Relative Age of Information: Maintaining Freshness while Considering the Most Recently Generated Information

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Abstract

A queueing system handling a sequence of message arrivals is considered where each message obsoletes all previous messages. The objective is to assess the freshness of the latest message/information that has been successfully transmitted, *i.e.*, "age of information" (AoI). We study a variation of traditional AoI, the *Relative AoI*, here defined so as to account for the presence of newly arrived messages/information to the queue to be transmitted.

I. Introduction

Certain types of communication involves a sequence of messages where each message obsoletes all previous ones [3], [2], [5], [4]. The generated messages are sent to a transmission server. For a simple example, a temperature sensor could periodically transmit a reading to a remote control system. The control system could prioritize the most recent temperature reading. The sensor could instead be an alarm such as a motion detector, which needs to be manually reset once tripped; any alarm message would render stale any queued or in-transmission "heartbeat" message that is periodically sent to indicate no intruder is present and that the sensor is properly functioning. Alternatively, the messages may be commands to a remote actuator of a control system.

For the transmission of such messages, an important performance criterion is the freshness (age) of the most recently successfully transmitted message (information), *i.e.*, Age of Information (AoI).

Consider a stream of messages to a transmission server with i.i.d. interarrival times $\sim X$ and i.i.d. message service-times $\sim S$. Again, a newly arrived message obsoletes all previous messages. This may imply that messages are of the same or comparable length (*i.e.*, dependent lengths) but there may be randomness associated with service, *e.g.*, time-varying noise or interference associated with the communication channel.

Let $D(t)$, respectively $A(t)$, be the largest message departure time from, respectively message arrival time to, the server that is $\leq t$. Let $\tilde{A}(t)$ be the largest arrival time among *successfully departed* messages such that $\tilde{A}(t) \leq t$. The AoI associated with the server at time t is [3],

$$\mathsf{AoI}(t) \;=\; t - (\tilde{A} \circ D)(t) \;=\; t - \tilde{A}(D(t)). \tag{1}$$

Previous work on AoI has considered a transmission server after a lossless FIFO queue wherein $A = \tilde{A}$ [3]. More recently, finite and lossy (e.g., bufferless) systems and server preemption have been considered for this definition of AoI [4]. For a bufferless/queueless system under push-out considered in the following, $A \circ D = \tilde{A} \circ D$. These identities will generally not hold for systems that may block some messages upon arrival (*i.e.*, that do not admit all messages).

Note that AoI (1) does not consider the arrival times of new messages since the last departure. The point is that most recently received message may be freshest possible. With this in mind, herein we are interested in the non-negative difference between the AoI and arrival time to the transmission system of the most recent message, *i.e.*, the *Relative AoI*,

$$\Delta(t) \;:=\; A(t) - \tilde{A}(D(t)). \tag{2}$$

That is, the question critical to performance which the definition of Δ is trying to address is: Has the most recently received message been transmitted? Note that Δ is zero if this is the case.

II. Busy periods of GI/GI/1/1-PO system

Consider messages sent at mean rate β with i.i.d. interarrival times $\sim X$ such that finite $\mathsf{E}X = 1/\beta > 0$ and $\mathsf{P}(X > 0) = 1$. Also, assume that the i.i.d. message service times S are such that finite $\mathsf{E}S = 1/\delta > 0$ and $\mathsf{P}(S > 0) = 1$ (and independent of the arrival process).

We consider a (queueless) GI/GI/1/1-PO server with service preemption and push-out (PO) when a new messge arrives. Again, in this case $\tilde{A} \circ D = A \circ D$ so that

$$\Delta(t) \quad := \quad A(t) - A(D(t)). \tag{3}$$

For this GI/GI/1/1-PO server, immediately after a departure time is a server idle period. Following that will be a busy period that concludes with a single departure. The arrival times are renewal points, as are the (successful) departure times.

In steady-state, let B be distributed as the length of a busy period and let N be the number of arrivals during the busy period not including the first arrival that starts it. An illustrative busy cycle is depicted in Fig. 1 for $N = 2$ (3 arrivals total), where J_0 is the idle period, busy period $B = J_1 + J_2 + J_3$, interarrival times J_1, J_2, and completed service time J_3. More generally, the completed service time is J_{N+1} and

$$B = \sum_{n=1}^{N+1} J_n.$$

Assume random variables X, S are independent for the following. The intervals

- $J_0 \sim X - S$ given $X > S$, *i.e.*, $\mathsf{E}J_0 = \mathsf{E}(X - S | X > S)$
- $\forall n \in \{1, ..., N\}$, $J_n \sim X$ given $S > X$, *i.e.*, $\mathsf{E}J_n = \mathsf{E}(X | S > X)$
- $J_{N+1} \sim S$ given $X > S$, *i.e.*, $\mathsf{E}J_{N+1} = \mathsf{E}(S | X > S)$.

First note that N is geometric with parameter

$$\zeta := \mathsf{P}(X > S),$$

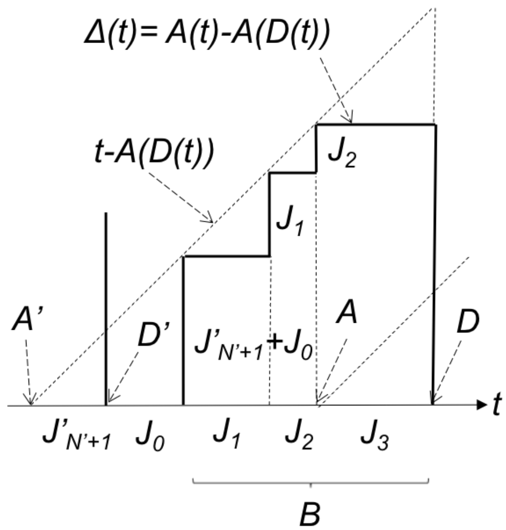

Fig. 1. Illustrative busy cycle (D', D) for bufferless server under push-out showing age of information (2) for $N = 2$ (3 arrivals), server idle period J_0, interarrival times J_1,J_2, and consecutive departure times D', D (and their corresponding arrival times A', A), and busy period $B = J_1 + J + 2 + J_2$.

i.e., $\mathsf{P}(N = n) = (1-\zeta)^n\zeta$ for integers $n \geq 0$ and

$$\mathsf{E}(B|N = n) \;\; = \;\; n\mathsf{E}(X|S > X) + \mathsf{E}(S|X > S).$$

So, the average busy cycle duration is

$$\begin{aligned}
&\mathsf{E}(D - D') = \mathsf{E}(J_0 + B) \\
&= \;\; \mathsf{E}(X - S|X > S) \\
&\quad\; +\mathsf{E}\left(\sum_{n=0}^{\infty}(1-\zeta)^n\zeta\mathsf{E}(B|N = n)\right) \\
&= \;\; \mathsf{E}(X - S|X > S) \\
&\quad\;\; + \mathsf{E}N\mathsf{E}(X|S > X) + \mathsf{E}(S|X > S) \\
&= \;\; \mathsf{E}(X|X > S) + \frac{1-\zeta}{\zeta}\mathsf{E}(X|S > X) \\
&= \;\; \mathsf{E}(X|X > S) + \frac{\mathsf{P}(S > X)}{\mathsf{P}(X > S)}\mathsf{E}(X|S > X) \\
&= \;\; \frac{\mathsf{E}X}{\mathsf{P}(X > S)}
\end{aligned}$$

Note that the rate at which messages are successfully transmitted is $1/\mathsf{E}(D - D')$.

A. D/M/1/1-PO example

So, for the case of exponential (memoryless) $S \sim \exp(\delta)$ and deterministic $X = 1/\beta$ a.s. (*i.e.*, D/M/1/1-PO server), the mean rate at which messages are successfuly transmitted is

$$\frac{1}{\mathsf{E}(D-D')} = \beta(1-\mathrm{e}^{-\delta/\beta}) = \delta(1-\frac{1}{2\rho}+\mathrm{o}(\frac{1}{\rho})),$$

where $\rho = \beta/\delta$.

Other quantities involved can be computed as, *e.g.*,

$$\begin{aligned} \mathsf{E}(S|X>S) &= \int_0^{1/\beta} s\frac{\delta \mathrm{e}^{-\delta s}}{1-\mathrm{e}^{-\delta/\beta}}\mathrm{d}s \\ &= \frac{1}{\delta}\left(1-\frac{1}{\rho}\cdot\frac{\mathrm{e}^{-1/\rho}}{1-\mathrm{e}^{-1/\rho}}\right) \end{aligned} \tag{4}$$

B. M/M/1/1-PO example

In the M/M/1/1-PO case where $S \sim \exp(\delta)$ and $X \sim \exp(\beta)$, we have

$$\zeta \;:=\; \mathsf{P}(X>S) = \frac{\delta}{\beta+\delta},$$

Also,

$$\begin{aligned} \mathsf{E}(S|X>S) &= \int_0^\infty\int_s^\infty s\frac{1}{\zeta}\beta\mathrm{e}^{-\beta x}\delta\mathrm{e}^{-\delta s}\mathrm{d}x\mathrm{d}s \\ &= \frac{1}{\beta+\delta} \end{aligned}$$

Similarly,

$$\begin{aligned} \mathsf{E}(X|S>X) &= \frac{1}{\beta+\delta} \\ \mathsf{E}(X|X>S) &= \frac{1}{\beta}+\frac{1}{\beta+\delta} \\ \Rightarrow \mathsf{E}(X-S|X>S) &= \frac{1}{\beta} \end{aligned}$$

So, by substitution,

$$\frac{1}{\mathsf{E}(D-D')} \;=\; \left(\frac{1}{\beta}+\frac{1}{\delta}\right)^{-1}$$

as expected.

III. Stationary average Δ (2) or (3) for GI/GI/1/1-PO system

In the following, we consider a bufferless server with preemption and push-out of the in-service message when a new message arrives.

Proposition 3.1: The stationary average relative age of information of the GI/GI/1/1-PO server is

$$\overline{\Delta} = \frac{\mathsf{E}(X \wedge S)}{\mathsf{P}(X > S)}$$

Proof: Using the Palm inversion formula [1],

$$\overline{\Delta} = \frac{\mathsf{E}\int_{D'}^{D=D'+J_0+B} \Delta(t)\mathrm{d}t}{\mathsf{E}(D - D')}.$$

Recall that $\mathsf{E}(D-D') = \mathsf{E}X/\mathsf{P}(X > S)$. Let $I_0 = J'_{N'+1} + J_0 \sim X$ given $X > S$. From Fig. 1, we see that

$$\begin{aligned}
&\mathsf{E}\left(\int_{D'}^{D=D'+J_0+B} \Delta(t)\mathrm{d}t \middle| N = n\right) \\
&= \mathsf{E}I_0\mathsf{E}B + \sum_{k=1}^{n} \mathsf{E}J_k \sum_{i=k+1}^{n+1} \mathsf{E}J_i \\
&= \mathsf{E}(X|X > S)(n\mathsf{E}(X|S > X) + \mathsf{E}(S|X > S)) \\
&\quad + \sum_{k=1}^{n}\Bigg((n-k)(\mathsf{E}(X|S > X))^2 \\
&\quad + \mathsf{E}(X|S > X)\mathsf{E}(S|X > S)\Bigg) \\
&= \mathsf{E}(X|X > S)\mathsf{E}(S|S > X) \\
&\quad + \mathsf{E}(X|S > X)\mathsf{E}(X + S|X > S)n \\
&\quad + (\mathsf{E}(X|S > X))^2\frac{n(n-1)}{2}
\end{aligned}$$

Substitute $\mathsf{E}N = (1-\zeta)/\zeta$, $\mathsf{E}N^2 = (1-\zeta)(2-\zeta)/\zeta^2$ and $\zeta := \mathsf{P}(X > S)$, and simplifying the Palm inversion formula gives

$$\begin{aligned}\overline{\Delta} &= \frac{\mathsf{P}(X>S)}{\mathsf{E}X}\Bigg(\mathsf{E}(X|X>S)\mathsf{E}(S|X>S)\\ &+ \mathsf{E}(X|S>X)\mathsf{E}(X+S|X>S)\frac{\mathsf{P}(S>X)}{\mathsf{P}(X>S)}\\ &+ (\mathsf{E}(X|S>X))^2\left(\frac{\mathsf{P}(S>X)}{\mathsf{P}(X>S)}\right)^2\Bigg)\end{aligned}$$

Let $\tilde{\mathsf{E}}(A|B) = \mathsf{E}(A|B)\mathsf{P}(B)$. Factoring $1/\mathsf{P}(X>S)^2$ gives

$$\begin{aligned}\overline{\Delta} &= \frac{1}{\mathsf{E}X\mathsf{P}(X>S)}\Bigg(\tilde{\mathsf{E}}(X|X>S)\tilde{\mathsf{E}}(S|X>S)\\ &\quad+\tilde{\mathsf{E}}(X|S>X)\tilde{\mathsf{E}}(X+S|X>S)\\ &\quad+(\tilde{\mathsf{E}}(X|S>X))^2\Bigg)\\ &= \frac{1}{\mathsf{E}X\mathsf{P}(X>S)}(\tilde{\mathsf{E}}(X|X>S)+\tilde{\mathsf{E}}(X|S>X))\\ &\quad\times(\tilde{\mathsf{E}}(S|X>S)+\tilde{\mathsf{E}}(X|S>X))\\ &= \frac{1}{\mathsf{E}X\mathsf{P}(X>S)}\mathsf{E}X\mathsf{E}(S\wedge X)\end{aligned}$$

□

Now consider the definition of age of information in Equ. (1) of [3], [4].

Corollary 3.1: For the GI/GI/1/1-PO server, the stationary average AoI (1) is

$$\overline{\mathsf{AoI}} = \overline{\Delta} + \frac{\mathsf{E}X^2}{2\mathsf{E}X}.$$

Proof: Note that $\mathsf{AoI}(t) = \Delta(t) + t - A(t)$, where simply by the Palm inversion formula, the stationary average age of the latest arrival $\mathsf{E}(t - A(t)) = \mathsf{E}\int_0^X t\mathrm{d}t/\mathsf{E}X$. □

A. D/M/1/1-PO and M/M/1/1-PO special cases for Δ

For the special case that $X = 1/\beta$ a.s. and $S \sim \exp(\delta)$, $\mathsf{E}(X|X > S) = \mathsf{E}(X|S > X) = 1/\beta$, and recall (4). So we have the following corollary of Prop. 3.1.

Corollary 3.2: $\overline{\Delta}$ for the D/GI/1/1-PO server is

$$\mathsf{E}(S|\frac{1}{\beta} > S) + \frac{1}{\beta} \cdot \frac{\mathsf{P}(S > \frac{1}{\beta})}{\mathsf{P}(\frac{1}{\beta} > S)}. \tag{5}$$

In particular, $\overline{\Delta}$ of a D/M/1/1-PO server is $1/\delta$.

The following corolloary states that $\overline{\Delta}$ is the same for the D/M/1/1-PO and M/M/1/1-PO special cases.

Corollary 3.3: $\overline{\Delta}$ for the M/M/1/1-PO server $1/\delta$.

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