P-1)]$ denote the received pilot matrix of size $N \times P$ at the receive antenna m. As the pilot design presented in Section III is orthogonal in the user domain, the pilot of each user is used as a projection matrix, to separate K CFOs and K TOAs. Define $\mathbf{R}_{m,k} = (1/P)\mathbf{Y}_{\text{pil},m}\mathbf{X}^H_{\text{pil},k}$ as the correlation matrix between the received mixture of pilots from K users at the m-th receive antenna of BS and the transmit pilot of user k averaged over P blocks. By using (1) and (2), $\mathbf{R}_{m,k}$ becomes

$$\mathbf{R}_{m,k} = \mathbf{C}(\phi_k)\tilde{\mathbf{H}}_{m,k} + \hat{\mathbf{z}}_m \tag{3}$$

where $\tilde{\mathbf{H}}_{m,k} = \mathbf{H}_{m,k}(\mathbf{I}_Q \otimes \mathbf{Z}_k)$ and $\hat{\mathbf{z}}_m$ is the noise term.

Hence, K $\mathbf{R}_{m,k}$ matrices at the m-th receive antenna have been separated successfully with no interference from other users. Since the TOA and CFO of the k-th user are embedded in $\mathbf{R}_{m,k}$, the K TOAs and K CFOs have been separated.

B. Multi-CFO Estimation

K CFOs of K users are separated by the separation of K $\mathbf{R}_{m,k}$ matrices, and can be estimated independently at the BS. A closed-form and null-subcarrier based CFO estimators are proposed for each CFO.

1) Closed-Form CFO Estimator: A closed-form (CF) based CFO estimator, referred to as CF-CFO, is derived by using the phase rotational invariance property of the CFO vector $\mathbf{e}_k$. It involves four small steps.

First, $\overline{\mathbf{E}}_{m,k}$ is obtained as $\overline{\mathbf{E}}_{m,k} = \mathbf{F}^H\mathbf{R}_{m,k}\mathbf{F}$. By using (3), $\overline{\mathbf{E}}_{m,k}$ can be expressed by $\overline{\mathbf{E}}_{m,k} = \mathbf{E}(\phi_k)\mathbf{F}^H\tilde{\mathbf{H}}_{m,k}\mathbf{F} + \tilde{\mathbf{z}}_m$, where $\tilde{\mathbf{z}}_m$ is the noise term. Since $\tilde{\mathbf{H}}_{m,k}$ is a diagonal matrix, $\mathbf{T}_{m,k} = \mathbf{F}^H\tilde{\mathbf{H}}_{m,k}\mathbf{F}$ would be a kind of circulant channel matrix with its first column vector being $\mathbf{t}_{m,k} = [t^0_{m,k}, \cdots, t^{N-1}_{m,k}]$. The truncated diagonal vector of $\overline{\mathbf{E}}_{m,k}$ would be $\hat{\mathbf{e}}^0_{m,k} = t^0_{m,k}\bar{\mathbf{e}}_k + \tilde{\mathbf{z}}^0_m$ where $\bar{\mathbf{e}}_k = \mathbf{e}_k(j_k : T : N, 1)$ and $\tilde{\mathbf{z}}^0_m$ is the noise vector. Its correlation matrix can be expressed as $\mathbf{R}^{m,k}_{\mathbf{ee}}(0) = \hat{\mathbf{e}}^0_{m,k}(\hat{\mathbf{e}}^0_{m,k})^H = |t^0_{m,k}|^2\bar{\mathbf{e}}_k\bar{\mathbf{e}}^H_k + \tilde{\mathbf{z}}^0_m(\tilde{\mathbf{z}}^0_m)^H$, however which depends on the unknown TOA.

Second, to be independent of the TOA, the elements of $\overline{\mathbf{E}}_{m,k}$ are right-shifted by one element, up to $(N-1)$ times, contributing to the $(N-1)$ new correlation matrices. Denote $\mathbf{R}^{m,k}_{\mathbf{ee}}(n)$ as the correlation matrix corresponding to the n-th ($n = 0, \cdots, N-1$) shift, and it can be given by $\mathbf{R}^{m,k}_{\mathbf{ee}}(n) = |t^n_{m,k}|^2\bar{\mathbf{e}}_k\bar{\mathbf{e}}^H_k + \tilde{\mathbf{z}}^n_m(\tilde{\mathbf{z}}^n_m)^H$. $n = 0$ means there is no shift. Summing the N correlation matrices, we can obtain $\mathbf{R}^{m,k}_{\mathbf{ee}} = \sum_{n=0}^{N-1}\mathbf{R}^{m,k}_{\mathbf{ee}}(n) = \| \mathbf{t}_{m,k} \|^2_{\text{F}} \bar{\mathbf{e}}_k\bar{\mathbf{e}}^H_k + \sum_{n=0}^{N-1}\tilde{\mathbf{z}}^n_m(\tilde{\mathbf{z}}^n_m)^H$. As $\| \mathbf{t}_{m,k} \|^2_{\text{F}}$ is the summed channel power and independent of TOA, $\mathbf{R}^{m,k}_{\mathbf{ee}}$ will be insusceptible to the TOA.

Third, M receive antennas are used to enhance the CFO estimation. The correlation matrix $\mathbf{R}_{\mathbf{ee},k}$ is computed by $\mathbf{R}_{\mathbf{ee},k} = \frac{1}{M}\sum_{m=0}^{M-1}\mathbf{R}^{m,k}_{\mathbf{ee}}$, and improved by the forward-backward (FB) averaging technique [6], [7], *i.e.*, $\mathbf{R}^{(\text{FB})}_{\mathbf{ee},k} = \frac{1}{2}(\mathbf{R}_{\mathbf{ee},k} + \mathbf{J}\mathbf{R}^*_{\mathbf{ee},k}\mathbf{J})$ where $\mathbf{J}$ is the $Q \times Q$ exchange matrix whose components are zero except for ones on the anti-diagonal.

Fourth, let $\mathbf{s}_k$ denote the signal eigenvector corresponding to the largest eigenvalue of $\mathbf{R}^{(\text{FB})}_{\mathbf{ee},k}$. $\mathbf{s}_{k,1}$ and $\mathbf{s}_{k,2}$ are defined as the first $(Q-1)$ and last $(Q-1)$ elements of $\mathbf{s}_k$ respectively. It can be verified that $\mathbf{s}_{k,2} = \mathbf{s}_{k,1}v_k$ with $v_k = e^{j2\pi T\phi_k/N}$. Thus, we can obtain $\hat{v}_k = (\mathbf{s}_{k,1})^{-1}\mathbf{s}_{k,2}$, and the CFO of user k is computed by

$$\hat{\phi}_{\text{CF},k} = \frac{N\angle\hat{v}_k}{2T\pi} \tag{4}$$

2) Null-Subcarrier Based CFO Estimator: A more accurate null-subcarrier (NS) based CFO estimator, referred to as NS-CFO, is derived by employing the null subcarriers of OFDM signals.

First, $\tilde{\mathbf{E}}_{m,k}$ is computed as $\tilde{\mathbf{E}}_{m,k} = \mathbf{F}^H\mathbf{R}_{m,k}$. Due to null subcarriers, $\check{\mathbf{E}}_{m,k} = \tilde{\mathbf{E}}_{m,k}(:, j_k : T : N)$ can be re-expressed by $\check{\mathbf{E}}_{m,k} = \mathbf{E}(\phi_k)\mathbf{F}_Q\overline{\mathbf{H}}_{m,k} + \check{\mathbf{z}}_m$ with $\overline{\mathbf{H}}_{m,k} = \text{diag}\{\tilde{\mathbf{H}}_{m,k}(j_k : T : N, j_k : T : N)\}$, $\mathbf{F}_Q = \mathbf{F}^H(:, j_k : T : N)$ and $\check{\mathbf{z}}_m$ being noise matrix.

In the absence of CFO ($\phi_k = 0$), $(\mathbf{F}^\perp_Q)^H\check{\mathbf{E}}_{m,k} = \mathbf{0}_{(N-Q)\times Q}$ where $\mathbf{F}^\perp_Q$ is the complementary part of $\mathbf{F}_Q$. However, this is not true for $\phi_k \neq 0$. Given a possible CFO search range of

$[e, f]$ and a trial value of ϕ_k, *i.e.*, $\tilde{\phi}_k$, we can obtain a compensated $\check{\mathbf{E}}_{m,k}$ as $\hat{\mathbf{E}}_{m,k}(\tilde{\phi}_k) = \mathbf{E}_k^H(\tilde{\phi}_k)\check{\mathbf{E}}_{m,k}$. A good trial value $\tilde{\phi}_k$ will make all the elements of $\mathbf{P}_1(\tilde{\phi}_k) = (\mathbf{F}_Q^{\perp})^H \hat{\mathbf{E}}_{m,k}(\tilde{\phi}_k)$ as zero. Thus, the CFO of user k is estimated by

$$\hat{\phi}_{\text{NS},k} = \arg \min_{\tilde{\phi}_k \in [e,f]} \frac{1}{M} \sum_{m=0}^{M-1} \| \mathbf{P}_1(\tilde{\phi}_k) \|_{\text{F}}^2 \tag{5}$$

To further enhance the CFO estimation performance, the C-FO estimates set $\hat{\boldsymbol{\phi}}_k = [\hat{\phi}_{k,0}, \hat{\phi}_{k,1}, \cdots, \hat{\phi}_{k,w-1}]$ corresponding to the w minimum values of $\| \mathbf{P}_1(\tilde{\phi}_k) \|_{\text{F}}^2$ is obtained. Then, an enhanced CFO estimate is obtained by

$$\hat{\phi}_{\text{NS},k} = \arg \min_{\tilde{\phi}_k \in \hat{\boldsymbol{\phi}}_k} \frac{1}{M} \sum_{m=0}^{M-1} \| \mathbf{P}_2(\tilde{\phi}_k) \|_{\text{F}}^2 \tag{6}$$

where $\mathbf{P}_2(\tilde{\phi}_k) = \overline{\mathbf{R}}_{m,k} \odot (\mathbf{1}_{N\times Q} - \tilde{\mathbf{R}}_{kk})$, $\overline{\mathbf{R}}_{m,k} = \tilde{\mathbf{R}}_{m,k}(:, j_k : T : N)$, $\tilde{\mathbf{R}}_{kk} = \mathbf{R}_{kk}(:, j_k : T : N)$ and $\tilde{\mathbf{R}}_{m,k} = \mathbf{C}^H(\tilde{\phi}_k)\mathbf{R}_{m,k}$. Note that $\mathbf{P}_2(\tilde{\phi}_k)$ would be a $N \times Q$ zero matrix if an accurate trial value $\tilde{\phi}_k$ is used. In this paper $w = 4$ is used.

C. Multi-TOA Estimation

K TOAs are separated by the separation of K $\mathbf{R}_{m,k}$ matrices in (3). A robust TOA (R-TOA) estimator against CFO is proposed, due to two features of the ICI matrix $\mathbf{C}(\phi_k)$.

Firstly, as a Toeplitz matrix, the ICI matrix $\mathbf{C}(\phi_k)$ has a structure like

$$\mathbf{C}(\phi_k) = \begin{bmatrix} c_0^k & c_{N-1}^k & \cdots & c_1^k \\ c_1^k & c_0^k & \cdots & c_2^k \\ \vdots & \vdots & \ddots & \vdots \\ c_{N-1}^k & c_{N-2}^k & \cdots & c_0^k \end{bmatrix} \tag{7}$$

Since $\tilde{\mathbf{H}}_{m,k}$ is a diagonal matrix, the diagonal vector of $\mathbf{C}(\phi_k)\tilde{\mathbf{H}}_{m,k}$ is also the diagonal vector of $\tilde{\mathbf{H}}_{m,k}$ with a scale ambiguity c_0^k. Let $\mathbf{d}_{m,k}^0 = \text{diag}\{\mathbf{R}_{m,k}(j_k : T : N, j_k : T : N)\}$ denote the truncated diagonal vector of $\mathbf{R}_{m,k}$. $\mathbf{d}_{m,k}^0$ can be written as $\mathbf{d}_{m,k}^0 = c_0^k \bar{\mathbf{h}}_{m,k} + \bar{\mathbf{z}}_m^0$ where $\bar{\mathbf{h}}_{m,k} = \mathbf{h}_{m,k}(j_k : T : N, 1)$ and $\bar{\mathbf{z}}_m^0$ is the noise vector. Then, the correlation matrix of $\mathbf{d}_{m,k}^0$, denoted as $\mathbf{R}_{\mathbf{dd}}^{m,k}(0) = \mathbf{d}_{m,k}^0(\mathbf{d}_{m,k}^0)^H$, can be given by $\mathbf{R}_{\mathbf{dd}}^{m,k}(0) = |c_0^k|^2 \bar{\mathbf{h}}_{m,k}\bar{\mathbf{h}}_{m,k}^H + \bar{\mathbf{z}}_m^0(\bar{\mathbf{z}}_m^0)^H$. As $|c_0^k|^2$ depends on the CFO, the TOA estimation based on $\mathbf{R}_{\mathbf{dd}}^{m,k}(0)$ will be susceptible to the CFO.

Secondly, the ICI matrix $\mathbf{C}(\phi_k)$ has the property of unitary power in each row/column, *i.e.*, $|c|^2 = |c_0^k|^2 + |c_1^k|^2 + \cdots + |c_{N-1}^k|^2 = 1$, which can be easily proved by considering $\mathbf{C}^H(\phi_k)\mathbf{C}(\phi_k) = \mathbf{F}\mathbf{E}^H(\phi_k)\mathbf{F}^H\mathbf{F}\mathbf{E}(\phi_k)\mathbf{F}^H = \mathbf{I}_N$. Therefore, in order to mitigate the impact of CFO on the TOA estimation, the elements of $\mathbf{R}_{m,k}$ are shifted up by one element, up to $(N-1)$ times, contributing to the $(N-1)$ new correlation matrices. Denote $\mathbf{R}_{\mathbf{dd}}^{m,k}(n) = \mathbf{d}_{m,k}^n(\mathbf{d}_{m,k}^n)^H$ as the correlation matrix corresponding to the n-th shift, with $\mathbf{d}_{m,k}^n = c_n^k \bar{\mathbf{h}}_{m,k} + \bar{\mathbf{z}}_m^n$. Summing them all, $\mathbf{R}_{\mathbf{dd}}^{m,k}$ can be obtained as $\mathbf{R}_{\mathbf{dd}}^{m,k} = |c|^2 \bar{\mathbf{h}}_{m,k}\bar{\mathbf{h}}_{m,k}^H + \sum_{n=0}^{N-1} \bar{\mathbf{z}}_m^n(\bar{\mathbf{z}}_m^n)^H$. Owing to $|c|^2 = 1$, the terms associated with CFO are removed.

To improve $\mathbf{R}_{\mathbf{dd}}^{m,k}$, the spatial correlation matrix [6], [7], [12] is applied. Defining $\mathbf{R}_{\mathbf{dd},k}$ as the averaged spatial correlation matrix of the k-th user, expressed by $\mathbf{R}_{\mathbf{dd},k} = \frac{1}{M}\sum_{m=0}^{M-1} \mathbf{R}_{\mathbf{dd}}^{m,k}$, and is further improved by the FB averaging technique [6], [7], *i.e.*, $\mathbf{R}_{\mathbf{dd},k}^{(\text{FB})} = \frac{1}{2}(\mathbf{R}_{\mathbf{dd},k} + \mathbf{J}\mathbf{R}_{\mathbf{dd},k}^*\mathbf{J})$. Lastly, $\mathbf{R}_{\mathbf{dd},k}^{(\text{FB})}$ is introduced to Step 4 of the ESPRIT algorithm [7] to obtain the robust TOA estimation of user k, *i.e.*, $\hat{\tau}_{k,0}$.

V. Performance Analysis

In this section, the CRLBs of multi-CFO and multi-TOA estimation are derived firstly. Then, the computational complexity of proposed algorithms is studied.

A. CRLB Derivation

The CRLBs of multi-CFO and multi-TOA estimation are derived with the proposed pilots $\mathbf{X}_{\text{pil},k}$.

Let $\mathbf{Y} = [\text{vec}\{\mathbf{Y}_0\}^T, \text{vec}\{\mathbf{Y}_1\}^T, \cdots, \text{vec}\{\mathbf{Y}_{M-1}\}^T]^T$ and $\bar{\mathbf{X}}_{\text{pil},k} = [\text{diag}\{\mathbf{x}_k(0)\}, \cdots, \text{diag}\{\mathbf{x}_k(P-1)\}]^T$. Thus, $\mathbf{Y}$ can be expressed as

$$\mathbf{Y} = (\mathbf{I}_M \otimes \mathbf{R})\mathbf{h} + \mathbf{Z} \tag{8}$$

where $\mathbf{R} = [\mathbf{r}_0, \mathbf{r}_1, \cdots, \mathbf{r}_{K-1}]$, $\mathbf{r}_k = (\mathbf{I}_P \otimes \mathbf{C}(\phi_k))\bar{\mathbf{X}}_{\text{pil},k}$, $\mathbf{h} = [\mathbf{h}_0^T, \mathbf{h}_1^T, \cdots, \mathbf{h}_{K-1}^T]^T$, $\mathbf{h}_k = [\mathbf{h}_{0,k}^T, \mathbf{h}_{1,k}^T, \cdots, \mathbf{h}_{M-1,k}^T]^T = (\mathbf{I}_M \otimes \mathbf{B}_k)\mathbf{A}_k$, and $\mathbf{Z}$ is the noise matrix. Denote $\mathbf{U} = (\mathbf{I}_M \otimes \mathbf{R})\mathbf{h}$, and $\mathbf{S}_k = \mathbf{I}_M \otimes \mathbf{r}_k$. The unknown variables are $\boldsymbol{\theta} = [\boldsymbol{\phi}, \boldsymbol{\tau}, \Re\{\mathbf{A}\}, \Im\{\mathbf{A}\}]$ with $\boldsymbol{\phi} = [\phi_0, \phi_1, \cdots, \phi_{K-1}]$, $\boldsymbol{\tau} = [\boldsymbol{\tau}_0, \boldsymbol{\tau}_1, \cdots, \boldsymbol{\tau}_{K-1}]$, $\mathbf{A} = [\mathbf{A}_0, \mathbf{A}_1, \cdots, \mathbf{A}_{K-1}]$. The Fisher information matrix is given by

$$\boldsymbol{\Phi} = \frac{2}{\sigma^2}\Re\Big[\frac{\partial \mathbf{U}^H}{\partial \boldsymbol{\theta}}\frac{\partial \mathbf{U}}{\partial \boldsymbol{\theta}^T}\Big] \tag{9}$$

where σ^2 is the variance of noise. Through some manipulations, we can obtain

$$\boldsymbol{\Phi} = \frac{2}{\sigma^2}\Re\begin{bmatrix} \mathbf{G}^H\mathbf{G} & -\mathbf{G}^H\mathbf{V} & -j\mathbf{G}^H\mathbf{W} & \mathbf{G}^H\mathbf{W} \\ -\mathbf{V}^H\mathbf{G} & \mathbf{V}^H\mathbf{V} & j\mathbf{V}^H\mathbf{W} & -\mathbf{V}^H\mathbf{W} \\ j\mathbf{W}^H\mathbf{G} & -j\mathbf{W}^H\mathbf{V} & \mathbf{W}^H\mathbf{W} & j\mathbf{W}^H\mathbf{W} \\ \mathbf{W}^H\mathbf{G} & -\mathbf{W}^H\mathbf{V} & -j\mathbf{W}^H\mathbf{W} & \mathbf{W}^H\mathbf{W} \end{bmatrix} \tag{10}$$

where $\mathbf{G} = [\mathbf{G}_0, \mathbf{G}_1, \cdots, \mathbf{G}_{K-1}]$ with $\mathbf{G}_k = \frac{2\pi}{N}\{\mathbf{I}_M \otimes [(\mathbf{I}_p \otimes \mathbf{F}\mathbf{E}(\phi_k)\mathbf{T}\mathbf{F}^H)\bar{\mathbf{X}}_{\text{pil},k}]\}\mathbf{h}_k$, $\mathbf{T} = \text{diag}\{\mathbf{t}\}$, $\mathbf{t} = [0, 1, \cdots, N-1]^T$, $\mathbf{V} = [\mathbf{V}_0, \cdots, \mathbf{V}_{K-1}]$, $\mathbf{V}_k = \frac{2\pi}{NT_s}\mathbf{S}_k(\mathbf{I}_M \otimes \mathbf{T}\mathbf{B}_k)\bar{\mathbf{A}}_k$, $\bar{\mathbf{A}}_k = [\text{diag}\{\mathbf{A}_{0,k}\}, \text{diag}\{\mathbf{A}_{1,k}\}, \cdots, \text{diag}\{\mathbf{A}_{M-1,k}\}]^T$, $\mathbf{W} = [\mathbf{W}_0, \mathbf{W}_1, \cdots, \mathbf{W}_{K-1}]$ with $\mathbf{W}_k = \mathbf{S}_k(\mathbf{I}_M \otimes \mathbf{B}_k)$. Detailed derivation of (10) is omitted here due to limited space.

The CRLB of $\boldsymbol{\theta}$ is given by the inverse of Fisher information matrix $\boldsymbol{\Phi}$, *e.g.*, $\boldsymbol{\Upsilon} = \boldsymbol{\Phi}^{-1}$. Therefore, the CRLBs of CFO and TOA are given by

$$\text{C}_{\text{RLB}}^{\text{CFO}} = \frac{1}{K}\sum_{k=0}^{K-1} \boldsymbol{\Upsilon}(k, k) \tag{11}$$

$$\text{C}_{\text{RLB}}^{\text{TOA}} = \frac{1}{K}\sum_{k=0}^{K-1} \boldsymbol{\Upsilon}(f(k), f(k)) \tag{12}$$

where $f(k) = K + kL$.

TABLE I

ANALYTICAL COMPUTATIONAL COMPLEXITY (N: SIZE OF AN OFDM SYMBOL, K: NUMBER OF USERS, Q: NUMBER OF SUBCARRIERS FOR PILOT DESIGN, M: NUMBER OF RECEIVE ANTENNAS, R: SIZE OF CFO SEARCH RANGE, Δ: STEP SIZE FOR CFO SEARCH, P_1 AND Q_1: CORRESPOND TO P AND Q IN [1], P_2 AND P_3: NUMBER OF SYMBOL BLOCKS OF [5] AND [7], SEP.: SEPARATION, EST.: ESTIMATION.)

Item		JCTE-1	JCTE-2	g1: iCFO [1]+fCFO [5]+ESPRIT [7]
Multi-CFO and Multi-TOA sep.		$O(2N^2KQM)$		$O(Q^2K^2M)$
Multi-CFO est.	Multi-iCFO est.	$O(K(4N^2Q+5Q^3+2Q^2NM))$	$O(KMR(2N-1)(2NQ-Q^2)/\Delta)$	$O(KQ_1(4N^2P_1+5M^2NP_1))$
	Multi-fCFO est.			$O(\frac{3}{4}KM^3P_2^3N\log_2 N)$
Multi-TOA est.		$O(K(9Q^3+2Q^2NM))$		$O(2Q^2MP_3K+5Q^3K)$

B. Complexity Analysis

In Table I, the computational complexity of multi-CFO and multi-TOA estimation for the proposed algorithms and the existing approaches are presented, in terms of the number of complex additions and multiplications. The proposed CFO and TOA estimators contribute to two joint CFO and TOA estimation (JCTE) schemes. JCTE-1 includes the CF-CFO estimator and the R-TOA estimator whereas JCTE-2 consists of the NS-CFO estimator and the R-TOA estimator. For comparison, the existing CFO estimation methods [1], [5] and the conventional ESPRIT based TOA estimation algorithm [7] are selected, referred to as g1: iCFO [1]+fCFO [5]+ESPRIT [7]. It is worth noting that the proposed JCTE1 and JCTE2 schemes could estimate iCFO and fCFO together whereas the existing methods [1], [5] need two separate algorithms for iCFO and fCFO estimation. Since the NS-CFO estimator is based on the direct search, JCTE-2 suffers from the high complexity, however contributes to high estimation accuracy, as shown in Fig. 2. With $N=64, K=2, Q=4, M=4, Q_1=8, P_1=8, R=6, P_2=2, P_3=2$ and $\Delta=0.01$, the numerical complexity of the proposed algorithms and the existing methods [1], [5], [7] can be computed. It is found the proposed JCTE-1 scheme is very computationally efficient, with complexity reduction around 7-fold than the existing methods [1], [5], [7].

VI. SIMULATION RESULTS

A simulation study is carried out to demonstrate the performance of the proposed multi-CFO and multi-TOA estimation algorithms, with $K=2$ users. System parameters are set as follows: each OFDM block contains $N=64$ subcarriers; the modulation scheme is quadrature phase shift keying; the CP length is $L_{\text{cp}}=16$; the two-ray channel model [10] is applied; the sampling time T_{s} is 5 ns. The root mean square error (RMSE) of CFO for the proposed CF-CFO estimator is defined as $\text{RMSE}_{\text{CFO}}^{\text{CF}}=\sqrt{\mathbb{E}\{\frac{1}{K}\sum_{k=0}^{K-1}(\hat{\phi}_{\text{CF},k}-\phi_k)^2\}}$ while that for the proposed NS-CFO estimator is $\text{RMSE}_{\text{CFO}}^{\text{NS}}=\sqrt{\mathbb{E}\{\frac{1}{K}\sum_{k=0}^{K-1}(\hat{\phi}_{\text{NS},k}-\phi_k)^2\}}$. The RMSE of TOA is $\text{RMSE}_{\text{TOA}}=\sqrt{\mathbb{E}\{\frac{1}{K}\sum_{k=0}^{K-1}(\hat{\tau}_{k,0}-\tau_{k,0})^2\}}$. The lower bounds of RMSEs of CFO and TOA are obtained from the square root of $\text{C}_{\text{RLB}}^{\text{CFO}}$ (11) and $\text{C}_{\text{RLB}}^{\text{TOA}}$ (12) respectively. The CFO of each user is randomly generated between -3 and 3. For fairness, the same number of symbols are used for the proposed JCTE-1, JCTE-2 schemes and the previous g1: iCFO [1]+fCFO [5]+ESPRIT [7] and ESPRIT [7] approaches.

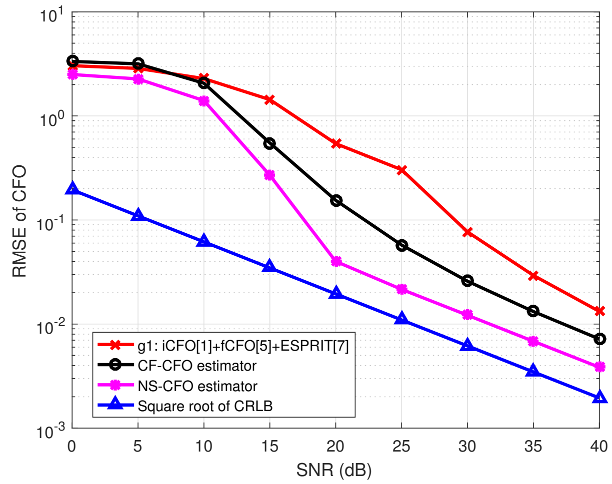

Fig. 2. RMSE performances of CFO of the proposed CF-CFO estimator, NS-CFO estimator and the existing methods [1], [5], [7].

Figs. 2 and 3 show the respective RMSE performances of CFO and TOA of the proposed algorithms with $Q=4$ and $M=4$, in comparison to the existing approaches [1], [5], [7]. First, it can be seen from Fig. 2 that both the proposed two CFO estimators outperform the existing methods [1], [5]. The proposed NS-CFO estimator has the best performance, whose performance approaches the CRLB from SNR $=20$ dB to SNR $=40$ dB. Second, as can be observed from Fig. 3, the proposed R-TOA estimator demonstrates a better performance than g1: iCFO [1]+fCFO [5]+ESPRIT [7] and ESPRIT [7]. The ESPRIT algorithm [7] has the worst performance, forming an error floor due to the presence of CFOs. Moreover, the proposed R-TOA estimator achieves a TOA RMSE performance close to the ESPRIT [7] with perfect CFO estimation and compensation and also to CRLB especially at high SNRs.

In Fig. 4, the impacts of the number of receive antennas M and number of subcarriers for pilot design Q on the RMSE performance of TOA are presented, with SNR $=25$ dB. First, it can be observed that the increase of M and Q can boost the TOA estimation performance of the proposed robust TOA estimator. Meanwhile, the proposed robust TOA estimator has a performance much closer to CRLB as M increases. Nevertheless, the ESPRIT algorithm [7] always

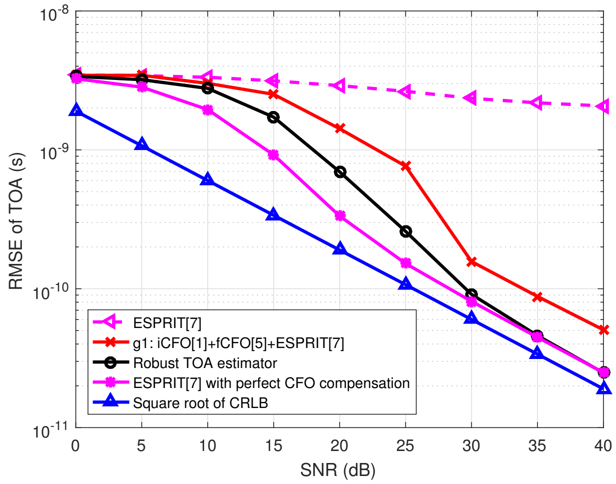

Fig. 3. RMSE performance of TOA of the proposed robust TOA estimator and the existing methods [1], [5], [7].

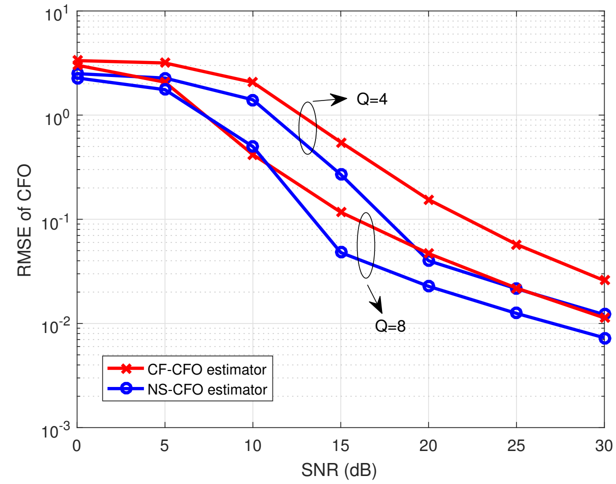

Fig. 5. Impacts of the number of subcarriers for pilot design Q on the RMSE performance of CFO, with $M = 4$.

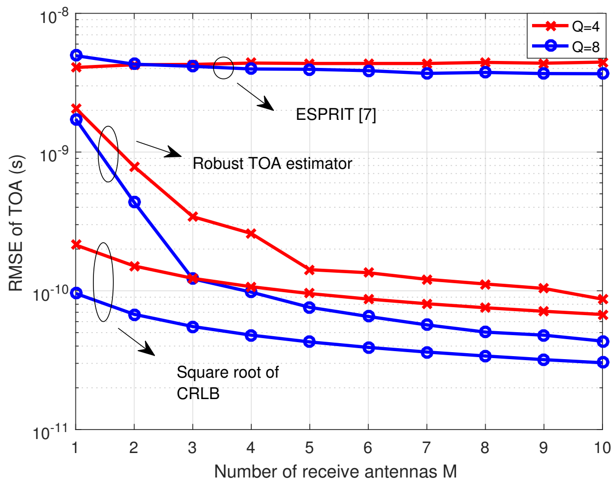

Fig. 4. Impact of the number of receive antennas M and number of subcarriers for pilot design Q on the RMSE performance of TOA at SNR$=25$ dB.

performs poorly, regardless of M and Q. Fig. 5 demonstrates the RMSE performances of CFO of the proposed two CFO estimators with $Q = 4$ and $Q = 8$ respectively. $M = 4$ receive antennas are used. It can be seen the NS-CFO estimator always outperforms the CF-CFO estimator, despite of Q. The performances of both two estimators are enhanced with Q.

VII. Conclusion

A joint multi-CFO and multi-TOA estimation scheme has been proposed for multiuser SIMO OFDM systems, including a CF-CFO estimator, a NS-CFO estimator and a R-TOA estimator. Both two CFO estimators provide better RMSE performances than the existing methods [1], [5]. Meanwhile, the NS-CFO estimator has a RMSE performance close to the CRLB from medium to high SNRs. The proposed CF-CFO estimator is very computationally efficient, with a complexity reduction of 7-fold over the methods in [1], [5], [7]. The proposed robust TOA estimator demonstrates the RMSE performance, close to the ideal case with perfect CFO compensation and the CRLB at high SNRs, without suffering from error propagations from the CFO estimation. Also, their performances can be enhanced with the increase in the number of receive antennas and the number of subcarriers for pilots design.

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