

Spinor–vector duality and sterile neutrinos in string derived models

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Abstract.

The MiniBooNE collaboration found evidence for the existence of sterile neutrinos, at a mass scale comparable to the active left-handed neutrinos. While sterile neutrinos arise naturally in large volume string scenarios, they are more difficult to accommodate in heterotic–string derived models that reproduce the GUT embedding of the Standard Model particles. Sterile neutrinos in heterotic–string models imply the existence of an additional Abelian gauge symmetry at low scales, possibly within reach of contemporary colliders. I discuss the construction of string derived Z' models that utilise the spinor–vector duality to guarantee that the extra $U(1)_{Z'}$ symmetry can remain unbroken down to low scales.

1. Introduction

Physics is first and foremost an experimental science. Be that as it may, the language that is used to parametrise experimental data is mathematics. Formulating mathematical models that can encompass a broader range of physical data is therefore a well defined scientific problem.

All contemporary experimental observations are accounted for by two fundamental theories. At the celestial, galactical and cosmological scales Einstein’s theory of general relativity reigns supreme. At the atomic and subatomic scales the Standard Model of particle physics accounts for all experimental data to an impressive and increasing precision. Yet the two theories are mutually incompatible. It is therefore mandated to seek mathematical frameworks that can accommodate the two theories in a mathematically consistent framework.

From an experimental point of view what is required is the continued precision measurement of prevailing mathematical parameters of contemporary experiments. These include: more precise measurements of the scalar sector and its couplings; more precise measurements of the neutrino sector and of its particle content; more refined measurement of the Cosmic Microwave Background Radiation; elucidation of the origin of the highest energy cosmic rays; further refined measurements of the cosmic microwave background radiation; further discovery and measurements of gravitational waves. Questions abound. Further afield can be listed the detection of the primordial cosmic neutrino and gravitational field backgrounds. Extending the energy threshold should be sought to test contemporary theories in new regimes and to explore possible signals that go beyond the current paradigms. Current theoretical extensions of the Standard Models do not provide an unambiguous indication for where experiments should look for detection of their signatures. In the absence of such unambiguous theoretical guidance experimentalists should focus on refined tests of the existing frameworks. The experimental

successes of the past decades have enabled an unprecedented level of understanding of the basic constituents of matter and interactions. Future world wide experimental facilities will extend the era of exploration, that started with Galileo’s telescope, to new horizons.

One exciting experimental result that requires resolution is the recent report by the MiniBooNE collaboration for further evidence for the existence of sterile neutrinos [1]. Evidence for sterile neutrinos was reported by the LSND collaboration in 1993 [2]. While the experimental and phenomenological analysis is far from settled [3], if the result is strengthened by future experimental data, it will have profound implications on string phenomenology. While sterile neutrinos may be naturally accommodated in large volume string scenarios, they are much harder to reconcile with the more favoured string GUT models.

From a theoretical perspective, contemporary interest is in the synthesis of quantum mechanics and general relativity. Observational phenomena at the highest energy and shortest distance scales probed to date, point to the Standard Model of particle physics as the viable theory. The observation of a scalar particle at the LHC is compatible with the Standard Model Higgs particle. All the observed data to date is consistent with the hypothesis that the Standard Model remains a viable perturbative parametrisation up to very high energy scales, *e.g.* the Grand Unified Theory (GUT), where all the gauge interactions are of comparable strength, or the Planck scale, where the magnitude of the gauge and gravitational couplings is comparable. Further evidence for this hypothesis stems from the logarithmic running of the Standard Model parameters, and the suppression of left–handed neutrino masses and proton decay mediating operators. Most tantalising is the fact that the Standard Model gauge charges of each family fit into spinorial **16** representation of $SO(10)$. The Standard Model gauge charges are experimental observable that were determined in the process of the experimental discovery of the Standard Model. Since the Standard Model matter sector consist of three generations; three group factors; and six left–handed Weyl multiplets (including the right–handed neutrino), fifty four independent parameters are required to account for the Standard Model gauge charges. The embedding of the Standard Model matter spectrum in $SO(10)$ representations reduces this number to one, *i.e.* the required number of spinorial **16** spinorial representations of $SO(10)$. An amazing coincidence indeed! However, the saga is not complete. Gravity exists! Without it we will all be floating up in space. String theory enables exploration of the augmentation of the Standard Model with gravity in a perturbatively consistent framework. Furthermore, while in the context of point quantum field theories the Standard Model may be augmented with an infinite number of continuous parameters, string theory severely constrains the allowed extensions, and may shed light on the origin of its existing parameters. A natural question is how can sterile neutrinos be accommodated in string vacua [4]?

2. Sterile neutrinos in large volume scenarios

Sterile neutrinos may be naturally accommodated in large volume scenarios. As depicted in the

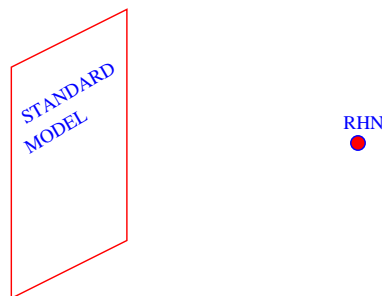


figure. In these scenarios the Standard Model states are confined to an observable brane, whereas the right–handed neutrino field is a bulk field. The couplings of these fields to the Standard

Model states are then suppressed by the volume of the extra dimensions, very much like the suppression of the gravitational couplings [5]. Assuming a five dimensional scenario, (x^μ, y) where $\mu = 0, \dots, 3$ and a y is a compactification circle with radius R . The Dirac spinor field in the bulk decomposes in the Weyl basis as $\Psi = (\nu_R, \bar{\nu}^c_R)$, and has the usual Fourier mode expansion

$$\nu_R^{(c)}(x, y) = \sum_n \frac{1}{\sqrt{2\pi r}} \nu_{Rn}^{(c)}(x) e^{iny/r} \quad (1)$$

The mass spectrum in four dimensions then contains a tower of Kaluza–Klein states with Dirac masses n/r and a free action for the lepton doublet, localized on the wall. The leading interaction term between the bulk and wall fermion fields is

$$S^{\text{int}} = \int d^4x \lambda l(x) h^*(x) \nu_R(x, y = 0) \quad (2)$$

with λ being a dimensionless parameter. The Yukawa coupling λ to all brane fields is rescaled like the dilaton and graviton couplings. The effective Yukawa on the four dimensional brane is given by

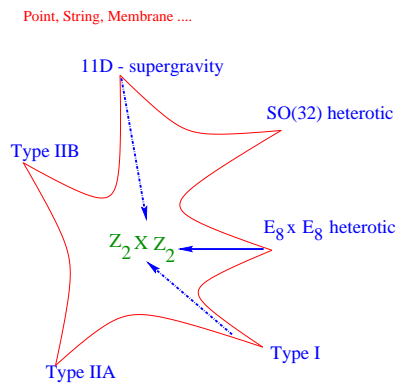
$$\lambda_{(4)} = \frac{\lambda}{\sqrt{r^n M_*^n}}, \quad (3)$$

which leads to very strong suppression of the Dirac mass. Thus, in large volume scenarios the bulk fields behave like sterile neutrinos with suppressed couplings to the left–handed neutrino brane fields with potentially observational effects [5]. In this scenarios, however, the appealing embedding of the Standard Model states in $SO(10)$ representations is abandoned.

3. String GUT models in the free fermionic construction

String phenomenology is in some respect an answer in search of a question. The answers are provided in the form of the Standard Models of particle physics and cosmology and their BSM extensions. One question that one may ask is what is the true string vacuum that reproduces the detailed features of these models, with possibly some additional predicted signatures beyond the Standard Models. Another question that one may ask is whether one can identify generic signatures of classes of string compactifications in the experimental particle and cosmological data. In this talk I argue that the existence of light sterile neutrinos in heterotic string GUT vacua mandates the existence of a light Z' vector boson, which in turn is hard to incorporate in these models.

The perturbative and non–perturbative duality symmetries among ten dimensional string theories, as well as eleven dimensional supergravity [6] reveals, as depicted in the figure below,



that the different string theories are limits of a more fundamental theory. If we take the embedding of the Standard Model states in $SO(10)$ representation as the primary guide in

seeking phenomenological signatures of string theory, then the string limit that should be used is the ten dimensional $E_8 \times E_8$ heterotic-string theory, because it is the only string theory that produces spinorial $SO(10)$ representations in its perturbative spectrum. In this respect we should anticipate that working in a perturbative limit of the more fundamental theory reveal some properties of the true string vacuum, but cannot fully characterise it. It may well be that other features can only be gleaned in a different limit. An example at hand is the stabilisation of the dilaton moduli that determines the strength of the gauge and gravitational interactions. In the perturbative heterotic limit the dilaton has a run away behavior and cannot be stabilised at a finite value. The underlying picture depicted in the figure above reveals that the dilaton can be interpreted as a moduli of an eleventh dimensions. Thus, to stabilise the dilaton at a finite value necessitates moving away from the perturbative heterotic limit. This illustrates the point that we should not expect any of the limits to fully describe the true string vacuum, but at best to capture some of its properties. On the other hand the true string vacuum will have some characteristics that are independent of the limit around which we expand. What those features are is a matter of research and debate, but we may hypothesise that they might be related to the underlying structure of the internal compactified space. This is suggested in the figure that exemplifies this notion with the case study of the $Z_2 \times Z_2$ toroidal orbifold in the different limits.

Phenomenological studies of string GUT theory proceed by compactifying the heterotic string from ten to four dimensions. The degrees of freedom of the internal six dimensional space can be represented as free or interacting world sheet conformal field theories. In simple cases the effective low energy quantum field theory is represented in the form of supergravity compactified on a complex internal manifold. However, at present our basic understanding of string theories is rudimentary and phenomenological studies should be regarded as exploratory. With this caveat in mind, simple classes of compactifications are orbifolds of six dimensional toroidal spaces. Of those the simplest orbifold is the Z_2 orbifold of a one dimensional torus, *i.e.* a circle. Taking the internal space to be a six torus, consistency dictates that a Z_2 orbifold can act on four internal coordinates at a time. The initial toroidal compactification gives rise to $N = 4$ spacetime supersymmetry and each Z_2 orbifold reduces the number of supersymmetries by two. Hence, to reduce the number of spacetime supersymmetries requires a $Z_2 \times Z_2$ orbifold of the six dimensional internal toroidal space. This class of string compactifications is among the simplest that one may construct, and it possesses several other appealing properties. One being that it naturally gives rise to three generations [7]. Heuristically, we may attribute the emergence of three generations in the $Z_2 \times Z_2$ orbifolds to the fact that we are dividing an internal six dimensional space into factors of two. The $Z_2 \times Z_2$ orbifold contains three twisted sectors and in many of the three generation models each one of the twisted sectors produces one generations. Another important property of the phenomenological $Z_2 \times Z_2$ orbifold models is that they preserve the $SO(10)$ embedding of the weak hypercharge, hence facilitating the construction of string GUT models. The $Z_2 \times Z_2$ orbifold produces an abundance of phenomenological three generation models that serves as a case study to explore possible relations between string theory and experimental data.

The $Z_2 \times Z_2$ orbifold models have been most amply studied by using the free fermionic formulation of the heterotic-string in four dimensions. The equivalence of two dimensional fermions and bosons entails that this formulation is entirely equivalent to the toroidal orbifold construction. Indeed, for any free fermion model one can find the bosonic equivalent [8]. Since the late eighties the free fermion models served as a laboratory to study phenomenological issues in the Standard Model and unification, including: the construction of string models that produced solely the spectrum of the Minimal Supersymmetric Standard Model in the observable charged matter sector [9]; fermion mass hierarchy and mixing [10]; neutrino masses [11]; and more [12]. The $Z_2 \times Z_2$ orbifold models represent a case study and other approaches are being pursued [13]. The task of string phenomenology is to develop the tools to discern between the

different cases and identify their signatures in the experimental data.

In the fermionic formulation of the heterotic-string in four dimensions all the worldsheet degrees of freedom required to cancel the conformal anomaly are represented in terms of two dimensional free fermions on the string worldsheet. In the standard notation the 64 worldsheet fermions the lightcone gauge are denoted as:

$$\begin{array}{l} \text{Left-Movers:} \\ \text{Right-Movers} \end{array} \quad \psi^\mu, \chi_i, y_i, \omega_i \quad (\mu = 1, 2, i = 1, \dots, 6)$$

$$\bar{\phi}_{A=1, \dots, 44} = \begin{cases} \bar{y}_i, \bar{\omega}_i & i = 1, \dots, 6 \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1, \dots, 5} \\ \bar{\phi}_{1, \dots, 8} \end{cases}$$

Of these the $\{y, \omega | \bar{y}, \bar{\omega}\}^{1, \dots, 6}$ correspond to the six compactified dimensions of the internal manifold; $\bar{\psi}^{1, \dots, 5}$ generate the $SO(10)$ GUT symmetry; $\bar{\phi}^{1, \dots, 8}$ generate the hidden sector gauge group; and $\bar{\eta}^{1, 2, 3}$ generate three $U(1)$ symmetries. Models in the free fermionic formulation are written in terms of boundary condition basis vectors, which specify the transformation properties of the fermions around the noncontractible loops of the worldsheet torus, and the Generalised GSO projection coefficients of the one loop partition function [14]. The free fermion models correspond to $Z_2 \times Z_2$ orbifolds with discrete Wilson lines [8].

4. Sterile neutrinos in free fermion models

In the heterotic-string GUT models the $SO(10)$ gauge symmetry is broken at the string scale to one of its subgroups by discrete Wilson lines. Due to the underlying $SO(10)$ symmetry the models possess some common features, irrespective of the unbroken $SO(10)$ subgroup, The neutrino mass matrix in these models takes the generic form

$$(\nu_i, N_i, \phi_i) \begin{pmatrix} 0 & (M_D)_{ij} & 0 \\ (M_D)_{ij} & 0 & \langle \bar{\mathcal{N}} \rangle_{ij} \\ 0 & \langle \bar{\mathcal{N}} \rangle_{ij} & \langle \phi \rangle_{ij} \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j \\ \phi_j \end{pmatrix}, \quad (4)$$

where M_D is the Dirac mass matrix and is proportional to the up-quark mass matrix due to the underlying $SO(10)$ symmetry [15]. Taking the mass matrices to be nearly diagonal the physical eigenvalues are

$$m_{\nu_j} \sim \left(\frac{k M_u^j}{\langle \bar{\mathcal{N}} \rangle} \right)^2 \langle \phi \rangle, \quad m_{N_j}, m_\phi \sim \langle \bar{\mathcal{N}} \rangle. \quad (5)$$

where k is a renormalisation factor due to RGE evolution, and $\langle \bar{\mathcal{N}} \rangle \gtrsim 10^{14} \text{ GeV}$, as required to produce sufficiently suppressed left-handed tau neutrino mass [15]. This mass scale is dictated by the $SO(10)$ relations between the top quark and tau neutrino Dirac mass terms. The result is that the mass eigenvalues consist of one light left-handed neutrino and two heavy sterile states at the $\langle \bar{\mathcal{N}} \rangle$ scale. There is no light sterile neutrino. This result is borne out in detailed studies of neutrino masses in free fermionic models [11, 16]. The conclusion of these studies is that it is hard enough to generate sufficiently suppressed left-handed neutrino masses, let alone any additional light sterile neutrinos. Furthermore, we generally expect any non-chiral state to receive mass of the order of the GUT or string scales. Only states that are chiral with respect to some gauge symmetry remain massless down to lower scales. This outcome is supported by detailed analysis in concrete models. Some mass terms of vector-like states may arise from

nonrenormalisable terms, and therefore suppressed compared to the leading terms. They will appear at intermediate scale, a few orders of magnitude below the GUT or Planck scales, but not at a scale required for sterile neutrinos. The conclusion is that existence of sterile neutrinos at low scales requires that they are chiral with respect to an additional $U(1)$ gauge symmetry beyond the Standard Model. The extra $U(1)$ symmetry may be broken at intermediate scale above the TeV scale, and may be within reach of contemporary experiments.

5. Low scale Z' in free fermionic models

The interest in string inspired Z' models followed from the observation that string inspired effective field theory models give rise to E_6 GUT like models. Extra $U(1)$ symmetries in these string inspired models therefore possess an E_6 embedding and have generated multitude of papers since the mid-eighties. However, the construction of string derived models that admit an unbroken extra $U(1)$ symmetry down to low scales proves to be more of a challenge. The combination of $U(1)_{B-L}$ and $U(1)_{T_{3R}}$ given by

$$U(1)_{Z'} = \frac{3}{2}U(1)_{B-L} - 2U(1)_{T_{3R}}$$

is embedded in $SO(10)$ and orthogonal to the weak hypercharge combination $U(1)_Y = \frac{1}{2}U(1)_{B-L} + U(1)_{T_{3R}}$ [17]. Preserving this combination down to low scales guarantees that dimension four operators that may induce proton decay are sufficiently suppressed [17]. However, as discussed above the resulting seesaw neutrino mass scale is too low to produce sufficient suppression of left-handed neutrino masses [15]. On the other hand, the symmetry breaking pattern in the string models $E_6 \rightarrow SO(10) \times U(1)_A$ entails that $U(1)_A$ is anomalous and cannot be part of a low scale unbroken $U(1)_{Z'}$ [18]. String derived models with low scale $U(1)_{Z'} \notin E_6$ were studied in [19]. However, agreement with the measured values of $\sin^2(\theta)_W(M_Z)$ and $\alpha_s(M_Z)$ favours Z' models with E_6 embedding [20]. A $Z' \in E_6$ string derived model was presented in [21].

The construction of the string derived Z' model in [21] utilises the spinor–vector duality [22] that was observed in the classification of free fermionic heterotic–string $SO(10)$ models [23]. The classification was extended to models in which the $SO(10)$ symmetry is broken to $SO(6) \times SO(4)$ [24]; $SU(5) \times U(1)$ [25]; $SU(3) \times SU(2) \times U(1)^2$ [26]; and $SU(3) \times U(1) \times SU(2)^2$ [27]. The free fermionic classification method uses a fixed set of boundary condition basis vectors and the enumeration of vacua is obtained by varying the GGSO projection coefficients of the one loop partition function¹. The basis typically contains thirteen or fourteen boundary condition basis vectors. Correspondingly there are up to 91 modular invariant independent binary phases. Some of the free phases are fixed by physical requirements, such as the requirement that the vacua possess $N = 1$ spacetime supersymmetry. The space of scanned vacua therefore correspond to some 10^{15} independent $Z_2 \times Z_2$ orbifold heterotic–string vacua. The entire physical spectrum is analysed by expressing the GGSO projection conditions of the massless states arising in all the states producing sectors in the form of algebraic equations, which are coded in a computer program. In this manner a large space of models and their entire physical spectrum is spanned and analysed according to prescribed physical criteria.

An example of the utility of the method is provided by spinor–vector duality [22]. The duality is under the exchange of the total number of $(16 + \overline{16})$ representations of $SO(10)$ with the total number of 10 representations. For every model with a number $\#_1$ of $(16 + \overline{16})$ and $\#_2$ of 10 there is another models with a $\#_2$ of $(16 + \overline{16})$ and $\#_1$ of 10. The duality is easy to understand if we consider the extension of $SO(10) \times U(1)_A$ to E_6 . The chiral and anti-chiral representations of E_6 decompose under $SO(10) \times U(1)$ as $27 = 16 + 10 + 1$ and

¹ See *e.g.* talk by Benjamin Percival at this conference

$\overline{27} = \overline{16} + 10 + 1$. In this case $\#_1 = \#_2$. The E_6 enhanced symmetry point therefore correspond to the self-dual point under the exchange of the total number of $SO(10)$ spinor plus anti-spinor, with the total number of vectors. The compactifications with E_6 symmetry correspond to heterotic-string compactifications with $(2, 2)$ worldsheet supersymmetry, and the breaking of $E_6 \rightarrow SO(10) \times U(1)_A$ correspond to the breaking of the $(2, 2)$ worldsheet supersymmetry to $(2, 0)$ (or $(2, 1)$ [18]). The important observation is that the spectral flow operator of the right-moving $N = 2$ worldsheet supersymmetry is the operator that induces the map between the spinor-vector dual vacua. This duality was initially observed from the classification of the large spaces of fermionic $Z_2 \times Z_2$ orbifold and subsequent deeper understanding ensued in terms of the worldsheet properties.

The classification method provides a fishing method to extract models with specified physical properties. In order to remain unbroken down to low scales an extra $U(1)$ symmetry has to be anomaly free. The $Z_2 \times Z_2$ orbifold allows the existence of self-dual models under spinor-vector duality, in which the gauge symmetry is, however, not enhanced to E_6 . In these models the total number of $(16 + \overline{16})$ is equal to the total number of 10 representations. In such models the chiral spectrum still forms complete E_6 representations, but the gauge symmetry is not enhanced to E_6 . This is possible in the $Z_2 \times Z_2$ orbifolds if the different components that form complete E_6 multiplets arise from different fixed points of the $Z_2 \times Z_2$ orbifold. That is if both a spinorial and vectorial states are massless at the same fixed point, the symmetry is necessarily enhanced to E_6 . However, if they are obtained from different fixed points, the spectrum may be self-dual and form complete E_6 multiplets without enhancement of the $SO(10) \times U(1)_A$ symmetry to E_6 . In this case, by virtue of the fact that the massless states form complete E_6 multiplets $U(1)_A$ is anomaly free and may remain unbroken to low scales. In ref. [21] such a spinor-vector self dual model was extracted with subsequent breaking of the $SO(10)$ symmetry to $SO(6) \times SO(4)$, which preserves the spinor-vector self-duality. This model is a string derived model in which an extra $U(1)$ with E_6 embedding may remain unbroken down to low scales. Alternative scenarios to the self-dual string derived model of ref. [21] are models with different E_8 symmetry breaking patterns [28].

The extra $U(1)_{Z'}$ symmetry requires the existence of additional matter states that obtain mass at the $U(1)_{Z'}$ breaking scale. The additional matter states ensure that $U(1)_{Z'}$ is anomaly free. The chiral spectrum forms complete E_6 multiplets, which includes an $SO(10)$ singlet state that can serve as a sterile neutrino. The neutrino mass matrix for one generation takes the form

$$\begin{array}{c} L_i \quad S_i \quad H_i \quad \overline{H}_i \quad N_i \\ \begin{array}{c} L_i \\ S_i \\ H_i \\ \overline{H}_i \\ N_i \end{array} \begin{pmatrix} 0 & 0 & 0 & \lambda n & \lambda v \\ 0 & 0 & \lambda v_2 & \lambda v_3 & 0 \\ 0 & \lambda v_2 & 0 & z' & 0 \\ \lambda n & \lambda v_3 & z' & 0 & 0 \\ \lambda v & 0 & 0 & 0 & \mathcal{N}^2/M \end{pmatrix}, \end{array}$$

A plausible numerical scenario per generation is given by

$$\begin{aligned} \lambda v &= 1\text{GeV}; & \lambda v_2 &= 5 \times 10^{-4}\text{GeV} \approx m_e; & \lambda v_3 &= 5 \times 10^{-4}\text{GeV} \approx m_e; \\ \lambda n &= 5 \times 10^{-4}\text{GeV} \approx m_e; & z' &= 5 \times 10^4\text{GeV} = 50\text{TeV}; & \overline{\mathcal{N}} &= 5 \times 10^{14}\text{GeV}, \end{aligned}$$

In this scenario there are two light eigenstates which are mixture of ν_L^i and S_i , with $\sin \theta \approx 0.98$ and $m_{1,2} = \{10^{-2}\text{eV}, 10^{-3}\text{eV}\}$; two nearly degenerate eigenstates of H_i and \overline{H}_i with $m_{3,4} = \{50\text{TeV}, 50\text{TeV}, \}$ and a heavy eigenstate N_i with $m_5 = 2.5 \times 10^{11}\text{GeV}$.

6. Conclusions

Sterile neutrinos are hard to accommodate in heterotic-string derived models, unless their lightness is protected by an additional $U(1)$ gauge symmetry under which they are chiral.

Further substantiation of the observations of the LSND and MiniBooNE collaborations may imply the existence of additional matter and interaction at a scale within reach of contemporary colliders, ushering a new era of discovery. Point quantum field theory models will continue to be used to parametrise the experimental data. However, the synthesis of gravity with the gauge interactions mandates a departure from the point idealisation of elementary particles. String theory models, of which the fermionic $Z_2 \times Z_2$ orbifolds serves as a fertile crescent over the past 30 years, provide a self consistent framework to explore this synthesis.

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