# Intangible Capital and the Rise in Wage and Hours Volatility

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**Abstract**

In a standard real business cycle model extended to include intangible capital (IC) I show that a rise in the income share of IC in the production function, in line with data can account for a significant share of the increase in real wage volatility (both absolute and relative to income) and labor input volatility (relative to income) observed in the U.S. since the mid 1980's even as volatility of output declined. Intangible capital accumulates stochastically and similar to final goods requires physical capital, intangible capital and labor to produce. Under these conditions an increase in the share of IC in production increases the propagation of the IC-specific shock which raises (absolute and relative) wage and labor input volatility. The higher propagation of the IC shock also accounts for the large decline in the pro-cyclicality of labor productivity (relative to both output and labor) observed during this period.

# 1 Introduction

Recent literature documented substantial changes in the dynamics of key labor market aggregates that accompanied the large drop in output volatility in the post-1984 period in the U.S. These changes are:

i) Volatility of the real average wage rate, both relative to output and in absolute terms, increased markedly (Ohanian and Raffo, 2012; Champagne and Kurmann, 2013; Gali and Van Rens, 2014),

ii) Volatility of labor input relative to output increased (Ohanian and Raffo, 2012; Gali and Van Rens, 2014) and

iii) The pro-cyclicality of labor productivity relative to output and labor declined significantly with the latter turning strongly negative (Stiroh, 2009; Gali and Gambetti, 2009; Gali and Van Rens, 2014).

I argue in this paper that a rise in the importance of intangible capital (IC) in production in recent decades, can jointly account for the observed shifts in labor market dynamics along with the decline in aggregate output volatility that characterized the so called Great Moderation of this period.

Intangible capital, as defined in McGrattan and Prescott (2012) is "accumulated know-how from investing in research and development, brands, and organizations, which is for the most part expensed by companies rather than capitalized". Hall (2000, 2001) attributes the majority of the increase in the valuation of corporations in the 1990's to what he calls e-capital. Nakamura (2001) reports, using different estimates of intangible investments that the rate of such investments, and its economic value, accelerated significantly beginning around 1980. U.S. private gross investment in intangibles was at least

$1 trillion by the end of 1999, same as business investment in traditional, tangible capital. This finding is matched by Corrado, Hulten and Sichel (2005) while Corrado, Hulten and Sichel (2009) find that IC's share in income increased from 9.4% in the period 1973-1995 to 14.6% in 1995-2003. The ratio of intangible to tangible assets increased from 20% in the pre-1980 period to around 60% in 2010 (Corrado et al., 2009, 2010; Falato et al, 2014 and Dottling and Perotti, 2015).

As expected, the share of employment in occupations that are predominantly associated with production of intangibles, as a fraction of total employment, also increased substantially during this period. Using occupation data from the Department of Labor's March Current Population Survey (CPS), I split workers into two groups, (a) workers engaged in the creation of innovative property like engineers, architects, scientists, artists, entertainers and IT workers, and (b) organizational workers namely managers, marketers and human resource specialists all of whom are associated with developing economic competencies[1]. I term the total employment of the two groups together as IC employment. This is similar to the classification of IC related employment in Nakamura (2001). McGrattan and Prescott (2012) also use occupation data to show the shift in employment to IT sectors, that occurred in the 1990's. Here I focus more broadly on the intangible capital revolution that began in the years leading up to the Great Moderation, and which includes but is not limited to the IT sector[2].

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[1]See Flood, King, Ruggles, and Warren (2015) for details on the micro data series. Data was generated online at IPUMS-CPS, University of Minnesota, www.ipums.org. Occupation codes used for IC employment: 000-200, 200-225, 229, 233, 256, 258

[2]Nakamura (2001) gives an account of the reasons for the increased use of intangible capital from around this period.
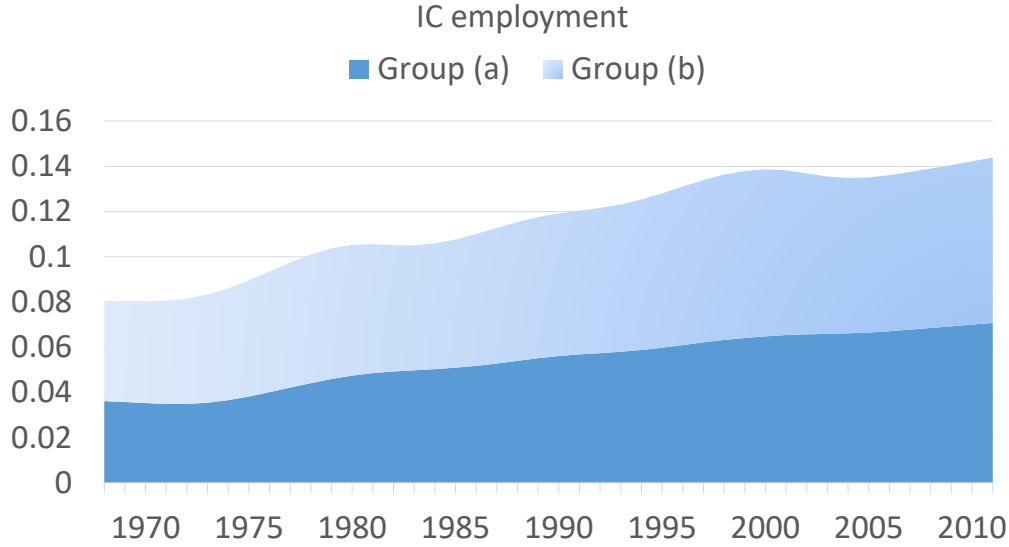
Figure 1: Employment shares in IC related occupations. Group (a)-engineers, architects, scientists, artists, entertainers and IT workers; Group (b)-managers, marketers and human resource specialists. Data is from Occupation series, 1990 basis (occ90) from IPUMS-CPS

Figure 1 plots the evolution of employment of the two groups of IC workers in recent decades. Between 1970-2010, employment of group (a) doubled while group (b)'s employment increased by 65%. Total IC employment rose by a marked 80% during this period.

Two key features of IC investment that distinguishes it from investment in physical capital are, the former is associated with additional risks and, it requires labor, physical capital and intangible capital to produce (see for example Brynjolfsson, 2002; Eisfeldt and Papanikolaou, 2013 and McGrattan and Prescott, 2012). In addition, IC itself is non-rival, in the sense that the same IC can be used for final good production as well as to produce more IC. I incorporate these features into an otherwise standard Real Business Cycle (RBC) framework by introducing an additional IC investment producing sector with an IC sector specific productivity shock (interchangeably called the IC shock throughout this paper).

The main result of this paper is driven by two observations. Firstly, a rise in the importance of IC in production increases the propagation of IC shocks in the model relative

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to neutral technology shocks. In other words, as IC's share in production increases, the same change in IC has a larger impact on output. Secondly, output of the final goods sector falls upon impact of a positive IC shock. This is because more productive IC causes an increase in IC generation which requires labor to produce. Labor is reallocated from the final goods to the IC sector leading to a fall in final goods output upon impact of the IC shock. Thus a permanent increase in IC productivity causes an initial decline in final output, followed by a recovery leading to a higher level of output than before the shock.

The results of the paper follow from these two elements. As the share of IC rises, business cycles are progressively more driven by IC-specific shocks. This reduces the volatility of output compared to labor input (as output initially falls in response to the IC shock which increases labor input) and weakens the positive correlation between productivity and labor input, even making the correlation negative. Finally, although the cyclicality of productivity falls, it does not imply a decline in the volatility of real wage. In fact real wage volatility rises since the marginal product of labor, and therefore the wage, internalizes the effect of building up the IC stock for future production.

*Related Literature* - The literature exclusively attributes the shift in labor market dynamics to a rise in US labor market flexibility around this time. Gali and Van Rens (GVR from now on) (2014), show that these changes can be caused by a reduction in hiring costs arising from an increase in labor market turnover. Champagne and Kurmann (2013) and Nucci and Riggi (2013) both argue that a shift towards performance-pay contracts played an important role in the changing U.S. labor market dynamics. The former also use micro-data to establish the empirical evidence and show that changes in workforce composition did not play a role in the rising wage volatility of this period. Comin, Groshen and Rabin (2008) associate the higher wage volatility with a general increase in firm level (profit-to-sales ratio or the growth rate of sales, employment or sales per worker) volatility. They too rule out any role played by compositional changes of the workforce and observe that the relationship between sales and wage volatility at the firm level is stronger since the 1980's and for services rather than manufacturing firms. To my knowledge, the current paper is the first to focus on the link between the rising importance of intangible capital and changes in labor market dynamics.

The mechanism in this paper can be likened to the "productivity slowdown" literature of the 1990's. The latter contends that exogenous technological progress initially reduces measured productivity through an increase in mismeasurement of one or more of, learning and quality (Hornstein and Krusell, 1996), investment for setting up and learning new technologies (Greenwood and Yorukoglu, 1997) or the aggregate capital stock (Mukoyama, 2005). My paper is most closely related to Greenwood and Yorukoglu (1997) who argue that new technology requires investment in costlier skilled labor and particularly in learning which is expensed rather than capitalized, similar to IC investments in the current paper. More specifically, an exogenous technological progress in their paper, causes learning to increase which is unmeasured while benefits of the new technology are not completely realized leading to a slowdown in productivity. In the current environment, an exogenous increase in the importance of IC in production, leads to an increase in its investment which is unmeasured, like investment in learning in the previous paper. A key difference, however, is that benefits of learning dissipate with time in Greenwood and Yorukoglu (1997) as knowledge of the new technology becomes widespread, whereas the benefits of IC investment, that is IC output, accumulates over time. As the importance of IC rises in my model, the higher accumulated stock of IC ensures an increase in output growth and productivity. Arguably however, productivity growth would be higher if IC investments were capitalized and not expensed. Finally, the mismeasurement in investment in the previous framework corrects itself over time since the new technology benefits become better measured and the cost of investing in learning (skill premium) falls as more people become familiar with the technology. In the current framework the mismeasurement in IC investment is permanent.

This paper is also related to the recent literature that includes IC in standard RBC models to shed light on otherwise puzzling business cycle phenomena. McGrattan and Prescott (2010), in such a framework, generate the observed boom of the 1990's. Without IC their model predicts a depressed economy in the 90's. McGrattan and Prescott (2012), using the same model, reassess the Great Recession of 2008-2009 and the slow recovery period from 2009-2011 and show that the inclusion of IC and non-neutral technology change in the production of final goods and services can account for the fact that labor productivity

rose during the Great Recession even as GDP crashed. Thus these authors are the first to my knowledge to attribute the fall in the pro-cyclicality of labor productivity to a rise in the productivity of intangible capital. However, they do not consider the role of an IC shock and they focus on the productivity boom of the 1990's whereas my focus is on the period generally associated with the Great Moderation beginning in the mid 1980's. I choose this break date following common practice in the literature (Gali and Gambetti (2009), Barnichon (2010), Champagne and Kurmann, 2012 and GVR (2014)) of dating the changes in labor market dynamics, including the vanishing pro-cyclicality of productivity, from the start of the Great Moderation, regarding the timing of which there is some consensus (McConell and Perez-Quiros, 2000; Stock and Watson, 2003). Finally, Gourio and Rudanko (2014) are able to account for the counter-cyclical and highly volatile labor wedge (ratio of the marginal rate of substitution of households and the marginal product of labor of firms) when they incorporate complementary IC production into a simple RBC framework.

The rest of the paper is organized as follows, Section 2 provides a summary of the changes in labor market dynamics documented in the literature for the pre and post-84 periods, Section 3 presents the model with IC and highlights the key channels through which labor market and other aggregates are affected, Section 4 analyzes quantitatively the impact of a rising share of IC in the model economy and Section 5 concludes.

## 2 Changes in labor market dynamics

In this section I review the evidence provided in the literature of the key changes in labor market dynamics in the post-1984 period. Different authors using varied data sets, lengths of time series and filtering methods find largely similar and statistically significant changes in key labor market moments. I especially focus on and compare my model generated results to GVR (2014) since their empirical study jointly focuses on the three main changes in labor market dynamics that I seek to understand in this paper. In this section however, I discuss results from a wide range of studies in the literature, all

of whom report similar changes in labor market trends in post-84 U.S. data.

**The rising relative volatility of labor input** For BP filtered log data, Gali and Gambetti (2009), using an estimated structural vector autoregression (SVAR) with time-varying coefficients and stochastic volatility, report an increase in hours volatility relative to output from 0.79 in the pre-84 to 1.10 in the post-84 period. GVR (2014) report labor market moments for both the private sector and the total economy. The former uses data from the BLS labor productivity and cost program (LPC) while the latter uses an unpublished series of economy-wide hours constructed by the BLS, also used in Francis and Ramey (2009). For BP filtered data, GVR (2014) find an increase in relative hours volatility for the private sector from 0.86 to 1.06 for the pre and post-84 periods while for the total economy volatility increased from 0.71 to 0.76. When using HP filtered data they find relative hours volatility increased from 0.80 to 1.20 or by 50% for the private sector and from 0.70 to 0.89 or by 27% for the total economy. They also report slightly smaller but statistically significant increases in relative employment volatility for the same time periods for the different filtering methods.

**The rising volatility of real wage** GVR (2014) find that volatility of compensation per hour for the private sector in the National Income and Product Accounts (NIPA) increased from 0.71 to 1.38 in absolute terms and from 0.30 to 0.88 relative to GDP from their pre-84 to post-84 sample using BP filtered data. For HP-filtered data wage volatility increased from 0.85 to 1.03 or by 21% (absolute) and 0.35 to 0.86 or by 46% (relative to GDP) between the pre and post-84 periods. Combining NIPA and the unpublished economy-wide series for hours constructed by BLS, they report volatilities of compensation per hour for the total economy as well. For this measure of the wage rate, volatility increased from 0.84 to 0.95 or by 13% in absolute terms and 0.34 to 0.80 or by 35% relative to GDP for the HP filtered series. They also report (smaller) increases in absolute and relative volatility of earnings per hour using a slightly smaller data set from the Current Employment Statistics (CES) across the different filtering methods. Gourio (2007) and Champagne and Kurmann (2013) find similar increases in absolute and relative real wage volatility between the pre-84 and post-84 periods.

**The fall in pro-cyclicality of labor productivity** Stiroh (2009) reports that correlation of labor productivity growth and hours growth declined substantially during the period after the mid-80s. Gali and Gambetti (2009) find that the unconditional correlation of labor productivity and output (logged and BP-filtered) fell close to zero in the post-84 period from a high of 0.61 in their pre-84 sample whereas unconditional correlation between labor productivity and hours went from 0.18 to -0.46. When a first difference transformation of the data is used instead of a BP-filter they find a similar although weaker (and statistically significant) change in the correlations of these variables. GVR (2014) report similar declines in correlations of labor productivity across their alternative definitions of variables and filtering methods. Specifically, for the private sector, using HP filtered data and hours as the measure of labor input they find the correlation of labor productivity with GDP fell from 0.61 in pre-84 to 0.04 in post-84 or by 57%. Correlation of labor productivity with hours went from 0.17 to -0.56, a fall of 73% in the same period.

# 3   Model

The model is a two-sector variant of the standard RBC framework with a final goods and an intangible capital investment sector. A representative firm combines physical capital, intangible capital and labor to produce final goods and IC. Both final goods and IC sectors are subject to productivity shocks. The firm accumulates physical and intangible capital while labor is supplied by a representative household.

**Firm**

The firm solves the following problem,

$$\text{Max} \, E_t \sum_{t=0}^{\infty} M_{0,t}[y_t - w_t l_t - x_{k,t}], \tag{1}$$

subject to,

$$y_t = A_t k_{y,t}^{\alpha} z_t^{\gamma} (l_{y,t})^{1-\alpha-\gamma}, \tag{2}$$

$$x_{z,t} = B_t k_{z,t}^{\alpha} z_t^{\gamma} (l_{z,t})^{1-\alpha-\gamma}, \tag{3}$$

$$k_{t+1} = (1 - \delta_k)\, k_t + x_{k,t} - \zeta_k \left( \frac{x_{k,t}}{k_t} \right) k_t, \tag{4}$$

$$z_{t+1} = (1 - \delta_z)\, z_t + x_{z,t} - \zeta_z \left( \frac{x_{z,t}}{z_t} \right) z_t. \tag{5}$$

where $M_{0,t}$ is the stochastic discount factor, in equilibrium equal to the marginal rate of substitution of households. $y_t$ is total output in the final goods sector and $l_t$ is the total labor employed by the representative firm. $k_{i,t}, z_t$ and $l_{i,t}$ where $i = \{y, z\}$, are the physical capital, intangible capital and labor inputs in the final good and IC sectors respectively in period $t$. $x_{i,t}$ are investments in physical capital and IC with $\delta_k$ and $\delta_z$ their respective depreciation rates. $x_{ki,t}$ are physical capital investments in the final good and IC sectors such that $x_{ky,t} + x_{kz,t} = x_{k,t}$ and $k_{y,t} + k_{z,t} = k_t$. Equations (2) and (3) are the production functions for final goods and IC investments respectively. Both physical and intangible capital investments are associated with convex adjustment costs specified by the functions $\zeta_i(.)$.

$A_t$ is a productivity shock in the final goods sector. It follows a first order autoregressive process,

$$\log A_t = \rho_A \log A_{t-1} + e_t^A,$$

where $e_t^A$ are zero-mean, i.i.d. innovations. $B_t$ is a productivity shock to the IC investment sector, which also follows an AR(1) process given by

$$\log B_t = \rho_B \log B_{t-1} + e_t^B$$

such that $e_t^B$ are zero-mean, i.i.d. innovations. Note that this is a shock to the productivity of investing in IC, not to the productivity of IC in final goods production. In this sense

$B_t$ is more akin to a neutral technology shock than to an investment specific technology (IST) shock. I discuss the effects of this shock in more detail in Section 4.5, comparing it to both neutral technology and IST shocks in the literature. Finally equations (4) and (5) give the laws of motion for physical and intangible capital accumulation respectively.

My aim in this paper is to study the effects of an increase in the share of intangible capital ($\gamma$) relative to physical capital and labor in the production process. Corrado et al (2005) show in their empirical work that IC's share in income increased from 9.4% in the period 1973-1995 to 14.6% in 1995-2003. Particularly for 2000-2003, the share of income earned by the owners of intangible capital reached 15%, while the owners of physical capital received 25%; the remaining 60% was absorbed by labor. Their calculations are complemented by results from other studies. For example, Karabarbounis and Neiman, (2014) show that labor's share in output declined substantially from around 67% in the early 1980's to 60% in 2012. More recently, Koh, Santaeulalia-Llopis and Zheng (2016), using updated national income and product accounts (NIPA) data from Bureau of Economic Analysis show that the labor share in fact declined from 68% in 1947 to 60% in 2013. They further document that this secular decline in the U.S. labor share is entirely driven by the increasing importance of software, R&D and artistic originals in national income through this period.

In light of the above studies and following Giglio and Severo (2012) I assume that the majority of the increase in IC's income share comes from a decline in labor's share in income and, a smaller fraction from the income share of physical capital, $\alpha$. This methodology helps maintain the constant returns to scale in production. Note, again in keeping with the evidence above, this implies that while labor's share in income declines with a rise in $\gamma$, there is an overall increase in the income share of capital, $\alpha + \gamma$, in the model. Specifically, I assume that an increase in $\gamma$ causes $\alpha$ to change in the following way,

$$\alpha_1 = \alpha_0 - \tau(\gamma_1 - \gamma_0), \tag{6}$$

where the subscripts 0 and 1 refer to the pre-84 and post-84 income shares of factor inputs respectively and $\tau < 1$ is the fraction of the increase in $\gamma$ that is deducted from $\alpha$. The

remaining, $1 - \tau$, is then deducted from the income share of labor. Thus an increase in $\gamma$ leads to a less than proportionate decline in the income shares of both labor and physical capital.

The firm's optimality condition with respect to IC is given by,

$$E_t \left( M_{t+1} \gamma \frac{y_{t+1}}{z_{t+1}} + \lambda_{t+1} \left( 1 - \delta_z + \gamma \frac{x_{z,t+1}}{z_{t+1}} - \zeta_z'(\frac{x_{z,t+1}}{z_{t+1}}) - \zeta_z(\frac{x_{z,t+1}}{z_{t+1}}) \right) \right) = \lambda_t, \qquad (7)$$

where $\lambda$ measures the "shadow value" of the IC constraint to the firm and equation (7) gives an intuitive expression for it. $\lambda_t$ equals the expected discounted value of the marginal benefit from having an extra unit of $z_{t+1}$ which is the sum of two components. The first is IC's contribution to an increase in output of final goods by the amount of its discounted marginal productivity. The second is the change in the expected shadow value of the IC constraint due to an increase in IC investment, by the amount of its marginal productivity in the IC sector, along with the un-depreciated amount of IC.

Labor demand in the final good and IC sectors are given by the respective sectoral first order conditions with respect to labor,

$$(1 - \alpha - \gamma)\, A_t k_{y,t}^\alpha l_{y,t}^{-\alpha-\gamma} = w_t, \qquad (8)$$

$$\lambda_t \,(1 - \alpha - \gamma)\, B_t k_{z,t}^\alpha l_{z,t}^{-\alpha-\gamma} = w_t. \qquad (9)$$

In both equations (8) and (9) the firm equates the marginal cost of employing an additional unit of labor, or the real wage, on the right hand side, to its marginal benefit on the left. In the final goods sector in equation (8), the marginal benefit of an extra unit of labor is simply its marginal product. In the IC sector in equation (9), the marginal benefit of any additional labor is its marginal product multiplied by the shadow value of the IC constraint to the firm, $\lambda_t$[3]. That is, the effect of an increase in IC accumulation

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[3]See Appendix for a discussion of the remaining first order conditions of the model.

by the firm is internalized by the marginal benefit of additional labor employed in the IC sector.

## Households

The representative household maximizes consumption,

$$\text{Max} \qquad E_t \sum_{t=0}^{\infty} \beta^t (c_t - \psi \frac{l_t^{1+\frac{1}{\eta}}}{(1+\frac{1}{\eta})}),$$

subject to the following budget constraint,

$$c_t = w_t l_t, \tag{10}$$

where $c_t$ is the household's consumption and $l_t$ is total labor supplied by the household. $\psi$ represents the disutility derived from working and $\eta$ is the Frisch elasticity of labor supply. The first order condition with respect to labor supply is then given by,

$$l_t = \left(\frac{w_t}{\psi}\right)^{1/\eta}. \tag{11}$$

We assume preferences of the form described in Greenwood, Hercowitz and Hoffman (1988) (hereafter GHH) in this section. As is well known in the literature these preferences do not take into account the wealth effect of a change in the wage rate on labor supply as represented by the household's optimality condition above. From equation (11), labor supply is a function of the wage rate alone (and not of household consumption). Therefore any change in the wage rate causes labor input to change in the same direction and by an equivalent amount.

Results using log preferences are presented in our sensitivity analysis section (Appendix A.3). While the qualitative results of the model remain unchanged under log preferences, the responsiveness of labor supplied changes more than the wage rate does with an increase in $\gamma$ under that specification. I include a detailed discussion of these effects in the Appendix. Empirically, however, the change in labor input volatility is lower than that

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in the wage rate. The GHH specification allows for changes in both labor and wage to be of a similar magnitude. I therefore consider GHH preferences in this section and present results using log preferences in the Appendix.

**Definition of equilibrium**

An equilibrium in this economy is defined in the usual way. That is, an equilibrium is a sequence of wages, $\{w\}_{t=0}^{\infty}$, and corresponding labor inputs in the two sectors $\{l_{y,t}, l_{z,t}\}_{t=0}^{\infty}$ such that (i) firms maximize profits subject to equations (2)-(5) and households maximize their utility subject to equation (12) taking as given the exogenous and endogenous states $\{A_t, B_t\}, \{k_{y,t}, k_{z,t}, z_t\}$ and the price sequence $\{w\}_{t=0}^{\infty}$ for labor, and (ii) the capital, labor and goods markets clear as follows:

$$k_{y,t} + k_{z,t} = k_t, \tag{12}$$

$$l_{y,t} + l_{z,t} = l_t, \tag{13}$$

$$c_t + x_t = y_t, \tag{14}$$

.

where $k_t$ is the aggregate physical capital stock in the economy.

# 4　The impact of a rise in IC

Using steady state versions of the equations in Section 3 it can be shown that,

$$\frac{l_z}{l} = \frac{\beta \gamma}{1 - \gamma(1 - \beta)}. \tag{15}$$

Equation (15) is the share of IC employment in total labor supplied. It is a positive function of $\gamma$, the income share of IC in the production function (full derivation of equation

(15) is in Appendix A.2). Intuitively this is straightforward, since an increasing share of IC in the production process implies a larger emphasis on production of IC investments and hence greater employment in the IC sector. Thus an increase in $\gamma$ in the model is directly associated with a rising intangible sector employment share. I use this equation to derive the pre- and post-84 values of $\gamma$ for the calibration exercise in the next section, given the documented rise in IC's employment share in Figure 1 (Section 1).

I next examine the interaction between final goods and the IC sector quantitatively with the aim to understand the aggregate consequences of an increase in $\gamma$. I particularly focus on changes in labor market variables comparing the effects of the IC shock to those arising from fluctuations in the pure productivity shock before studying their joint effects. An analysis of the sensitivity of the model's results to changes in the key parameters especially those related to the IC sector shock is provided in Appendix A.3.

## 4.1    Calibration

The model is calibrated to the U.S. economy with the time period $t$ representing a quarter. I set the discount factor of the households, $\beta = 0.99$ corresponding to a quarterly interest rate of 1%. I assume $\eta = 2.5$ for the Frisch elasticity of labor supply which lies in between the range of 2 to 4 typically estimated by macroeconomic studies. The results of the model remain unchanged for reasonably higher or lower values of $\eta$. The disutility of labor parameter $\psi$ is a constant and is set to equal 2.64 in order that total steady state hours worked is 1/3 or $l_t = 0.33$ in the pre-84 period in the model. Like all parameters of the model (except $\gamma$), I do not allow $\psi$ to change when I consider a higher value of $\gamma$, however, unlike other parameters of the model $\psi$ has no quantitative impact on the model's results.

The depreciation rate of physical capital, $\delta_k$ is set to the standard quarterly value of 0.025 implying a yearly depreciation rate of 10%. The depreciation rate of intangible capital, $\delta_z$ is more difficult to calibrate. Corrado et al (2009) use limited information available for different types of IC to compute the annual depreciation rate for each type. The corresponding quarterly depreciation rates for the different types of IC are 5% for scientific

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and non-scientific R&D, 8.25% for computerized information other than software, 10% for firm-specific resources and 12% for brand equity. A simple average yields a depreciation rate for IC of around 8%. McGrattan and Prescott (2009, 2012) assume benchmark annual depreciation rates for IC between 0-7%, which imply much smaller quarterly depreciation rates of between 0-1.75%. I assume a benchmark quarterly depreciation rate for IC of 6.5%, which implies an annual rate of 26% and lies in between the values reported by Corrado et al (2009) and those used by McGrattan and Prescott (2009, 2012). I report results for both higher and lower values of $\delta_z$ used in the literature, in the Appendix.

The convex adjustment cost function for investment in physical capital is of the form, $\zeta = \frac{\phi_k}{2} \left( \frac{x_{k,t}}{k_t} - \delta \right)^2 k_t$, such that the cost of adjustment depends on the ratio of investment to capital and scales up with the level of capital. $\phi_k$, the capital adjustment cost parameter is chosen to match a volatility of investment in physical capital that is about three times that of output. $\gamma$, the income share of IC and the parameter of interest in the model, requires values for the periods before and after the Great Moderation. I allow the value of $\gamma$ to shift in a way that causes the share of employment in the IC sector to go from a targeted pre-84 value in the occupation data analyzed in Section 1, to a post-2010 target. In Section 1 the employment share of IC at the beginning of the period under consideration is 8% rising to 16% in 2016. Using these values and the steady state equation (15), gives us the pre- and post-84 values of $\gamma$ of 0.08 and 0.16 respectively. These are in line with Corrado et al (2005) who estimate that the income share of IC rose from an average 9.4% in the period 1973-1995, to an average of 14% in 1995-2003.

$\alpha_0$, which is the Pre-84 value of physical capital's income share in equation (10), is set to 0.28 such that the total elasticity of the two types of capital taken together is equal to 0.35 in the Pre-84 period. This implies that labor's income share is 0.65 to begin with, which is in line with the findings of Karabarbounis and Neiman (2014) and Koh et al. (2016) for the pre-84 period. Given the strong evidence in favor of a significant decline in labor's income share that accompanied the period of increase in IC's income share in the literature (Karabarbounis and Neiman, 2014; Koh et al, 2016; Corrado et al, 2009), I set $\tau$, the percentage increase in $\gamma$ that is deducted from $\alpha$ (in equation (6)), equal to

30% implying 70% of the rise in IC's income share is deducted from the share of labor. Labor's income share thus declines to 59% in the post-84 period in the model, in line with the estimates in these studies, causing the total income share of (intangible and physical) capital to rise to 41%.

I set the standard deviation of innovation to the productivity shock in final goods to 0.01 to match the pre-84 average output volatility in GVR with a persistence $\rho$ of 0.95 following the business cycle literature. There is not much information available in the literature about selecting parameters governing the dynamics of the IC shock. I calibrate the volatility of the IC shock $\sigma_b = 0.0285$ to approximately match the volatility of the average rate of investment in IC calculated as the ratio of gross fixed capital formation of intangible assets divided by the stock of IC, from Corrado, Haskel, Jona-Lasinio and Iommi (2012). Their data is available from 1995-2010. I set the persistence of the IC shock to $\rho_b = 0.88$ and conduct sensitivity analysis using both higher and lower values of $\rho_b$ relative to $\rho$. I assume an adjustment cost function for IC similar to that of physical capital, $\zeta_z = \frac{\phi_z}{2} \left( \frac{x_{z,t}}{z_t} - \delta \right)^2 z_t$ and set $\phi_z = 1.45$ in order to target the pre-84 correlation between labor productivity and output in GVR. Finally, I assume the shocks to be highly correlated in this section and present results with uncorrelated shocks in the Appendix.

| Parameter | Explanation | Value | Target |
|---|---|---|---|
| $\beta$ | Discount rate | 0.99 | Quarterly interest rate=0.01 |
| $\eta$ | Labor supply elasticity | 2.5 | Literature |
| $\psi$ | Disutility of labor parameter | 2.64 | Hours worked=0.3 |
| $\delta_k$ | Depreciation rate of $k$ | 0.025 | Literature |
| $\delta_z$ | Depreciation rate of IC | 0.065 | Literature |
| $\phi_k$ | Adj. cost parameter for k | 5 | Rel. investment volatility=3 |
| $\phi_z$ | Adj. cost parameter for IC | 1.45 | Pre-84 $Corr(LP_t, y_t)$ (GVR) |
| $\alpha_0$ | Income share of $k$ ($\alpha$ pre-84) | 0.28 | Literature |
| $\gamma_0$ | Pre-84 IC income share | 0.08 | Pre-84 IC employment share |
| $\gamma_1$ | Post-84 IC income share | 0.16 | Post-2010 IC employment share |
| $\rho$ | Persistence of prod. shock | 0.95 | Literature |
| $\sigma$ | St. dev. of prod. shock | 0.01 | Pre-84 output volatility (GVR) |
| $\sigma_b$ | St. dev. of IC shock | 0.0285 | Volatility of IC investment rate |
| $\rho_b$ | Persistence of IC shock | 0.88 | |
| corr($e, e_b$) | Correlation of shocks | 0.9 | |
| Model Determined | | | |
| $\alpha_1$ | Post-84 income share of $k$ | 0.25 | From equation (10) |

Table 1: Calibrated parameter values

## 4.2 Response of labor, real wage and output to a productivity shock

In this section, I study the response of key labor market aggregates to a one standard deviation shock to the productivity of final goods alone. In other words, the innovation to the IC shock is set to zero throughout this section. I start the model at steady state and simulate it for one thousand periods. I drop the first two hundred observations and HP-filter the model generated time series. The results are presented in Figure 2.
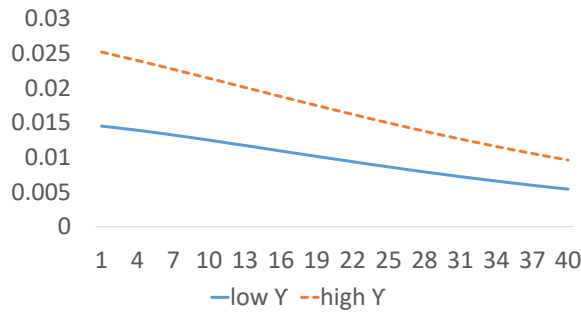
(a) Output
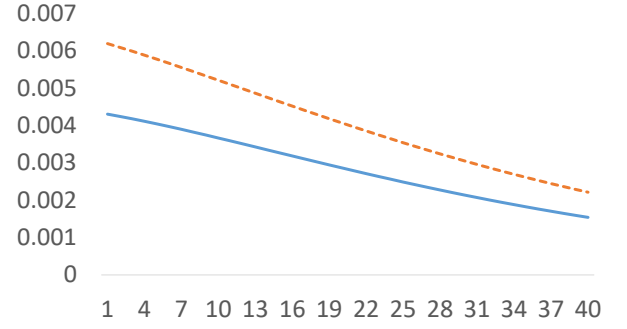
(b) Hours

(c) Wage
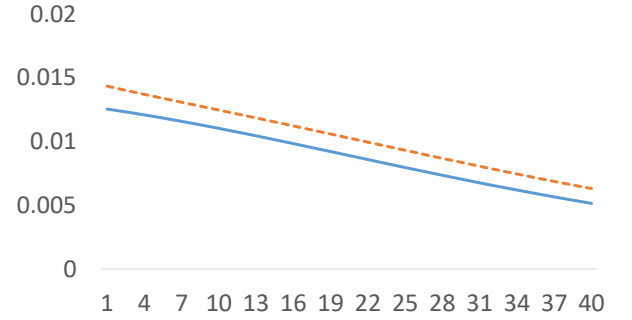
(d) Labor productivity
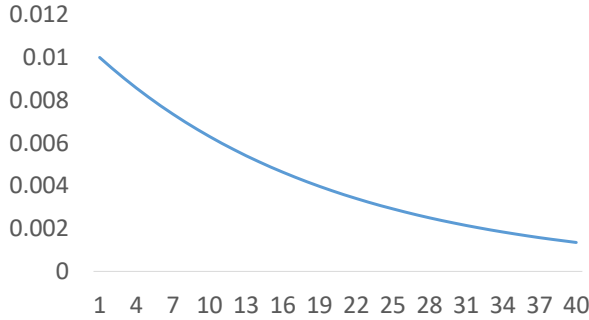
(e) Productivity shock
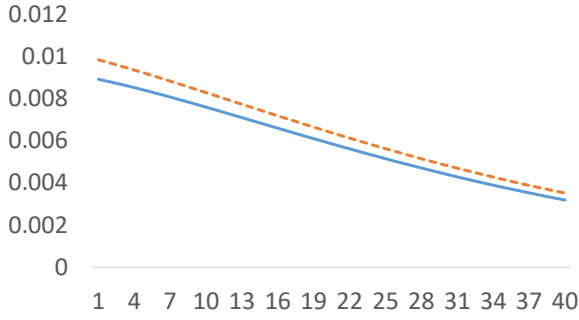
Figure 2: The figures show impulse responses to a "pure" (i.e. I leave the innovation to the IC shock unaffected) productivity shock. The responses are percent deviations from steady state.

As expected, a positive productivity shock increases final output, employment, labor productivity and real wage upon impact as in standard RBC models. Unlike the standard RBC framework however, labor input volatility relative to output is higher in this model. This is because unlike in the standard setting, an increase in final good's productivity here, increases the marginal productivity of IC. This causes labor demand and consequently labor input in the IC sector to rise in addition to the original increase in labor input in final goods. Thus aggregate hours are more volatile relative to output in this framework,

making this extension an improvement over the standard RBC framework which is known to generate too little volatility of employment. This increased responsiveness of labor input to a technology shock due to the inclusion of IC in an otherwise standard RBC model is also highlighted in Gourio and Rudanko (2014). Finally, in Figure 2, there is a strong positive correlation between labor productivity and both output and labor in response to the productivity shock. These correlations are about 0.99 (see Table 2), which is again standard in simple RBC models.

|  | Low $\gamma$ | High $\gamma$ | Relative |
|---|---|---|---|
| $vol(y)$ | 2.53 | 2.35 | 0.93 |
| $vol(l)$ | 1.8 | 1.66 | 0.92 |
| $vol(w)$ | 0.71 | 0.65 | 0.92 |
| $vol(l)/vol(y)$ | 0.71 | 0.71 | 1 |
| $vol(w)/vol(y)$ | 0.28 | 0.28 | 1 |
| $Corr(lp, y)$ | 0.99 | 0.99 | |
| $Corr(lp, l)$ | 0.99 | 0.99 | |

Table 2: The table reports moments of model implied output, hours, wages, and labor productivity in response to a pure (i.e. we leave the innovation to the IC shock unaffected) productivity shock. All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

In Table 2, the absolute volatilities of output, employment and real wage fall as $\gamma$ rises[4]. At higher $\gamma$ the IC constraint faced by the firm is stronger, in other words, the firm needs to raise its investment in IC more in order to increase the production of final goods in response to the productivity shock. Thus output volatility in response to the productivity shock is lower at higher $\gamma$ causing labor and wage to respond less as well to the same shock. The relative volatility of hours and wage however, remains unchanged as $\gamma$ rises. Finally the correlation of productivity with both labor and output is not affected by the rise in $\gamma$.

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[4]Volatility of a variable $x$ in the model, is measured by its coefficient of variation, such that $vol(x) = \frac{[var(x_t)]^{1/2}}{mean(x_t)}$.

## 4.3  Response of labor, real wage and output to a IC shock

In this section, I repeat the simulation exercise above with an IC shock alone. That is, I shut down the productivity shock to final goods and allow only for a one standard deviation shock to the IC sector. As before, I simulate the model for one thousand periods, drop the first two hundred observations, HP-filter the model generated time series and report the impulse responses for high and low values of $\gamma$.



(a) Output

(b) Hours

(c) Wage
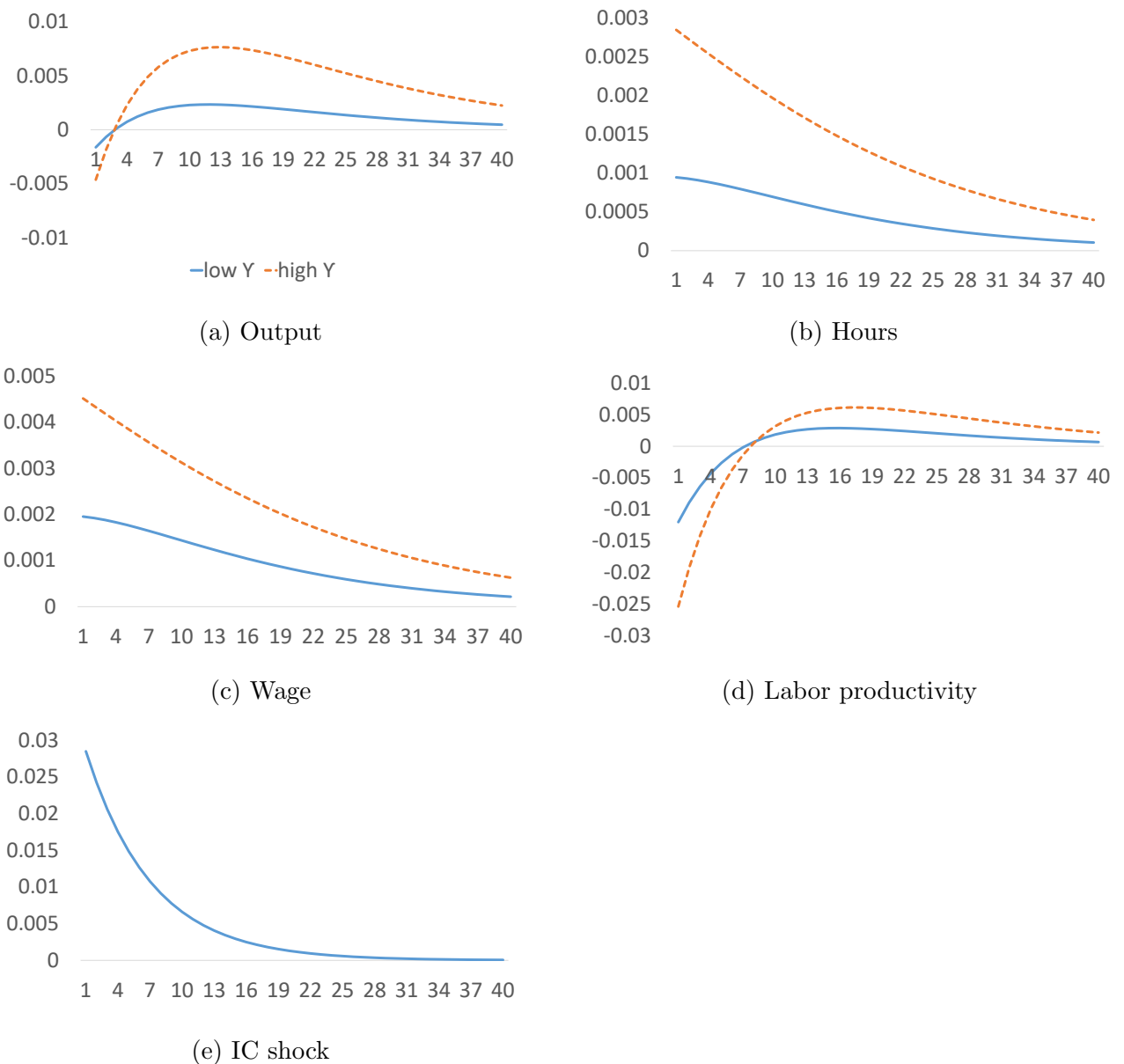
(d) Labor productivity

(e) IC shock

Figure 3: The figures show impulse responses to a pure IC shock. The responses are percent deviations from steady state.

Figure 3 presents the impulse responses of labor input, wages, output and productivity

to a pure IC shock. A positive IC shock causes reallocation of labor from the final goods to the IC sector upon impact as the productivity of the latter increases relative to the former. Thus output falls upon impact of the IC shock while IC investment rises. Higher IC investment demand drives up labor demand in IC causing total labor input to rise in turn. Thus measured output and aggregate labor input move in opposite directions as the IC shock hits, causing measured labor productivity to fall upon impact in Figure 3. This generates a negative correlation between labor input and measured labor productivity as observed in Figure 3. Measured output and labor productivity are, however, positively correlated since both fall upon impact of the IC shock. The wage increases upon impact of the IC shock driven by the higher labor demand of the IC sector. As the initial impact of the IC shock passes, the stock of IC in the economy increases, final good's output rises and employment in the IC sector falls causing the wage rate and total employment to climb back down while labor productivity recovers.

|  | Low $\gamma$ | High $\gamma$ | Relative |
|---|---|---|---|
| $vol(y)$ | 0.44 | 0.7 | 1.59 |
| $vol(l)$ | 0.4 | 0.75 | 1.88 |
| $vol(w)$ | 0.16 | 0.3 | 1.88 |
| $vol(l)/vol(y)$ | 0.91 | 1.07 | 1.18 |
| $vol(w)/vol(y)$ | 0.36 | 0.43 | 1.18 |
| $Corr(lp, y)$ | 0.81 | 0.79 | |
| $Corr(lp, l)$ | -0.76 | -0.81 | |

Table 3: The table reports moments of model implied output, hours, wages, and labor productivity in response to a pure IC shock. All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

At higher $\gamma$ final output falls more initially due to the IC shock than when $\gamma$ is lower, as observed in Figure 3. Moreover, final output eventually recovers after the initial impact of the IC shock passes at the higher $\gamma$. Final output volatility is thus higher at higher $\gamma$ as seen in Table 3. This can be explained as follows: a higher $\gamma$ necessitates a larger reallocation of resources from final goods to the IC sector upon impact of the IC shock,

since IC investments themselves require IC for production and the elasticity of IC is given by $\gamma$ in both sectors. As more IC is accumulated final goods output eventually increases more than before due to the higher $\gamma$.

The larger increase in IC investment (upon impact) and final output (eventually) at higher $\gamma$, also causes labor demand to rise in both sectors causing real wage and (from equation 13) total labor input to respond more to the IC shock. Thus labor and real wage volatility increase with $\gamma$ in response to a pure IC shock. From Table 3, the relative volatility of labor and wage also rise in this case (unlike in case of the productivity shock above). This is because, the increased responsiveness of labor input and real wage in both sectors is measured but only the rise in final goods output (and not intangible investment) is measured causing the volatility of wage and labor input relative to final output to be higher at higher $\gamma$.

In the presence of the pure IC shock, the larger drop in measured output as $\gamma$ increases, accompanied by the larger increase in labor input due to the increased IC sector labor demand, causes the negative correlation between labor input and measured labor productivity to be stronger at higher $\gamma$ (see Table 3). Finally, note that the level of volatility generated by the IC shock is much lower compared to the productivity shock in Section 4.2. However, the *changes* in volatilities are much more substantial. This latter exerts quite a large influence on labor market and aggregate output dynamics as $\gamma$ rises in the presence of both shocks. We study the joint effects of the two shocks together next.

## 4.4 Effect of intangible capital when both productivity and IC shocks are present

In this section, I allow both shocks to be jointly present as $\gamma$ increases. The model is solved similarly to the above two sections. As emphasized earlier, the aim is to investigate if the increase in IC's importance in production in recent decades can move several macroeconomic moments in the direction observed in the data. Table 4 presents the correlations generated by the complete model and compares them to the different empirical

results discussed in Section 2.

| | Correlation Productivity | | | | | |
|---|---|---|---|---|---|---|
| | with output | | | with employment | | |
| Data | Pre-84 | Post-84 | Relative | Pre-84 | Post-84 | Relative |
| U.S. (GVR) | 0.61 | 0.04 | -0.57 | 0.17 | -0.56 | -0.73 |
| 1949-2007, HP filtered | | | | | | |
| U.S. (OR) | 0.77 | 0.67 | -0.1 | 0.27 | -0.03 | -0.30 |
| 1960-2007, BP filtered | | | | | | |
| U.S. (CK) | 0.65 | 0.01 | -0.64 | 0.21 | -0.50 | -0.71 |
| 1964-2006, HP filtered | | | | | | |
| Model | 0.6 | 0.03 | -0.57 | 0.4 | -0.31 | -0.71 |
| HP filtered | | | | | | |

Table 4: Correlation Productivity. GVR=Gali and VanRens (2014), OR=Ohanian and Raffo (2012) and CK=Champagne and Kurmann (2012)

Firstly note that the model generated correlation of measured labor productivity with output is much lower in Table 4 than in the above sections. This is because, as shown in Sections 4.2 and 4.3, a positive productivity shock increases output and labor input while a positive IC shock reduces output and increases labor input. Thus in the presence of an IC shock, output rises less while labor input rises more, than if only a pure technology shock was present. Measured labor productivity therefore rises much less than in the case of the pure technology shock causing pro-cyclicality of productivity with respect to output to be lower (recall that I used the pre-84 correlation between output and productivity in GVR as a target in my calibration). As $\gamma$ rises, the increasing share of IC increases the propagation of the IC shock implying, the increase in output is even lower while the rise in hours accompanying the higher $\gamma$ is higher still. This causes the pro-cyclicality of measured productivity with respect to output to fall substantially as $\gamma$ rises in Table 4, similar to the changes reported by the empirical studies for this period.

Labor input and measured productivity move strongly positively in response to a pure technology shock and strongly negatively in response to a pure IC shock. In the presence

of both shocks therefore, the (positive) correlation between hours and productivity generated by the model is lower than in the case of a pure technology shock. As $\gamma$ rises to its higher value, the higher propagation of the IC shock implies the negative correlation between labor input and measured productivity generated by this shock plays a larger role. In other words, an increase in $\gamma$ causes larger increases in labor input (driven by the IC shock) to be associated with smaller increases in measured output (due to higher reallocation from final goods to IC investment). This causes a sharp decline in the correlation of productivity with labor input such that productivity turns strongly counter-cyclical in Table 4, as observed in the data.

|  | Standard Deviation | | | Relative standard deviation | | |
|---|---|---|---|---|---|---|
|  | 1) Pre-84 | 2)Post-84 | 3)Post-84/Pre-84 | 4)Pre-84 | 5)Post-84 | 6)Post-84/Pre-84 |
| **Data** | | | | | | |
| $\sigma(y)$ | 2.47 | 1.19 | 0.48 | 1 | 1 | 1 |
| $\sigma(l)$ | 1.71 | 1.06 | 0.62 | 0.70 | 0.89 | 1.27 |
| $\sigma(w)$ | 0.84 | 0.95 | 1.14 | 0.34 | 0.80 | 2.33 |
| **Model** | | | | | | |
| $\sigma(y)$ | 2.4 | 2.2 | 0.91 | 1 | 1 | 1 |
| $\sigma(l)$ | 2.15 | 2.36 | 1.10 | 0.89 | 1.07 | 1.20 |
| $\sigma(w)$ | 0.87 | 0.96 | 1.10 | 0.36 | 0.44 | 1.20 |

Table 5: The table reports volatilities of model implied quantities of output, hours, wages, and labor productivity. All series were HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments. Moments of HP-filtered (total economy) data for sample period 1949-2007 is from Gali and Van Rens (2014).

| $vol(x_k)/vol(y)$ | $vol(c)/vol(y)$ | $Corr(x_k, y)$ | $Corr(c, y)$ | $Corr(l, y)$ | $Corr(c, x_k)$ |
|---|---|---|---|---|---|
| 3 | 0.62 | 0.92 | 0.86 | 0.98 | 0.6 |

Table 6: The table reports moments of other key variables implied by the model. $x_k$ is investment in physical capital, $c$ is consumption, $l$ is total labor supplied and $y$ is total output. All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

As argued by Stiroh (2009) and Gali and Gambetti (2009), a substantial fraction of the decline in output volatility characterizing the Great Moderation can be explained

by the sizeable decline in the correlation between labor productivity and hours. From Table 4, an increase in $\gamma$ does indeed lead to a large decline in the correlation of hours and productivity in the model as output responds less and labor responds more to a combination of technology and IC shocks. Thus output volatility declines while labor input volatility rises with $\gamma$. From Table 5, real wage volatility also rises with $\gamma$, both absolutely and relative to output.

Note that while the model predicts an increase in the absolute volatility of labor input, both GVR (2014) and Gali and Gambetti (2009) report a small decline in the volatility of this variable in the data during this period. However, the decline in absolute volatility of labor input in the data is a small one and models trying to explain one or more of these labor market features often find an increase in absolute labor input volatility. Nucci and Riggi (2013) and Champagne and Kurmann (2012) are two such examples. However, labor input volatility relative to output increased substantially in the data as it does in the current framework. The model generated 22% increase in labor input volatility relative to output represents over 80% of the increase in relative hours volatility documented by GVR (2014).

Table 6 presents some other key business cycle statistics generated by the benchmark model with IC. Other than investment in physical capital, none of these moments were targeted in the calibration process. It is clear from the table that the model does a good job of reproducing standard business cycle moments. Moreover, it generally improves labor market related results along several lines, like generating a higher volatility of hours relative to output and a more realistic positive correlation between measured productivity and output. Thus the model provides a framework within which the recent shifts in labor market dynamics arise jointly as a result of the rising importance of IC, without sacrificing, and often improving upon, key business cycle moments.

## 4.5 On the nature of the IC shock

In this section, I briefly compare the IC shock of my model with both investment specific technology (IST) shocks of Greenwood, Hercowitz and Krusell (1997, 2000) (or GHK) and

neutral or multi-factor productivity shocks (MFP). This is especially important because, on the surface, some effects of an IC shock resemble that of an IST shock, but upon careful analysis it becomes clear that the mechanisms involved are in fact more similar to an MFP shock to the IC investment sector. In this sense, the setting of the IC shock in this paper can be likened to the MFP shock to investment in physical capital in Guerrieri et al (2014)[5].

Similar to an MFP shock, the IC shock in my model raises output, investment (in both types of capital) and employment in the IC sector. A differentiating feature of IC is that intangible investment is not measured, implying, an increase in the production of IC investment brought about by the IC shock does not raise aggregate (measured) output. In fact, output falls as resources reallocate from the final goods to the IC sector in response to the IC shock. Consumption falls with output but investment in both types of capital increase due to the higher productivity of the IC sector which uses both types of capital in its production. Thus the IC shock, working alone, generates a positive correlation between measured output and consumption but a negative correlation between measured output and investment giving rise to a negative correlation between consumption and investment (of both types). It is well established in the literature that an IST shock generates negative co-movement between consumption and investment. Guerrieri et al (2014) show for instance that expansionary MFP shocks boost consumption in every period, whereas expansionary IST shocks cause consumption to fall substantially for many periods generating the negative correlation between investment and consumption commonly associated with IST shocks. This happens because IST shocks make consumption more expensive relative to investment causing agents to substitute in favor of investment and away from consumption. Unlike this mechanism for IST shocks, the negative correlation due to an IC shock arises in the current framework because IC investments are unmeasured. Hence although IC investments rise, output and hence consumption falls in response to the IC shock. Had IC investments been measured, the IC shock would raise total output, total labor supplied, investment and consumption similar to an MFP shock

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[5]The authors provide a good account of the conditions under which an aggregate IST shock can approximate an MFP shock to the investment sector (in physical capital).

and generate the observed positive correlations between these variables.

A second effect of the IC shock that resembles that of an investment specific shock is that both shocks cause labor productivity to fall upon impact. In case of an IST shock however, productivity falls because hours rise immediately but investment takes time to adjust. In case of the IC shock, productivity falls due to the drop in measured output upon impact although there is also an immediate increase in hours. Thus under an IC shock the negative impact on measured productivity is stronger than under an investment specific shock - but as before, if IC investments were measured, both output and hours would rise, reversing the effect of the IC shock on labor productivity and making it resemble a neutral productivity shock to the IC (investment producing) sector.

In sum, there are similarities between the effects of the IC shock in the current model and the IST shocks of GHK, however, the likeness does not stem from the similar nature of the two shocks, but from the assumption that investment in intangibles is unmeasured.

# 5    Conclusion

I study the effects of a rise in the importance of intangible capital in the production process since the mid 1980's, on labor market dynamics. I show that an increase in the share of IC in production, where IC accumulation is subject to additional volatility, causes wage and labor input volatility to rise, both absolutely and relative to income while measured output volatility falls as observed during this period. Additionally, as the propagation of the IC shock relative to the productivity shock increases there is a significant decline in the pro-cyclicality of measured labor productivity relative to both output and labor.

The main effect of an increase in intangible capital in the model, is to lower the responsiveness of wages and hours to the productivity shock in final goods and to raise it to the shock to intangible investments causing volatility of both wage and labor input to rise as the importance of IC and hence the propagation of IC shock rises. Output volatility however, falls because as the share of IC in income increases, more intangible investments

need to be produced before final output can increase in response to the productivity shock.

The fact that measured output increases less in the presence of an IC shock while labor input increases more gives rise to a lower pro-cyclicality of measured labor productivity relative to both output and labor input compared to standard RBC models. The increased propagation of the IC shock, as IC becomes more important, causes the pro-cyclicality of productivity to decline further, with the correlation of productivity relative to labor turning strongly negative as in the data.

The current framework can be extended to study several relevant questions in macroeconomics and finance. One such question relates to the higher volatility of financial variables in the data that generic RBC models fail to replicate. Gomme, Ravikumar and Rupert (2011) for instance document that the rate of return on equity (RoE) is six times more volatile than the return on business capital. The current paper uses a constant returns to scale production function in order to stay close to the canonical RBC framework and therefore generates no profits or firm earnings. To the extent IC is expensed, however, I expect the current framework augmented with decreasing returns to scale or imperfect competition to give rise to firm profits, through equation (1), that are more volatile than output due to the higher volatility of labor and wage in the presence of shocks to IC investment. Since calculations of RoE are based on profits (or dividends) of the firm, it should in turn imply a higher RoE volatility.

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# A    Appendix

## A.1    Firm's optimality conditions

The first order condition for physical capital in the final goods sector is,

$$M_{t+1}E_t\left(\frac{\alpha y_{t+1}}{k_{y,t+1}}+1-\delta-\zeta_k\left(\frac{x_{ky,t+1}}{k_{y,t+1}}\right)+\zeta_k'\left(\frac{x_{ky,t+1}}{k_{y,t+1}}\right)\frac{x_{ky,t+1}}{k_{y,t+1}}\right)=M_t(1+\zeta_k'\left(\frac{x_{ky,t}}{k_{y,t}}\right))\text{(A.1)}$$

The right hand side is the marginal cost of having an extra unit of $k_{t+1}$ which is one unit of output (given up today) plus the associated adjustment cost of the added unit of investment, $\zeta'(.)$. The left hand side gives the marginal benefit of an additional unit of $k_{t+1}$ which is composed of the discounted marginal product of physical capital, the value to the firm of undepreciated future capital and the contribution of the new unit of capital to the marginal decline in installation costs in the future.

The first order condition with respect to physical capital in the IC sector is similarly given by,

$$M_{t+1}E_t\left(\left(1-\delta-\zeta_k\left(\frac{x_{kz,t+1}}{k_{z,t+1}}\right)+\zeta_k'\left(\frac{x_{kz,t+1}}{k_{z,t+1}}\right)\frac{x_{kz,t+1}}{k_{z,t+1}}\right)+\lambda_{t+1}\frac{\alpha x_{z,t+1}}{k_{z,t+1}}\right)=M_t(1+\zeta_k'\left(\frac{x_{kz,t}}{k_{z,t}}\right))\text{(A.2)}$$

Here $\lambda$ is the lagrange multiplier associated with the law of motion for IC given by equation (5). Similar to the final goods sector the marginal benefit of an additional unit of $k_{t+1}$ in IC on the left is equated to its marginal cost on the right. However, unlike the final goods sector, the marginal product of an extra unit of $k_{t+1}$ in the IC sector on the left hand side, is weighted by $\lambda_{t+1}$, the future value of the lagrange multiplier associated with the IC constraint. That is, the contribution to marginal revenue generated from an additional unit of $k_{t+1}$ in the IC sector depends on the expected value to the firm of its future IC constraint. The rest of the terms in equation (8) are similar in meaning to the corresponding terms in equation (7).

## A.2 Steady state analysis

Using steady state versions of equations (5) and (7) gives $\lambda$ as a function of $\frac{y}{z}$ at steady state,

$$\lambda = \left(\frac{\beta\gamma}{\phi(1-\gamma)}\right)\frac{y}{z} \tag{A.3}$$

Substituting (A.3) into the optimality condition for labor in the IC sector (equation 9) and using the steady state of equation (5) once again, we arrive at the following condition for steady state employment in the IC sector,

$$w = \left(\frac{\beta\gamma(1-\alpha-\gamma)}{1-\gamma}\right)\frac{y}{l_z} \tag{A.4}$$

The optimality condition for employment in final goods sector or equation (8) similarly gives us the following steady state expression,

$$w = (1-\alpha-\gamma)\frac{y}{l_y} \tag{A.5}$$

Equating (A.4) and (A.5) above, we get $l_y$ as a function of $l_z$ at steady state,

$$l_y = \frac{1-\gamma}{\beta\gamma}l_z, \tag{A.6}$$

implying a total labor supply $(l_y + l_z)$ of,

$$l = \frac{1-\gamma(1-\beta)}{\beta\gamma}l_z. \tag{A.7}$$

The labor share in the IC sector, or equation (15) of Section 4 is then given by,

$$\frac{l_z}{l} = \frac{\beta\gamma}{1-\gamma(1-\beta)}.$$

## A.3    Sensitivity Analysis

In this section I test the sensitivity of my model's results to some key parameters. The results discussed are presented in Tables 1 and 2.

*Log preferences:* I first substitute the GHH preferences of Section 3 with a standard or log utility function as follows: $u = log(c_{h,t}) - \psi \frac{l_t^{1+\frac{1}{\eta}}}{(1+\frac{1}{\eta})}$. As expected, the wealth effect of a wage change now comes into play under these preferences. Recall that with GHH preferences, the household's first order condition with respect to labor (equation (13)) gives labor as an increasing function of the wage alone. Under log preferences, the same first order condition becomes, $l_t = \left(\frac{w_t}{\psi c_t}\right)^{1/\eta}$, that is, changes in $w_t$ now also affect current consumption, $c_t$.

The substitution effect of a wage change causes labor supplied by households to rise in response to an increase in wage rate as leisure becomes more costly. The wealth effect on the other hand, implies labor supplied falls with wage increases due to an increase in household's consumption (including leisure).

In the context of the current model, an increase in $\gamma$ raises the importance of (intangible) capital in the economy. Thus the incentive to save and invest in IC in an economy with higher $\gamma$, is higher. This greater saving motive causes agents to increase their current labor supply more in response to an increase in the wage rate. Only, since there is no actual saving by households in the model, the households ensure higher consumption tomorrow in a high-$\gamma$ economy by supplying more labor to the IC investment sector today since marginal productivity of future IC stock is higher implying higher wages and hence higher consumption next period. Thus, in a high-$\gamma$ scenario, an increase in the wage rate causes $c_t$ to rise less and $l_t$ to rise more leading to larger changes in labor input for a given change in the wage rate. That is, under log preferences, the increase in labor input volatility due to a rise in $\gamma$, is heightened while the rise in wage volatility is subdued as seen in Panel 1 of Table 1[6]. Under the GHH preferences of Section 3, both

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[6]I re-parametrize the model to reflect the same targets as in Section 4.1 and then keep them constant while raising $\gamma$ in the same way as before. For instance, the standard deviation of the productivity shock is set to match the average output volatility in the pre-84 period in GVR as before while the IC shock is re-calibrated to match the volatility of IC investment rate.

hours and real wage volatility rise by similar amounts as $\gamma$ increases (Table 5). In sum, the results of the benchmark model go through under standard preferences but the rise in relative wage volatility is less pronounced than under GHH preferences. Also in Table 2, the correlations are higher than that observed in the data or under GHH preferences although the pro-cyclicality of productivity relative to both output and labor declines as before.

*IC's depreciation rate ($\delta_z$):* As explained in Section 4.1, the depreciation rate of IC is a difficult parameter to pin down given the dearth of empirical estimates and the wide range of values used in the literature. In Section 4, I use a quarterly depreciation rate of 6.5% (26% annual depreciation rate). In this section I consider both higher and lower values of $\delta_z$ - that is $\delta_z = 0.075$ and $\delta_z = 0.01$ respectively.

From Table 1, the changes in the relative volatilities of labor input and the wage rate due to an increase in $\gamma$ is unaffected across the different values of $\delta_z$ considered. Only, when $\delta_z$ is larger, the fall in output volatility due to an increase in $\gamma$ is smaller and changes in the absolute volatilities of employment and wages are higher as $\gamma$ increases. From equation (9), a higher $\delta_z$ makes the IC constraint less important for the firm (as $\lambda_t$ falls) causing the firm to respond less to the IC-shock. Therefore there is a lower reallocation of labor and other resources from the final goods to the IC sector causing output and hence its volatility to fall less as $\gamma$ rises. Correlations of measured productivity with labor and output in Table 2 for both higher and lower values of $\delta_z$ considered, are largely along the lines of the benchmark model and falls substantially as $\gamma$ increases, with the decline in correlations increasing in the magnitude of $\delta_z$. This is because, when IC depreciates at a higher rate, an increase in $\gamma$ necessitates larger increases in IC investments to account for the higher rates of depreciation in the future IC stock causing the fall in correlations of output and labor with productivity to be more pronounced.

*Correlation of shocks ($\chi$):* In this section I allow the shocks to be uncorrelated ($\chi = 0$). From Tables 1 and 2, the qualitative results remain unchanged, with labor and wage volatility increasing, output volatility declining and the pro-cyclicality of measured productivity falling. The quantitative strength of these results wane however with declining

correlations between the shocks. The model generated results are one of the weakest for the uncorrelated shock with labor and wage volatility rising only 4% and output falling by the same proportion as $\gamma$ rises.

Thus a rising $\gamma$ has a larger effect on labor market dynamics, when the two shocks are more strongly correlated since the higher correlation between the shocks implies the productivity of the two sectors increase almost simultaneously, causing the reallocation effect (from final goods to IC) to be stronger. Recall from our discussions above, that as $\gamma$ increases the IC constraint becomes more important for the firm. At a higher $\gamma$ therefore the firm depends on and responds more to the IC shock. When the shocks are more strongly correlated, there is a tighter tradeoff between the two sectors and the firm's response due to the IC shock affects the economy more strongly.

When the shocks are less correlated, the firms face less of a tradeoff in choosing to reallocate resources between the two sectors. They accumulate as much IC stock as possible while productivity in the IC sector is high (due to the IC shock), in order to be able to increase production of final goods as much as possible when the shock to the latter sector hits. Thus the effects of the IC shock does not have a strong influence on how the firm reacts to the productivity shock in final goods and all the general effects of the IC shock discussed in Section 4.3, become less pronounced. That is, output volatility falls less and labor input and wage volatilities increase less (panel 5, Table 1).

The degree of pro-cyclicality generated, between measured labor productivity and output in Table 2 is much higher for uncorrelated shocks and while the correlations are in line with the data and the benchmark model in case of productivity and labor input, there is no further decrease in these correlations as $\gamma$ rises. This is once again due to the IC shock (which moves measured output and labor input in opposite directions) having a lower impact, when uncorrelated to the productivity shock. As $\gamma$ increases output's responsiveness to the IC shock rises less causing less of a fall in output (due to less reallocation), thus although correlation of output with productivity falls it is less pronounced than in the benchmark model. Labor however, is much less positively correlated with productivity at the lower $\gamma$ as before, since labor input increases quite a bit in response

to the IC shock even though output does not fall by as much. However, a rise in $\gamma$ has an ambiguous effect on this correlation, since the strong positive response of labor input due to the combined impact of the joint shocks vanishes. Now the IC shock does not generate the additional increase in labor input through the otherwise strong reallocation of labor from final goods to IC and hence labor input rises by less at higher $\gamma$.

*Persistence of shocks $(\rho, \rho_b)$:* I also experiment with higher values of persistence of the IC shock $(\rho_b)$ relative to the persistence of the productivity shock $(\rho)$ such that in panel 5 (of Tables 1 and 2) both shocks are equally persistent while in panel 6 the IC shock's persistence is higher.

Note that in Table 1 as the persistence of the IC shock is raised relative to that of the productivity shock, the relative volatility results are not much affected. In fact the rise in relative volatility of wages and labor with the increase in $\gamma$ is the same as that in the benchmark model with output volatility falling by a similar amount. Moreover, from Table 2 - the drop in correlations between productivity and both output and labor, as $\gamma$ increases, are much larger for larger values of $\rho_b$ relative to $\rho$. Employment of the IC sector is measured but its output is not, while a highly correlated IC shock means increased reallocation from the final goods to the IC sector as explained above. Therefore a more persistent IC shock implies, an increase in $\gamma$ causes employment to rise more in the IC sector and measured output therefore falls more thus generating a larger decline in the correlations of productivity relative to both output and labor as in Table 2.

| | Standard Deviation | | |
| | 1) Pre-84(Low $\gamma$ ) | 2)Post-84(High $\gamma$ ) | 3)Post-84/Pre-84 |
| --- | --- | --- | --- |
| *Panel 1: Log preferences* | | | |
| $\sigma(y)$ | 2.43 | 2.34 | 0.96 |
| $\sigma(l)/\sigma(y)$ | 0.51 | 0.58 | 1.14 |
| $\sigma(w)/\sigma(y)$ | 0.79 | 0.84 | 1.06 |
| *Panel 2: $\delta_z = 0.075$* | | | |
| $\sigma(y)$ | 2.44 | 2.31 | 0.95 |
| $\sigma(l)/\sigma(y)$ | 0.89 | 1.03 | 1.16 |
| $\sigma(w)/\sigma(y)$ | 0.36 | 0.42 | 1.16 |
| *Panel 3: $\delta_z = 0.01$* | | | |
| $\sigma(y)$ | 2.27 | 2.04 | 0.9 |
| $\sigma(l)/\sigma(y)$ | 0.89 | 1.04 | 1.17 |
| $\sigma(w)/\sigma(y)$ | 0.36 | 0.42 | 1.17 |
| *Panel 4: corr(e,$e_b$)=0* | | | |
| $\sigma(y)$ | 2.47 | 2.38 | 0.96 |
| $\sigma(l)/\sigma(y)$ | 0.75 | 0.78 | 1.04 |
| $\sigma(w)/\sigma(y)$ | 0.30 | 0.32 | 1.04 |
| *Panel 5: $\rho_b = 0.95$* | | | |
| $\sigma(y)$ | 2.32 | 2.11 | 0.91 |
| $\sigma(l)/\sigma(y)$ | 0.89 | 1.07 | 1.20 |
| $\sigma(w)/\sigma(y)$ | 0.36 | 0.44 | 1.20 |
| *Panel 6: $\rho = 0.9, \rho_b = 0.95$* | | | |
| $\sigma(y)$ | 2.27 | 2.02 | 0.89 |
| $\sigma(l)/\sigma(y)$ | 0.88 | 1.05 | 1.20 |
| $\sigma(w)/\sigma(y)$ | 0.37 | 0.43 | 1.20 |

Table 1: Sensitivity analysis of the effect of IC output and labor market volatilities.)

|  | Correlation Productivity | | | | | |
|  | with output | | | with employment | | |
|  | Low $\gamma$ | High $\gamma$ | Relative | Low $\gamma$ | High $\gamma$ | Relative |
|---|---|---|---|---|---|---|
| (1) *Log preference* | 0.92 | 0.85 | -0.07 | 0.64 | 0.42 | -0.22 |
| (2) $\phi = 0.075$ | 0.59 | 0.13 | -0.46 | 0.4 | -0.25 | -0.65 |
| (3) $\phi = 0.01$ | 0.68 | 0.02 | -0.66 | 0.55 | -0.26 | -0.81 |
| (4) corr$(e, e_b)$=0 | 0.82 | 0.65 | -0.17 | 0.18 | 0.19 | 0.01 |
| (5) $\rho_b = 0.95$ | 0.73 | -0.05 | -0.78 | 0.62 | -0.32 | -0.94 |
| (6) $\rho = 0.90, \rho_b = 0.95$ | 0.81 | -0.08 | -0.89 | 0.74 | -0.3 | -1.04 |

Table 2: Sensitivity analysis of the correlation of productivity with output and labor