From Solow to Romer: Teaching Endogenous Technological Change in Undergraduate Economics

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Abstract

Undergraduate students learn economic growth theory through the seminal Solow model, which takes the growth rate of technology as given. To understand the origin of technological progress, we need a model of endogenous technological change. The Romer model fills this important gap in the literature. However, given its complexity, undergraduate students often find the Romer model difficult. This paper proposes a simple method of teaching the Romer model. We add three layers of structure (one at a time) to extend the familiar Solow model into the less familiar Romer model. First, we incorporate a competitive market structure into the Solow model. Then, we modify the competitive market structure into a monopolistic market structure. Finally, we introduce an R&D sector that invents new products.

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1 Introduction

Economic growth is an important topic in economics. As Acemoglu (2013) argues, "economics instructors should spend more time teaching about economic growth and development at the undergraduate level because the topic is of interest to students, is less abstract than other macroeconomic topics, and is the focus of exciting research in economics." Undergraduate students often learn the theory of economic growth through the seminal Solow model originated from Solow (1956). This elegant model provides the following important insight: in the long run, economic growth must come from technological progress instead of capital accumulation. However, the Solow model takes the growth rate of technology as given, and hence, it does not provide any insight on the determinants of technological progress. Taylor (2010) writes that "teaching beginning students the Solow model, augmented with endogenous technology, is the first step toward teaching them modern macroeconomics."

Romer (1990) greatly enhances economists' understanding of endogenous technological change by developing a growth model known as the Romer model in which technological progress is driven by the invention of new products, which in turn is due to research and development (R&D) by profit-seeking entrepreneurs. Unfortunately, the Romer model is relatively complicated, and undergraduate students often find it difficult. In particular, although it is not difficult to demonstrate the key assumption that technological change is driven by R&D, it is much more difficult to demonstrate how the level of R&D is determined in the market equilibrium within the Romer model. Therefore, macroeconomic textbooks at the intermediate level, such as Jones (2016) and Barro *et al.* (2017), often assume a given level of R&D when presenting the Romer model without showing how the equilibrium level of R&D is determined in the market economy. However, Jones (2016) writes that "[i]n Romer's original model, he set up markets for labor and output, introduced patents and monopoly power to deal with increasing returns, and let the markets determine the allocation of labor. What Romer discovered is fascinating [...] but unfortunately beyond the scope of this text."

To address the above issue, this paper proposes a simple method to solve the market equilibrium level of R&D in the Romer model by adding three layers of structure (one at a time) to extend the familiar Solow model into the less familiar Romer model. First, we incorporate into the Solow model a competitive market structure in which final goods are produced by competitive firms that employ labor and rent capital from households. Then, we modify the competitive market structure into a monopolistic market structure in which differentiated intermediate goods are produced by monopolistic firms. Finally, we introduce to the monopolistic Solow model an R&D sector, which invents new varieties of intermediate goods and gives rise to endogenous technological progress. Once we derive the endogenous growth rate of technology, we can then perform experiments in this mathematical laboratory by using comparative statics to explore the determinants of technological progress. All mathematical derivations are based on simple calculus and algebra at the level of intermediate microeconomics. We hope that by presenting it as a step-by-step extension of the Solow model, we have made the Romer model more accessible, at least to advanced undergraduate students in economics.

The rest of this paper is organized as follows. Section 2 presents the step-by-step transformation of the Solow model into the Romer model. Section 3 offers concluding thoughts.

2 From Solow to Romer

Section 2.1 reviews a basic version of the Solow model with *exogenous* technological progress. Section 2.2 discusses a simple version of the Romer model with *exogenous* R&D. Section 2.3 incorporates a competitive market structure into the Solow model.[1] Section 2.4 modifies the competitive market structure into a monopolistic market structure. Section 2.5 introduces an R&D sector to the monopolistic Solow model, which becomes the Romer model with *endogenous* R&D and *endogenous* technological progress.

2.1 The Solow model

In this subsection, we consider a basic version of the Solow model with exogenous technological progress. Output Y is produced by an aggregate production function $Y = K^{\alpha}(AL)^{1-\alpha}$, where A is the level of technology that grows at an exogenous rate $g > 0$, K is the stock of capital, and L is the size of a constant labor force. The parameter $\alpha \in (0, 1)$ determines capital intensity α and labor intensity $1 - \alpha$ in the production process. The key equation in the Solow model is the capital-accumulation equation given by $\Delta K = I - \delta K$, where the parameter $\delta > 0$ is the depreciation rate of capital. Investment I is assumed to be a constant share $s \in (0, 1)$ of output Y. Substituting the investment function $I = sY$ and the production function $Y = K^{\alpha}(AL)^{1-\alpha}$ into the capital-accumulation equation yields

$$\frac{\Delta K}{K} = \frac{sY}{K} - \delta = s\left(\frac{AL}{K}\right)^{1-\alpha} - \delta. \tag{1}$$

Equation (1) can then be used to explore the transition dynamics of an economy from an initial state to the steady state, which is a common analysis in macroeconomic textbooks at the intermediate level. In the long run, the economy is on a balanced growth path, along which capital K grows at a constant rate implying that Y/K and A/K are constant in the long run. This in turn implies that in the long run, output Y and capital K grow at the same rate as technology A; i.e.,

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K} = \frac{\Delta A}{A} \equiv g.$$

This is an important insight of the Solow model, which shows that in the long run, economic growth comes from technological progress (i.e., $g > 0$), without which the growth rate of the economy would converge to zero due to decreasing returns to scale of capital in production (i.e., $\alpha < 1$).[2]

[1] Solow (1956) also discusses the implications of his model in a competitive market.

[2] If the production function features constant returns to scale of capital (i.e., $\alpha = 1$), then the long-run growth rate of output and capital would be $\Delta Y/Y = \Delta K/K = s - \delta$. However, this is an unrealistic way to generate endogenous growth in the long run.

2.2 A simple Romer model with exogenous R&D

To understand the origin of technological progress in the Solow model, we can follow Romer (1990) to assume that the technology growth rate g is determined by R&D labor L_R. For example, we specify $g = \theta L_R$, where $\theta > 0$ is a parameter that determines the productivity of R&D labor. Then, to close the model, we modify the production function to $Y = K^\alpha (AL_Y)^{1-\alpha}$, where L_Y denotes production labor. Finally, the total amount of labor in the economy is $L_Y + L_R = L$. The rest of the model is the same as above. If we assume that production labor is given by $L_Y = (1-l)L$ and R&D labor is given by $L_R = lL$, where $l \in (0,1)$ is the exogenous R&D share of labor, then we can proceed to explore the implications of changes in l as in Jones (2016). While this analysis is interesting, it has a similar limitation as the analysis in Solow (1956). Solow (1956) assumes an exogenous technology growth rate g and is silent on its origin. The simple Romer model here assumes an endogenous technology growth rate $g = \theta L_R$ but an exogenous R&D labor share $l = L_R/L$, and hence, it is silent on the determination of R&D in the market economy. Unfortunately, solving for the market equilibrium level of R&D is often difficult for undergraduate students.[3] In the following subsections, we use a step-by-step approach to extend the Solow model in Section 2.1 into a fully-specified Romer model in Section 2.5.

2.3 The Solow model with a competitive market structure

The basic Solow model in Section 2.1 does not feature any market structure. Here we embed a market economy into the model in which competitive firms produce goods Y by employing labor L and renting capital K from households, which devote a constant share s of income to accumulate capital. The capital-accumulation equation is given by

$$\Delta K = s(WL + RK) - \delta K, \tag{2}$$

where W is the real wage rate and R is the real rental price of capital. Competitive firms produce goods Y to maximize real profit Π. The production function $Y = K^\alpha (AL)^{1-\alpha}$ is the same as before, whereas the profit function is given by $\Pi = Y - WL - RK$. From profit maximization, the first-order conditions that equate the real wage rate to the marginal product of labor and the real rental price to the marginal product of capital are given by

$$W = (1-\alpha)\frac{Y}{L}, \tag{3}$$

$$R = \alpha \frac{Y}{K}. \tag{4}$$

Substituting (3) and (4) into (2) yields the same capital-accumulation equation as in (1) because wage income and capital income add up to the level of output. Therefore, dynamics and long-run growth in the two versions of the Solow model are the same.

[3] See Aghion and Howitt (2009) and Jones and Vollrath (2013) for an excellent treatment of the Romer model at the advanced undergraduate level.

2.4 The Solow model with a monopolistic market structure

In this subsection, we further introduce a monopolistic sector of differentiated intermediate goods into the Solow model. The production of final goods Y now requires the combination of labor L and a number of differentiated intermediate goods as follows:

$$Y = L^{1-\alpha} \sum_{i=1}^{N} X_i^{\alpha}, \tag{5}$$

where X_i denotes intermediate goods $i \in [1, N]$ and N denotes the number of intermediate goods which increases at the rate g. The profit function of competitive firms that produce final goods is

$$\Pi = Y - WL - \sum_{i=1}^{N} P_i X_i.$$

From profit maximization, the first-order conditions are given by (3) and

$$P_i = \alpha L^{1-\alpha} X_i^{\alpha-1}, \tag{6}$$

for $i \in [1, N]$.

Each industry i is dominated by a monopolist who owns the technology of the differentiated product i. The monopolistic firm in industry i produces X_i units of intermediate goods by renting X_i units of capital from households. The profit function of the monopolistic firm in industry i is

$$\pi_i = P_i X_i - R X_i. \tag{7}$$

The monopolistic firm chooses X_i to maximize π_i subject to the conditional demand function in (6). The profit-maximizing price of X_i is $P_i = R/\alpha$. For a more general treatment, we assume that firms may not be able to charge this profit-maximizing price due to price regulation as in Evans *et al.* (2003) or incomplete patent protection as in Goh and Olivier (2002). In either case, the price of X_i is given by $P_i = \mu R$, where $\mu \in (1, 1/\alpha)$. Substituting $P_i = \mu R$ into (6) shows that $X_i = X$ for $i \in [1, N]$. Then the resource constraint on capital requires that $NX = K$; i.e., the usage of capital by all intermediate goods firms equals the total supply of capital.

Imposing symmetry $X_i = X$ and then substituting $X = K/N$ into the production function in (5) yield

$$Y = L^{1-\alpha} N X^{\alpha} = K^{\alpha} (NL)^{1-\alpha}, \tag{8}$$

which is the same production function as in the basic Solow model when $A = N$. In other words, the level of technology A in the basic Solow model is interpreted as the number of differentiated intermediate goods N in this monopolistic Solow model. As a result, the growth rate of technology A is determined by the growth rate of N given by g. As before, households devote a constant share s of income to accumulate capital. The capital-accumulation equation is[4]

$$\Delta K = s(WL + RK + N\pi) - \delta K. \tag{9}$$

[4] Alternatively, one can assume that monopolistic profit does not contribute to capital investment; i.e., $I = s(WL + RK) = s(1 - \alpha + \alpha/\mu)Y$, where $\mu > 1$ captures the monopolistic distortion that leads to a lower *level* of investment and capital. In this case, output and capital would still grow at g in the long run.

From (3), we have $WL = (1-\alpha)Y$ whereas we can derive $N\pi = (\mu-1)RK$ from (7). Finally, using (6), one can show that $RK = \alpha Y/\mu$. Therefore, $WL + RK + N\pi = Y$, which together with (8) implies that (9) is the same capital-accumulation equation as in (1). Intuitively, wage income, capital income and monopolistic profit add up to the level of output, such that investment is once again a constant share of output. As a result, dynamics and long-run growth in the three versions of the Solow model are the same.

2.5 The Romer model

In the monopolistic Solow model, the growth rate of technology is given by the growth rate of the number of differentiated intermediate products. However, this growth rate g is assumed to be exogenous. To endogenize technological progress, we introduce an R&D sector to the monopolistic Solow model, which then becomes a version of the Romer model with an exogenous saving rate s. Aside from the R&D sector, the Romer model is the same as the monopolistic Solow model except that labor is now allocated between R&D and the production of final goods. Equation (5) becomes

$$Y = L_Y^{1-\alpha} \sum_{i=1}^{N} X_i^{\alpha},$$

where L_Y denotes production labor. From the profit maximization of competitive firms that produce final goods, the first-order conditions are given by (6) and

$$W = (1-\alpha)\frac{Y}{L_Y}. \tag{10}$$

We follow Romer (1990) to specify the following innovation equation for the creation of new differentiated intermediate goods:

$$\Delta N = \theta N L_R, \tag{11}$$

where the parameter $\theta > 0$ captures R&D productivity. L_R is R&D labor, and the resource constraint on labor is given by $L_R + L_Y = L$. Therefore, the most important part of solving the Romer model is to determine the equilibrium level of R&D labor in the market economy, which in turn determines the growth rate of the number of differentiated intermediate goods given by $\Delta N/N = \theta L_R$.

Given that the R&D sector is perfectly competitive, zero profit implies that R&D revenue $\Delta N v$ equals R&D cost WL_R; i.e.,

$$\Delta N v = WL_R \Leftrightarrow \theta N v = W, \tag{12}$$

where v is the value of an invention. The value of an invention is the present value of future monopolistic profits. Let's use r to denote the exogenous real interest rate.[5] In the steady

[5] In the original Romer model, the interest rate is endogenous and determined by the household's optimal consumption path. To avoid using dynamic optimization, we assume an *exogenous* interest rate. Under this assumption, the no-arbitrage condition $r = R - \delta$ may not hold given the investment rate s is also exogenous.

state, the value v of an invention is given by

$$v = \frac{\pi}{r} = \frac{1}{r}\left(\frac{\mu-1}{\mu}\right)\frac{\alpha Y}{N}, \tag{13}$$

where the second equality uses $N\pi = (\mu - 1)RK$ and $RK = \alpha Y/\mu$ from the previous subsection. Rewriting (10) yields $W/Y = (1-\alpha)/L_Y$ whereas rewriting (12) yields $W/Y = \theta N v/Y = \frac{\theta}{r}\left(\frac{\mu-1}{\mu}\right)\alpha$, where the second equality uses (13). Equating these two equations yields the steady-state equilibrium level of production labor L_Y^* as shown in Figure 1.

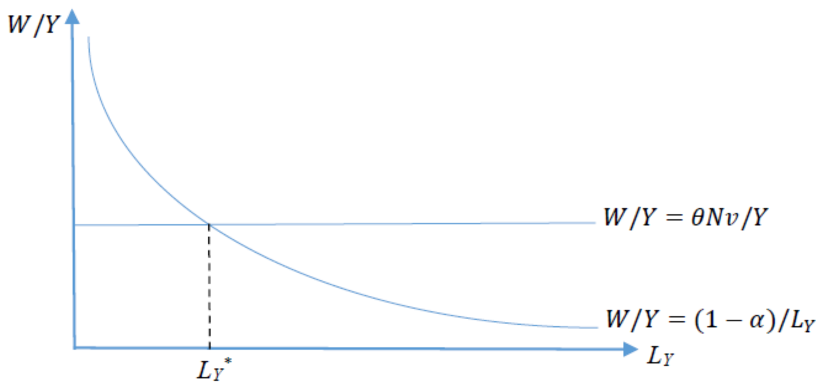

Figure 1: Equilibrium production labor

Substituting L_Y^* into the resource constraint on labor yields the steady-state equilibrium level of R&D labor L_R^* given by[6]

$$L_R^* = L - L_Y^* = L - \frac{1-\alpha}{\alpha}\left(\frac{\mu}{\mu-1}\right)\frac{r}{\theta}.$$

Therefore, the long-run growth rate of technology in the Romer model is

$$g^* \equiv \frac{\Delta N}{N} = \theta L_R^* = \theta L - \frac{1-\alpha}{\alpha}\left(\frac{\mu}{\mu-1}\right) r. \tag{14}$$

This endogenous technology growth rate g^* is also the long-run growth rate of output and capital. To see this, we use the following capital-accumulation equation:[7]

$$\frac{\Delta K}{K} = \frac{sY}{K} - \delta = s\left(\frac{NL_Y}{K}\right)^{1-\alpha} - \delta,$$

[6] Labor force L is assumed to be sufficiently large such that $L_R^* > 0$.

[7] To derive this equation, we assume households devote $s(WL_Y + RK + N\pi)$ to capital investment I, and R&D wage income WL_R is invested in intangible capital (i.e., the value of new inventions $\Delta N v$). As in footnote 4, one could assume $I = s(WL_Y + RK) = s(1 - \alpha + \alpha/\mu)Y$ to allow for monopolistic distortion on capital accumulation, in which case the long-run growth rate of output and capital is still g^* in (14).

where production labor L_Y is stationary in the long run. Therefore, a constant growth rate of capital K on the balanced growth path implies that Y/K and N/K are also stationary in the long run, which in turn implies that

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K} = \frac{\Delta N}{N} = \theta L_R^*(\theta, \mu, r, \alpha, L).$$

Given the expression for the endogenous growth rate in (14), we can now perform experiments in this mathematical laboratory by using comparative statics to explore the determinants of economic growth in the Romer model. Equation (14) shows that the equilibrium growth rate g^* is increasing in $\{\theta, \mu, \alpha, L\}$ and decreasing in r. The intuition of these comparative statics results can be explained as follows.

2.5.1 Experiment 1: changing R&D productivity

An improvement in R&D productivity θ increases the growth rate of technology for a given level of R&D labor and also makes R&D more productive, which in turn increases R&D labor in the economy. Therefore, g^* is increasing in θ. To see this, recall that the growth rate of the number of differentiated products is $g^* = \theta L_R^*(\theta)$. Therefore, an increase in R&D productivity θ has a direct positive effect on g^* by increasing R&D productivity and also an indirect positive effect by increasing R&D labor L_R^*. The parameter θ captures the importance of human capital on the innovation capacity of an economy. To stimulate economic growth, policymakers could consider devoting more resources to education that improves the innovative capacity of entrepreneurs, scientists and engineers.

2.5.2 Experiment 2: changing the monopolistic price

A larger μ enables monopolistic firms to raise their price and earn more profits, which in turn provide more incentives for R&D. Therefore, g^* is increasing in μ. To see this, recall that the growth rate of the number of differentiated products is $g^* = \theta L_R^*(\mu)$. Therefore, an increase in the markup ratio μ increases the growth rate of N by increasing R&D labor L_R^*. The parameter μ captures the effects of the underlying economic institutions, such as antitrust and patent policies, on R&D and economic growth. To stimulate economic growth, policymakers could consider reforming antitrust and patent policies to enable innovators to earn more profits.

2.5.3 Experiment 3: changing the interest rate

A higher interest rate r reduces the present value of future monopolistic profits and the value of inventions, which in turn decreases R&D in the economy. Therefore, g^* is decreasing in r. To see this, recall that the growth rate of the number of differentiated products is $g^* = \theta L_R^*(r)$. Therefore, an increase in the interest rate r decreases the growth rate of N by decreasing R&D labor L_R^*. The parameter r captures the effects of financial frictions, such as credit constraints, on innovation. To stimulate economic growth, policymakers could consider policies that relax credit constraints faced by innovators.

2.5.4 Experiment 4: changing capital and labor intensity in production

An increase in α increases capital intensity and reduces labor intensity in the production process, allowing more labor to be devoted to R&D. Therefore, g^* is increasing in α. To see this, recall that the growth rate of the number of differentiated products is $g^* = \theta L_R^*(\alpha)$. Therefore, an increase in capital intensity α increases the growth rate of N by increasing R&D labor L_R^*. The parameter α captures the effect of structural transformation of an economy from a labor-intensive production process to a capital-intensive production process. However, the positive effect of α on R&D is based on the assumption that R&D does not require the use of capital. If R&D also uses capital, then the effect of α on growth may be reversed depending on the degree of capital intensity in the R&D process.

2.5.5 Experiment 5: changing the size of the labor force

Finally, a larger labor force L increases the supply of labor in the economy, which in turn increases R&D labor and the growth rate. Therefore, g^* is increasing in L. To see this, recall that the growth rate of the number of differentiated products is $g^* = \theta L_R^*(L)$. Therefore, an increase in the labor force L increases the growth rate of N by increasing R&D labor L_R^*. In the literature, this is known as the scale effect, which however is often viewed as a counterfactual implication of the Romer model.[8]

3 Conclusion

Since the seminal work of Solow (1956), there has been much progress in the research of economic growth. However, the teaching of economic growth in undergraduate macroeconomic courses is still mostly based on the Solow model. Although this seminal model provides important insights, it takes the growth rate of technology as given. To understand the origin of technological progress, we need a model of endogenous technological change. The Romer model provides a useful framework for this purpose. However, given its complexity, undergraduate students often find the Romer model difficult. In this paper, we have proposed a method that serves as a bridge between the Solow model and the Romer model in three steps. Furthermore, the mathematical derivations involve only basic calculus and algebra. We hope that by providing a bridge with the Solow model, we have made the Romer model more accessible to undergraduate students in economics.

[8] To remove this scale effect, one can follow Jones (1995) to modify (11) to $\Delta N = \theta N^{\phi} L_R$, where the parameter $\phi < 1$ captures the degree of intertemporal knowledge spillovers. For example, suppose $\phi = 0$. Then, $\Delta N/N = \theta L_R/N$. In this case, a constant growth rate of N on the balanced growth path implies that L_R/N is stationary, which in turn implies $\Delta N/N = \Delta L_R/L_R$ in the long run. In other words, it is the *growth rate* of R&D labor, instead of the *level* of R&D labor, that determines the long-run growth rate of technology in the Jones model. This implication is robust to $-\infty < \phi < 1$.

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