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Stochastic modelling for hysteretic bit-rock interaction of a drill string under torsional vibrations

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Abstract

This paper aims at constructing a stochastic model for the hysteretic behaviour of the nonlinear bit-rock interaction of a drill string under torsional vibrations. The proposed model takes into account the fluctuations of the stick-slip oscillations observed during the drilling process. These fluctuations are modelled by introducing a stochastic process associated with the variations of the torque on bit, which is a function of the bit speed. The parameters of the stochastic model are calibrated with field data. The response of the proposed stochastic model, considering the random bit-rock rock interaction, is analyzed, and statistics related to the stability of the drill string are estimated.

Keywords

bit-rock stochastic interaction model, drillstring dynamics, hysteretic friction, experimental identification

1 Introduction

Drill string is a slender structure used for exploitation of oil reserves. A top drive rotates the system at the top, which transmits the torque to the bit that drills the rock. The design and analysis of drill strings are usually performed using a computational model. Several drill string models have been proposed in the literature. The complete dynamics of a drill string includes axial, torsional, and lateral vibrations Tucker and Wang C (1999); Ritto et al. (2009). In addition, in the development of directional drilling, advanced features taking into account the horizontal dynamics analysis might be necessary Ritto et al. (2013); Cunha et al. (2015). Torsional control is also an issue Patil et al. (2013); Ritto and Ghandchi-Tehrani (2018).

The present paper focuses on the analysis of the torsional vibration of the drill string, which is nonlinear due to the bit-rock interaction. Field data (Pavone and Desplans (1994); Ritto et al. (2017); Real et al. (2018)) show that the torque on bit varies nonlinearly with the bit speed. Furthermore, these observations show hysteretic cycles which fluctuate during the drilling process, Real et al. (2018). Torsional vibration is a problem in drill string dynamics, especially when its length reaches thousands of meters. In some conditions, the lateral and axial vibrations are small, and the torsional vibration is the main concern (Ritto et al. (2017)); it might lead to stick-slip oscillations, Kyllingstad nd Halsey (1988); Leine et al. (1998); Tucker and Wang C (1999); Richard and Detournay (2004); Khulief et al. (2007); Kreuzer and Steidl (2012); Patil et al. (2013); Hong et al. (2010). In this severe conditions, the bit sticks (zero speed) then slips (high speed), and that might cause, for instance, measurement equipment failure, low rate of penetration, bit damage, and fatigue Wu et al. (2012).

In Real et al. (2018), the authors have observed fluctuations of the nonlinear bit-rock interaction law during the drilling. These fluctuations, which are affected by the variation of soil mechanical properties during the drilling, have to be taken into account in order to study the robust stability of the drill string. The goals of the present paper are (1) to characterize and construct a probabilistic model of these fluctuations and then (2) to estimate the probability of instability of the drill string including the stochastic non-linear bit-rock interaction model. In the literature, only few papers are concerned with the probabilistic modelling of the bit-rock interaction. In Spanos et al. (2002), a stationary random process is considered to model lateral forces at the bit. Recently, Qiu et al. (2016, 2017) analysed the random drill string dynamics considering random white noise for the axial force and for the bit-rock interaction coefficients. In Ritto and Sampaio (2012)), uncertainties are considered in the bit-rock interaction parameters. In Ritto et al. (2009), an adaptation of the nonparametric probabilistic approach Soize (2000) is proposed to model globally the uncertainties in the bit-rock interaction model. For the two later referred papers, the probabilistic models are not time-dependent, i.e., the bit-rock interaction model is random but does not vary during the drilling. Constructing such stochastic computational models including the stochastic fluctuations of the bit-rock interaction forces would be helpful for robust optimization of the drill string Ritto et al. (2010).

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In the present work, a continuous nominal model for the torsional dynamics of a drill string is discretized by means of the finite element method (Jansen (1993); Khulief and Al-Naser (2005); Ritto and Sampaio (2012)), and a reduced-order model is constructed using the elastic modes of the linear structure. The bit-rock interaction is modelled as a non-linear torque applied at the bit. This simple model enables the analysis of the stability of the system. Then a stochastic non-linear bit-rock interaction model is proposed by introducing a multiplicative stationary Gaussian stochastic process. Field data are used to calibrate the power-spectral density function of this stochastic process. Once calibrated, independent realizations are generated and statistics of the drill string dynamical response are estimated using the Monte Carlo method.

The paper is organized as follows. Section 2 describes the nominal drill string computational model, including the nonlinear bit-rock interaction. Section 3 presents the proposed probabilistic model for the bit-rock interaction, which is modelled as a nonstationary stochastic process to take into account its stochastic fluctuations during the drilling. The numerical results are presented in Section 4, where deterministic and stochastic dynamical responses are analysed. The last section brings the concluding remarks.

2 Deterministic model for the drill string

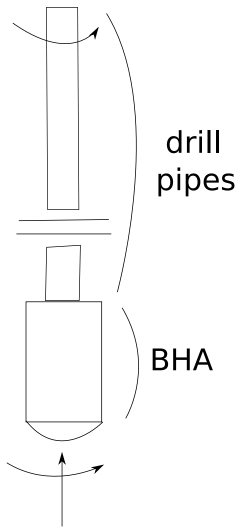

Figure 1. General scheme of a drill string.

Figure 1 presents a scheme of the torsional system considered in the analysis. A vertical wellbore, with only torsional vibrations, is taken into account. The system is composed of drill pipes (about 5,000 meters) and the bottom hole assembly (BHA, about 400 meters). In the present paper lateral and axial vibrations are not modelled. A pure torsional model might give good results if axial and lateral vibrations are small, Ritto et al. (2017).

A constant angular speed Ω is imposed at the top and a reaction torque acts at the bottom due to the bit-rock interaction. In this paper, the torsional dynamics of the drill string is analysed by solving the differential equation:

$$\rho J \frac{\partial^2 \boldsymbol{\theta}(x,t)}{\partial t^2} - GJ \frac{\partial^2 \boldsymbol{\theta}(x,t)}{\partial x^2} = \boldsymbol{\mathcal{T}}(x,t), \qquad (1)$$

where the space x and the time t are the independent variables, $\boldsymbol{\theta}(x,t)$ is the angular rotation, $\boldsymbol{\mathcal{T}}(x,t)$ is the torque per unit length, J is the cross sectional polar area moment of inertia, and ρ and G are the mass density and shear modulus of the material of the column. The boundary conditions related to the imposed angular speed at the top are given by

$$\begin{cases} \boldsymbol{\theta}(0,t) = \Omega t \\ \dot{\boldsymbol{\theta}}(0,t) = \Omega \end{cases}, \qquad (2)$$

and the initial conditions are

$$\boldsymbol{\theta}(x,0) = 0, \quad \dot{\boldsymbol{\theta}}(x,0) = \Omega \ , \qquad (3)$$

As proposed by Real et al. (2018), the rotational displacements about a rotating frame is considered. Let $\boldsymbol{\theta}^{rel}(x,t)$ be the relative torsional degree of freedom in the rotating frame associated to the top sectional area (at $x=0$). We introduce the absolute rotational displacement as

$$\boldsymbol{\theta}(x,t) = \Omega t + \boldsymbol{\theta}^{rel}(x,t). \qquad (4)$$

The system is discretized by means of the finite element model, where linear shape functions are applied. Let $\mathbf{u}(t)$ be the vector of $\boldsymbol{\theta}^{rel}(x,t)$ nodal values related to the drill string mesh. Note that, in the rotating frame, the angular displacement is fixed at the top because there is no relative displacement between the top drive and the first node, at the top of the drill string. Adding a proportional damping to the system, the vector $\mathbf{u}(t)$ is solution of the matrix equation

$$[M]\ddot{\mathbf{u}}(t) + [D]\dot{\mathbf{u}}(t) + [K]\mathbf{u}(t) = \mathbf{T}(\dot{\mathbf{u}}(t), \ddot{\mathbf{u}}(t))), \qquad (5)$$

where $[M]$ is the mass matrix, $[D]$ is the damping matrix, $[K]$ is the stiffness matrix, and $\mathbf{T}(\dot{\mathbf{u}}(t), \ddot{\mathbf{u}}(t))$ is the torque vector. According to Eqs. (3) and (4), the initial conditions in the rotating frame read

$$\mathbf{u}(0) = \mathbf{0}, \quad \dot{\mathbf{u}}(0) = -\Omega \mathbf{1}. \qquad (6)$$

where $\mathbf{1} = [I_{n\times 1}]$ a vector with all entries equal to one. All the components of the torque vector are zero except the one corresponding to the drill bit node (at $x = L$). For this node, the nonlinear torque applied to the bit is denoted by $T_{\text{bit}}(\dot{\theta}_{\text{bit}}(t), \ddot{\theta}_{\text{bit}}(t))$ and will be described in the next section.

The normal modes of the conservative homogeneous system are used to construct a reduced-order model. The m first eigenvalues $0 < \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_m$ associated with the elastic modes $\{\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots, \boldsymbol{\varphi}_m\}$ are solution of the generalized eigenvalue problem

$$[K]\,\boldsymbol{\varphi} = \lambda [M]\,\boldsymbol{\varphi}\,. \qquad (7)$$

The reduced-order model is obtained by projecting the full computational model on the subspace spanned by the m first elastic modes calculated using Eq. (7). Let $[\Phi]$ be $n \times m$ matrix whose columns are the m first elastic modes. We can then introduce the following approximation

$$\mathbf{u}(t) = [\Phi]\,\mathbf{q}, \qquad (8)$$

in which $\mathbf{q}$ is the vector of the m generalized coordinates which are solution of the reduced-order system

$$[\widetilde{M}]\ddot{\mathbf{q}}(t) + [\widetilde{D}]\dot{\mathbf{q}}(t) + [\widetilde{K}]\mathbf{q}(t) = \tilde{\mathbf{T}}(\dot{\mathbf{q}}(t), \ddot{\mathbf{q}}(t)), \qquad (9)$$

with the initial conditions

$$\mathbf{q}(0) = \mathbf{0}, \quad \dot{\mathbf{q}}(0) = -\Omega\,[\widetilde{M}]^{-1}[\Phi]^T[M]\mathbf{1}. \qquad (10)$$

In these equations, $[\widetilde{M}] = [\Phi]^T\,[M]\,[\Phi]$, $[\widetilde{D}] = [\Phi]^T\,[D]\,[\Phi]$ and $[\widetilde{K}] = [\Phi]^T\,[K]\,[\Phi]$ are $m \times m$ mass, damping and stiffness reduced-order matrices, and $\widetilde{\mathbf{T}}(\dot{\mathbf{q}}(t), \ddot{\mathbf{q}}(t)) = [\Phi]^T\mathbf{T}([\Phi]\dot{\mathbf{q}}(t), \Phi]\ddot{\mathbf{q}}(t))$ is the vector of the reduced-order generalized torque. The set of equations (8), (9) and (10) can be solved using commonly used integration schemes, such as the Euler or the Runge-Kutta schemes, for instance.

3 Probabilistic model for the bit-rock interaction with hysteretic effects

Figure 2 shows the field data (Ritto et al. (2017); Real et al. (2018)) that will support the proposed model presented in this section. The downhole information used in this paper was acquired using a downhole mechanics measurement unit capable of providing both real-time measurement through mud telemetry and continuously recorded high-frequency data throughout the run. The sub, installed at the BHA above the bit, contains a suite of 19 sensors sampled at 10,000 Hz and downsampled and filtered prior to recording at 50 Hz. Following is a list of 50-Hz data recorded in this sub: triaxial accelerations; gyro rpm; magnetometer rpm; axial loading; torque; bending moment (Shi et al. (2016)).

The field data shown in Fig. 2 was measured at the BHA, very close to the drill bit, Shi et al. (2016). A window average smoothing was used to filter measurements noise and obtain the stick-slip cycles shown in the figure. The figure shows the variation of the torque on bit with respect to the bit speed; and also the bit speed as a function of time. A deterministic bit-rock interaction model that takes into account the observed hysteresis (non-reversibility) effect was proposed by Real et al. (2018). The bit-rock interaction mechanism is complex, but its overall behaviour has similarities with friction models; Hess and Soom (1990); Olsson et al. (1998); Wojewoda et al. (2008). The deterministic hysteretic bit-rock interaction model considered in the present paper reads (Real et al. (2018))

$$\underline{T}_{\rm bit}(\dot\theta_{\rm bit}, \ddot\theta_{\rm bit}) = b_0 \left(\tanh(b_1\dot\theta_{\rm bit}) + \frac{b_2\dot\theta_{\rm bit}^{b_4}}{1 + b_3\dot\theta_{\rm bit}^{b_5}}(1 + H(\dot\theta_{\rm bit}\ddot\theta_{\rm bit})\right), \qquad (11)$$

for $\dot\theta_{\rm bit} > 0$, in which $b_0, b_1, b_2, b_3, b_4, b_5$ are positive parameters to be fitted such that $0 < b_4 < b_5$. The bit-rock interaction model contains a hysteretic function H, which is a function of the bit angular acceleration (to separate forward and backward phases) and which is defined by

$$H(\dot\theta_{\rm bit}, \ddot\theta_{\rm bit}) = \beta_1 \tanh(\beta_2\ddot\theta_{\rm bit})\,, \qquad (12)$$

where β_1 and β_2 are two positive parameters. Note that the hysteretic cycle is limited $(1 \pm \beta_1)$, and if H equals to zero there is no hysteresis.

In Real et al. (2018), the parameters of this bit-rock interaction model have been calibrated such that it fits

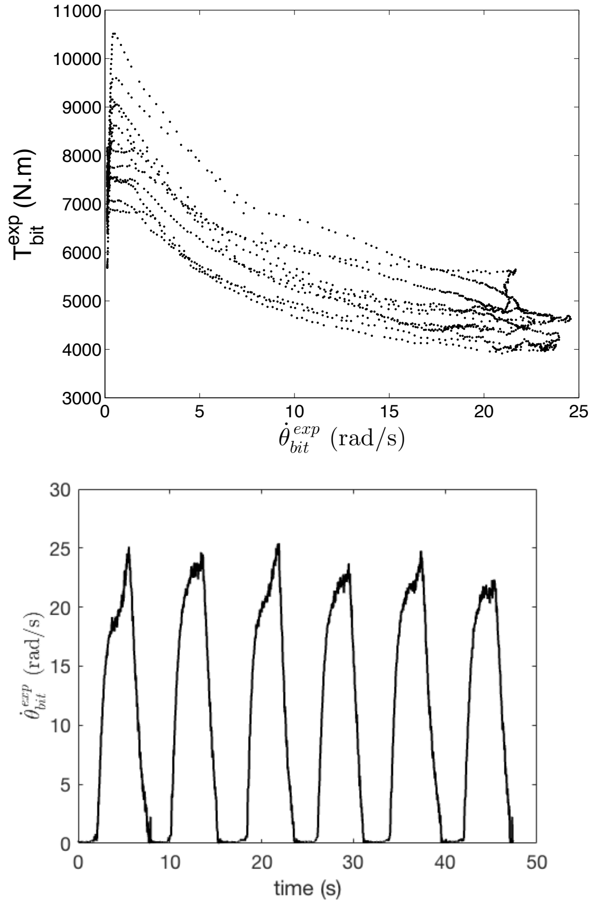

Figure 2. Field data stick-slip cycles: measured torque $T_{\rm bit}^{\rm exp}$ versus the bit speed $\dot\theta_{\rm bit}$ (top), and bit speed versus time (bottom).

with the mean field data cycle. The optimal parameters are $b_0 = -3478, b_1 = 938, b_2 = 2.56, b_3 = 0.38, b_4 = 0.78$, and $b_5 = 1.1$, $\beta_1 = 14\%$, $\beta_2 = 10.6$.

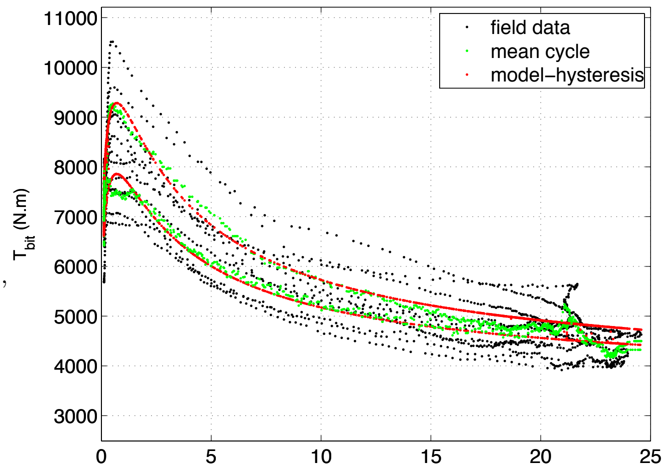

Figure 3. Bit-rock interaction: field data (black), mean field data cycle (green) and bit-rock interaction model with hysteresis (red).

Figure 3 compares the field data with the deterministic model. It can be seen that, in average, the bit-rock interaction model, Eq.(11), presents a reasonable good fit comparing to the field data mean cycle. Nevertheless, it is noticed a considerable time fluctuation of the field data cycles. Part of this fluctuation is explained by the variation of the soil mechanical properties during the drilling. The probabilistic modelling of this fluctuation is the objective of this paper.

If we take a close look at the field data, we notice that the torque on bit varies from about 7 to 10.5 kNm, when the bit speed is close to 1 rad/s. For higher speeds, 20 rad/s, the torque on bit varies from about 4 to 5.8 kNm. The distance between the higher and lower values is very different, depending on the bit speed: 3.5 and 1.8 kNm. But, for both speeds the torque on bit is varying plus or minus 20%. Therefore, the present paper proposes to take into account the stochastic fluctuations of the bit-rock interaction model by including a multiplicative stochastic process to the deterministic model:

$$T_{\mathrm{bit}}(\dot{\theta}_{\mathrm{bit}}(t), \ddot{\theta}_{\mathrm{bit}}(t)) = \underline{T}_{\mathrm{bit}}(\dot{\theta}_{\mathrm{bit}}(t), \ddot{\theta}_{\mathrm{bit}}(t))(1+\eta(t)), \tag{13}$$

where $\eta(t)$ is a centred stochastic process $\eta(t)$ which can be rewritten as

$$\eta(t) = \frac{T_{\mathrm{bit}}(\dot{\theta}_{\mathrm{bit}}(t), \ddot{\theta}_{\mathrm{bit}}(t))}{\underline{T}_{\mathrm{bit}}(\dot{\theta}_{\mathrm{bit}}(t), \ddot{\theta}_{\mathrm{bit}}(t))} - 1. \tag{14}$$

The experimental stochastic process $\eta^{\exp}(t)$ can be computed using Eq. (14) and the field data related to the torque on bit. This stochastic process is shown in Fig. 4. Unfortunately, there is not enough field data to completely characterize this stochastic process. We will assume here that $\eta(t)$ is a centred stationary Gaussian stochastic process. This assumption should be verified in future works, using more experimental data. The power spectral density (PSD) is estimated using the periodogram method Priestley (1981). Figure 5 shows the estimated field data PSD.

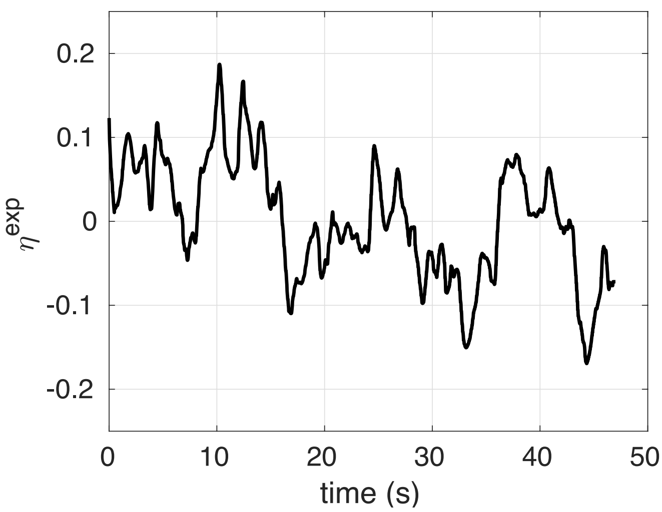

Figure 4. Stochastic process $\eta^{\exp}(t)$ obtained experimentally with field data.

Regarding Figure 5, the PSD is constant until a critical frequency and then decreases linearly (in log-log scale). This type of PSD is often encountered when addressing turbulent forces Batou and Soize (2009). Then the proposed PSD model $S(f)$ is written as

$$\begin{aligned} \log(S(f)) &= A_0 \quad \text{for} \quad \mathrm{f} < \mathrm{f}_0\,, \\ \log(S(f)) &= a\log(f) + b \quad \text{for} \quad \mathrm{f} \geq \mathrm{f}_0\,, \end{aligned} \tag{15}$$

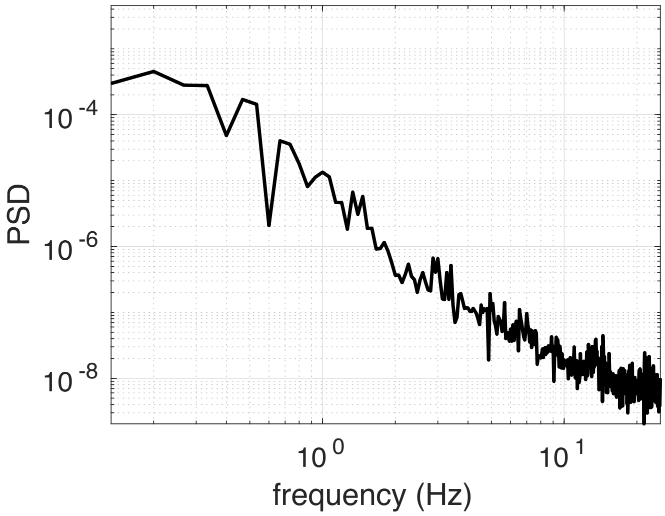

Figure 5. Field data PSD.

where f_0, A_0, a and b are the parameters of the model. These parameters are calibrated using the experimental PSD such that $f_0 = 0.27$, $A_0 = -7.6$, $a = -3.13$ and $b = -11.67$ (with appropriate units). Figure 6 compares the calibrated PSD model with field data PSD, where a reasonable agreement is observed.

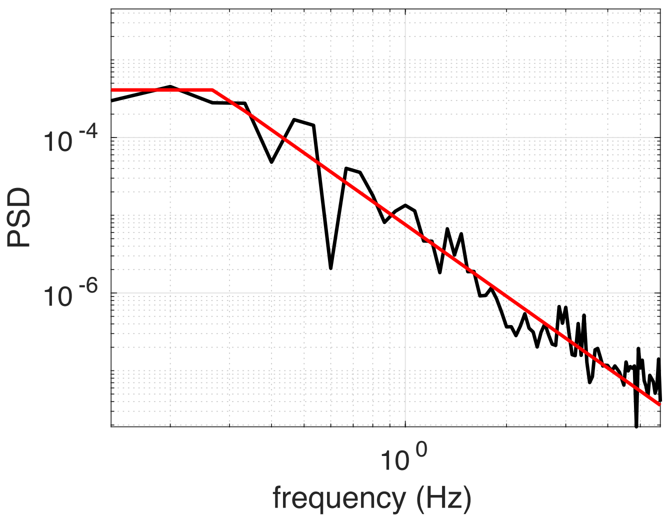

Figure 6. Comparison between the calibrated PSD (red line) and the field data PSD (black line).

With the PSD $S(f)$ in hands one can generate independent realizations of the stochastic process $\eta(t)$ using a classical generator of Gaussian process, Benaroya (2005). Figure 7 shows two independent trajectories of $\eta(t)$, which again give reasonable agreement with the observed process $\eta^{\exp}(t)$ shown in Figure 4.

4 Simulation of the stochastic drill string dynamics

The computational time to perform a deterministic run in a computer Quad-Core Processor (1GHz), 4 GB RAM, is about 3 seconds. The computational time to perform the stochastic samples is about 13 minutes.

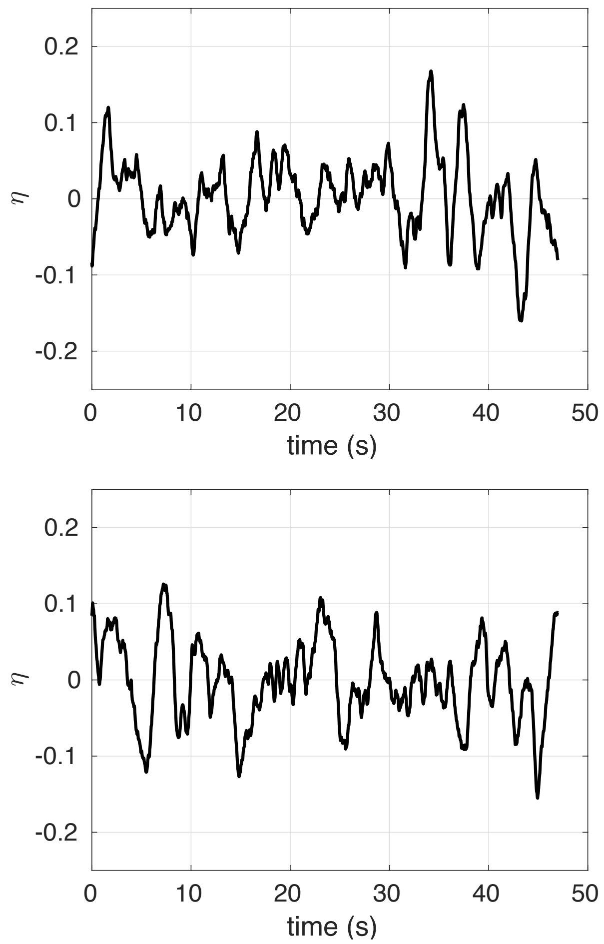

Figure 7. Two independent realizations of the stochastic process $\eta(t)$.

Algorithm 1: Simulation of the drill string dynamics.

INITIALIZATION:

Generate a realization of stochastic process $\eta(t)$;

$\mathbf{q}_0 = \mathbf{0}$;

$\dot{\mathbf{q}}_0 = -\Omega\,[\widetilde{M}]^{-1}[\Phi]^T[M]\mathbf{1}$;

LOOP: for $k = 1, \ldots, (n_t)$ **do**

  Update the angle and angular speed (depending on the integration scheme):

  $(\dot{\mathbf{q}}_{i-1}, \dot{\mathbf{q}}_{i-1}) \rightarrow (\dot{\mathbf{q}}_i, \dot{\mathbf{q}}_i)$;

  $\dot{\mathbf{u}}_i = [\Phi]\,\dot{\mathbf{q}}_i$;

  $\dot{\theta}_{\text{bit,i}} = \dot{u}_{\text{bit,i}} + \Omega$;

  $\ddot{\theta}_{\text{bit,i}} = \ddot{u}_{\text{bit,i}}$;

  Calculate the torque on bit:

  $T_{\text{bit,i}}(\dot{\theta}_{\text{bit,i}}, \ddot{\theta}_{\text{bit,i}}) = \underline{T}_{\text{bit}}(\dot{\theta}_{\text{bit,i}}, \ddot{\theta}_{\text{bit,i}})(1 + \eta_i)$;

  Calculate the torque vector $\mathbf{T}_i$;

  Calculate the reduced-order torque vector

  $\tilde{\mathbf{T}}_i = [\Phi]^T\mathbf{T}_i$

	DP	BHA
Elastic Modulus [GPa]	220	220
Poisson's coefficient	0.29	0.29
Volumetric mass density [kg/m^3]	$7,800$	$7,800$
Length [m]	$4,733.60$	466.45
Inner radius [m]	0.0595	0.0363
Outer radius [m]	0.070	0.0803

Table 1. Drill string characteristics.

4.1 *Analysis of one realization of the stochastic drill string dynamical response*

The previous section was concerned with the construction of a stochastic bit-rock interaction model including hysteretic cycles fluctuations. A stationary stochastic process was introduced for the computation of the torque on bit, Eq.(13). The stochastic bit-rock interaction model is then added to the torsional drill string model, Eq.(9), and the stochastic non-linear dynamical response of the drill string is computed and analysed. Note that Eq.(9) becomes random because of the random bit-rock interaction, Eq.(13). The general integration scheme for one realization of the stochastic bit-rock interaction model is presented in Algorithm 1.

Table 1 contains the parameters of the drill string used for the simulation.

The mass and stiffness matrices are constructed using 100 finite elements, after convergence check. The generalized damping matrix is diagonal with damping ratios equal to 0.005 for the first mode, 0.03 for the second and third modes, and 0.005 for all the other modes. The first five natural frequencies computed for the system are: 0.13, 0.42, 0.74, 1.07, 1.41 Hz.

The non-linear equation (9) is solved using a modified Euler scheme with a time step 0.512 ms. For one realization of the stochastic bit-rock interaction model, Figure 8 shows the stochastic response of the drill string in the stationary regime.

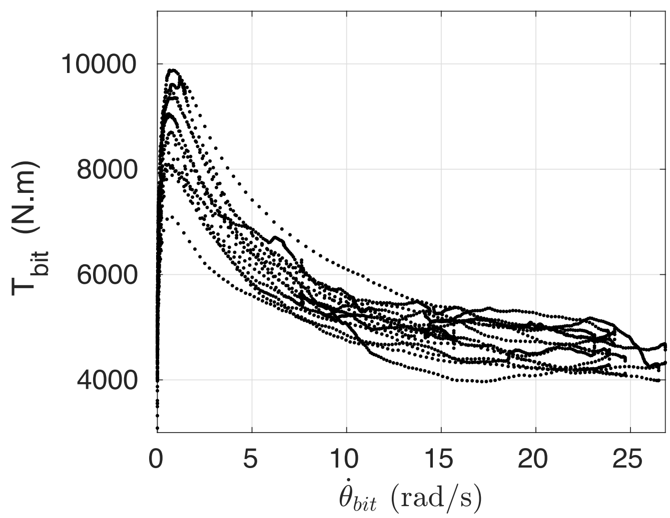

Figure 8. One realization of the simulated bit-rock interaction.

In comparison with Figure 2, it can be observed the same stochastic behaviour but with a slightly larger value of the maximum angular speed. Figure 9 compares the steady-state of random bit response using the proposed stochastic bit-rock interaction model with the corresponding time range response obtained using the deterministic model described by Eq. (11). The simulation was computed up to 2000 s, and the figure shows the steady state response from 1200 to 1300 s. While a 3-cycles periodic regime is reached for the deterministic case, no periodicity is observed in the stochastic case.

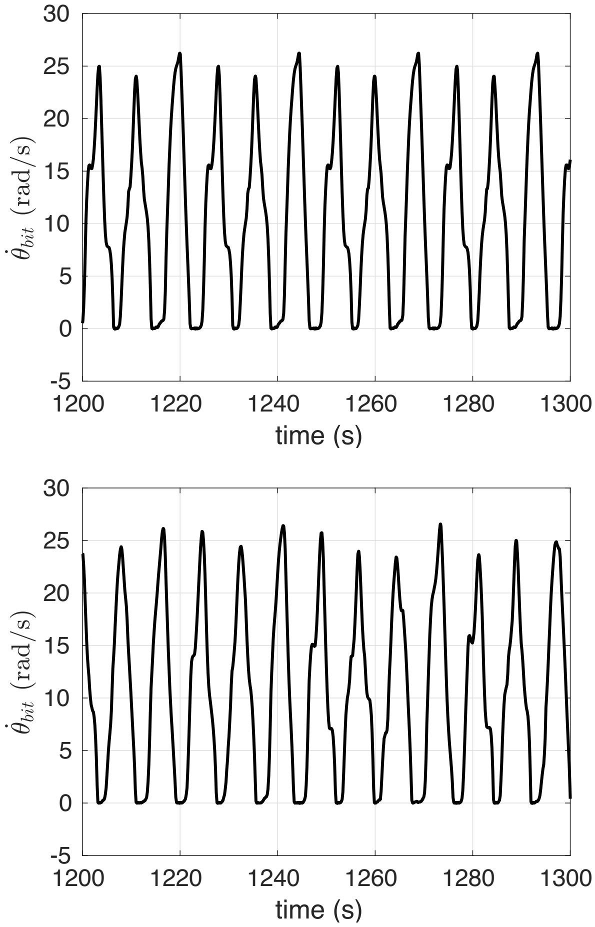

Figure 9. Steady-state response of the bit angular speed: deterministic bit-rock interaction model (top, 3-cycles periodic regime is observed) and stochastic bit-rock interaction model (bottom, no periodicity is observed).

For the deterministic case, the 3-cycles periodic sequence for the duration of the stick and slip phases are respectively $(1.34, 2.53, 2.48)$ s and $(5.4, 6.28, 6.41)$ s. As a reference, the mean stick and slip duration of the six cycles registered from the field are, respectively, 2.82 s and 5.31 s. For the stochastic case, these durations are random and their associated probability distribution (obtained statistically using 750 cycles) are plotted in Fig. 10. We can see in these figures the variability of the stick and slip durations. The values calculated with the deterministic bit-rock interaction model fall inside the support of the probability distributions.

4.2 Stochastic stability analysis

Now the analysis is extended to quantify statistics on the stability threshold of the system, as the imposed speed at the top varies. The torsional stability of a drill string can be quantified through the stick-slip severity factor, defined by (Ritto et al. (2017))

$$\gamma_{\mathrm{SS}}(\Omega) = \frac{\dot{\theta}_{\mathrm{bit}}^{\max}(\Omega) - \dot{\theta}_{\mathrm{bit}}^{\min}(\Omega)}{2\,\Omega}, \tag{16}$$

where $\dot{\theta}_{\mathrm{bit}}^{\max}(\Omega)$ and $\dot{\theta}_{\mathrm{bit}}^{\min}(\Omega)$ are the minimum and maximum bit speed in the steady-state regime. In the case of

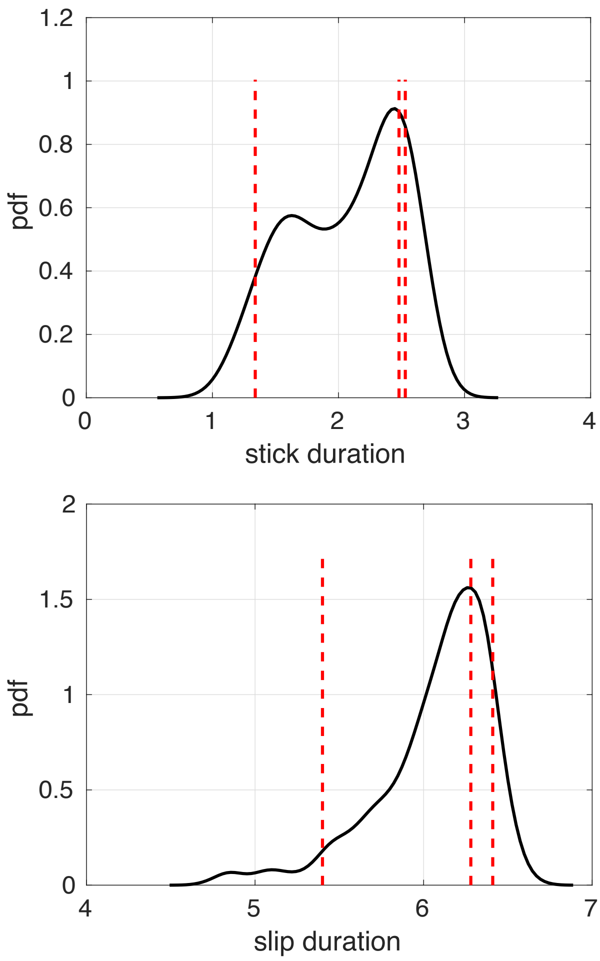

Figure 10. Random stick and slip phases duration: probability density functions (black line) and corresponding 3-cycles periodic values calculated with a deterministic bit-rock interaction model (red vertical lines).

no torsional oscillations, $\gamma_{\mathrm{SS}} = 0$. If there are torsional oscillations then $\gamma_{\mathrm{SS}} > 0$. As a reference, the available field data presents stick-slip oscillations with $\gamma_{\mathrm{SS}} = 0.99$, in which the top drive speed is 12.6 rad/s.

First, the deterministic system is analysed. The stick-slip severity factor in the frequency range $B = [6; 27]$ rad/s is plotted in Fig. 11. As expected, the stick-slip severity factor decreases when the imposed rotation at the top increases. If the speed at the top is lower than 16 rad/s, γ_{SS} is greater than 0.5.

For the stochastic bit-rock interaction model, the stick-slip severity factor becomes random and its statistics are estimated using the Monte Carlo simulation method with $n_s = 500$ samplings. For each Monte Carlo simulation, a realization of the stochastic bit-rock interaction model is generated, a realization of the stochastic angular speed is calculated, and the corresponding stick-slip severity factor is determined on the stationary regime. Figure 13 shows statistics on the random stick-slip severity factor. The convergence with respect to the number of samplings n_s is analysed by introducing the convergence function

$$CV(n_s) = \frac{1}{n_s}\sum_{i=1}^{n_s}\int_B \gamma_{\mathrm{SS}_i}(\omega)^2 d\omega, \tag{17}$$

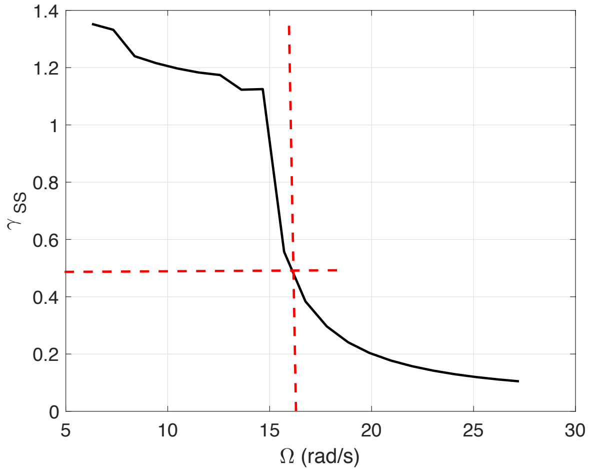

Figure 11. Stick-slip severity factor for a deterministic bit-rock interaction. Red dashed line indicates the stability threshold of γ_{SS}.

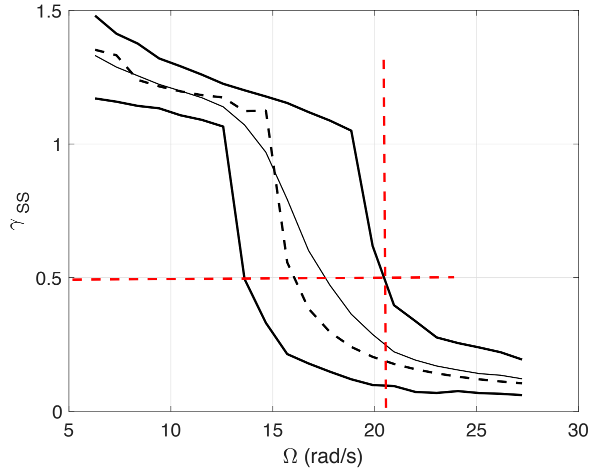

Figure 13. Random stick-slip severity factor for a stochastic bit-rock interaction. Solid lines: mean response and 90% confidence region. Dashed line: deterministic case (black), stability threshold (red).

where γ_{SS_i} corresponds to the i^{th} calculated realization of the stick-slip severity factor, and B is the integration domain. The convergence function is plotted in Figure 12; where a reasonable convergence is achieved with 500 realizations.

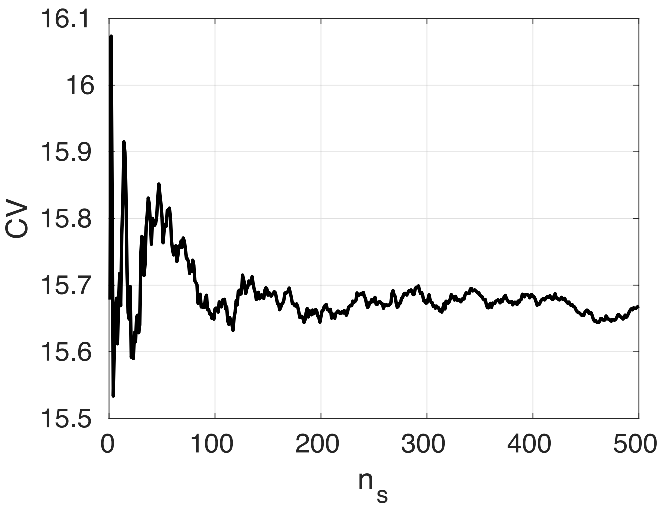

Figure 12. Convergence function $n_s \mapsto CV(n_s)$.

The statistical envelope shown in Fig. 13, due to the stochastic bit-rock interaction, yields large fluctuation in the random stick-slip severity factor. This means that bit-rock interaction variability has a direct impact on the drill string stability and should therefore be taken into account for a robust analysis of the drill string dynamics. The results show that when the top speed is about 20.5 rad/s the system has 5% probability of having the value of γ_{SS} greater than 0.5. The probability of instability increases as the speed decreases. This result brings much more information comparing with the deterministic result, where, for the same threshold of $\gamma_{\mathrm{SS}} < 0.5$, the limit speed of 16 rad/s was obtained.

5 Concluding Remarks

In the present paper, a new probabilistic model for the bit-rock interaction model is proposed. This model includes a multiplicative stochastic process to take into account fluctuations of the torque on bit during the drilling. The stochastic model was calibrated with field data, and it takes into account hysteretic cycles and their stochastic fluctuations. The proposed bit-rock probabilistic model can be constructed independently from the computational model of the column.

The deterministic and stochastic torsional dynamics of a drill string are analysed and a reasonable agreement between model predictions and field data is observed. The statistics of the stick and slip duration were also analysed.

A considerable impact of the proposed stochastic model on the torsional stability of the system was observed. For this end, the stick-slip severity factor is computed and statistical envelopes are plotted for varying imposed speed at the top. A robust estimation of the minimum top drive speed is estimated using these plots.

In future works, additional experimental data will be collected in order to validate our assumptions on the probability distribution of the stochastic process $\eta(t)$ introduced in the stochastic non-linear bit rock interaction model. Also, the PSD is likely to depend on the imposed angular speed. Future experimental results will enable the characterization of this angular speed dependency.

Conflict of interest

The authors declare no conflict of interest in preparing this article

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