Growth Accounting and Endogenous Technical Change

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Abstract

This study explores growth accounting under endogenous technological progress. It is well known that the Solow approach overstates (understates) the contribution of capital accumulation (technological progress) to economic growth and the Mankiw-Romer-Weil approach addresses this issue. However, we find that the Mankiw-Romer-Weil approach is inconsistent (consistent) with the lab-equipment (knowledge-driven) specification for technological progress. We also examine the importance of capital accumulation on growth in China under the two approaches.

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1 Introduction

The traditional approach to growth accounting, introduced by Solow (1957), decomposes economic growth into the growth rates of factor inputs and technological progress measured by total factor productivity (TFP); see Barro (1999) for a review. Interpreting these accounting relationships as causal relationships however requires an assumption that the growth rates of factor inputs, e.g., physical capital, are independent from technological progress. An important result from the seminal Solow growth model is that long-run growth in output and capital is driven by technological progress. Therefore, interpreting the accounting relationships from the Solow approach as causal relationships may overstate (understate) the contribution of capital accumulation (technological progress) to growth; see e.g., Aghion and Howitt (2007) for this critique. An alternative approach to growth accounting, originated from Mankiw, Romer and Weil (1992),[1] addresses this issue by essentially scaling up the importance of technological progress and measuring the contribution of capital by the growth rate of the capital-output ratio, rather than the growth rate of capital.

This study examines the validity of these two approaches to growth accounting under endogenous technical change.[2] We consider two common specifications for technological progress: the *knowledge-driven* and *lab-equipment* specifications. As Hsieh and Klenow (2010) write, "in contrast to the well-understood endogeneity of physical capital in the neoclassical growth model, the determinants of [...] TFP are much less well understood." We find that the Mankiw-Romer-Weil approach is consistent with the knowledge-driven specification that features labor as input in innovation. Under this knowledge-driven specification, technological progress does not require physical capital, so the Mankiw-Romer-Weil approach that scales down (up) the contribution of capital accumulation (technological progress) is valid. However, under the lab-equipment specification that features final goods as input in innovation, the Mankiw-Romer-Weil approach understates the contribution of capital accumulation to growth because capital accumulation contributes to technological progress. Intuitively, because innovation indirectly uses research capital, growth is increasing in capital investment. Finally, we also examine the importance of capital accumulation on growth in China under the two approaches and discuss their different implications in the conclusion.

2 Review of growth accounting

This section briefly reviews the two approaches to growth accounting. Let's start with the following aggregate production function:

$$Y = K^{\alpha}(AL)^{1-\alpha}, \tag{1}$$

where Y denotes output, A denotes technology, K denotes physical capital, and L denotes effective labor, which includes human capital and raw labor. The parameter $\alpha \in (0,1)$ determines capital intensity in production. In the following subsections, we present the

[1] See also Klenow and Rodriguez-Clare (1997), Hall and Jones (1999) and Hayashi and Prescott (2002).

[2] See also Barro (1999), Aghion and Howitt (2007) and Hsieh and Klenow (2010).

Solow and Mankiw-Romer-Weil approaches and show their different implications on the contribution of capital to growth.

2.1 The Solow approach

We take the log of (1) and differentiate it with respect to time to obtain

$$\frac{\dot{Y}}{Y} = (1-\alpha)\frac{\dot{A}}{A} + \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L}, \tag{2}$$

where $\dot{x}/x$ denotes the growth rate of variable $x \in \{Y, A, K, L\}$. In other words, (2) decomposes the growth rate of output into the growth rates of technology, physical capital and effective labor. Given that our focus is on the relative importance of technological progress and capital accumulation, we consider a constant effective labor L for simplicity.[3] Under the Solow approach, the share of growth that capital is responsible for is measured by $\alpha(\dot{K}/K)/(\dot{Y}/Y)$. On the balanced growth path, the capital-output ratio is constant, which implies that capital is responsible for the share α of long-run growth in output.

As an illustration, we consider China's data to explore the importance of capital accumulation on growth in China. From Brandt, Hsieh and Zhu (2008), the average value of capital share in China is about 0.5.[4] From Zhu (2012), the average growth rates of output and physical capital have been roughly the same since 1978.[5] Therefore, we consider the following stylized facts for China: $\alpha = 1/2$, and a constant K/Y since the late 1970's. Under the Solow approach to growth accounting, one would conclude that capital accumulation $\dot{K}/K$ has been responsible for about half of the growth in China. To see this, the contribution of capital to growth in China under the Solow approach is

$$\text{Solow approach: } \frac{\alpha\dot{K}/K}{\dot{Y}/Y} \approx \alpha \approx \frac{1}{2}.$$

However, this Solow approach may overstate (understate) the contribution of capital accumulation (technological progress). The reason is that capital accumulation is partly driven by technological progress. As the seminal Solow growth model shows, long-run growth in output and capital is driven by technological progress. In the next subsection, we consider an alternative approach to growth accounting that addresses this issue.

2.2 The Mankiw-Romer-Weil approach

Mankiw, Romer and Weil (1992) consider an alternative approach to growth accounting. In essence, it involves dividing both sides of (1) by Y^{α} to obtain

$$Y^{1-\alpha} = A^{1-\alpha}(K/Y)^{\alpha}L^{1-\alpha}. \tag{3}$$

[3] Extending the analysis by allowing for growth in effective labor L would not change our results.

[4] Given innovation under imperfect competition, capital intensity α differs from capital share, which however is a reasonable proxy under a small aggregate markup.

[5] The average annual growth rate of the capital-output ratio K/Y in China from 1978 to 2007 was 0.04%.

Then, taking the log of (3) and differentiating it with respect to time yield

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \frac{\alpha}{1-\alpha}\frac{\dot{(K/Y)}}{(K/Y)}, \tag{4}$$

where we have assumed $\dot{L}/L = 0$. An interpretation of (4) is that capital accumulation is driven by technological progress. Therefore, we should scale up the importance of A by a factor of $1/(1-\alpha)$. If capital has made an additional contribution to output growth, then K should have grown at a faster rate than Y in the short run. On the balanced growth path, the capital-output ratio is constant, so that capital does not contribute to long-run growth.

Using the Mankiw-Romer-Weil approach, Zhu (2012) concludes that economic growth in China is mainly driven by growth in technology A because K/Y has been roughly constant since 1978; formally, the contribution of capital to growth in China is

$$\text{Mankiw-Romer-Weil approach: } \frac{\alpha}{1-\alpha}\frac{\dot{(K/Y)}}{(K/Y)}\frac{1}{\dot{Y}/Y} \approx 0.$$

Therefore, according to the Mankiw-Romer-Weil approach, capital has made almost zero contribution to growth in China, whereas according to the Solow approach, capital has contributed to as much as half the growth in China. Given the very different implications, we next examine these two approaches under endogenous technological progress.

3 Growth accounting under endogenous technical change

The previous section reviews that the two approaches to growth accounting have different implications on the contribution of capital to growth. The reason is that the Solow approach does not consider the underlying determinant that drives capital accumulation, whereas the Mankiw-Romer-Weil approach assumes that capital accumulation is driven by technological progress but not vice versa. In reality, technological progress is an endogenous process. In this section, we consider two common specifications for technological progress and explore the validity of the Solow and Mankiw-Romer-Weil approaches under each specification.

3.1 Knowledge-driven technological progress

We now modify the aggregate production function as follows:

$$Y = K^{\alpha}(AL_Y)^{1-\alpha}, \tag{5}$$

where $L_Y = (1-s_A)L$ denotes production labor and $s_A \in (0,1)$ is the share of labor devoted to improving technology A. The law of motion for technology is given by

$$\dot{A} = \overline{\theta} A L_R, \tag{6}$$

where $L_R = s_A L$ denotes R&D labor.[6] The term $\overline{\theta} \equiv \theta/L$ denotes R&D productivity, where $\theta > 0$ is a productivity parameter and $1/L$ captures a dilution effect that removes a counterfactual scale effect from the model.[7] The term A on the right hand side of (6) captures intertemporal knowledge spillovers from existing technologies A to new technology $\dot{A}$ as in the knowledge-driven R&D specification in Romer (1990). Let's denote the steady-state growth rate of technology as $g_A \equiv \dot{A}/A = \theta s_A$.[8]

The law of motion for capital accumulation is given by

$$\dot{K} = I - \delta K, \tag{7}$$

where I denotes capital investment and the parameter $\delta \in (0, 1)$ denotes the capital depreciation rate. Manipulating (7) yields

$$\frac{\dot{K}}{K} = \frac{I}{K} - \delta. \tag{8}$$

In the long run, the steady-state capital growth rate g_K is constant, which in turn implies a constant steady-state investment-capital ratio I/K. Together with a constant investment-output ratio I/Y in the long run, we have established that the steady-state capital-output ratio K/Y must be constant, which in turn implies that output and capital have the same steady-state growth rate (i.e., $g_Y = g_K$).

Taking the log of (5) and differentiating it with respect to time yield

$$\frac{\dot{Y}}{Y} = (1-\alpha)\frac{\dot{A}}{A} + \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}_Y}{L_Y}. \tag{9}$$

We assume that s_A is constant, which in turn implies $\dot{L}_Y/L_Y = 0$. Finally, we substitute the long-run condition $g_Y = g_K$ into (9) to obtain

$$g_Y = g_K = g_A = \theta s_A. \tag{10}$$

Therefore, although technological progress is endogenous in this model, it is independent of capital accumulation. In contrast, capital accumulation is driven by technological progress.

We now examine the validity of the Solow and Mankiw-Romer-Weil approaches to growth accounting within the context of this model. Under the Solow approach, we have the following condition in the long run:

$$\frac{\dot{Y}}{Y} = (1-\alpha)\frac{\dot{A}}{A} + \alpha\frac{\dot{K}}{K} \Rightarrow g_Y = (1-\alpha)g_A + \alpha g_K. \tag{11}$$

[6] In Section 3.2, we discuss the potential determinants of s_A. In a market economy, resources are channelled into the R&D sector through the financial sector, where the interest rate plays a fundamental role and determines the rate of return on financial assets in the form of firm equity.

[7] In an online appendix, we sketch out a second-generation R&D-based growth model that provides a microfoundation for this dilution effect; see Laincz and Peretto (2006) and Ha and Howitt (2007) for empirical evidence that supports this model.

[8] Without the dilution effect $1/L$, g_A would be increasing in L, which is inconsistent with empirical evidence; see for example Jones (1995). In other words, the dilution effect removes the strong scale effect, under which a larger population implies a higher *growth rate* of technology, but not the weak scale effect, under which a larger population causes a higher *level* of technology; see the online appendix.

As we can see, the Solow approach assigns the share α of growth to capital accumulation g_K, which should in fact be assigned to technological progress g_A as (10) shows.

Under the Mankiw-Romer-Weil approach, we have the following long-run condition:

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \frac{\alpha}{1-\alpha}\frac{\dot{(K/Y)}}{(K/Y)} \Rightarrow g_Y = g_A. \tag{12}$$

Therefore, the Mankiw-Romer-Weil approach correctly assigns the entire long-run growth in output to technological progress g_A. We summarize these results below.

Proposition 1 *The Mankiw-Romer-Weil approach to growth accounting is consistent with the knowledge-driven technological progress under which the Solow approach overstates the contribution of capital accumulation to economic growth and understates the contribution of technological progress.*

3.2 Lab-equipment technological progress

We now consider another specification for technological progress. The production function is

$$Y = K^{\alpha}(AL)^{1-\alpha}. \tag{13}$$

The law of motion for technology is modified to capture the lab-equipment R&D specification in Rivera-Batiz and Romer (1991) as follows:

$$\dot{A} = \overline{\theta}R = \frac{\theta R}{L}, \tag{14}$$

where $R = s_A Y$ and $s_A \in (0,1)$ is now the share of output devoted to improving technology. Substituting $R = s_A Y$ and (13) into (14) yields

$$\frac{\dot{A}}{A} = \theta s_A \left(\frac{K}{AL}\right)^{\alpha}, \tag{15}$$

which in turn implies that in the case of a constant steady-state growth rate of technology, the capital-technology ratio K/A must be constant in the long run.

The law of motion for capital is the same as in (7). For simplicity, we define $s_K \in (0,1)$ as the constant share of output devoted to capital accumulation (i.e., capital investment net of depreciation). Formally,

$$s_K Y \equiv \dot{K} = I - \delta K, \tag{16}$$

which in turn implies that

$$\frac{\dot{K}}{K} = s_K \frac{Y}{K} = s_K \left(\frac{AL}{K}\right)^{1-\alpha}. \tag{17}$$

Therefore, we can combine (15) and (17) to obtain

$$\frac{\dot{A}}{A} = \frac{\dot{K}}{K} \Leftrightarrow \theta s_A \left(\frac{K}{AL}\right)^{\alpha} = s_K \left(\frac{AL}{K}\right)^{1-\alpha}. \tag{18}$$

Then, we derive the steady-state capital-technology ratio as

$$\frac{K}{A} = \frac{s_K}{\theta s_A} L. \tag{19}$$

Substituting (19) into (15) yields the steady-state growth rate of technology given by

$$g_A = (\theta s_A)^{1-\alpha} (s_K)^{\alpha}, \tag{20}$$

which in turn determines the steady-state growth rate of output and capital as $g_Y = g_K = g_A$. If we take an approximation of (20), we have[9]

$$\ln g_A = (1-\alpha)\ln(\theta s_A) + \alpha \ln(s_K) \Rightarrow g_A \approx (1-\alpha)(\theta s_A) + \alpha s_K. \tag{21}$$

In this model, technological progress and capital accumulation follow a two-way process: technological progress drives capital accumulation (i.e., $g_K = g_A$) but capital accumulation also drives technological progress (i.e., g_A depends on s_K). Therefore, the causal determinants of the long-run growth rate of output and technology in this model are the technology-investment rate s_A and the capital-investment rate s_K. Although we have assumed constant investment rates $\{s_A, s_K\}$, they need not be exogenous. In a market equilibrium, $\{s_A, s_K\}$ are determined by household preference, market structure and government policies, etc.

We now examine the two approaches within the context of this model. Under the Mankiw-Romer-Weil approach, we have the following long-run condition:

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \frac{\alpha}{1-\alpha} \frac{(\dot{K/Y})}{(K/Y)} \Rightarrow g_Y = g_A, \tag{22}$$

where the technology growth rate $g_A \approx (1-\alpha)(\theta s_A) + \alpha s_K$ depends on the capital-investment rate s_K. In other words, capital investment s_K contributes to technological progress g_A and output growth g_Y. Intuitively, because innovation indirectly uses research capital, growth is increasing in capital investment. However, as (22) shows, all the growth in output is wrongly attributed to technological progress under the Mankiw-Romer-Weil approach.

Under the Solow approach, we have the following long-run condition:

$$\frac{\dot{Y}}{Y} = (1-\alpha)\frac{\dot{A}}{A} + \alpha \frac{\dot{K}}{K} \Rightarrow g_Y = (1-\alpha) g_A + \alpha g_K, \tag{23}$$

where $(1-\alpha) g_A + \alpha g_K = g_A \approx (1-\alpha)(\theta s_A) + \alpha s_K$. The Solow approach correctly assigns some of the growth in output to growth in capital g_K, which captures the effect of s_K in (21). As (17) shows, the capital growth rate $\dot{K}/K$ is determined by the capital-investment rate s_K for a given capital-technology ratio K/A. Similarly, the Solow approach correctly assigns some of

[9] Here we use a first-order approximation $\ln(1+x) \approx x$ of the Mercator series.

the growth in output to growth in technology g_A, which captures the effect of θs_A in (21). As (15) shows, the technology growth rate $\dot{A}/A$ is determined by the technology-investment rate s_A for a given capital-technology ratio K/A. Therefore, under the lab-equipment specification that features final goods as input in the innovation process, the Solow approach is more valid than the Mankiw-Romer-Weil approach because the former captures the contribution of capital investment to technological progress and economic growth via the growth rate of capital in the aggregate production function.

Proposition 2 *Under the lab-equipment technological progress, the Solow approach to growth accounting is more valid than the Mankiw-Romer-Weil approach, which understates the contribution of capital accumulation to economic growth and overstates the contribution of technological progress.*

4 Conclusion

In this letter, we have explored the validity of two conventional approaches to growth accounting under two common specifications for endogenous technical change. We find that if the innovation process is captured by the lab-equipment (knowledge-driven) technological progress, then the Solow (Mankiw-Romer-Weil) approach to growth accounting is more appropriate in which case capital accumulation would have contributed to almost half (none) of the growth in China. In an earlier version of this study,[10] we consider a more general innovation specification with different degrees of capital intensity in production and innovation. We propose a weighted average of the Solow and Mankiw-Romer-Weil approaches for growth accounting in which capital intensity in innovation determines the relative weight of the two approaches.

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Online Appendix

In this appendix, we provide a microfoundation for the dilution effect on R&D productivity using a variant of the second-generation R&D-based growth model. The aggregate production function of final goods is given by

$$Y = \int_0^N K_Y^\alpha(i)[A(i)L_Y(i)]^{1-\alpha} di, \tag{A1}$$

where $\{A(i), K_Y(i), L_Y(i)\}$ are the technology level, capital and labor inputs of intermediate goods $i \in [0, N]$. The variable N denotes the number of varieties of these intermediate goods. The law of motion for technology of intermediate goods $i \in [0, N]$ is given by

$$\dot{A}(i) = \widetilde{\theta} K_R^\beta(i)[A(i)L_R(i)]^{1-\beta}, \tag{A2}$$

where $\{K_R(i), L_R(i)\}$ are the capital and labor inputs devoted to improving the technology of intermediate goods $i \in [0, N]$ and $\widetilde{\theta} > 0$ is a productivity parameter.

We consider a symmetric equilibrium in which $L_R(i) = s_A L/N$, $L_Y(i) = (1 - s_A)L/N$, $K_R(i) = s_A K/N$, $K_Y(i) = (1 - s_A)K/N$ and $A(i) = A$ for all $i \in [0, N]$. Substituting these conditions into (A1) and (A2) yields

$$Y = N \left[\frac{(1 - s_A)K}{N}\right]^\alpha \left[\frac{A(1 - s_A)L}{N}\right]^{1-\alpha} = (1 - s_A)K^\alpha (AL)^{1-\alpha}, \tag{A3}$$

$$\dot{A} = \widetilde{\theta} \left(\frac{s_A K}{N}\right)^\beta \left(\frac{A s_A L}{N}\right)^{1-\beta} = \frac{\widetilde{\theta}}{N} s_A K^\beta (AL)^{1-\beta}. \tag{A4}$$

Equation (A4) shows that R&D productivity $\widetilde{\theta}/N$ is diluted by the number of varieties of intermediate goods. The law of motion for N is given by

$$\dot{N} = \phi L - \delta_N N, \tag{A5}$$

where $\phi > 0$ measures the efficiency of the society in creating new varieties and $\delta_N > 0$ is the obsolescence rate of varieties. In the steady state, we have $N = \phi L/\delta_N$, which shows that N is increasing in L. Substituting this condition into (A4), we have

$$\dot{A} = \frac{\theta}{L} s_A K^\beta (AL)^{1-\beta}, \tag{A6}$$

where we have defined $\theta \equiv \delta_N \widetilde{\theta}/\phi$. Setting $\beta = 0$ in (A6) yields (6). Setting $\beta = \alpha$ in (A6) yields (14). Taking the log of (A3) and differentiating the resulting expression with respect to time yield

$$\frac{\dot{Y}}{Y} = (1 - \alpha)\frac{\dot{A}}{A} + \alpha \frac{\dot{K}}{K}. \tag{A7}$$

The law of motion for capital is given by (16), which in turn implies a constant capital-output ratio K/Y in the long run. Therefore, the steady-state growth rate of output and capital is given by $g_Y = g_K = g_A$ as before.