Modal Analysis and Damage Detection for Suspension Bridges: A Numerical and Experimental Investigation.

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**Abstract**. This paper addresses the problem of damage detection in suspension bridges hangers, with an emphasis on the modal flexibility method. Specifically, suspension bridges’ hanger have a small contribution to the modal strain energy of the low frequency modes, and their damage has a little impact on the variability of the modal properties This paper aimed at evaluating the capability and the accuracy of the modal flexibility method to detect and locate single and multiple damages in suspension bridges hangers, with different level of severity and various locations. The study is conducted numerically and experimentally on a laboratory suspension bridge mock-up. First, the covariance-driven stochastic subspace identification is used to extract the modal parameters of the bridge from experimental data, using only output measurements data from ambient vibration. Then, the method is demonstrated for several damage scenarios and compared against other classical methods, such as: Coordinate Modal Assurance Criterion (COMAC), Enhanced Coordinate Modal Assurance Criterion (ECOMAC), Mode Shape Curvature (MSC) and Modal Strain Energy (MSE). The paper demonstrates the relative merits and shortcomings of these methods which play a significant role in the damage detection of suspension bridges.

**Keywords:** suspension bridge, stochastic subspace identification, modal flexibility, damage detection.

**1. Introduction**

Over the recent years, the improvement in construction materials and construction technology, computing capability, and above all a better understanding of the physics of the complex phenomena which control the external loads acting on structures, have revolutionized the civil engineering community, enabling the construction of large and elegant civil infrastructures. With the rise of large-scale civil engineering structures, structural health monitoring (SHM) problems have become a crucial scientific issue for the last two decades (Mei *et al*. 2017, Wang *et al*. 2016, Wu and Casciati 2014). The goal is to be able to detect, locate and assess the extent of damage in a structure so that its remaining life can be known and possibly extended. As an alternative to the current local detection methods, the vibration-based methods have been widely applied over the years (Doebling *et al*. 1998, Alvandi and Cremona 2006, Deraemaeker *et al*. 2008, Casciati *et al*. 2015).

For health monitoring of suspension bridges, it is necessary to identify a modal model which consists of eigenfrequencies, mode shapes and damping ratios from vibration data. These model parameters can be obtained by a variety of modal identification techniques (Kim *et al*. 2013). Due to the difficulty to apply an artificial force on large civil structures, the measurement has to rely upon available response induced by ambient excitation sources. However, it is practically impossible to measure this ambient excitation and the outputs are the only information that can be used to the modal identification methods. In such cases, output-only measurement becomes the most practical means for identifying the structure. This type of test makes use of ambient environment effects such as wind, traffic loads, and environmental loads as excitation forces (Siringoringo and Fujino 2008). Many output-only algorithms have been proposed in the last decades, including Ibrahim time domain ITD method (Ibrahim 1977), eigensystem realization algorithm ERA (Juang and Pappa 1985), natural excitation technique NExT (Farrar and James III 1997) and stochastic subspace identification SSI (Van Overschee and De Moor 2012).

As a robust and accurate system identification algorithm, the stochastic subspace identification (SSI) method is very efficient to apply for high noise and harsh operating environments (Wu *et al*. 2016). Two types of implementation are available: the covariance-driven implementation, SSI-cov (Peeters and Ventura 2003) and the data-driven implementation, SSI-data (Peeters and De Roeck 1999). For the SSI-cov, three methods can be implemented: the balanced realization (BR), the principal component (PC), and the canonical variate analysis (CVA).

The effectiveness of detection often depends on the extent of damage, but for suspension bridges, as large civil engineering structures, the structure modal data are little sensitive to tiny damages. In such a case, finding an effective indicator for damage detection or damage location becomes particularly important. For this purpose, many traditional modal parameters-based methods have been successfully applied, such as Coordinate Modal Assurance Criterion COMAC (Wahab and De Roeck 1999), Enhanced Coordinate Modal Assurance Criterion ECOMAC (Hunt 1992), Mode Shape Curvature MSC (Pandey *et al*. 1991), Modal Strain Energy MSE (Stubbs *et al*. 1995, Li *et al*. 2006) and Modal Flexibility method MF (Pandey and Biswas 1994).

In this paper, we use a modal flexibility based technique to detect and locate single and multiple damages with varied locations and severity on a laboratory mockup of a suspension bridge. Similar studies have been conducted numerically (Talebinejad et *al*. 2011, Ni et *al*. 2008, Wickramasinghe *et al*. 2016), or using the dynamically measured flexibility matrix method (Gul and Catbas *et al*. 2008, Catbas *et al*. 2004). Martinez-Castro et *al.* (2014) and Bernagozzi et *al.* (2017) consider a MF approach to detect damage in a steel frame structure of multi-storey laboratory mock-ups, where the damage is simulated by reducing the shear stiffness of the storey, or by removing structural elements. Koo et *al.* (2008) used the MF method to detect cracks in a 10m bridge model with a steel box-girder. In all these studies, an excitation system is used with a known input, and the damage is located in structural elements with high modal strain energy. In the present paper, the MF method is employed for damage detection in suspension bridges’ hangers. Particularly, the hangers have a little contribution to the modal strain energy of the low frequency modes, and their stiffness variation has a little extent on the structure dynamics. Moreover, the method is combined with the SSI-cov method for modal identification, using only output measurements data, from ambient vibration.

We present a synthetic study on modal identification and damage detection procedures for structural health monitoring of suspension bridges’ hangers, using only vibration measurements. For the applicability of damage detection in a real structure, an operational modal analysis method is applied and evaluated in this study, namely covariance-driven stochastic subspace identification (SSI-cov). The problems of damage detection are treated using the modal flexibility-based damage index which can be obtained directly from SSI-cov method. The following parts of this paper are organized as follows: Section 2 presents the SSI-cov and MF methods for damage index estimation. Section 3 mainly focuses on the numerical validation of the SSI-cov and the damage index in the presence of noise; in addition, the vibration characteristics of the bridge mock-up are shown in this section. Section 4 mainly focuses on the experimental verification of the proposed damage index in single and multiple damages with varied locations and severity in the hangers of the suspension bridge. Section 5 gives an experimental comparison between the modal flexibility method and the traditional modal parameters-based methods. Finally, findings and conclusions of the study are summarized at the end.

**2. Methodology and algorithm**

The Modal Flexibility (MF) method is a widely used technique in damage detection; it associates the damage to the change in the modal parameters of the structure (assuming an experimental modal analysis can be conducted). For large civil structures, such as suspension bridges, only ambient vibration, induced by the environment, can be measured. Hence, in order to use the MF method, one should first identify the modal parameters using existing blind methods, based on output-only measurements.

*2.1 Covariance-driven stochastic subspace identification (SSI-cov)*

The use of output-only measurements (ambient vibration) to extract the modal parameters of civil structures is a well-established practice, which has been widely used in the past few decades (Rainieri and Fabbrocino 2014). For the time-domain signal, the covariance-driven stochastic subspace identification method (SSI-cov) is one of the most robust and accurate system identification methods for output-only modal analysis of mechanical structures. In this section, the SSI-cov is briefly described.

Consider a linear time-invariant structure governed with its equation of motion:

 (1)

where *M*, *C*d, and *K* are constant mass, damping, and stiffness matrices, respectively; *f* (*t*) is the external force vector, and *q*(*t*) = [*q*1(*t*), . . ., *qm*(*t*)]*T* is the displacement vector.

Through using model reduction, sampling and considering the noise, Eq(1) can be converted to the discrete-time stochastic state-space model without the input term:

 (2)

where *xk* is the discrete-time state vector, *A* is the discrete state matrix, *C* is the output matrix. *wk* is the process noise from the disturbances and modelling inaccuracies and *vk* is the measurement noise from the sensor inaccuracy. They are both unmeasurable vector signals but can be assumed stationary with zero mean , . Further the stochastic process *xk* is stationary with zero mean , where the state covariance matrix is independent of the time *k*. *wk* and *vk* are independent of the actual state , . The output covariance matrices *Ri* are defined as

 (3)

The next state-output covariance matrix *G* is defined as

 (4)

From these definitions the relationship between *A*, *C* and *Ri* (*i*=1,2···) is easily deduced:

 (5)

Eq. (5) is the core expression of the SSI-cov method. It can be seen from this formula that the output covariance matrices *Ri* is directly related to the structure system matrices *A* and *C*, which is the key step to identify the modal parameters of the structures.

By applying the output covariance matrices, constructing the Hankel matrix and Toeplitz matrix (see Van Overschee and De Moor 2012 for a detailed derivation), the modal parameters are obtained from the discrete state matrix *A* and the output matrix *C*. Through conversion, an eigenvalue decomposition of the discrete-time state matrix can be obtained by

 (6)

where Λ*d* is a diagonal matrix containing the discrete-time complex eigenvalues and *Ψ* contains the eigenvectors as columns. The eigenvalues of *A* occur in complex conjugated pairs and can be written as:

 (7)

where *ξi* are the modal damping ratios and *ωi* are the eigenfrequencies (rad/s). The mode shapes at the sensor locations, defined as columns of *Φ*, are found as:

 (8)

Finally, for further improving the accuracy of the SSI-cov method, stabilization diagrams are often applied to distinguish spurious modes. A whole detailed derivation of the identification procedure can be found in (Van Overschee and De Moor 2012).

*2.2 Modal flexibility-based damage index*

Based on structural flexibility matrix, Pandey and Biswas (1994) proposed the MF method to detect the damage of a wide-flange steel beam. Modal flexibility of a structure converges rapidly as the frequency increases, it can be calculated using only a few lower-order modes. Unlike the traditional methods, which require an analytical model of the structures to evaluate the flexibility, the MF method can use only the experimental data collected from the structure. However, for the large civil structures, the mass-normalized mode shapes data are not available. In order to apply the MF method in large civil structures, one of the simplest methods is used to calculate the MF with ambient vibration measurements. The modal flexibility at a position *x* of the damaged and undamaged structure can be defined as:

 (9)

 (10)

where *i* (*i*=1, 2, 3 …*m*) is the considered mode number; *ωi* and *ωi*\* are the *i*th frequencies, while and are the *i*th mode shapes of the damaged and undamaged structure at position *x,* respectively. All of the above modal parameters are assumed to be obtained from the SSI-cov method.

The stiffness of a damaged structures is, in general, smaller compared to the healthy structure, and inversely for the flexibility. The variation of the flexibility matrix, with respect to the nominal structure, can be obtained as:

 (11)

In this study, Δ*F* is normalized by the *Fx* and hence the damage index for locating damage in a structure is defined as in Eq. (12). The normalized damage index is determined by the relative change of the modal flexibility:

 (12)

Generally, the dynamic characteristics of a suspension bridge often include with lateral, vertical, torsional and coupled modes (Huang *et al*. 2005). The lateral and vertical mode shapes as the significant components can be directly used to detect the damage in suspension bridges. In Wickramasinghe’s method, they define and test two corresponding damage indices. One index is based on vertical components of mode shapes and the other is based on its lateral components. Their results confirm that using the vertical components of the mode shapes are much better than the lateral ones. Eq. (12) is hence rewritten as Eq. (13) to accommodate the damage index (DI), where the subscripts *v* denotes the vertical components of the mode shapes.

 (13)

**3. Numerical implementation**

In this paper, a finite element model of a laboratory suspension bridge was used to simulate various damage scenarios. It was used to obtain the vibration properties in both the damaged and undamaged states and thereby to verify the proposed damage detection methodology.

*3.1 Numerical model*

Consider the suspension bridge of Fig.1, the experimental mock-up is shown in Fig.12. It has a span of 2.2m and two articulated towers (pylons) of 0.62m, the main steel cables (catenary) have a diameter of 1mm and the 2×10 hangers have a diameter of 0.5mm; the deck is free to rotate at both ends and is attached to the catenary by the two rows of hangers.

The mock-up consists of a flexible deck supported with a set of prestressed cables. We assume that the dynamics of the cables can be neglected and their interaction with the deck is restricted to the longitudinal tension (this assumption has been verified in previous studies). Note that this assumption doesn't affect the result, while it simplifies the modelling. Assuming a classical finite element formulation, the equation governing the dynamic response of the system is:

 (14)

where *X* is the vector of global coordinates of the finite element model, *M* and *K* are respectively the mass and stiffness matrices of the structure, including a linear model of the cables; the geometric stiffness due to the prestress in the hanger is included in the model and the structural damping is neglected to simplify the presentation. *fd* is the vector of external disturbances (expressed in global coordinates).

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| Fig. 1 CAD view of the studied suspension bridge |

The model of the bridge of Fig.1 is analyzed using SAMCEF Finite Element software, and exported to MATLAB software, where a state space model is built. The deck is modelled with finite elements of beams, while the hangers and the main cable (catenary) are modelled with bars.

Table 1 Comparison between the numerical and the experimental natural frequencies and mode shapes. The natural damping has been estimated between 0.1% and 0.8%.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Mode  # | Numerical[Hz] | Experimental[Hz] | Numerical  Mode shape | Experimental  Mode shape |
| 1st B | 4.7 | 5.1 |  |  |
| 2nd B | 6.5 | 6 |  |  |
| 1st T | 9.8 | 10.1 |  |  |
| 3rd B | 12.3 | 11.8 |  |  |
| 2nd T | 12.1 | 13.2 |  |  |
| 4th B | 17.6 | 19.1 |  |  |

Table 1 shows the first few vibration modes of the structure and compares them to those measured experimentally (only the deck is shown). The agreement between the model and the experiment is fairly good. From the above comparisons, it can be concluded that the FE model is a good representation of the mock-up.

*3.2 Modal identification*

The bridge is excited on its deck using a point force, as indicated in Fig.2; a stationary Gaussian white noise (GWN) is used. The vertical acceleration of the deck is measured at 20 points, close the hangers. The numbering of the hangers from 1 to 10 on one side, starting from the right, and from 11 to 20 on the opposite side.

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| Fig. 2 Sensor layout and excitation locations in the bridge deck |

In order to use the SSI-cov method for modal identification, the ambient vibration of the structure is measured during at least 1000 to 2000 periods of the lowest natural frequency:

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where *f*min is the lowest natural frequency in Hz (Cantieni 2005, Wickramasinghe *et al*. 2016). Based on the preliminary FRF analysis, *f*min is close to 5 Hz. Therefore, the recording time was selected as 200sec. A zero-mean Gaussian white noise is added to the accelerometer outputs, where a standard deviation of 1% to 5% of the signal level is considered.

In order to minimize the effect of the uncertainties on the identified modes, the concept of stabilization diagram is introduced to help in the manual selection of the modes that are more likely to represent the physical modes (Goi and Kim 2016) and eliminate the spurious modes by using the stabilization criteria (Reynders *et al*. 2008). The frequencies 𝑓𝑖 and damping 𝜉𝑖 ratios are expected to be obtained in certain ranges 𝑓*min* ≤ 𝑓𝑖 ≤ 𝑓*max*and 𝜉*min* ≤ 𝜉𝑖≤ 𝜉*max*; all the modes out of these ranges will be discriminated. Only modes verifying these criteria are plotted in the stabilization diagram. In the classical implementation of the stabilization diagram, the typical stability criteria values are as follows: 𝜀𝑓= 0.5 % for frequency, 𝜀𝜉= 10 % for damping. Two modes identified in certain two orders, “𝑖” and “𝑖 + 1,” will be plotted in the stabilization diagram if (Mrabet *et al*. 2014)

 (15)

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| Fig. 3 Stabilization diagram. The criteria are: 0.5% for frequencies, 10% for the damping ratios |

Table 2 Identification results by SSI-cov in noisy random vibration (1%-5% root mean square noise)

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| --- | --- | --- | --- | --- | --- |
| Mode # | **Frequency (Hz)** | | **Damping ratio** | | **MAC** |
| Theoretical | Identified | Theoretical | Identified |
| 1 | 4.69 | 4.65 | 0.01 | 0.013 | 0.9967 |
| 2 | 6.51 | 6.55 | 0.01 | 0.013 | 0.9997 |
| 3 | 9.83 | 9.80 | 0.01 | 0.014 | 1 |
| 4 | 12.07 | 11.97 | 0.01 | 0.012 | 0.8399 |
| 5 | 12.31 | 12.30 | 0.01 | 0.011 | 0.9977 |
| 6 | 17.60 | 17.59 | 0.01 | 0.009 | 0.9994 |
| 7 | 19.07 | 19.12 | 0.01 | 0.012 | 0.9971 |
| 8 | 23.35 | 23.36 | 0.01 | 0.012 | 0.9993 |

A typical stabilization diagrams is shown in Fig.3. In the frequency range from 0 to 30 Hz, at least 9 modes are present. The identified eigenfrequencies and damping ratios are given in Table 2. There are two pairs of rather close modes: 11–13 Hz. Although these modes are too closely spaced, SSI-cov method is successful to identify both modes in such a pair. Another advantage is that the SSI-cov method can also identify the damping ratios directly. Compared the other seven identified mode shapes with high MAC (Modal Assurance Criterion) values, the lower MAC value in the fourth mode is primarily because it is not reasonably excited. This is seen from the stabilization diagram of the system responses that the fourth mode is rarely present, whereas the other seven modes are quite active. On the other hand, the general requirement of the output-only method is that the modes should be reasonably excited. Therefore, the other seven quite active modes will be used to calculate the vertical damage index (DI*v*) in the following section.

*3.3 Damage detection: Sensitivity*

Damage can be simulated by changing the Young’s modulus, changing the cross-section area or simply removing the elements at the damage location. In this study, we simulate damage in the hanger cables by reducing their Young’s modulus. It is worth noting that a bad signal and a mistaken identification affects the damage index DI and may lead to wrong damage detection results. In order to figure out the extent of the signal noise and the identification error, we applied the damage detection method on a healthy bridge.

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| Fig. 4 Effect of modal identification error (with SSI-cov) on the Damage Index |

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| (a) DI*v* with 1% RMS noise | (b) DI*v* with 5% RMS noise |
| Fig. 5 Effect of measurement noise on the damage index (for a healthy structure) | |

Fig.4 illustrates the effect of modal identification error on the damage index DI level, where the SSI-cov method has been used to extract the modal parameters (natural frequencies and mode shapes); it shows that a damage index of 1.5% could be measured due to the identification error. Fig.5 shows the effect of measurement noise, where two levels of the noise are considered. It is evident that these two figures have clear deviations and demonstrate the influence of the noise and identification error on the damage detection accuracy. Therefore, one can consider the average value of the damage index DI in Fig.4 and Fig.5 as a threshold, above which one or several hangers are likely to be damaged. In this study, it is set to 3%.

To verify the feasibility of the MF method, a 95% damage is simulated numerically in hanger 3 and 5, where two levels of measurement noise are considered. Then the damage indices DIs are constructed by using the SSI-cov method to extract the modal parameters of the damaged and undamaged model. The results are shown as follow:

* Test-case 1: damage cable at the mid-span

In this damage case, the stiffness of the hanger cable 5 (mid-span) is reduced by 95%, then the modal parameters are identified from ambient vibration. The damage indices (a) DI*v* with 1% standard deviation measurement noise and (b) DI*v* with 5% standard deviation noise are shown in Fig.6. The DI*v* for the damaged cable reaches its maximum value at the nodes of the damaged location. In Fig. 6(b), although the noise level is relatively high, the DI*v* still shows its maximum value at the damaged location.

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| (a) DI*v* with 1% RMS noise | (b) DI*v* with 5% RMS noise |
| Fig. 6 Damage indices DI for Test-case 1 (damage in hanger 5) | |

* Test-case 2: damage cable at one third of the span

Figure 7 illustrates the DI*v* for the second damage case, after reducing by 95% the stiffness of cable 3 (at one third of the span). Observe that the plot of DI*v* peaks at the exact damage location for both levels of noise.

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| (a) DI*v* with 1% RMS noise | (b) DI*v* with 5% RMS noise |
| Fig. 7 Damage indices for DI Test-case 2 (damage in hanger 3) | |

Based on the examination of the two examples, it can be concluded that incorporating the vertical components of the mode shapes for detecting damage in a hanger for this bridge structure is a successful approach. The performance of DI*v* in single and multiple damage scenarios with different damage levels is evaluated through experimental study in next section.

**4. Experimental implementation**

*4.1 Experimental setup*

The laboratory mock-up of the suspension bridge (Fig.8) is used to investigate the capability of the modal flexibility method. It consists of two articulated towers of 0.62m distant of 2.2m; the deck is free to rotate at both ends and is attached to the catenary by two rows of 10 hangers. This mock-up was used previously for demonstrating active damping with stay cables (Preumont *et al*. 2016).

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| Fig. 8 Laboratory mock-up of the suspension bridge |

The catenary consists of a steel cable with a diameter of 1mm and the hangers are made of steel cables of 0.5 mm; the tension *T*0 in the catenary and the hangers can be adjusted with screws. It is measured indirectly from the lateral bending natural frequency *fs* according to the string formula:

 (16)

*fs* being measured by a non-contact custom made laser sensor (Achkire and Preumont 1998), *L* is the length of the cable, *ϱ* is its mass density and *A* is its cross section. In this way, it was possible to distribute the tension in the hangers uniformly. As shown in Fig.8, a small magnet is attached to the deck and a voice coil is used to apply a disturbance to the structure (band-limited white noise). Prior to vibration measurement, the data acquisition system was established, which involves 4 single-axial accelerometers, positioned to measure vertical accelerations. Layout of accelerometers is illustrated in Fig.9. The sample setting is the same as the section 3.2. The output data are obtained by repeated measurements at different positions on the deck, Fig.9.. The modal parameters of the damaged and undamaged bridge are estimated by the same method (SSI-cov) as in section 3. The experimental frequencies and mode shapes of the healthy bridge are shown in Table 1.

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| (a) Accelerometer | (b) Position on the deck |
| Fig. 9 Detail information of accelerometer | |

Table 3 Different damage scenarios

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| Damage scenarios | Location | Damage Intensity |
| Single damage scenario | | |
| Case 1 | Cable 5 | Reduce 95% tension |
| Case 2 | Cable 3 | Reduce 95% tension |
| Case 3 | Cable 3 | Reduce 80% tension |
| Case 4 | Cable 7 | Reduce 90% tension |
| Case 5 | Cable 19 | Reduce 90% tension |
| Multiple damage scenario | | |
| Case 6 | Cable 15 and Cable 16 | Reduce 90% tension |
| Case 7 | Cable 4  Cable 18 | Reduce 80% tension  Reduce 90% tension |
| Case 8 | Cable 6  Cable 9 | Reduce 90% tension  Reduce 95% tension |

*4.2 Damage scenarios*

The damage in the hanger cables is simulated by reducing the pre-stress in the cables in the mock-up. Cable numbers are shown in Fig.2. The numbering of the hangers from 1 to 10 on one side, starting from the right, and from 11 to 20 on the opposite side. Table 3 below presents the details of the damage scenarios considered in this section. For each damage case, the SSI-cov method is used for modal identification, and the damage index is computed.

*4.3 Results and discussion*

*4.3.1 Single damage scenario*

* Damage Case 1

Experiment results of the damage index for Case 1 are shown in Fig.10 (damage at cable 5). In Fig.10, the vertical damage index DI*v* for the damaged cable reaches its maximum value at the damaged location. In this damage case, the vertical damage index is able to detect the 95% damage of the cable successfully at the middle of span and confirm the actual location of the damage.

* Damage Case 2 and 3

Figure 11 illustrates the DI*v* for Case 2 and Case 3, where the tension of cable 3 is reduced (95%). The damage is easily distinguished from DI*v* peak for Case 2. However, when the damage intensity reduces to 80% in Case 3, the DI*v* shows its maximum values at various locations, and hence the results are unreliable.

Based on the examination of these three cases, it can be concluded that using the vertical damage index is a successful approach to detect severe damages in the hanger cables.

* Damage Case 4 and 5

In this scenario, there are two 90% tension reductions near the middle and four-fifths of the span (cable 7 and cable 19), respectively. From Fig.12 (a), it is evident that the damage index (DI*v*) gives a correct prediction of the damage location. Similar trends are observed in Fig.12 (b). Therefore, the results of these two cases demonstrate the effectiveness of this approach for damage detection, with severe intensities.

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| Fig. 10 Damage indices for damaged cable 5 (95%) |

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| (a) Case 2 (95%) | (b) Case 3 (80%) |
| Fig. 11 Damage indices for damaged cable 3 | |

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| --- | --- |
|  |  |
| (a) Damage indices for damaged cable 7 (90%) | (b) Damage indices for damaged cable 19 (90%) |
| Fig. 12 Results of Case 4 and 5 | |

*4.3.2 Multiple damage scenario*

* Multiple damage Case 6

In this damage case, the hanger tension is reduced by 90% at the middle span (cable 15 and cable 16); the results are shown in Fig.13 (a) and it can be seen that DI*v* detects accurately the damage.

* Multiple damage Case 7

In damage Case 7, there are two damage locations, one near the mid-span (cable 4) with 80% tension reduction, another one near two-thirds of the span (cable 18) with 90% tension reduction. The results are shown in Fig.13 (b). It is evident that the method is able to detect accurately the presence of the damage, but fails to detect it accurately.

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| (a) Damage indices for damaged cable 15 and 16 | (b) Damage indices for damaged cable 4 and 18 |
| Fig. 13 Results of Case 6 and 7 | |

* Multiple damage Case 8

In this damage case, 90% tension reduction is made near the mid-span (cable 6) and 95% tension reduction is also made near four-fifths of the span (cable 9). The results are shown in Fig.14 and it can be seen that DI*v* locates the damage accurately. However, the figure suggests also the presence of damage in cable 2, which calls to search for a new damage index, dedicated for locating the damage.

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| Fig. 14 Damage indices for damage Case 8 (damage at cable 6 and 9) |

**5. Comparison with traditional methods**

In this section, the traditional modal parameters-based damage detection methods: Coordinate Modal Assurance Criterion COMAC (Wahab and De Roeck 1999), Enhanced Coordinate Modal Assurance Criterion ECOMAC (Hunt 1992), Mode Shape Curvature MSC (Pandey *et al*. 1991) and Modal Strain Energy MSE (Stubbs *et al*. 1995, Li *et al*. 2006) are introduced to detect the damage for the scenarios of Table 3. The experimental results are summarized in Table 4.

As one observes in Table 4, most of the methods cannot detect the low-level damages, corresponding to a loss of 80% of the hangers (Case 3 and Case 7). When the damage intensity is high, up to 95% tension loss, and close to the mid-span (Case 1), three traditional methods: ECOMAC, MSC and MSE are able to detect the damage successfully and confirm the actual location of the damage. The detailed results are shown in Fig.15 for Case 1. However, through comparing the experimental results (Table 4) of these four traditional methods we can find that once the damage intensity down to 90% and the damage position deviates from the mid-span, the performance of the MSC and MSE methods are better than the COMAC and ECOMAC methods (Case 2, Case 4 and Case 6). A reasonable explanation for this outcome is the fact that both the COMAC and ECOMAC methods depend on the mode shape amplitudes of the damaged structure, while the MSC and MSE methods are calculated based on the second derivative of the mode shapes, which is more sensitive to local the damage (Talebinejad *et al*. 2011). As a result, these two methods are relatively successful to detect the damage. For the 90% damage level near the mid-span (Case 4 and Case 6), the MSE method has the same performance as MF method. However, when the damage occurs near the edge of the span (Case 5 and Case 8), the performance of the MSC and MSE methods are inferior to the MF method. Therefore, the MF method is more applicable than others.

Table 4 Comparison of damage detection methods

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Case # | COMAC | ECOMAC | MSC | MSE | MF |
| 1 | X | O | O | O | O |
| 2 | X | X | OO | O | O |
| 3 | X | X | OO | OO | OO |
| 4 | X | OO | OO | O | O |
| 5 | X | X | X | X | O |
| 6 | X | OO | O | O | O |
| 7 | X | OO | X | OO | OO |
| 8 | X | OO | OO | OO | O |

\***O**: damage detected; X: damage not detected; OO: damage detected not accurately

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| --- | --- |
|  |  |
| (a) COMAC | (b) ECOMAC |
|  |  |
| (c) MSC | (d) MSE |
| Fig.15 Detailed results for Case 1 | |

**6. Conclusions**

This study compared numerically and experimentally several damage detection method on a laboratory suspension bridge mock-up, with an emphasis on the Modal Flexibility method. It presented a successful, numerical and experimental implementation of the method for damage detection in the hangers of suspension bridges, where only ambient vibration data has been used. The effectiveness is demonstrated by a number of numerical simulations and experiments. The comparison between the MF method and the traditional methods (COMAC, ECOMAC, MSC and MSE) is carried out for eight experimental cases. Modal parameters including natural frequencies and mode shapes of the healthy structure and the damaged structure are extracted by the SSI-cov method, using output only measurements. Based on the obtained results and their interpretations, the main findings are presented below:

* The DI*v* was capable for successfully detecting the presence of single and multiple damage in the hangers of suspension bridges (more than 90% intensity). The hangers are likely to be damaged if the average value of the damage index exceeds a predefined threshold about 3%, which represents the effect of the measurement noise and identification error.
* The damage localization has been made in a heuristic way, where the hanger with the highest damage index is considered as damaged. Particularly, this is due to the fact that the damage index value is proportional to the damage level and very sensitive to its location in the structure (cfr. e.g. Fig.11 and Fig.13), hence, setting a minimum detection threshold for each hanger would lead to misleading results and the DI would fail to detect low levels of damages. A more objective and robust criterion must be find to locate accurately the damage, particularly for multiple damages. The use of an external excitation system will be considered in a separate study.
* The results obtained with the DI*v* showed its superior capability compared with the traditional modal parameters-based methods.
* The performance of the MSC and MSE methods were better than the COMAC and ECOMAC methods. Once the damage occurs at the edge of the span, the performance of the MSC and MSE methods were inferior to the MF method. However, only high-intensity damages were detectable by the modal flexibility method. This is due to the very low contribution of the hanger tension to the global stiffness matrix.

Finally, the findings of this study must be extended and validated to field tests on a real and full-scale suspension bridge.

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