

# Iterative Semi-Blind CFO Estimation, SI Cancellation and Signal Detection for Full-Duplex Systems

Yujie Liu<sup>\*†</sup>, Xu Zhu<sup>\*</sup>, Eng Gee Lim<sup>†</sup>, Yufei Jiang<sup>‡</sup> and Yi Huang<sup>\*</sup>

<sup>\*</sup>Department of Electrical Engineering and Electronics, University of Liverpool, Liverpool, UK  
Email: {yujieliu, xuzhu, yufei.jiang, yi.huang}@liverpool.ac.uk

<sup>†</sup>Department of Electrical and Electronic Engineering, Xi'an Jiaotong-Liverpool University, Suzhou, P. R. China  
Email: enggee.lim@xjtlu.edu.cn

<sup>‡</sup>School of Electronic and Information Engineering, Harbin Institute of Technology, Shenzhen, P. R. China

**Abstract**—We propose an iterative semi-blind carrier frequency offset (CFO) estimation, self-interference (SI) cancellation and signal detection scheme for full-duplex (FD) orthogonal frequency division multiplexing (OFDM) systems. To the best of our knowledge, this is the first work to consider signal detection of FD systems in the presence of both CFO and SI. The CFO estimation, SI cancellation and signal detection are performed initially by a subspace based semi-blind method, which are then enhanced significantly by performing iterations among them. Its CFO compensation is performed on the desired signal estimate, avoiding the introduction of CFO to the SI. The pilots for the desired signal and SI are carefully designed to enable simultaneous transmission of them to achieve FD training mode. Simulation results show that, the proposed iterative scheme, with much lower training overhead, demonstrates a significant performance enhancement over the existing methods. By utilizing the second order statistics of the received signal, a much superior bit error rate (BER) performance can be achieved compared to the case with perfect SI cancellation and CFO compensation. Its output signal-to-interference-and-noise-ratio (SINR) is close to that with perfect SI cancellation, and robust against the input signal-to-interference ratio (SIR).

## I. INTRODUCTION

Full-duplex (FD) transmission, allowing the simultaneous transmission and reception over the same time and frequency band, can double the transmission rate [1]–[8]. Orthogonal frequency division multiplexing (OFDM) is a key modulation scheme for various communications standards, due to its high spectrum efficiency. The combination of FD and OFDM will be a potential technology in next generation communication systems. However, the FD system performance is heavily limited by the strong self-interference (SI) from its transmitter to its receiver, whereas the OFDM system is highly sensitive to the carrier frequency offset (CFO) incurred by the mismatch between local oscillators at the transmitter and receiver or a Doppler frequency shift.

In FD OFDM systems, there are mainly three techniques to cancel the SI for detection of the desired signal, namely passive cancellation [1], [2], analog cancellation [1], [3] and digital cancellation [1], [4]–[8]. In this paper, we mainly focus on digital cancellation, which estimates SI channel, creates a

replica of the received SI and then cancels it from the received signal. A least square (LS) based SI channel estimator was proposed in [1], however, which treats the desired signal as additive noise, degrading the system performance. A two-stage LS (TS-LS) cancellation scheme was presented in [4], which iterates between SI cancellation and signal detection. However, its performance requires a good initial estimate of the desired channel. A maximum-likelihood (ML) channel estimator was described for both SI and desired channels estimation in [5], by utilizing the known SI, the pilots and unknown data symbols of the desired signal. Nevertheless, it has high training overhead, due to the consecutive pilot transmission for a long period. Koohian proposed a superimposed signaling technique to cancel the SI and detect the desired signal without requiring the procedure of unknown channels estimation [6]. However, its SI channel is assumed to be flat fading only. Meanwhile, all the aforementioned approaches [1], [4]–[6] have not considered the estimation of CFO between the SI and desired signal.

CFO estimation and compensation are well-researched problems in half-duplex (HD) OFDM systems [9]–[11]. However, it is not straightforward to apply them to the FD OFDM systems, since the compensation of CFO based on the desired signal would introduce a CFO to the SI. Only a few works in the literature have investigated the CFO in FD systems. [7] developed a novel receiver architecture for FD systems. Nevertheless, the pilots of the desired signal and SI are non-overlapping, i.e., sent in different time slots, resulting in reduced spectral efficiency. [8] proposed a two-step synchronization structure that synchronizes based on the SI firstly and then the desired signal. However, the first synchronization step treats the desired signal as noise, resulting in poor performance, and it considers fractional CFO only. Meanwhile, both [7] and [8] did not consider the detection of the desired signal.

In this paper, an iterative semi-blind CFO estimation, SI cancellation and signal detection (ISCIS) scheme is proposed for FD OFDM systems. Our work is different in that, this, to the best of our knowledge, is the first work to consider

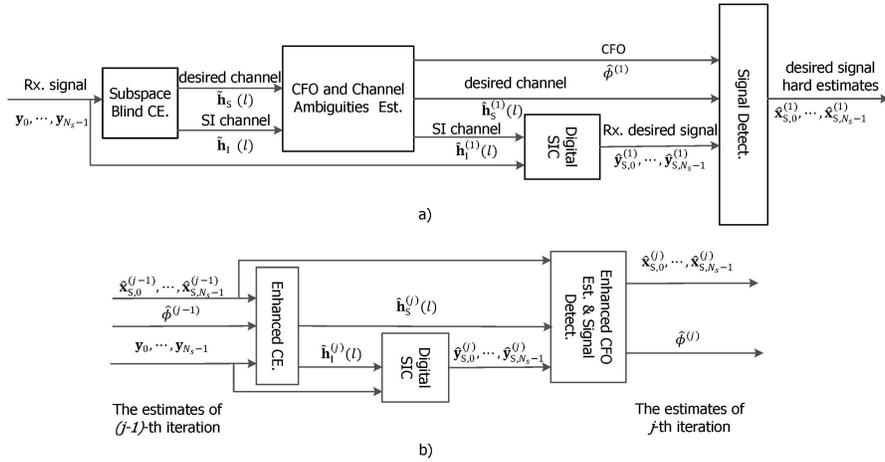


Fig. 1. a) initialization stage b) the  $j$ -th ( $j \geq 2$ ) iteration of the proposed ISCIS scheme (Rx.: received, CE.: channel estimation, Est.: estimation, SIC: SI cancelation and Detect.: detection).

signal detection of FD systems in the presence of both CFO and SI, while the existing work on FD systems have either focused on signal detection by assuming no presence of CFO only [1], [4]–[6] or considered CFO compensation only [7], [8]. Based on the equivalent system model, CFO estimation, SI cancelation and signal detection are performed initially by a subspace based semi-blind method, whose performance is then enhanced significantly by performing iterations among them. Its CFO compensation is performed on the desired signal estimate, avoiding the introduction of CFO to the SI by the existing methods [7], [8]. Also, the pilots for the desired signal and SI are carefully designed to enable simultaneous transmission of them to achieve FD training mode, while the pilots of the existing work [1], [4]–[8] are sent in different time slots, resulting in reduced spectral efficiency. Simulation results show that the proposed iterative scheme converges fast and contributes to a significant bit error rate (BER) performance enhancement. Besides, the proposed scheme, with much lower training overhead, always outperforms the existing methods [4], [5], [8]–[11], especially under the medium to high signal-to-noise-ratios (SNRs). By utilizing the second order statistics of the received signal, a much superior BER performance can be achieved compared to the case with perfect SI cancelation and CFO compensation. The output signal-to-interference-and-noise-ratio (SINR) is close to that with perfect SI cancelation, and robust against the input signal-to-interference ratio (SIR).

In the following, Section II describes the system model. The proposed ISCIS scheme is proposed in Section III. Complexity analysis and simulation results are given in Sections IV and V. Section VI draws conclusion.

*Notations:* Bold symbols represent vectors/matrices, and superscripts  $T$ ,  $*$ ,  $H$  and  $\dagger$  denote the transpose, complex conjugate, complex conjugate transpose and pseudo inverse of a vector/matrix.  $\text{diag}\{\mathbf{a}\}$  is a diagonal matrix with vector  $\mathbf{a}$  on its diagonal.  $\mathbf{I}_N$  is a  $N$ -dimensional identity matrix.

$\mathbf{0}_{M \times N}$  and  $\mathbf{1}_{M \times N}$  are  $M \times N$  all-zero matrix and all-one matrix respectively.  $\mathbb{E}\{\}$  is the expectation operator.  $\otimes$  is the Kronecker product. Matlab notations for matrices and vectors are used throughout this paper.

## II. SYSTEM MODEL

We consider a bidirectional FD OFDM wireless communication system, where two transceivers, namely transceiver-1 and transceiver-2, operate in FD scheme. Each transceiver is equipped with a single transmit antenna and  $N_r$  receive antennas. Due to the inherent symmetry, transceiver-2 is chosen as our research object, as the same performance can be observed at transceiver-1. The signal transmitted from transceiver-1 to transceiver-2 is referred to as the desired signal while the signal transmitted from transceiver-2 to transceiver-2 is SI. In the following, we focus on digital cancelation of the residual SI after passive and analog cancelation.

A data frame consists of  $N_s$  OFDM blocks with  $N$  subcarriers each. The transmit vectors of data symbols corresponding to the  $i$ -th ( $i = 0, \dots, N_s - 1$ ) OFDM symbol block for the desired signal and SI are given by  $\mathbf{x}_{S,i} = [x_{S,i}(0), x_{S,i}(1), \dots, x_{S,i}(N-1)]^T$  and  $\mathbf{x}_{I,i} = [x_{I,i}(0), x_{I,i}(1), \dots, x_{I,i}(N-1)]^T$  respectively, where  $x_{S,i}(n)$  and  $x_{I,i}(n)$  are the corresponding symbols on subcarrier  $n$  ( $n = 0, 1, \dots, N-1$ ). Each OFDM symbol block  $\mathbf{x}_{S,i}$  or  $\mathbf{x}_{I,i}$  is processed by Inverse Discrete Fourier Transform (IDFT), and zero padded with  $L$  zeros and then transmitted. The transmitted time-domain (TD) desired signal is denoted as  $s_{S,i}(0), \dots, s_{S,i}(M-1)$ , while the transmitted TD SI is  $s_{I,i}(0), \dots, s_{I,i}(M-1)$ , with  $M = N + L$ .

Define  $\mathbf{h}_S^{n_r} = [h_S^{n_r}(0), h_S^{n_r}(1), \dots, h_S^{n_r}(L_c-1)]^T$  and  $\mathbf{h}_I^{n_r} = [h_I^{n_r}(0), h_I^{n_r}(1), \dots, h_I^{n_r}(L_c-1)]^T$  as the respective desired and SI channel impulse responses (CIRs) for the  $n_r$ -th receive antenna, with  $L_c$  being the length of CIR. The channels are assumed to exhibit quasi-static block fading and the CIRs remain constant for a frame's duration. Define  $\phi$  ( $\phi \in (-N/2, N/2]$ ) as the CFO between the transceiver-1 and transceiver-2. The

SI does not experience CFO assuming all the transmit and receive antennas of transceiver-2 share one local oscillator [7], [8]. The TD received signal in the  $i$ -th block at the  $n_r$ -th ( $n_r = 0, 1, \dots, N_r - 1$ ) receive antenna can be written as

$$y_i^{n_r}(m) = e^{j2\pi\phi(m+iM)/N} \sum_{l=0}^{L_c-1} h_S^{n_r}(l) s_{S,i}(m-l) + \sqrt{\frac{1}{\rho}} \sum_{l=0}^{L_c-1} h_I^{n_r}(l) s_{I,i}(m-l) + w_i^{n_r}(m) \quad (1)$$

where  $\rho$  is the input average desired signal-to-SI-ratio, denoted as SIR, before digital cancelation, and  $w_i^{n_r}(m)$  ( $m = 0, 1, \dots, M-1$ ) is the noise term.

### III. ITERATIVE SEMI-BLIND CFO ESTIMATION, SI CANCELATION AND SIGNAL DETECTION

In this section, the proposed ISCIS scheme is presented. Thanks to the equivalent system model shown in Section III-A, the CFO estimation, SI cancelation and signal detection are performed initially by the proposed subspace based semi-blind method requiring only one pilot block in FD training mode, and then their performance can be enhanced significantly by performing iterations among them. The block diagram of the proposed ISCIS scheme is illustrated in Fig. 1.

#### A. Equivalent System Model

An equivalent system model is described where the system model with CFO (1) can be formulated as a system model without CFO, by incorporating CFO into the transmitted desired signal and desired channel respectively. In this way, the CFO compensation is performed on the transmitted desired signal estimate instead of the received signal, avoiding the CFO introduction to the SI.

Inspired by [9], (1) can be equivalent to

$$y_i^{n_r}(m) = \sum_{l=0}^{L_c-1} \bar{h}_S^{n_r}(l) \bar{s}_{S,i}(m-l) + \sqrt{\frac{1}{\rho}} \sum_{l=0}^{L_c-1} h_I^{n_r}(l) s_{I,i}(m-l) + w_i^{n_r}(m) \quad (2)$$

where  $\bar{h}_S^{n_r}(l) = e^{j2\pi\phi l/N} h_S^{n_r}(l)$  and  $\bar{s}_{S,i}(m) = e^{j2\pi\phi(m+iM)/N} s_{S,i}(m)$  are denoted as the modified desired channel and the modified desired signal respectively. Denote  $\mathbf{y}_i = [y_i^0(0), y_i^1(0), \dots, y_i^{N_r-1}(0), \dots, y_i^0(M-1), y_i^1(M-1), \dots, y_i^{N_r-1}(M-1)]^T$ . We can obtain

$$\mathbf{y}_i = \bar{\mathbf{H}}_S \bar{\mathbf{s}}_{S,i} + \mathbf{H}_I \mathbf{s}_{I,i} + \mathbf{w}_i \quad (3)$$

where

$$\bar{\mathbf{H}}_S = \begin{bmatrix} \bar{\mathbf{h}}_S(0) & \mathbf{0}_{N_r \times 1} & \cdots & \mathbf{0}_{N_r \times 1} \\ \bar{\mathbf{h}}_S(1) & \bar{\mathbf{h}}_S(0) & \cdots & \mathbf{0}_{N_r \times 1} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\mathbf{h}}_S(L) & \bar{\mathbf{h}}_S(L-1) & \cdots & \bar{\mathbf{h}}_S(0) \\ \mathbf{0}_{N_r \times 1} & \bar{\mathbf{h}}_S(L) & \cdots & \bar{\mathbf{h}}_S(1) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{N_r \times 1} & \mathbf{0}_{N_r \times 1} & \cdots & \bar{\mathbf{h}}_S(L) \end{bmatrix} \quad (4)$$

$\bar{\mathbf{h}}_S(l) = [\bar{h}_S^0(l), \bar{h}_S^1(l), \dots, \bar{h}_S^{N_r-1}(l)]^T$ ,  $\mathbf{H}_I$  is defined as the same form to  $\bar{\mathbf{H}}_S$  but with  $\bar{\mathbf{h}}_S(l)$  replaced

by  $\mathbf{h}_I(l) = \sqrt{\frac{1}{\rho}} [h_I^0(l), h_I^1(l), \dots, h_I^{N_r-1}(l)]^T$ ,  $\bar{\mathbf{s}}_{S,i} = [\bar{s}_{S,i}(0), \bar{s}_{S,i}(1), \dots, \bar{s}_{S,i}(N-1)]^T$  and  $\mathbf{s}_{I,i} = [s_{I,i}(0), s_{I,i}(1), \dots, s_{I,i}(N-1)]^T$ . Note that  $\bar{\mathbf{h}}_S(l) = \mathbf{h}_I(l) = \mathbf{0}_{N_r \times 1}$  for  $l = L_c, \dots, L$ .

In the following, we propose an ISCIS scheme, which are summarized in two steps. First, a semi-blind CFO estimation, SI cancelation and signal detection (SCIS) scheme is proposed, which provides initial estimates of the modified desired and SI channels, the CFO and the desired signal. Then, to further enhance the system performance, we perform iterations among them until a satisfactory performance is obtained.

#### B. Initial Stage of the ISCIS Scheme

The proposed SCIS scheme is described as follows. First, the subspace based blind channel estimator is exploited to estimate the modified desired and SI channels but with some ambiguities. Second, the pilots for the desired signal and SI are carefully designed to enable simultaneous transmission of them to achieve FD training mode, and the corresponding channel ambiguities and CFO can be extracted by the parametric channel estimation methods. Third, based on the modified desired channel and SI channel estimates, the received SI is generated and canceled from the received signal, and then the desired signal can be easily detected.

1) *Blind Channel Estimation*: The subspace based blind channel estimator [9] is applied to estimate the channels  $\bar{\mathbf{h}}_S(l)$  and  $\mathbf{h}_I(l)$ , which is summarized in four steps.

*Step 1.* Considering a data frame, the correlation matrix of the received signal is computed by  $\mathbf{R}_y = \frac{1}{N_s} \sum_{i=0}^{N_s-1} \mathbf{y}_i \mathbf{y}_i^H$

*Step 2.*  $Q = MN_r - 2N$  coorthogonal eigenvectors,  $\boldsymbol{\gamma}_q = [\gamma_q^T(0), \gamma_q^T(1), \dots, \gamma_q^T(M-1)]^T$  ( $q = 0, 1, \dots, Q-1$ ), corresponding to the smallest  $Q$  eigenvalues of the matrix  $\mathbf{R}_y$ , are obtained, where  $\boldsymbol{\gamma}_q(m)$  is a vector of size  $N_r$ .

*Step 3.*  $\mathbf{V}$  is constructed from  $\boldsymbol{\gamma}_q$ , i.e.,  $\mathbf{V} = \sum_{q=0}^{Q-1} \mathbf{V}_q^H \mathbf{V}_q$ , with

$$\mathbf{V}_q = \begin{bmatrix} \gamma_q^H(L) & \gamma_q^H(L-1) & \cdots & \gamma_q^H(0) \\ \gamma_q^H(L+1) & \gamma_q^H(L) & \cdots & \gamma_q^H(1) \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_q^H(M-1) & \gamma_q^H(M-2) & \cdots & \gamma_q^H(N-1) \end{bmatrix} \quad (5)$$

2 eigenvectors corresponding to the 2 smallest eigenvalues of  $\mathbf{V}$  are found, denoted as  $\mathbf{U}$ .

*Step 4.* The modified desired and SI channels are estimated by  $\bar{\mathbf{h}}_S(l) = \mathbf{U}(lN_r : (l+1)N_r - 1, 1)$  and  $\bar{\mathbf{h}}_I(l) = \mathbf{U}(lN_r : (l+1)N_r - 1, 2)$ . Denote  $\bar{\mathbf{h}}(l) = [\bar{\mathbf{h}}_S(l), \bar{\mathbf{h}}_I(l)]$  of size  $N_r \times 2$ .

However, there exist channel ambiguities between the estimated and true channels. For example,  $\bar{\mathbf{h}}_S(l) = \bar{\mathbf{h}}(l) \mathbf{b}_S$  and  $\bar{\mathbf{h}}_I(l) = \bar{\mathbf{h}}(l) \mathbf{b}_I$ , where  $\mathbf{b}_S$  and  $\mathbf{b}_I$  of size  $2 \times 1$ , as the corresponding desired and SI channel ambiguity vectors, should be estimated in the following.

2) *Pilot Design*: A TD pilot block is well designed to enable simultaneous transmission of them to achieve FD training mode, and the corresponding channel ambiguities and CFO can be extracted by the parametric channel estimation methods.

With the channel estimates by the subspace based blind method, (3) can be rewritten as

$$\mathbf{y}_i = \mathbf{H}\mathbf{B}_S\bar{\mathbf{s}}_{S,i} + \mathbf{H}\mathbf{B}_I\mathbf{s}_{I,i} + \mathbf{w}_i \quad (6)$$

where  $\mathbf{H}$  is defined as the same form to  $\bar{\mathbf{H}}_S$  with  $\bar{\mathbf{h}}_S(l)$  replaced by  $\tilde{\mathbf{h}}(l)$ ;  $\mathbf{B}_S = \mathbf{I}_N \otimes \mathbf{b}_S$  and  $\mathbf{B}_I = \mathbf{I}_N \otimes \mathbf{b}_I$ . By multiplying the received signal  $\mathbf{y}_i$  with the pseudoinverse of  $\mathbf{H}$ , we can obtain

$$\mathbf{r}_i = \mathbf{B}_S\bar{\mathbf{s}}_{S,i} + \mathbf{B}_I\mathbf{s}_{I,i} + \bar{\mathbf{w}}_i \quad (7)$$

where  $\bar{\mathbf{w}}_i = \mathbf{H}^\dagger \mathbf{w}_i$ .  $\mathbf{r}_i$  is divided into  $N$  vectors of length 2 as  $\mathbf{r}_i = [\mathbf{r}_i^T(0), \dots, \mathbf{r}_i^T(N-1)]^T$ , and  $\mathbf{r}_i(n)$  is given by

$$\mathbf{r}_i(n) = \mathbf{b}_S e^{j2\pi\phi(n+iM)/N} s_{S,i}(n) + \mathbf{b}_I s_{I,i}(n) + \bar{\mathbf{w}}_i(n) \quad (8)$$

where  $\bar{\mathbf{w}}_i(n)$  is defined similarly to  $\mathbf{r}_i(n)$ .

According to (8), it can be observed that as long as  $s_{S,i}(0) = \dots = s_{S,i}(N-1) = a$  and  $s_{I,i}(0) = \dots = s_{I,i}(N-1) = b$ , (8) can be rewritten as

$$\mathbf{r}_{\text{pil},i}(n) = \mathbf{b} e^{j2\pi\phi(n+iM)/N} + \bar{\mathbf{w}}_i(n) \quad (9)$$

where  $\mathbf{b} = [\mathbf{b}_S, \mathbf{b}_I]$  and  $\phi = [\phi, 0]^T$ . For simplicity,  $a$  and  $b$  have been specified as 1 in (9). It can be noticed that (9) looks like the channel frequency response model in [12]. Therefore, parametric channel estimation methods, e.g., estimation of signal parameters via rotational invariance technique (ESPRIT) and LS, can be exploited here to estimate the CFO and channel ambiguities respectively.

It is noteworthy that two pilot patterns have also been designed in [9] to jointly estimate the CFOs and channel ambiguities for multiple users. However, to avoid the multiuser interferences, the pilots of different users should be non-overlapping, resulting in reduced spectral efficiency. In contrast, our proposed pilot design can be overlapping, which is indeed designed for FD systems.

3) *CFO and Channel Ambiguities Estimation*: As discussed earlier, with the pilot design, the problem of CFO and channel ambiguities estimation relates to the parametric channel estimation problem. Thus, the time delay estimator, e.g., ESPRIT, in the parametric channel estimation, can be exploited here to extract the unknown CFO, while the path amplitude estimator, e.g., LS, can be used to determine the channel ambiguities.

Utilizing the ESPRIT algorithm, the CFO estimation is summarized in four steps below:

*Step 1.* Form the matrix  $\mathbf{r}_{\text{pil},i} = [\mathbf{r}_{\text{pil},i}(0), \dots, \mathbf{r}_{\text{pil},i}(N-1)]^T$  with size  $N \times 2$ , and it can be expressed as

$$\mathbf{r}_{\text{pil},i} = \mathbf{V}_i \mathbf{b}^T + \tilde{\mathbf{w}}_i \quad (10)$$

where  $\mathbf{V}_i = [\mathbf{V}(\phi), \mathbf{V}(0)]$  with  $\mathbf{V}(\phi) = [e^{j2\pi\phi iM/N}, \dots, e^{j2\pi\phi(N-1+iM)/N}]^T$  and  $\mathbf{V}(0) = \mathbf{1}_{N \times 1}$ , and  $\tilde{\mathbf{w}}_i = [\tilde{\mathbf{w}}_i(0), \dots, \tilde{\mathbf{w}}_i(N-1)]^T$ .

*Step 2.* Compute the auto-correlation matrix of  $\mathbf{r}_{\text{pil},i}$ ,  $\mathbf{R}_r = \frac{1}{2} \mathbf{r}_{\text{pil},i} \mathbf{r}_{\text{pil},i}^H$ . It is worth noting the auto-correlation matrix has been averaged by 2, thanks to the ambiguities incurred by the blind channel estimation.  $\mathbf{R}_r$  is further improved by the forward-backward (FB) averaging technique [12], obtaining

$\mathbf{R}_{\text{FB},r} = \frac{1}{2}(\mathbf{R}_r + \mathbf{J}\mathbf{R}_r^*\mathbf{J})$ , where  $\mathbf{J}$  is the  $N \times N$  matrix whose components are zero except for ones on the anti-diagonal.

*Step 3.* 2 eigenvectors corresponding to the largest 2 eigenvalues of  $\mathbf{R}_{\text{FB},r}$  are found, denoted as  $\mathbf{u}$  of size  $N \times 2$ . Due to the phase rotational invariance property, there exists the following relationship  $\mathbf{u}_2 = \mathbf{u}_1 \text{diag}\{e^{j2\pi\phi/N}\}$  where  $\mathbf{u}_1$  and  $\mathbf{u}_2$  are the first  $(N-1)$  and last  $(N-1)$  rows of  $\mathbf{u}$  respectively.

*Step 4.* The CFO can be extracted by

$$\hat{\phi}^{(1)} = \frac{\angle \delta N}{2\pi} \quad (11)$$

where  $\delta$  is the eigenvalues of  $\mathbf{u}_1^\dagger \mathbf{u}_2$ . It is worth noticing that  $\hat{\phi}^{(1)}$  consists of two CFOs. The one with the largest absolute value is the unknown CFO  $\phi^{(1)}$ , i.e.,  $\hat{\phi}^{(1)} = \max |\phi^{(1)}|$ , since the CFO of SI is always 0.

The channel ambiguity vectors are then computed by LS method, with the CFO estimate  $\hat{\phi}^{(1)}$ . It involves two steps.

*Step 1.* Form the matrix  $\hat{\mathbf{V}}_i = [\mathbf{V}(\hat{\phi}^{(1)}), \mathbf{V}(0)]$ .

*Step 2.* According to (10), the channel ambiguity vectors are estimated by the LS method, i.e.,  $\hat{\mathbf{b}} = (\hat{\mathbf{V}}_i^\dagger \mathbf{r}_{\text{pil},i})^T$ . Thus, the desired and SI channel ambiguity vectors are obtained as  $\hat{\mathbf{b}}_S = \hat{\mathbf{b}}(:, 1)$  and  $\hat{\mathbf{b}}_I = \hat{\mathbf{b}}(:, 2)$  respectively. Therefore, the modified desired and SI channel estimates are obtained as

$$\hat{\mathbf{h}}_S^{(1)}(l) = \tilde{\mathbf{h}}(l) \hat{\mathbf{b}}_S, \quad \hat{\mathbf{h}}_I^{(1)}(l) = \tilde{\mathbf{h}}(l) \hat{\mathbf{b}}_I \quad (12)$$

Define  $\hat{\mathbf{H}}_S^{(1)}$  and  $\hat{\mathbf{H}}_I^{(1)}$  as the circulant desired and SI channel matrices, following the same form to  $\bar{\mathbf{H}}_S$ , with  $\bar{\mathbf{h}}_S(l)$  replaced by  $\hat{\mathbf{h}}_S(l)$  and  $\hat{\mathbf{h}}_I(l)$  respectively. Throughout this paper, we assume the first OFDM block  $i = 0$  is transmitted for training.

4) *SI Cancellation and Signal Detection*: With the SI channel estimate, the received SI can be generated and canceled from the received signal, obtaining  $\hat{\mathbf{y}}_{S,i}^{(1)} = \mathbf{y}_i - \hat{\mathbf{H}}_I^{(1)} \mathbf{s}_{I,i}$ . With the CFO estimate  $\hat{\phi}^{(1)}$ , the TD desired signal is estimated by

$$\hat{\mathbf{s}}_{S,i}^{(1)} = \mathbf{E}(\hat{\phi}^{(1)}) (\hat{\mathbf{H}}_S^{(1)})^\dagger \hat{\mathbf{y}}_{S,i}^{(1)} \quad (13)$$

$\mathbf{E}(\hat{\phi}^{(1)}) = \text{diag}\{[e^{-j2\pi\hat{\phi}^{(1)}iM/N}, \dots, e^{-j2\pi\hat{\phi}^{(1)}(N-1+iM)/N}]\}$ . Note that CFO compensation is performed on the modified desired signal estimate. By performing Discrete Fourier Transform (DFT) on  $\hat{\mathbf{s}}_{S,i}^{(1)}$ , the frequency-domain desired signal is detected, and the hard detection is obtained as  $\hat{\mathbf{x}}_{S,i}^{(1)}$ .

### C. Iterative Stages of the ISCIS Scheme

To further improve the performance, we perform iterations among signal detection, SI cancellation and CFO estimation. First, a more accurate SI cancellation is performed, based on the enhanced estimates of the modified desired channel and SI channel by the previous desired signal estimates and known SI. Second, an enhanced CFO estimate is obtained, thanks to the inherent relationship between the previous desired signal estimates and the newly estimated modified desired signal.

TABLE I

ANALYTICAL COMPUTATIONAL COMPLEXITY ( $N$ : SIZE OF AN OFDM SYMBOL,  $N_s$ : NUMBER OF OFDM BLOCKS WITHIN ONE FRAME,  $L_c$ : CHANNEL LENGTH,  $N_r$ : NUMBER OF RECEIVED ANTENNAS,  $I$ : NUMBER OF ITERATIONS,  $M$ :  $M = N + L$  AND  $T$ : NUMBER OF SUBCARRIERS FOR PILOT IN [5]).

Algorithm	ISCIS	g1: [10]+ [11]+ [5]	g2: [10]+ [11]+ [4]
CFO estimation	$5N^3I + (2N^2N_s + N_sN)(I - 1)$	$4N^3 + 5N_r^2N^2 + 6N_r^3N\log_2N$	$4N^3 + 5N_r^2N^2 + 6N_r^3N\log_2N$
Channel estimation	$N_r^3M^3 + 2N_sN_r^2M^2 + (16N^2N_r + 2N_r^3L_c^2N)M + 8N^3 + 16N_sN_r^3L_c^2N(I - 1)$	$(8N_r^3L_c^2NN_s + 4N_r^2NL_cN_s + 8N_r^3N^2L_cN_s + 2N^2N_r^2N_s)I$	$(4N_r^2NL_cN_s + 5N_r^3L_c^2NN_s)I$
SI cancelation	$2N_rMN_sNI$	$(N_s + 1)N\log_2N + 2N_rNN_s$	$(N_s + 1)N\log_2N + 2N_rNN_s$
Signal detection	$(4N^2N_rM + N^3 + 2NN_sN_rM + N_sN\log_2N)I$	$N_sN_r(N - T) + N\log_2N$	$N_sN_rN + N\log_2N$

### 1) Enhanced Channel Estimation and SI Cancelation:

Define  $\tilde{\mathbf{y}}_i = \mathbf{y}_i(1: N_rN, 1)$ , which can be written as

$$\tilde{\mathbf{y}}_i = \mathbf{S}_{S,i}\tilde{\mathbf{h}}_S + \mathbf{S}_{I,i}\mathbf{h}_I + \tilde{\mathbf{w}}_i \quad (14)$$

where

$$\mathbf{S}_{S,i} = \begin{bmatrix} \tilde{\mathbf{s}}_{S,i}(0) & \mathbf{0}_{N_r \times N_r} & \cdots & \mathbf{0}_{N_r \times N_r} \\ \tilde{\mathbf{s}}_{S,i}(1) & \tilde{\mathbf{s}}_{S,i}(0) & \cdots & \mathbf{0}_{N_r \times N_r} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{s}}_{S,i}(N-1) & \tilde{\mathbf{s}}_{S,i}(N-2) & \cdots & \tilde{\mathbf{s}}_{S,i}(N-L_c) \end{bmatrix} \quad (15)$$

$\tilde{\mathbf{s}}_{S,i}(n) = \text{diag}\{\tilde{s}_{S,i}(n), \dots, \tilde{s}_{S,i}(n)\}$  is of size  $N_r \times N_r$ ;  $\mathbf{S}_{I,i}$  is defined as the same way to  $\mathbf{S}_{S,i}$  but with  $\tilde{\mathbf{s}}_{S,i}(n)$  replaced by  $\tilde{\mathbf{s}}_{I,i}(n) = \text{diag}\{s_{I,i}(n), \dots, s_{I,i}(n)\}$ ;  $\tilde{\mathbf{h}}_S = [\tilde{\mathbf{h}}_S(0)^T, \dots, \tilde{\mathbf{h}}_S(L_c - 1)^T]^T$  and  $\mathbf{h}_I = [\mathbf{h}_I(0)^T, \dots, \mathbf{h}_I(L_c - 1)^T]^T$ ;  $\tilde{\mathbf{w}}_i = \mathbf{w}_i(1: N_rN, 1)$ .

According to (14), the modified desired channel and SI channel can be obtained by LS method with the previous modified desired signal matrix estimate  $\hat{\mathbf{S}}_{S,i}^{(j-1)}$  and known SI  $\mathbf{S}_{I,i}$ . By performing IDFT on the previous desired signal estimate  $\hat{\mathbf{x}}_{S,i}^{(j-1)}$ , the TD desired signal is obtained as  $\hat{\mathbf{s}}_{S,i}^{(j-1)}$ , and its modified version is calculated by  $\hat{\tilde{\mathbf{s}}}_{S,i}^{(j-1)} = \mathbf{E}(-\hat{\phi}^{(j-1)})\hat{\mathbf{s}}_{S,i}^{(j-1)}$ . Then  $\hat{\mathbf{S}}_{S,i}^{(j-1)}$  can be easily computed by replacing  $\tilde{s}_{S,i}(n)$  in (15) with  $\hat{\tilde{\mathbf{s}}}_{S,i}^{(j-1)}(n)$ . Denote  $\mathbf{A}_i = [\hat{\mathbf{S}}_{S,i}^{(j-1)}, \mathbf{S}_{I,i}]$ , and the modified desired and SI channels are enhanced by  $[\hat{\mathbf{h}}_{S,i}^{(j)}; \hat{\mathbf{h}}_{I,i}^{(j)}] = \mathbf{A}_i^\dagger \tilde{\mathbf{y}}_i$ . If  $N_s$  symbols are used, the temporal averaged modified desired and SI channels can be computed as  $\hat{\mathbf{h}}_S^{(j)} = \frac{1}{N_s} \sum_{i=0}^{N_s-1} \hat{\mathbf{h}}_{S,i}^{(j)}$  and  $\hat{\mathbf{h}}_I^{(j)} = \frac{1}{N_s} \sum_{i=0}^{N_s-1} \hat{\mathbf{h}}_{I,i}^{(j)}$  respectively. Similarly, the circulant desired and SI channel matrices  $\hat{\mathbf{H}}_S^{(j)}$  and  $\hat{\mathbf{H}}_I^{(j)}$  can be obtained by following the same form of  $\hat{\mathbf{H}}_S$ . With the new circulant SI channel matrix estimate  $\hat{\mathbf{H}}_I^{(j)}$ , the SI can be canceled from the received signal, obtaining  $\hat{\mathbf{y}}_{S,i}^{(j)} = \mathbf{y}_i - \hat{\mathbf{H}}_I^{(j)} \mathbf{s}_{I,i}$ .

2) *Enhanced CFO Estimation and Signal Detection:* The modified desired signal is reestimated by the new modified desired channel estimate as  $\hat{\tilde{\mathbf{s}}}_{S,i}^{(j)} = (\hat{\mathbf{H}}_S^{(j)})^\dagger \hat{\mathbf{y}}_{S,i}^{(j)}$ . In the ideal case, the reestimated modified desired signal and the previous desired signal estimate meets,

$$\hat{\tilde{\mathbf{s}}}_{S,i}^{(j-1)} = \mathbf{E}(\phi)\hat{\tilde{\mathbf{s}}}_{S,i}^{(j)} \quad (16)$$

Therefore, the CFO can be further enhanced by the ESPRIT algorithm as follows.

*Step 1.* The CFO vector  $\mathbf{e}_i = [e_i(0), e_i(1), \dots, e_i(N-1)]^T$  with  $e_i(n) = \hat{\tilde{\mathbf{s}}}_{S,i}^{(j)}(n)/\hat{\tilde{\mathbf{s}}}_{S,i}^{(j-1)}(n)$  is computed.

*Step 2.* Calculate the temporal averaged correlation matrix of the CFO vector by  $\mathbf{R}_e = \frac{1}{N_r} \sum_{i=0}^{N_s-1} \mathbf{R}_{e,i}$  with  $\mathbf{R}_{e,i} = \mathbf{e}_i \mathbf{e}_i^H$ . It is further enhanced by the FB averaging technique, obtaining  $\mathbf{R}_{FB,e} = \frac{1}{2}(\mathbf{R}_e + \mathbf{J}\mathbf{R}_e^*\mathbf{J})$ .

*Step 3.* The remaining of the CFO estimation keeps the same to Steps 3 and 4 of the CFO estimator in Section III-B, but selecting one eigenvector corresponding to the largest eigenvalue only. Denote the new CFO estimate as  $\phi^{(j)}$ .

The desired signal is then compensated by

$$\hat{\mathbf{s}}_{S,i}^{(j)} = \mathbf{E}(\phi^{(j)})\hat{\tilde{\mathbf{s}}}_{S,i}^{(j)} \quad (17)$$

Similarly, a new desired signal hard estimate is obtained as  $\hat{\mathbf{x}}_{S,i}^{(j)}$ . The above procedures are repeated until a satisfactory performance is obtained. Define  $I$  as the number of iterations.

## IV. COMPLEXITY ANALYSIS

In Table I, the computational complexity of the proposed scheme and the existing methods [4], [5], [10], [11] are presented, in terms of the number of complex additions and multiplications. They are compared in four aspects, namely CFO estimation, channel estimation, SI cancelation and signal detection. The integer CFO and fractional CFO estimators in HD systems [10], [11], referred to as ICFO-HD and FCFO-HD respectively, are selected for comparison in CFO estimation. The ML [5] and TS-LS [4] SI cancelation methods are chosen for comparison in performances of SI cancelation and channel estimation. They are arranged in two groups, namely g1: ICFO-HD [10]+FCFO-HD [11]+ML [5] and g2: ICFO-HD [10]+FCFO-HD [11]+TS-LS [4]. Note that the existing CFO estimators [10], [11] are performed in HD mode to compensate the CFO in the desired signal, and the SI cancelation and signal detection [4], [5] are then implemented in FD mode. For the proposed scheme, channel estimation, consisting of the subspace based blind channel estimation, channel ambiguities estimation and enhanced channel estimation, is the dominant term contributing to the overall computational complexity.

Their relative computational complexity are illustrated in Table II, with  $N = 32$ ,  $N_s = 150$ ,  $L_c = 14$ ,  $N_r = 3$ ,  $I = 3$ ,  $M = 48$  and  $T = 1$ . It is worth noticing that the proposed scheme requires only one pilot whereas g1 and g2 need 6 and 2 equivalent pilots separately. The existing methods g1 are found to have the highest computational complexity. The existing methods g2 have the lowest complexity, which is around half of that of the proposed scheme. Nevertheless, as stated in Section V, the proposed scheme, with much lower

training overhead, provides much better performances than the existing methods g1 and g2 [4], [5], [8]–[11].

TABLE II  
NORMALIZED COMPUTATIONAL COMPLEXITY.

Proposed ISCIS scheme	1
g1: ICFO-HD [10]+FCFO-HD [11]+ML [5]	2.18
g2: ICFO-HD [10]+FCFO-HD [11]+TS-LS [4]	0.46

## V. SIMULATION RESULTS

A simulation study is carried out to demonstrate the performance of the proposed ISCIS scheme. System parameters are set as follows: each OFDM block contains  $N = 32$  subcarriers; the modulation scheme is quadrature phase shift keying; the number of receive antennas is  $N_r = 3$ ; the Long Term Evolution Extended Pedestrian A Model with sampling rate  $f_s = 30.72$  MHz [13] is applied; the zero-padding length is  $L = 16$ ; the CFO value is randomly generated in  $[-N/2, N/2)$ ; each data frame contains  $N_s = 150$  OFDM symbols. Unless otherwise stated, the averaged input SIR  $\rho$  before digital cancellation is assumed to be  $-20$  dB. The root mean square errors (RMSEs) of CFO and channel estimation for the  $j$ -th iteration are defined as  $\text{RMSE}_{\text{CFO}}^{(j)} = \sqrt{\mathbb{E}\{(\hat{\phi}^{(j)} - \phi)^2\}}$  and  $\text{RMSE}_{\text{Channel}}^{(j)} = \sqrt{\mathbb{E}\{\frac{1}{2N_r L_c} [(\hat{\mathbf{h}}_s^{(j)} - \bar{\mathbf{h}}_s)^2 + (\hat{\mathbf{h}}_l^{(j)} - \bar{\mathbf{h}}_l)^2]\}}$  respectively. The output SINR is defined as the averaged desired signal to the residual SI and noise power ratio after the digital cancellation.

Fig. 2 shows the BER performances of the proposed ISCIS scheme and the existing methods [4], [5], [10], [11]. It is easily observed that the proposed ISCIS schemes regardless of the number of iterations all outperform the existing methods [4], [5], [10], [11], especially from the medium to high SNRs. They even provide higher BER performance than the case with perfect SI cancellation and CFO compensation using the same number of training symbols from SNR=15 dB to SNR=40 dB. This is because apart from the training symbol, the proposed scheme also exploits the second order statistics of unknown data symbols to enhance the channel estimation performance.

Fig. 3 demonstrates the convergence behaviors of the BER performances at SNR=13 dB and SNR=19 dB of the proposed ISCIS scheme. We can observe that the proposed scheme converges fast and its speed depends on the SNR. At low SNRs, e.g., SNR=13 dB, due to the poor initial BER performance and the error propagations, further iterations would not greatly enhance the performance. At medium SNRs, e.g., SNR=19 dB, the initial BER performance is improved a lot with around 20 dB increment, after 3 iterations only. At high SNRs, e.g., SNR=25 dB, the proposed ISCIS scheme provides a rather good initial BER performance (approaching 0), and no further iterations are needed. Since it is hard to plot BER with 0, the BER performance against the number of iterations at SNR=25 dB is not shown in Fig. 3.

Fig. 4 compares the proposed ISCIS scheme with the existing both FD and HD CFO estimators [8]–[11] in the

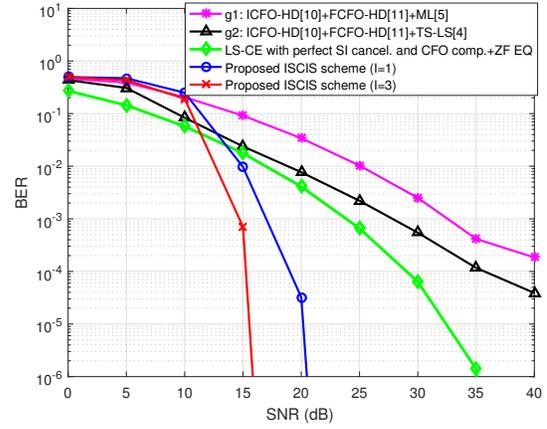


Fig. 2. BER performances of the proposed ISCIS scheme and the existing methods [4], [5], [10], [11] (LS-CE: least square channel estimation, ZF-EQ: zero-forcing equalization, cancel.: cancellation, comp.: compensation).

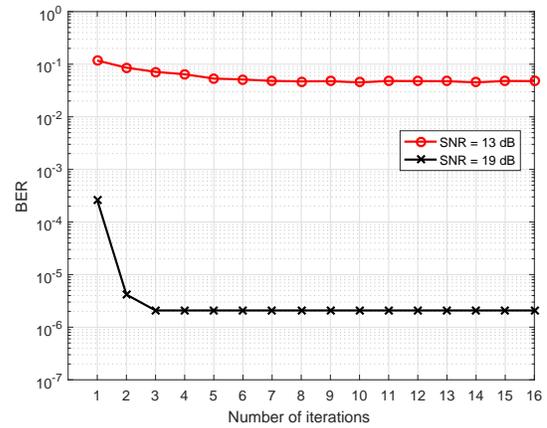


Fig. 3. Convergence behaviors of BER performance at SNR=13 dB and SNR=19 dB of the proposed ISCIS scheme.

RMSE performances of CFO. Note that the existing HD CFO estimators [9]–[11] are performed with the perfect SI cancellation. Thanks to the iterations between CFO estimation and signal detection, the RMSE performance of the proposed scheme is always far superior to those by the existing methods [8]–[11]. Among the existing CFO estimators [8]–[11], ICFO-HD [10] and FCFO-HD [11] with perfect SI cancellation contribute to the best performance. The two-step synchronization scheme for FD systems [8] has the poorest performance, as its first stage treats the desired signal as noise.

Fig. 5 shows that the proposed scheme presents a much better performance in terms of RMSE of channel estimates than the existing methods [4], [5], [10], [11] from medium to high SNRs, with approximately 10 dB improvement. It also outperforms the case with perfect SI cancellation and CFO compensation, thanks to the utilization of both training and data symbols.

In Fig. 6, the output SINRs of the proposed scheme and the existing methods [4], [5], [10], [11] are demonstrated as

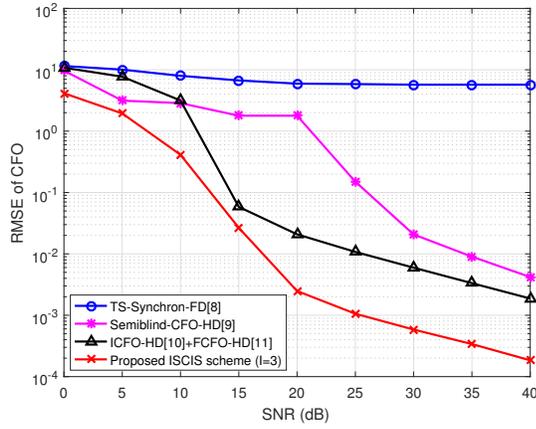


Fig. 4. RMSE of CFO of the proposed ISCIS scheme and the existing methods [8]–[11].

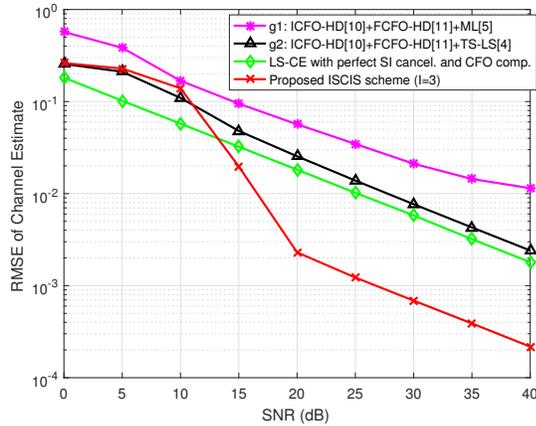


Fig. 5. RMSE of channel of the proposed ISCIS scheme and the existing methods [4], [5], [10], [11].

a function of input SIRs at SNR=30 dB. Regarding perfect cancellation, the output SINR should equal to the input SINR, as the SI has been canceled completely. It is found that, regardless of the input SIR, the output SINR of the proposed scheme always approaches to the ideal case with perfect cancellation. Meanwhile, the proposed scheme can cancel more SI, around 12 dB and 5 dB than g1: ICFO-HD [10]+FCFO-HD [11]+ML [5] and g2: ICFO-HD [10]+FCFO-HD [11]+TS-LS [4] respectively.

## VI. CONCLUSION

An iterative semi-blind CFO estimation, SI cancellation and signal detection scheme has been proposed for FD OFDM systems. It converges within 1-3 iterations for the BER performance. It is also very spectral efficient, requiring one pilot only. The proposed scheme demonstrates significant performance enhancements in terms of both BER and RMSEs of CFO and channel estimates over the existing methods [4], [5], [8]–[11], especially under the medium to high SNRs. Meanwhile, thanks to the utilization of both training symbols and unknown

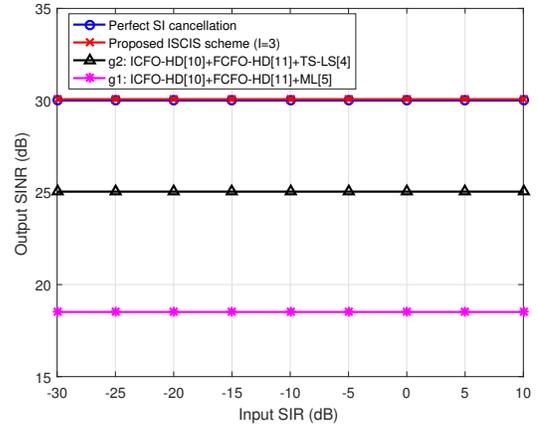


Fig. 6. Output SINR versus input SIR at SNR= 30 dB.

data, its BER and RMSE performance of channel are far superior to the case with perfect SI cancellation and CFO compensation. Besides, despite of the input SIR, the ISCIS scheme can cancel much more SI than the existing methods [4], [5], whose output SINR is always close to the ideal case with perfect SI cancellation at high SNRs.

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