

# Perturbation Observer based Adaptive Passive Control for Nonlinear Systems with Uncertainties and Disturbances

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## Abstract

This paper develops a perturbation observer based adaptive passive control (POAPC) scheme to provide great robustness of nonlinear systems against to the unpredictable uncertainties and disturbances therein. The proposed POAPC scheme includes a high-gain perturbation observer (HGPO) and a robust passive controller. The HGPO is designed to online estimate the perturbation aggregated from the combinatorial effect of system nonlinearity, parameter uncertainty, unmodelled dynamics and fast time-varying external disturbances. Then the robust passive controller, using the estimated perturbation, can produce just minimal control effort to compensate the magnitude of the current actual perturbation. Furthermore, the convergence of estimation error of HGPO and the closed-loop system stability are analyzed theoretically. Finally, two practical examples are given to show the effectiveness and advantages of the proposed approach over the accurate model-based passive control scheme and the linearly parametric estimation based adaptive passive control scheme.

## 1 Introduction

Generally speaking, most practical systems should be modelled as nonlinear systems. The concept of passivity has played a crucial role in the analysis and control of nonlinear systems in the framework of Lyapunov stability theory Khalil et al. (2001). A passive system is characterized by the property that at any time the amount of energy which the system can conceivably supply to its environment cannot exceed the amount of energy that has been supplied to it Isidori (1995). Passivity views a dynamical system as an energy-transformation device, which decomposes complex nonlinear systems into simpler subsystems that, upon interconnection, add up their local energies to determine the full system's behaviour. The action of a controller connected to the dynamical system may also be regarded, in terms of energy, as another separate dynamical system. Thus the control problem can then be treated as finding an interconnection pattern between the controller and the dynamical system, such that the changes

of the overall energy function can take a desired form. This ‘energy shaping’ approach is the essence of passive control (PC), which takes the system energy into account and gives a clear physical meaning Ortega et al. (2002); Fujimoto et al. (2003); Loria et al. (2001); Kenji et al. (2012). The conventional PC usually requires an accurate system model for controller design. However, no matter how detailed and complex the nonlinear model is, there still exists an inevitable gap between the developed nonlinear model and the practical system. That is, many unpredictable factors, such as system nonlinearity, parameter uncertainty, unmodelled dynamics and fast time-varying external disturbances, should be considered in the nonlinear model during the investigation of practical control systems Yao et al. (2011, 2014). Therefore, the development of advanced PC schemes with robustness against to those unpredictable factors is an important issue, which is very crucial and challenging in both theory and practice.

In order to handle the parameter uncertainty, an adaptive passive control (APC) was proposed by Zhou et al. (2012); Mermoud et al. (2001); Bobtsov et al. (2014), in which an uncertain nonlinear system was assumed to have explicit linearly parametric uncertainties, and then a linearly parametric estimation based adaption law was developed to approximate these unknown parameters. However, these laws might not be trivial and the resulting adaptive system is highly nonlinear, with slow system responses to parameter variations. Robust passive control (RPC) is another effective method Lin et al. (1999); Joshia et al. (2001); Wu et al. (2013), in which the structural uncertainty was assumed to be a nonlinear function of the state and bounded by a known function. In general, such a function may not be easily found, and it would give an over-conservative result as well. On the other side, functional approximators have also been proposed to estimate uncertainties of nonlinear systems recently. For example, the adaptive neural network, which has the capability of nonlinear function approximation, learning, and fault tolerance, was applied to estimate uncertain fast time-varying nonlinear dynamics Du et al. (2011); Mou et al. (2010); Ge et al. (2007). Adaptive dynamic programming (ADP) was applied in Fu et al. (2011); Zhang et al. (2011) to directly approximate, through learning and online measurement, the optimal controllers with guaranteed stability for dynamical systems with the unknown dynamics. Robust adaptive dynamic programming designed in Jiang et al. (2013) extended it into uncertain systems with unmeasured state variables and unknown system order. A Fradkov theorem based passification approach was proposed to stabilize a nonlinear system with functional and parametric uncertainties Bobtsov et al. (2005), which was also extended to the nonlinear delay systems with an unmodelled dynamics Bobtsov et al. (2011). However, the aforementioned literatures have mainly discussed one or a few types of uncertainties. The functional estimation for model uncertainties, parameter uncertainties and fast time-varying external disturbances existed in nonlinear passive systems has not yet been fully investigated.

When a reliable model for a part of the system uncertainties is available, the robust performance of nonlinear passive systems can be improved through the readily compensation based on model-based techniques. However, most of the model uncertainties is arbitrary and cannot be readily identified, so a robust

compensation method is needed to cope with it in real-time without depending on a direct perturbation model. In the past decades, an enormous variety of elegant approaches based on observers have been developed to estimate system uncertainties, such as the unknown input observer (UIO) Johnson et al. (1971), disturbance observer (DOB) Chen et al. (2000), uncertainty and disturbance estimator (UDE) Zhong et al. (2004), generalized proportional integral observer (GPIO) Chen et al. (2016), enhanced decentralized PI control via advanced disturbance observer Sun et al. (2015), observer-based adaptive fuzzy tracking control Li et al. (2015), hybrid fuzzy adaptive output feedback control via adaptive fuzzy state observer Li et al. (2015), adaptive fuzzy output feedback control via state observer Li et al. (2016), ect. Meanwhile, perturbation observer (PO) has also been widely employed which can dynamically estimate the lumped uncertainty not considered in the nominal model based on an extended state. It can be regarded as a model regulator which drives the physical system with uncertainty to the nominal model Kwon et al. (2004). A typical PO is formulated directly for nonlinear systems in time domain by using the derivative of state variable and time-delay Youcef-Toumi et al. (1992), which is the origin of different type of PO based control system. The first type is the sliding-mode control with perturbation estimation, which uses sliding-mode PO (SMPO) to reduce the conservativeness of the sliding-mode control Kwon et al. (2004); Yang et al. (2016). The SMPO frequently suffers the discontinuity of high-speed switching. The second type is the active disturbance rejection control (ADRC) Han et al. (2009), which designs nonlinear disturbance observers. The nonlinear observer is usually too complex for stability analysis. In the authors' previous work Jiang et al. (2004); Wu et al. (2004); Chen et al. (2014); Yang et al. (2015); Yang, Jiang et al. (2016), a high-gain perturbation observer (HGPO) was proposed. Compared with SMPO and the nonlinear observer, HGPO can provide almost the same performance but has merits of easy design and implementation.

Motivated by the above discussion, this paper aims to develop a new type of APC for nonlinear passive systems in consideration of model uncertainties, parameter uncertainties and fast time-varying external disturbances by extending HGPO developed in our previous research. The main contributions of this paper are summarized as follows:

- 1) An HGPO is designed to online estimate the perturbation including system nonlinearity (variables of a polynomial of degree higher than one or in the argument of a function which is not a polynomial of degree one), parameter uncertainty, unmodelled dynamics (ignored dynamics during the approximation from the original high order system to the simplified lower order system) and fast time-varying external disturbances (some type of extraneous signals during system operation which usually includes some high-frequency signals that vary rapidly along with the time). A passive controller with the estimated perturbation from HGPO as an input, called as perturbation observer based adaptive passive control (POAPC), is developed to provide desired robustness against to unpredictable uncertainties and disturbances, of which the physical property of the system

can be fully exploited via energy shaping.

- 2) Both the convergence of estimation error of HGPO and the stability of the closed-loop nonlinear system are analyzed theoretically.
- 3) The proposed POAPC is evaluated in two practical examples and its advantages over some existing APCs and PC are verified.

The rest of the paper is organized as follows. Section 2 formulates the problem to be discussed. Section 3 presents the controller design and necessary theoretical analysis. Example results are exposed in Section 4. Finally, concluding remarks are contained in Section 5.

## 2 Problem Formulation and Preliminaries

Consider a normal passive system as follows Isidori (1995)

$$\begin{cases} \dot{\mathbf{y}} = a(\mathbf{y}, \mathbf{z}) + \bar{\mathbf{B}}\mathbf{u} + \zeta(\mathbf{y}, \mathbf{z}, t) \\ \dot{\mathbf{z}} = f_0(\mathbf{z}) + p(\mathbf{y}, \mathbf{z})\mathbf{y} \end{cases} \quad (1)$$

where output  $\mathbf{y} \in \mathbb{R}^m$  and input  $\mathbf{u} \in \mathbb{R}^m$ , such that system (1) is of the relative degree  $\{1, 1, \dots, 1\}$ .  $a(\mathbf{y}, \mathbf{z}) \in \mathbb{R}^m$  is nonlinear which includes the structural and parametric uncertainty,  $\zeta(\mathbf{y}, \mathbf{z}, t) \in \mathbb{R}^m$  includes the equivalent effect of unmodelled dynamics and fast time-varying external disturbances.  $\mathbf{z} \in \mathbb{R}^{n-m}$  is the internal dynamics, with  $f_0(\mathbf{z}) \in \mathbb{R}^{n-m}$  represents the zero dynamics and  $p(\mathbf{y}, \mathbf{z})$  is a known smooth function of dimension  $(n - m) \times m$ . The unknown control gain  $\bar{\mathbf{B}} \in \mathbb{R}^{m \times m}$  is written as

$$\bar{\mathbf{B}} = \begin{bmatrix} b_{11}(y, z) & \cdots & b_{1m}(y, z) \\ \vdots & \vdots & \vdots \\ b_{m1}(y, z) & \cdots & b_{mm}(y, z) \end{bmatrix} \quad (2)$$

**Remark 1.** The same number of inputs and outputs, a known  $p(\mathbf{y}, \mathbf{z})$ , and the relative degree  $\{1, 1, \dots, 1\}$  are basic assumptions for such particular normal passive system Mermoud et al. (2001). Note that several work has been done to relax these fundamental assumptions by Bobtsov et al. (2014, 2005, 2011). In particular, a perturbation estimation based coordinated adaptive passive control (PECAPC) has been designed if the input number is larger than the output number Yang et al. (2015), while the relative degree has also been relaxed to any arbitrary number Yang, Jiang et al. (2016).

Assume system (1) is zero-state detectable and locally weakly minimum-phase Khalil et al. (2001), i.e., there exists a positive differentiable function  $W_0(\mathbf{z})$ , with  $W_0(0) = 0$ , such that

$$\frac{\partial W_0(\mathbf{z})}{\partial \mathbf{z}^T} f_0(\mathbf{z}) \leq 0 \quad (3)$$

for all  $\mathbf{z}$  in the neighbourhood of  $\mathbf{z}(t) = 0$ .

The known part of function  $a(\mathbf{y}, \mathbf{z})$  and  $\bar{\mathbf{B}}$  are assumed to be zero for the simplification of formulations. In fact, one can assume the nominal part is known, and only use the perturbation to represent the uncertain part. Define a fictitious state as

$$\Psi(\mathbf{y}, \mathbf{z}, u, t) = a(\mathbf{y}, \mathbf{z}) + (\bar{\mathbf{B}} - \mathbf{B}_0)\mathbf{u} + \zeta(\mathbf{y}, \mathbf{z}, t) \quad (4)$$

where  $\Psi(\cdot) \in \mathbb{R}^m$  is called the perturbation.  $\mathbf{B}_0 = \text{diag}[b_{10}, b_{20}, \dots, b_{m0}]$  with  $b_{i0}$  the nominal control gain. Extend the output dynamics of system (1), yields

$$\begin{cases} \dot{y}_i = \Psi_i(\cdot) + b_{i0}u_i \\ \dot{y}_{ei} = \dot{\Psi}_i(\cdot) \\ \dot{\mathbf{z}} = f_0(\mathbf{z}) + p(\mathbf{y}, \mathbf{z})\mathbf{y} \end{cases}, \quad i = 1, \dots, m \quad (5)$$

where  $y_{ei} = \Psi_i(\cdot)$  is the extended state to represent the perturbation.

**Assumption 1.** Jiang et al. (2004); Wu et al. (2004); Chen et al. (2014); Yang et al. (2015)  $b_{i0}$  is chosen to satisfy:

$$\left| \frac{\|\bar{\mathbf{B}}\|}{|b_{i0}|} - 1 \right| \leq \theta_i < 1 \quad (6)$$

where  $\theta_i$  is a positive constant, and  $\|\cdot\|$  is the Euclidean norm.

**Assumption 2.** Jiang et al. (2004); Wu et al. (2004); Chen et al. (2014); Yang et al. (2015) The function  $\Psi_i(\mathbf{y}, \mathbf{z}, u, t) : \mathbb{R}^m \times \mathbb{R}^{n-m} \times \mathbb{R}^m \times \mathbb{R}^+ \rightarrow \mathbb{R}$  and  $\dot{\Psi}_i(\mathbf{y}, \mathbf{z}, u, t) : \mathbb{R}^m \times \mathbb{R}^{n-m} \times \mathbb{R}^m \times \mathbb{R}^+ \rightarrow \mathbb{R}$  are locally Lipschitz in their arguments over the domain of interest and are globally bounded:

$$|\Psi_i(\mathbf{y}, \mathbf{z}, u, t)| \leq \gamma_{i1}, \quad |\dot{\Psi}_i(\mathbf{y}, \mathbf{z}, u, t)| \leq \gamma_{i2} \quad (7)$$

where  $\gamma_{i1}$  and  $\gamma_{i2}$  are positive constants. In addition,  $\Psi_i(0, 0, 0, 0) = 0$  and  $\dot{\Psi}_i(0, 0, 0, 0) = 0$ , such that the origin is an equilibrium point of the open-loop system.

An HGPO Jiang et al. (2004); Wu et al. (2004); Chen et al. (2014); Yang et al. (2015) is designed for extended system (5) as

$$\begin{cases} \dot{\hat{y}}_i = \hat{\Psi}_i(\cdot) + h_{i1}(y_i - \hat{y}_i) + b_{i0}u_i \\ \dot{\hat{\Psi}}_i(\cdot) = h_{i2}(y_i - \hat{y}_i) \end{cases}, \quad i = 1, \dots, m \quad (8)$$

where  $\hat{y}_i$  and  $\hat{\Psi}_i(\cdot)$  are the estimates of  $y_i$  and  $\Psi_i(\cdot)$ . Positive constants  $h_{i1}$  and  $h_{i2}$  are the observer gains.

**Remark 2.** HGPO is used in this paper for its relatively easy design and implementation. Note that other types of observer, i.e., SMPO Kwon et al. (2004) and nonlinear observer Han et al. (2009) can be used for PO design. They can provide almost the same estimation performance.

The control problem is described as follows: For an uncertain nonlinear system (1), design an HGPO (8) to estimate perturbation (4), and find out a robust passive controller  $u$  such that the origin of uncertain nonlinear system (1) is stable.

### 3 Design of POAPC and its Analysis

#### 3.1 Estimation error convergence analysis of HGPO

Define the estimation error as  $\tilde{\mathbf{y}}_{\text{po}_i} = [\tilde{y}_i, \tilde{y}_{ei}]^T$ , where  $\tilde{y}_i = y_i - \hat{y}_i$  and  $\tilde{y}_{ei} = y_{ei} - \hat{y}_{ei}$  symbolize the estimation error of  $y_i$  and  $y_{ei}$ , respectively. System (5) can be written as follows

$$\begin{bmatrix} \dot{y}_i \\ \dot{y}_{ei} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_i \\ y_{ei} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \dot{\Psi}_i(\cdot) + \begin{bmatrix} b_{i0} \\ 0 \end{bmatrix} u_i \quad (9)$$

The HGPO (8) can be written as

$$\begin{bmatrix} \dot{\hat{y}}_i \\ \dot{\hat{y}}_{ei} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{y}_i \\ \hat{y}_{ei} \end{bmatrix} + \begin{bmatrix} h_{i1} \\ h_{i2} \end{bmatrix} \tilde{y}_i + \begin{bmatrix} b_{i0} \\ 0 \end{bmatrix} u_i \quad (10)$$

The estimation error can be obtained by subtracting (10) from (9) as

$$\begin{bmatrix} \dot{\tilde{y}}_i \\ \dot{\tilde{y}}_{ei} \end{bmatrix} = \begin{bmatrix} -h_{i1} & 1 \\ -h_{i2} & 0 \end{bmatrix} \begin{bmatrix} \tilde{y}_i \\ \tilde{y}_{ei} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \dot{\Psi}_i(\cdot) \quad (11)$$

HGPO estimation error (11) can be rewritten in the following matrix form:

$$\dot{\tilde{\mathbf{y}}}_{\text{po}_i} = \mathbf{A}_{\text{po}_i} \tilde{\mathbf{y}}_{\text{po}_i} + \mathbf{B}_{\text{po}_i} \dot{\Psi}_i(\cdot) \quad (12)$$

Which shows that the estimation error  $\tilde{\mathbf{y}}_{\text{po}_i}$  is driven by  $\dot{\Psi}_i(\cdot)$ . Matrices  $\mathbf{A}_{\text{po}_i}$  and  $\mathbf{B}_{\text{po}_i}$  are given as

$$\mathbf{A}_{\text{po}_i} = \begin{bmatrix} -h_{i1} & 1 \\ -h_{i2} & 0 \end{bmatrix}, \mathbf{B}_{\text{po}_i} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (13)$$

where  $\mathbf{A}_{\text{po}_i}$  is a Hurwitz matrix.

As in any asymptotic observer, the observer gain  $\mathbf{H}_{\text{po}_i} = [h_{i1}, h_{i2}]^T$  should be chosen to achieve an asymptotic error convergence, that is

$$\lim_{t \rightarrow \infty} \tilde{\mathbf{y}}_{\text{po}_i}(t) = 0 \quad (14)$$

In the absence of  $\dot{\Psi}_i(\cdot)$ , the estimation error will asymptotically convergence to the origin as  $\mathbf{A}_{\text{po}_i}$  is Hurwitzian for any positive constants  $h_{i1}$  and  $h_{i2}$ .

In the presence of  $\dot{\Psi}_i(\cdot)$ , one needs to determine the observer gain with an additional goal of rejecting the effect of  $\dot{\Psi}_i(\cdot)$  on the estimation error  $\tilde{y}_{\text{po}_i}$ . This can be ideally achieved, for any  $\dot{\Psi}_i(\cdot)$ , if the transfer function from  $\dot{\Psi}_i(\cdot)$  to  $\tilde{y}_{\text{po}_i}$  is identically zero.

Rewrite (11) into the equations form and rearrange the equations, yields:

$$\begin{cases} \tilde{y}_{ei} = (s + h_{i1})\tilde{y}_i \\ \dot{\Psi}_i(\cdot) = s\tilde{y}_{ei} + h_{i2}\tilde{y}_i \end{cases} \quad (15)$$

where  $s$  is the Laplace variable.

By replacing  $\tilde{y}_{ei}$  by  $\tilde{y}_i$  in the second equation of (15), one can have

$$\dot{\Psi}_i(\cdot) = (s^2 + h_{i1}s + h_{i2})\tilde{y}_i \quad (16)$$

Similarly, by replacing  $\tilde{y}_i$  by  $\tilde{y}_{ei}$  in (16), one can obtain

$$\dot{\Psi}_i(\cdot) = \frac{(s^2 + h_{i1}s + h_{i2})}{s + h_{i1}}\tilde{y}_{ei} \quad (17)$$

To this end, one can obtain the transfer function from  $\dot{\Psi}_i(\cdot)$  to  $\tilde{y}_{po_i}$  in the following matrix form:

$$\mathbf{H}_{po_i}(s) = \begin{bmatrix} \frac{\tilde{y}_i}{\dot{\Psi}_i(\cdot)} \\ \frac{\tilde{y}_{ei}}{\dot{\Psi}_i(\cdot)} \end{bmatrix} = \frac{1}{s^2 + h_{i1}s + h_{i2}} \begin{bmatrix} 1 \\ s + h_{i1} \end{bmatrix} \quad (18)$$

By calculating the  $\|\mathbf{H}_{po_i}\|_\infty$ , it can be seen that the norm can be arbitrarily small by choosing  $h_{i2} \gg h_{i1} \gg 1$ . Define  $h_{i1} = \alpha_{i1}/\epsilon_i$  and  $h_{i2} = \alpha_{i2}/\epsilon_i^2$ , where  $\alpha_{i1}$ ,  $\alpha_{i2}$ , and  $\epsilon_i \ll 1$  are some positive constants. It can be shown that

$$\mathbf{H}_{po_i}(s) = \frac{\epsilon_i}{(\epsilon_i s)^2 + \alpha_{i1}\epsilon_i s + \alpha_{i2}} \begin{bmatrix} \epsilon_i \\ \epsilon_i s + \alpha_{i1} \end{bmatrix} \quad (19)$$

Hence,  $\lim_{\epsilon_i \rightarrow 0} \mathbf{H}_{po_i}(s) = 0$ . As an infinite observer gain is impossible in practice, one can determine the observer gain such that the estimation error  $\tilde{\mathbf{y}}_{po_i}$  will exponentially converge to a small neighbourhood which is arbitrarily close to the origin. The result is summarized as the following Proposition 1.

**Proposition 1.** Jiang et al. (2004); Wu et al. (2004) *Consider system (5), and design an HGPO (8). If Assumptions 1-2 hold for some values  $\gamma_{i1}$ ,  $\gamma_{i2}$ , and  $b_{i0}$ . Then given any positive constant  $\delta_{po_i}$ , from the initial estimation error  $\tilde{\mathbf{y}}_{po_i}(0)$ , the observer gain  $\mathbf{H}_{po_i}$  can be chosen such that the estimation error will exponentially converge into the neighbourhood*

$$\|\tilde{\mathbf{y}}_{po_i}(t)\| \leq \delta_{po_i} \quad (20)$$

HGPO (8) is basically an approximate differentiator. This can be readily seen in the special case when the perturbation  $\Psi_i(\cdot)$  and control  $u_i$  are chosen to be zero and thus the observer is linear. The transfer function from  $y_i$  to  $\hat{y}_{po_i}$  for system (8) is given by

$$\frac{\alpha_{i2}}{(\epsilon_i s)^2 + \alpha_{i1}\epsilon_i s + \alpha_{i2}} \begin{bmatrix} 1 + (\epsilon_i \alpha_{i1}/\alpha_{i2})s \\ s \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ s \end{bmatrix} \quad \text{as } \epsilon_i \rightarrow 0 \quad (21)$$

Thus, on a compact frequency interval, HGPO approximates  $\Psi_i(\cdot) = \dot{y}_i$  for a sufficiently small  $\epsilon_i$ .

**Remark 3.** It should be mentioned that during the design procedure,  $\epsilon_i$  is only required to be some relatively small positive constant and the performance of HGPO is not sensitive to the observer gain, as it is determined based on the bound of perturbation estimation error. Moreover, the bound of estimates used in HGPO can be obtained through the analysis of the system within a predetermined range of variation in system variables.

### 3.2 Controller structure and closed-loop system stability analysis

The estimate of perturbation  $\Psi(\cdot)$  is used to realize a robust passivation of the uncertain nonlinear system (1). After the lumped system uncertainties are estimated by HGPO, a robust passive controller called POAPC can be designed for the equivalent linear system as

$$\begin{cases} \mathbf{u} = \mathbf{B}_0^{-1} \left( -\hat{\Psi}(\cdot) - \mathbf{K}\mathbf{y} - \left( \frac{\partial W_0(\mathbf{z})}{\partial \mathbf{z}^T} p(\mathbf{y}, \mathbf{z}) \right)^T + \omega \right) \\ \omega^T = -\phi(\mathbf{y}) \end{cases} \quad (22)$$

where  $\mathbf{K} = \text{diag}[k_1, \dots, k_m]$ , with  $k_i \geq 1$ , is the feedback control gain,  $\omega \in \mathbb{R}^m$  is the additional input, where  $\phi : \mathbb{R}^m \rightarrow \mathbb{R}^m$  is any smooth function such that  $\phi(0) = 0$  and  $\mathbf{y}^T \phi(\mathbf{y}) > 0$  for all  $\mathbf{y} \neq 0$ .

Rewrite (12) into the singularly perturbed form by defining the scaled estimation error  $\eta_i = [\eta_{i1}, \eta_{i2}]^T = [\tilde{y}_i/\epsilon_i, \tilde{\Psi}_i(\cdot)]^T$ , which satisfies

$$\epsilon_i \dot{\eta}_i = \mathbf{A}_{i1} \eta_i + \epsilon_i \mathbf{B}_{i1} \dot{\Psi}_i(\cdot), \quad i = 1, \dots, m \quad (23)$$

with

$$\mathbf{A}_{i1} = \begin{bmatrix} -\alpha_{i1} & 1 \\ -\alpha_{i2} & 0 \end{bmatrix}, \mathbf{B}_{i1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (24)$$

where positive constants  $\alpha_{i1}$  and  $\alpha_{i2}$  are chosen such that  $A_{i1}$  is a Hurwitz matrix.

The closed-loop system dynamics obtained under HGPO (8) and POAPC (22) can be represented by

$$\begin{cases} \dot{\mathbf{y}} = -\mathbf{K}\mathbf{y} + \eta_{i2} - \left( \frac{\partial W_0(\mathbf{z})}{\partial \mathbf{z}^T} p(\mathbf{y}, \mathbf{z}) \right)^T + \omega \\ \dot{\mathbf{z}} = f_0(\mathbf{z}) + p(\mathbf{y}, \mathbf{z})\mathbf{y} \\ \epsilon_i \dot{\eta}_i = \mathbf{A}_{i1} \eta_i + \epsilon_i \mathbf{B}_{i1} \dot{\Psi}_i(\cdot) \end{cases}, \quad i = 1, \dots, m \quad (25)$$

The reduced system, obtained by substituting  $\eta_i = 0$  and  $z = 0$  in system (25), is calculated as

$$\dot{y}_i = -k_i y_i + \omega_i, \quad i = 1, \dots, m \quad (26)$$

The boundary-layer system, obtained by applying the change  $\tau_i = t/\epsilon_i$  to system (25) and setting  $\epsilon_i = 0$ , is given by

$$\frac{d\eta_i}{d\tau_i} = \mathbf{A}_{i1} \eta_i, \quad i = 1, \dots, m \quad (27)$$

**Theorem 1.** *Let Assumptions 1-2 hold, design HGPO (8) and POAPC (22); then there exists  $\epsilon_{i1}^*$ , with  $i = 1, \dots, m$ ,  $\epsilon_{i1}^* > 0$  such that,  $\forall \epsilon_i, 0 < \epsilon_i < \epsilon_{i1}^*$ , the closed-loop system (25) is passive and its origin is stable.*

**Proof.** For the proof of Theorem 1, the following corollary which is a special



case of Young's inequality will be used

**Corollary 1.**  $\forall a, b \in \mathbb{R}^+, \forall p > 1, \epsilon_0 > 0$  it has

$$ab \leq \frac{1}{\epsilon_0} a^p + (\epsilon_0)^{1/(p-1)} b^{p/(p-1)} \quad (28)$$

For the reduced system (26), define the Lyapunov function  $V(y_i) = (1/2)y_i^2$  over a ball  $B(0, r_i)$ , for some  $r_i > 0$ .  $\forall y_i \in B(0, r_i)$ , one can have

$$\dot{V}(y_i) = -k_i y_i^2 - y_i \phi(y_i) \leq -y_i^2 \quad (29)$$

Thus the origin of reduced system (26) is stable in a region  $\mathcal{R}_i$  which includes the origin.

The boundary-layer system (27) is exponentially stable in a region  $\Omega_i$  which includes the origin as  $\mathbf{A}_{i1}$  is Hurwitzian. Define the Lyapunov function  $W(\eta_i) = \eta_i^T \mathbf{P}_{i1} \eta_i$ , where  $\mathbf{P}_{i1}$  is the positive definite solution of the Lyapunov equation  $\mathbf{P}_{i1} \mathbf{A}_{i1} + \mathbf{A}_{i1}^T \mathbf{P}_{i1} = -I$ . This function satisfies

$$\lambda_{\min}(\mathbf{P}_{i1}) \|\eta_i\|^2 \leq W(\eta_i) \leq \lambda_{\max}(\mathbf{P}_{i1}) \|\eta_i\|^2 \quad (30)$$

$$\frac{\partial W(\eta_i)}{\partial \eta_i} \mathbf{A}_{i1} \eta_i \leq -\|\eta_i\|^2 \quad (31)$$

$$\left\| \frac{\partial W(\eta_i)}{\partial \eta_i} \right\| \leq 2\lambda_{\max}(\mathbf{P}_{i1}) \|\eta_i\| \quad (32)$$

Consider a storage function as follows

$$H(\mathbf{y}, \eta, \mathbf{z}) = \frac{1}{2} \mathbf{y}^T \mathbf{y} + \sum_{i=1}^m \beta_i W(\eta_i) + W_0(\mathbf{z}) \quad (33)$$

where  $\beta_i > 0$  is to be determined. Choose  $\xi_i < r_i$ , given Assumptions 1-2, for all  $(y_i, \eta_i) \in B(0, \xi_i) \times \{\|\eta_i\| \leq \xi_i\} = \Lambda_i$ , where  $B(\|\eta_i\|, \xi_i) \in \Omega_i$ ,  $\xi_i$  is a positive constant. Moreover, assume

$$|\dot{\Psi}_i(\cdot)| \leq L_{i1} \|y_i\| + L_{i2} \|\eta_i\| \quad (34)$$

where  $L_{i1}$  and  $L_{i2}$  are Lipschitz constants.

Differentiate  $H(\mathbf{y}, \eta, \mathbf{z})$  along system (25), using inequalities (30)~(34) and

Corollary 1, with  $p = 2$ ,  $a = \|y_i\|$  and  $b = \|\eta_i\|$ , it yields:

$$\begin{aligned}
\dot{H} &= \mathbf{y}^T \left( -\mathbf{K}\mathbf{y} + \eta_{i2} - \left( \frac{\partial W_0(\mathbf{z})}{\partial \mathbf{z}^T} p(\mathbf{y}, \mathbf{z}) \right)^T + \omega \right) + \sum_{i=1}^m \beta_i \frac{\partial W(\eta_i)}{\partial \eta_i} \left( \frac{\mathbf{A}_{i1} \eta_i}{\epsilon_i} + \mathbf{B}_{i1} \hat{\Psi}_i(\cdot) \right) \\
&+ \frac{\partial W_0(\mathbf{z})}{\partial \mathbf{z}^T} (f_0(\mathbf{z}) + p(\mathbf{y}, \mathbf{z})\mathbf{y}) \\
&\leq \sum_{i=1}^m \left( -k_i \|y_i\|^2 - \frac{\beta_i}{\epsilon_i} \|\eta_i\|^2 + 2\beta_i L_{i2} \|\mathbf{P}_{i1}\| \|\eta_i\|^2 + (1 + 2\beta_i L_{i1} \|\mathbf{P}_{i1}\|) \|y_i\| \|\eta_i\| \right) \\
&+ \frac{\partial W_0(\mathbf{z})}{\partial \mathbf{z}^T} f_0(\mathbf{z}) + \omega^T \mathbf{y} \\
&\leq \sum_{i=1}^m \left( -\|y_i\|^2 - \frac{\beta_i}{\epsilon_i} \|\eta_i\|^2 + 2\beta_i L_{i2} \|\mathbf{P}_{i1}\| \|\eta_i\|^2 + (1 + 2\beta_i L_{i1} \|\mathbf{P}_{i1}\|) \right. \\
&\times \left. \left( \frac{1}{\epsilon_{i0}} \|y_i\|^2 + \epsilon_{i0} \|\eta_i\|^2 \right) \right) + \omega^T \mathbf{y} \\
&\leq \sum_{i=1}^m \left( -\frac{1}{2} \|y_i\|^2 - \frac{\beta_i}{2\epsilon_i} \|\eta_i\|^2 - b_{i1} \|y_i\|^2 - b_{i2} \|\eta_i\|^2 \right) + \omega^T \mathbf{y}
\end{aligned}$$

where

$$b_{i1} = \frac{1}{2} - \frac{2}{\epsilon_{i0}} \left( \frac{1}{2} + \beta_i L_{i1} \|\mathbf{P}_{i1}\| \right), \quad b_{i2} = \frac{\beta_i}{2\epsilon_i} - 2\beta_i (\epsilon_{i0} L_{i1} + L_{i2}) \|\mathbf{P}_{i1}\| - \epsilon_{i0} \quad (35)$$

with  $\epsilon_{i0} > 0$ .

Now choose  $\beta_i$  small enough and  $\epsilon_{i0} \geq \epsilon_{i0}^* = 2 + 4\beta_i L_{i1} \|\mathbf{P}_{i1}\|$  such that  $b_{i1} > 0$ , and then choose  $\epsilon_{i1}^* = \beta_i / (\epsilon_{i0}^{*2} + 4\beta_i L_{i2} \|\mathbf{P}_{i1}\|)$ ,  $\forall \epsilon_i, \epsilon_i \leq \epsilon_{i1}^*$ , and choose an additional input  $\omega^T = -\phi(y)$ , where  $\phi(y)$  is a sector-nonlinearity satisfying  $\phi(y)y > 0$  for  $y \neq 0$  and  $\phi(0) = 0$ . It can be shown that

$$\dot{H} \leq \omega^T \mathbf{y} - \sum_{i=1}^m (\min(1/2, \beta_i / (2\epsilon_i)) (\|y_i\|^2 + \|\eta_i\|^2)) \leq \omega^T \mathbf{y} \leq -\phi(y)y \leq 0 \quad (36)$$

Therefore, one can conclude that the closed-loop system (25) is passive and its origin is stable. □

POAPC (22) does not require an accurate system model which structure is illustrated in Fig. 1. The nominal system is disturbed by the perturbation, POAPC can be decomposed as  $\mathbf{u} = \mathbf{B}_0^{-1}(\mathbf{u}_1 + \mathbf{u}_2 + \mathbf{u}_3 + \mathbf{u}_4)$ , where  $\mathbf{u}_1 = -\hat{\Psi}(\cdot)$  is the distinctive adaption mechanism based on online functional estimation of HGPO, which can provide significant robustness for lumped system uncertainties;  $\mathbf{u}_2 = -\mathbf{K}\mathbf{y}$  is the output feedback;  $\mathbf{u}_3 = \left( \frac{\partial W_0(\mathbf{z})}{\partial \mathbf{z}^T} p(\mathbf{y}, \mathbf{z}) \right)^T$  compensates the internal dynamics and reshapes the changes of storage function; and  $\mathbf{u}_4 = \omega$  helps to construct a passive system by introducing an additional input with the help of a sector-nonlinearity  $\phi(\mathbf{y})$ . Once the controller is set up for the problem within a predetermined range of variation in system variables, no tuning is

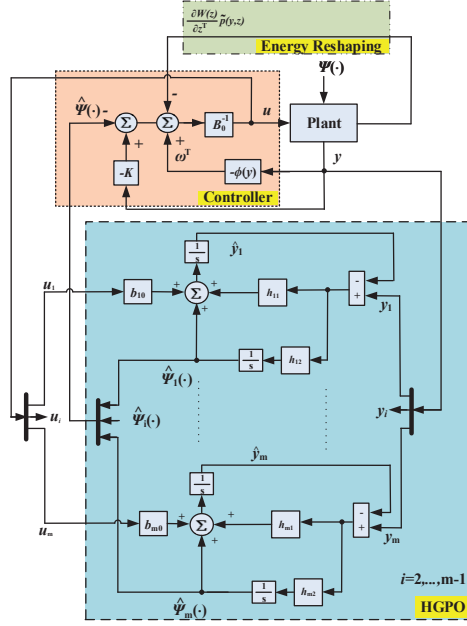


Figure 1: Control structure of POAPC

needed for start-up or compensation for changes in the system dynamics and fast time-varying external disturbance.

**Remark 4.** Compared to the conventional APC Zhou et al. (2012); Mermoud et al. (2001); Bobtsov et al. (2014) which can only estimate the linearly parametric uncertainties, POAPC can be regarded as a nonlinearly functional estimation method, as it can estimate the combinatorial effect of unknown parameters, unmodelled dynamics and fast time-varying external disturbances. If there does not exist uncertainties and fast time-varying external disturbances, and if the accurate system model is available, it can provide the same performance as that of the exact passive controller. Otherwise, it will perform much better than the exact passive controller. The use of HGPO leads to less concern over the measurement and identification of the unknown parameters, unmodelled dynamics and fast time-varying external disturbances. This tends to require less control efforts as the perturbation has already included all of this information. It can also avoid the issues of controller parameter initialization, learning coefficient optimization, and slow system response to the parameter variation existed in the conventional APC.

**Remark 5.** The original high-gain observer (HGO) gain must be chosen to be large enough to suppress the upper bound of the perturbation  $|\Psi_i(\cdot)|^{\text{sup}}$ , which will produce an undesirable peaking phenomenon Ahrens et al. (2009). In contrast, the proposed method introduces the perturbation estimation and

compensation, such that the HGO gain only needs to be chosen to suppress the perturbation estimation error  $|\tilde{\Psi}_i(\cdot)|$ . As  $|\Psi_i(\cdot)|^{\text{sup}}$  is always larger than  $|\tilde{\Psi}_i(\cdot)|$ , a relatively small gain value is needed thus the malignant effect of the peaking phenomenon can be partially reduced Jiang et al. (2004); Wu et al. (2004); Chen et al. (2014).

To this end, the overall design procedure of POAPC can be summarized as follows:

- Step 1: Define perturbation (4) for the original system (1);
- Step 2: Define a fictitious state  $y_{ei} = \Psi_i(\cdot)$  to represent perturbation (4);
- Step 3: Extend the original system (1) into the extended system (5);
- Step 4: Design HGPO (8) for the extended system (5) to obtain the output estimate  $\hat{y}_i$  and the perturbation estimate  $\hat{\Psi}_i(\cdot)$  by only the measurement of  $y_i$ ;
- Step 5: Design controller (22) for the original system (1).

## 4 Case Studies

This section will illustrate various features of the proposed POAPC and show its advantages over the existing results. In the first example, the control performance of POAPC is compared with that of APC for a magnetic levitator system. In the second example, POAPC is applied in an interconnected inverted pendulum system to verify its robustness against that of PC in the presence of system uncertainties.

### 4.1 Magnetic levitator system

Consider a single-input single-output (SISO) magnetic levitator system used in Mermoud et al. (2001), which is illustrated in Fig. 2. The levitation object is a ping-pong ball with a small permanent magnetic attached to it, by which an attraction force could be induced. The attraction force is generated by the electromagnet and controlled by an amplifier circuit. The height of the ball is determined by a photo emitter-detector. The system dynamics are presented in the following second-order differential equation

$$m\ddot{d} = F_c - mg + \zeta \quad (37)$$

where  $m$  is the mass of the ball,  $d$  is the distance of the ball from the reference line,  $g$  is the standard gravity and  $F_c$  is the magnetic control force,  $\zeta$  includes the system uncertainty and unmodelled dynamics.

The magnetic force characteristics is described as a function of voltage applied to the amplifier circuit and height of the ball, leading to the expression

$$F_c = V_c \hat{b}(d) = V_c \frac{1}{a_1 d^2 + a_2 d + a_3} \quad (38)$$

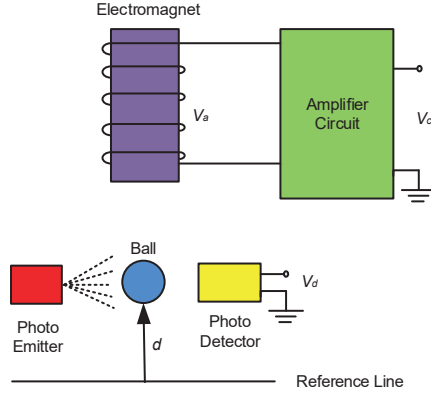


Figure 2: Schematic diagram of the magnetic levitator system.

Table 1: System parameters used in the magnetic levitator system

The mass of the ball	$m = 2.206 \text{ g}$
Real constant coefficients	$a_1 = 0.0231 \text{ mg}, a_2 = -2.4455 \text{ mg}, a_3 = 64.58 \text{ mg}$
The standard gravity	$g = 9.8 \text{ m/s}^2$
Nominal command voltage	$V_{c0} = 4.87 \text{ V}$

where  $V_c$  is the input control voltage applied to the amplifier, and  $\hat{b}(d) = 1/(a_1 d^2 + a_2 d + a_3)$ .  $a_1$ ,  $a_2$  and  $a_3$  are real constant coefficients.

The equilibrium points  $d_0$  and  $\dot{d}_0$  of system (37) without disturbances for a given nominal command voltage  $V_{c0}$  are obtained as

$$d_0 = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_1(a_3 - V_{c0}/mg)}}{2a_1}, \quad \dot{d}_0 = 0 \quad (39)$$

Define the output  $y = \dot{d} - \dot{d}_0$  and internal dynamics  $z = d - d_0$ , together with the control input  $u = V_c - V_{c0}$ . Define the system perturbation as follows

$$\Psi(\cdot) = \frac{\hat{b}(z)V_{c0}}{m} - g + \left(\frac{\hat{b}(z)}{m} - b_0\right)u + \zeta \quad (40)$$

System (37) can be obtained as

$$\begin{cases} \dot{y} = \Psi(\cdot) + b_0 u \\ \dot{z} = f_0(z) + p(y, z)y \end{cases} \quad (41)$$

where

$$b_0 = \frac{9}{10ma_3}, \quad f_0(z) = 0, \quad p(y, z) = 1. \quad (42)$$

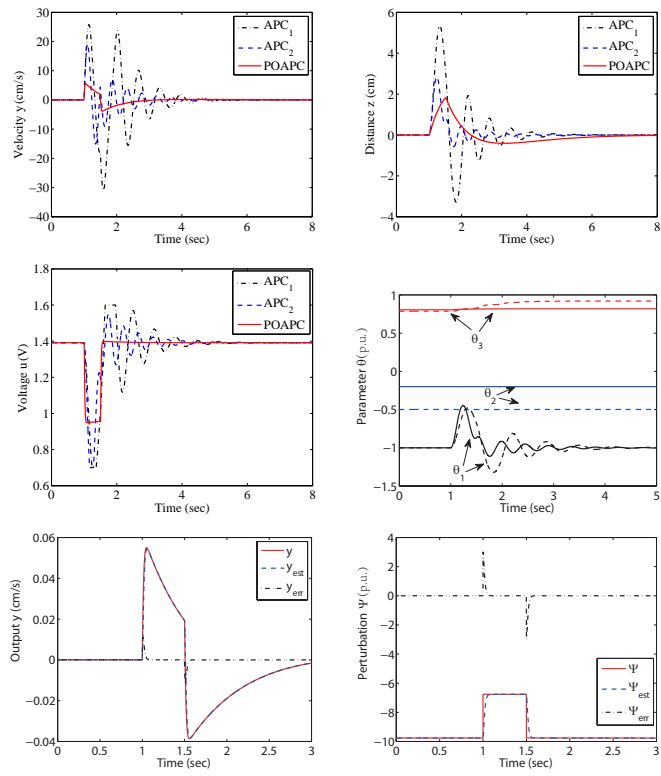


Figure 3: System responses obtained under the nominal model.

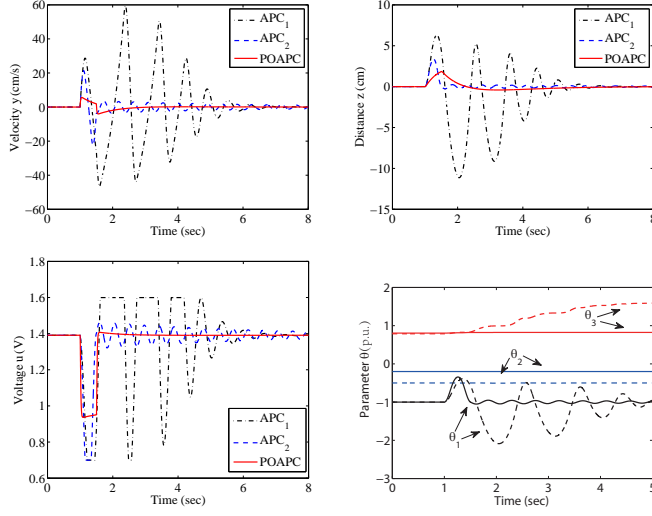


Figure 4: System responses obtained under the unmodelled dynamics  $\zeta = 1.5 \sin(y)$ .

An HGPO is designed to estimate the perturbation by only the measurement of  $y$  as

$$\begin{cases} \dot{\hat{y}} = \hat{\Psi}(\cdot) + h_1(y - \hat{y}) + b_0 u \\ \dot{\hat{\Psi}}(\cdot) = h_2(y - \hat{y}) \end{cases} \quad (43)$$

where  $h_1$  and  $h_2$  are the observer gains. During the most severe disturbance, the voltage of the amplifier circuit will reduce from its initial value to around zero within a short period of time,  $\Delta$ . Thus the boundary values of the system perturbation can be obtained as

$$|\hat{\Psi}(\cdot)| \leq \left( \frac{\hat{b}(z)V_{c0}}{m} - g + \max |\zeta| \right) \quad (44)$$

$$|\dot{\hat{\Psi}}(\cdot)| \leq \left( \frac{\hat{b}(z)V_{c0}}{m} - g + \max |\zeta| \right) / \Delta \quad (45)$$

The POAPC is designed as

$$\begin{cases} u = b_0^{-1}(-\hat{\Psi}(\cdot) - Ky - z + \omega) \\ \omega^T = -Dy \end{cases} \quad (46)$$

where a linear damping function  $\phi(y) = Dy$ ,  $D > 0$ , is chosen to construct a passive system. The zero dynamics of system are described by  $\dot{z} = 0$  which is Lyapunov stable. Choose  $H(y, \eta, z) = (1/2)y^2 + \beta W(\eta) + W_0(z)$  with  $W_0(z) = (1/2)z^2$ , it is easy to find that the output  $y$  and state  $z$  are locally zero-state detectable.

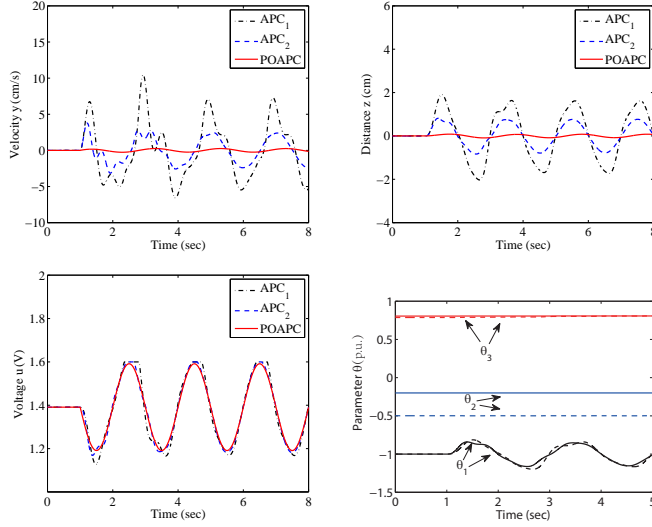


Figure 5: System responses obtained under the fast time-varying external disturbance  $\zeta = 1.4 \sin(\pi t)$ .

The conventional APC used in Mermoud et al. (2001) is

$$\begin{cases} u = (m/\hat{b}(d))\Theta^T \mathbf{u}_p \\ \dot{\Theta} = -\Lambda \mathbf{y} \mathbf{u}_p \end{cases} \quad (47)$$

where  $\Theta = [\theta_1, \theta_2, \theta_3]^T$  and  $\mathbf{u}_p = [\omega_1, z, \omega_2]^T$ ,  $\omega_1 = \hat{b}(d)V_{c0}/m - g$ ,  $\omega_2 = -\beta y$  with  $\beta = 2.9$ ,  $\Lambda = \text{diag}[\lambda_1, \lambda_2, \lambda_3]$  is the learning coefficient. Two sets of controller parameters are used, i.e., APC<sub>1</sub>:  $\Lambda_1 = [-1, -1, -1]^T$ ,  $\Theta_1(0) = [-1, -0.5, 0.8]^T$  (dashed line) and APC<sub>2</sub>:  $\Lambda_2 = [-2, -1.5, -0.5]^T$ ,  $\Theta_2(0) = [-1, -0.2, 0.8]^T$  (solid line). HGPO (43) parameters are  $h_1 = 300$ ,  $h_2 = 22500$ , and  $\Delta = 0.05$  s. Controller (46) parameters are  $K = 1.5$  and  $D = 0.5$ , while control input is bounded as  $0 \leq u \leq 1.6$  V. The values of system parameters are given in Table 1.

Fig. 3 shows the system responses obtained when an equivalent 3 N force applied at  $t = 1$  s and removed at  $t = 1.5$  s under the nominal model. One can find that APC consumes longer time for control parameter learning and requires larger control efforts. Moreover, its control performance is sensitive to controller parameters which can only be optimized locally at a particular operation point. In contrast, POAPC can effectively stabilize the disturbed system with less control costs as the system nonlinearity is globally removed. Note that a fast tracking of the state and perturbation can be obtained by HGPO, the relatively large estimation error at the instant of  $t = 1$  s and  $t = 1.5$  s is due to the discontinuity when the magnetic control force  $F_c$  is applied and removed.

System responses obtained under an unmodelled dynamics  $\zeta = 1.5 \sin(y)$  are given by Fig. 4. It shows that POAPC can maintain a satisfactory control per-



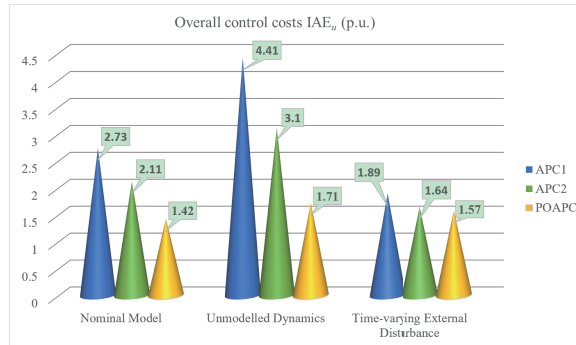


Figure 6: The overall control efforts obtained in the magnetic levitator system.

formance as the unmodelled dynamics is estimated and compensated online. In contrast, APC control performance degrades dramatically with a destabilizing effect as the system uncertainty is not in the assumed linearly parametric form. Fig. 5 demonstrates the system responses obtained when a fast time-varying external disturbance  $\zeta = 1.4 \sin(\pi t)$  occurs, in which POAPC can rapidly attenuate the disturbance thus a significant robustness can be resulted in. However, APC cannot find the optimal control parameters which causes an unsuppressed time-varying oscillation.

Table 2: IAE indices (in p.u.) obtained in the magnetic levitator system

Method \ Case	Nominal Model		Unmodelled Dynamics		Time-varying External Disturbance	
	IAE <sub>y</sub>	IAE <sub>z</sub>	IAE <sub>y</sub>	IAE <sub>z</sub>	IAE <sub>y</sub>	IAE <sub>z</sub>
APC <sub>1</sub>	34.76	5.27	48.76	9.17	26.24	10.57
APC <sub>2</sub>	18.52	3.14	24.97	4.56	18.98	4.73
POAPC	8.91	1.62	9.50	1.84	6.95	0.88

The integral of absolute error (IAE) indices of each approach calculated in different cases are tabulated in Table 2. Here  $IAE_x = \int_0^T |x - x^*| dt$  and  $x^*$  is the reference value of variable  $x$ . The simulation time  $T=8$  s. IAE<sub>y</sub> and IAE<sub>z</sub> are only 19.48% and 20.07% of those of APC<sub>1</sub> control, 38.05% and 30.35% of those of APC<sub>2</sub> with the unmodelled dynamics. Finally, the overall control efforts of different approaches are presented in Fig. 6, one can find POAPC needs less control efforts to that of APC but provides greater robustness.

## 4.2 Interconnected inverted pendulum system

Consider a multi-input multi-output (MIMO) system composed of two inverted pendulums on two carts, interconnected by a moving spring Da (2000), which is shown in Fig. 7. Assume that the pivot position of the moving spring is a

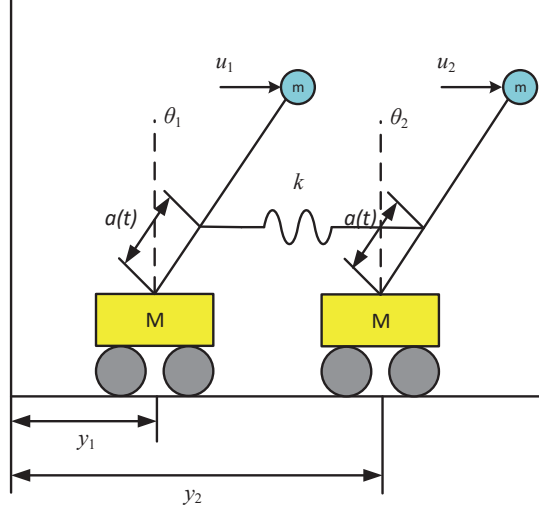


Figure 7: Schematic diagram of the interconnected inverted pendulum system.

function of time and it can change along the full length of the pendulums. In this example the motion of the carts is specified as a sinusoidal trajectory. The input to each pendulum is the torque  $u_i$ ,  $i = 1, 2$ , applied to the pivot point. It is desired to control each pendulum with mass independently so that each pendulum tracks its own trajectory reference while the connected spring and carts are moving. The system dynamics can be described as

$$\begin{cases} \dot{\theta}_1 = \omega_1 \\ \dot{\omega}_1 = \left( \frac{g}{cl} - \frac{ka(t)(a(t)-cl)}{cml^2} \right) \theta_1 + \frac{ka(t)(a(t)-cl)}{cml^2} \theta_2 - \beta_1 \omega_1^2 - \frac{k(a(t)-cl)}{cml^2} (s_1(t) - s_2(t)) \\ \quad + \frac{u_1}{cml^2} + \zeta_1 \\ \dot{\theta}_2 = \omega_2 \\ \dot{\omega}_2 = \left( \frac{g}{cl} - \frac{ka(t)(a(t)-cl)}{cml^2} \right) \theta_2 + \frac{ka(t)(a(t)-cl)}{cml^2} \theta_1 - \beta_2 \omega_2^2 - \frac{k(a(t)-cl)}{cml^2} (s_2(t) - s_1(t)) \\ \quad + \frac{u_2}{cml^2} + \zeta_2 \end{cases} \quad (48)$$

where  $\beta_1 = (m/M) \sin \theta_1$ ,  $\beta_2 = (m/M) \sin \theta_2$ ,  $c = m/(M + m)$ ,  $s_1(t) = \sin(2t)$ ,  $s_2(t) = L + \sin(3t)$ , and  $a = \sin(5t)$ . While the input to each pendulum is the torque  $|u_i| \leq 50$ ,  $i = 1, 2$ .  $\zeta_1$  and  $\zeta_2$  are the unmodelled dynamics. The definition of the symbols can be referred to Da (2000), and the values of system parameter are provided in Table 3.

Define the output  $\mathbf{y} = [\omega_1, \omega_2]^T$  and internal dynamics  $\mathbf{z} = [\theta_1, \theta_2]^T$ , together with the control input  $\mathbf{u} = [u_1, u_2]^T$ . The desired trajectories are chosen as  $\theta_{r1} = \sin(2t)$  and  $\theta_{r2} = \sin(t)$ . System initial states are  $\mathbf{z}(0) = [1, -1]^T$  and  $\mathbf{y}(0) = [0, 0]^T$ , respectively.

Table 3: System parameters used in the interconnected inverted pendulum system

The length of the pendulum	$l=1$	The spring constant	$k=1$
The gravity constant	$g=1$	The natural length of the spring	$L=2$
The mass of pendulum	$m=10$	The mass of cart	$M=10$

Define the system perturbation as follows

$$\Psi(\cdot) = \begin{bmatrix} \left( \frac{g}{cl} - \frac{ka(t)(a(t)-cl)}{cml^2} \right) z_1 + \frac{ka(t)(a(t)-cl)}{cml^2} z_2 - \beta_1 y_1^2 - \frac{k(a(t)-cl)}{cml^2} (s_1(t) - s_2(t)) + \zeta_1 \\ \left( \frac{g}{cl} - \frac{ka(t)(a(t)-cl)}{cml^2} \right) z_2 + \frac{ka(t)(a(t)-cl)}{cml^2} z_1 - \beta_2 y_2^2 - \frac{k(a(t)-cl)}{cml^2} (s_2(t) - s_1(t)) + \zeta_2 \end{bmatrix} \quad (49)$$

System (48) can be rewritten as

$$\begin{cases} \dot{\mathbf{y}} = \Psi(\cdot) + \mathbf{B}_0 \mathbf{u} \\ \dot{\mathbf{z}} = f_0(\mathbf{z}) + p(\mathbf{y}, \mathbf{z}) \mathbf{y} \end{cases} \quad (50)$$

where

$$\mathbf{B}_0 = \begin{bmatrix} b_{10} & 0 \\ 0 & b_{20} \end{bmatrix}, \quad f_0(\mathbf{z}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad p(\mathbf{y}, \mathbf{z}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (51)$$

During the most severe disturbance, both the angle and angular speed of the pendulum will reduce from their initial values to around zero within a short period of time,  $\Delta$ . Thus the boundary values of the system perturbation can be obtained as

$$\begin{aligned} |\hat{\Psi}_1(\cdot)| &= |(2 - 0.2 \sin(5t)(\sin(5t) - 0.5))z_1 + 0.2 \sin(5t)(\sin(5t) - 0.5)z_2 \\ &\quad - \sin(z_1)y_1^2 - 0.2(\sin(5t) - 0.5)(\sin(2t) - \sin(3t) - 2) + \zeta_1| \\ &\leq 2.1|z_1| + 0.3|z_2| + y_1^2 + 1.2 + \max |\zeta_1| \end{aligned} \quad (52)$$

$$|\hat{\dot{\Psi}}_1(\cdot)| \leq (2.1|z_1| + 0.3|z_2| + y_1^2 + 1.2 + \max |\zeta_1|)/\Delta \quad (53)$$

$$\begin{aligned} |\hat{\Psi}_2(\cdot)| &= |(2 - 0.2 \sin(5t)(\sin(5t) - 0.5))z_2 + 0.2 \sin(5t)(\sin(5t) - 0.5)z_1 \\ &\quad - \sin(z_2)y_2^2 - 0.2(\sin(5t) - 0.5)(2 + \sin(3t) - \sin(2t)) + \zeta_2| \\ &\leq 2.1|z_2| + 0.3|z_1| + y_2^2 + 1.2 + \max |\zeta_2| \end{aligned} \quad (54)$$

$$|\hat{\dot{\Psi}}_2(\cdot)| \leq (2.1|z_2| + 0.3|z_1| + y_2^2 + 1.2 + \max |\zeta_2|)/\Delta \quad (55)$$

Two identical HGPOs are designed to estimate the perturbation by only the measurement of  $y_i$ , where  $i = 1, 2$ , as follows

$$\begin{cases} \dot{\hat{y}}_i = \hat{\Psi}_i(\cdot) + h_{i1}(y_i - \hat{y}_i) + b_{i0}u_i \\ \dot{\hat{\Psi}}_i(\cdot) = h_{i2}(y_i - \hat{y}_i) \end{cases} \quad (56)$$

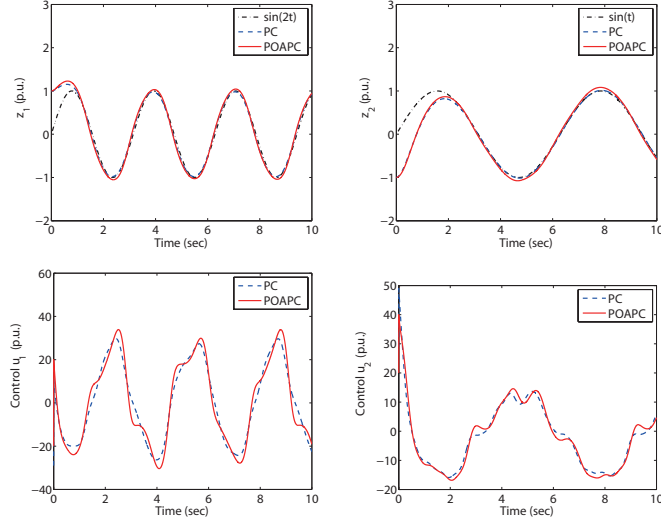


Figure 8: System responses obtained under the nominal model.

The POAPC is designed as

$$\begin{cases} \mathbf{u} = \mathbf{B}_0^{-1}(-\hat{\Psi}(\cdot) - \mathbf{K}\mathbf{y} - \mathbf{z} + \omega) \\ \omega^T = -\mathbf{D}\mathbf{y} \end{cases} \quad (57)$$

Two HGPOs (56) parameters are chosen as  $h_{i1} = 200$ ,  $h_{i2} = 10000$ ,  $b_{i0} = \frac{1}{cm/l^2} = 0.2$ . Controller (57) parameters are chosen as follows

$$\mathbf{K} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad (58)$$

Note that the above choice of the parameter values is only an example to study the control performance of POAPC, in fact, one can choose other values of them and it will only result in a different system damping.

In order to verify the effectiveness of POAPC, PC is used which assumes the accurate system model is completely known for comparison. Fig. 8 gives the tracking performance of PC and POAPC obtained under the nominal model, one can find that POAPC can achieve as satisfactory tracking performances as that of PC, their tiny differences are resulted from the estimation error. Fig. 9 demonstrates that POAPC can perform much better in the presence of the unmodelled dynamics  $\zeta_1 = 5 \sin(\omega_1)$  and  $\zeta_2 = 5 \cos(\omega_2)$ , as the real-time estimate of the system perturbation is used to compensate the actual system perturbation. Hence, it possesses a high adaptation capability and robustness in the presence of unknown dynamics with only one state measurement.

The IAE indices of each approach calculated in different cases are tabulated in Table 4, of which the simulation time  $T=10$  s. In particular, its  $IAE_{z_1}$  and

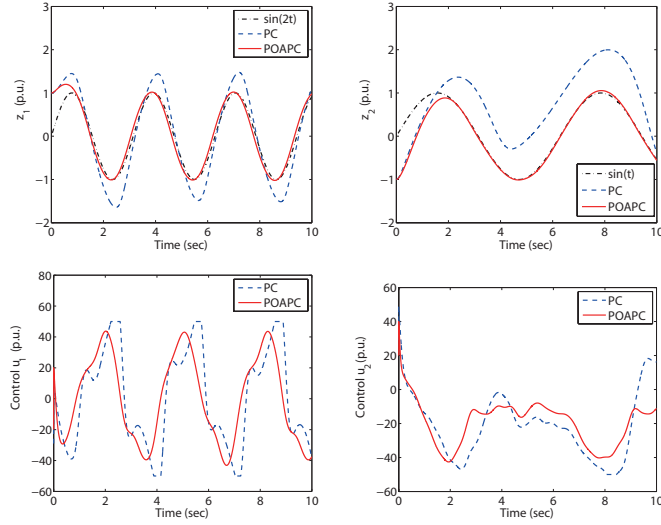


Figure 9: System responses obtained under the unmodelled dynamics  $\zeta_1 = 5 \sin(\omega_1)$  and  $\zeta_2 = 5 \cos(\omega_2)$ .

Table 4: IAE indices (in p.u.) obtained in the interconnected inverted pendulum system

Method \ Case	Nominal Model		Unmodelled Dynamics	
	IAE $_{z_1}$	IAE $_{z_2}$	IAE $_{z_1}$	IAE $_{z_2}$
PC	0.97	0.86	4.47	4.77
POAPC	1.01	0.89	1.03	0.92

IAE $_{z_2}$  are only 23.04% and 19.29% of those of PC with unmodelled dynamics. At last, the overall control efforts of different approaches are presented in Fig. 10, which shows POAPC needs similar control efforts to that of PC but provides greater robustness.

## 5 Conclusion

A nonlinearly functional estimation based POAPC approach is developed in this paper, which employs HGPO to make PC applicable in practice. It can effectively resolve the weakness of conventional APC or RPC when dealing with unmodelled dynamics and various system uncertainties. A proposition is given at first, which shows that for any given constant  $\delta_{p_{o_i}}$ , from the initial estimation error  $\tilde{y}_{p_{o_i}}(0)$ , the estimation error will exponentially converge into the neighbourhood  $\|\tilde{y}_{p_{o_i}}(t)\| \leq \delta_{p_{o_i}}$ . Then a theorem has been proved by the Lyapunov

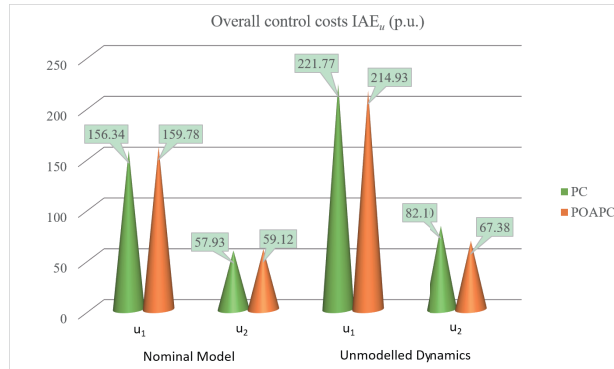


Figure 10: The overall control efforts obtained in the interconnected inverted pendulum system.

criterion, which shows that there exists  $\epsilon_{i1}^*$ , with  $i = 1, \dots, m$ ,  $\epsilon_{i1}^* > 0$  such that,  $\forall \epsilon_i, 0 < \epsilon_i < \epsilon_{i1}^*$ , the closed-loop system is passive and its origin is stable. Two practical examples have been studied, the simulation results verify that POAPC can provide significant robustness without the requirement of further assumptions/conditions made on system, and also achieve as satisfactory control performance as that of PC when an accurate system is available.

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