**A continuous review, (*Q*, *r*) inventory model for a deteriorating item with random demand and positive lead time**

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**Abstract**

In this paper, a single-product, single-location inventory system is considered. A fraction of the stock is lost every time unit, i.e., inventory experiences continuous decay. Demand is uncertain and replenishments require a positive lead time. Shortages are allowed and backorders-lost sales mixtures are considered. Inventory is reviewed continuously and a (*Q*, *r*) policy is applied. The problem is to find the order quantity and the reorder point that minimize the long-run expected total cost per time unit. It is known, in literature, that approaching this problem is a very difficult task. Moreover, very little research has been performed in this regard, although improvement of continuous review policy in an inventory system is a vital issue in ERP solutions, in particular for Industry 4.0. The cost model is developed by making some simplifying hypotheses and assuming that the dynamics of the inventory level (i.e., stock on hand minus backorders) is captured through an Itō diffusion. An iterative method is proposed to minimize the cost function. Numerical experiments are performed to investigate the efficiency of the proposed model. Comparisons with an estimate of the optimal policy and with a former heuristic model are given. Results show that the model developed in this work should provide a very good approximation of the optimal policy over a reasonably wide range of parameter values. A sensitivity analysis is finally carried out in order to draw some managerial insights.

*Keywords*: Inventory; Deteriorating; Stochastic; Heuristic; Optimization; Lead time

**1. Introduction**

The issue of managing deteriorating inventories has attracted the attention of numerous researchers since many decades ago. This interest is motivated by the observation that most physical goods lose their characteristics over time. Hence, if the rate of deterioration is not sufficiently small, its effect on the inventory cannot be neglected.

The term “deterioration” covers all cases related to damage, spoilage, dryness, vaporization, etc. that can take place during the normal storage of the units and that may affect their suitability for their original purpose. According to Goyal and Giri (2001), the products like foodstuff, green vegetables, human blood, photographic film, etc., which have a maximum usable lifetime, are known as perishable products, and the products like alcohol, gasoline, radioactive substances, etc., which have no shelf-life, are known as decaying products. A similar classification was proposed by Nahmias (2011) who gives the following definitions:

1. Inventory is said to be decaying when a fraction of the stock is lost every time unit;
2. Items are perishable when they are characterized by a constant utility up until an expiration date (which may be deterministic or stochastic), at which point the utility drops to zero.

This paper focuses on decaying inventories.

Inventory systems with deterioration have been extensively studied over the years. In a recent survey, Janssen et al. (2016) pointed out that, at the time of their paper, 17 literature reviews were available on the topic. They also observed that 393 academic articles on deteriorating inventory models were published between January 2012 and December 2015. These numbers give an idea about how much this topic is of interest to researchers.

The literature on decaying inventories can be subdivided into two macro-categories: deterministic demand and stochastic demand. Although the assumption of deterministic demand is little practical, the first group of researches is still attracting a lot of attention. This group can be further subdivided into many subclasses depending on the assumption made about lead time. Some works neglect lead time (Hsieh and Dye, 2017; Tavakoli and Taleizadeh, 2017; Banerjee and Agrawal, 2017; Pervin et al., 2018), while other works include a positive, deterministic lead time (Lashgari et al., 2018; Dey et al., 2008). We can also cite papers in which lead time is a fuzzy quantity (Rong et al., 2008), or a random variable (Sazvar et al., 2013, 2014). Finally, some authors investigated economic production quantity models (Valliathal and Uthayakumar, 2013; Wee and Widyadana, 2013; N. Li et al., 2017), or integrated vendor-buyer models (Chen and Sarker, 2017; Hemmati et al., 2017).

In literature, the class of decaying inventory models with deterministic demand is rather vast, and includes contributions considering a variety of aspects, such as ramp-type demand (Skouri et al., 2009; Skouri et al., 2011; Agrawal et al., 2013), permissible delay in payments (Skouri et al., 2011), trade credits (Wu et al., 2018), time value of money (Wu et al., 2016; Tavakoli and Taleizadeh, 2017; Wu et al., 2018), time-varying holding cost (Pervin et al., 2018), and expiration date-dependent deterioration rate (Wu et al., 2017; Wu et al., 2018). Each model in this class is essentially an extension of the original model developed by Ghare and Schrader (1963), from which the modelling approach is inherited, i.e., the use of an ordinary differential equation to describe the inventory dynamics.

A more realistic assumption is to allow the demand per period to take random values. However, this makes the decay problem much more complex than in the deterministic case, especially when a positive order lead time is considered (Nahmias, 1982). The question greatly simplifies if one adopts the assumption of zero lead time. The contributions of Benkherouf et al. (2003), Pang (2011), Lan et al. (2011), Maihami and Karimi (2014), and Y. Li et al. (2017) are examples of works with stochastic demand that include this assumption. Note that periodic review models with zero lead time are useful when lead times are sufficiently small (Nahmias, 2011). Assuming zero lead time is, however, little realistic.

If a positive lead time is introduced, the problem becomes much more complex. According to Nahmias (1982, 2011), the reason is that the decay can be only applied to the on-hand inventory and not to the on-order inventory, which means that it is not possible to identify the state vector with the inventory position (i.e., on-hand plus on-order stock minus backorders). In this case, the state vector should include each outstanding order as a separate variable. Moreover, the question is even more difficult for a continuous review inventory system than for its periodic review counterpart, since the state vector becomes a possibly infinite dimensional vector of on hand and on order stocks (Nahmias, 2011). This makes the computation of the exact model impractical and a heuristic, approximate approach is thus required (Nahmias, 1981, 1982; Kouki et al., 2016).

The work of Nahmias and Wang (1979) was the first attempt to investigate the management of decaying inventories under demand uncertainty in the presence of a positive lead time taking into account a continuous review policy. They used similar arguments to Hadley and Whitin (1963) to develop a heuristic lot size-reorder point model including exponential decay.

In his survey, Nahmias (2011) affirmed: “the only study known to this writer that considers decay in the context of stochastic demand and positive lead times is that of Nahmias and Wang (1979)”. By reviewing the publications on decaying inventories after Nahmias’ survey, it seems that this research problem is well under-studied. To the best of our knowledge, only Jha and Shanker (2009) consider the problem in the case of a single vendor-multiple buyers supply chain with a service level constraint and controllable lead time. However, their work is essentially an application of Nahmias and Wang’s model. Researches developing a conceptually new model, i.e., a new approach to the problem, seem to be not present in literature. Further effort is thus needed to address this challenging problem as the existing literature has not addressed the question sufficiently, although improvement of continuous review policy in an inventory system is a vital issue in ERP solutions, in particular for Industry 4.0. An overview of some key literature about decaying inventory is given in Table 1.

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TABLE 1 HERE

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This paper proposes a novel heuristic lot size-reorder point model for decaying inventories subject to uncertain demand in presence of a positive deterministic or stochastic lead time. Backorders-lost sales mixtures and shortage costs are included. The dynamics of the inventory level (i.e., stock on hand minus backorders) is captured through a stochastic differential equation (SDE) representing an Itō diffusion. The distribution of the lead-time demand is obtained exploiting the properties of the stochastic process satisfying this SDE. The efficiency of the model is investigated by means of numerical experiments. In these tests, comparisons with the approximate model of Nahmias and Wang (1979) and with an estimate of the optimal policy obtained using a simulative approach are provided. A sensitivity analysis is finally carried out in order to draw some managerial insights.

The main contributions of this work can be summarized as follows:

* It provides a reasonably complete, quantitative discussion about how the deterministic decaying inventory model can be obtained, showing the link with a perishable inventory system with random lifetimes, and how it can be extended to the stochastic demand case.
* It gives and formally demonstrates some properties of the Itō diffusion arising from the inclusion of the random component into the deterministic decaying inventory model.
* It develops a novel heuristic lot size-reorder point model for the decay problem, in a continuous review system, with random demand and positive lead time using the previously derived Itō diffusion to characterize the dynamics of the inventory level. The model is, to the best of our knowledge, the first and only attempt, after Nahmias and Wang (1979), to provide a new solution approach to the considered decay problem.
* It presents extensive numerical experiments to verify the efficiency of the proposed model, and to give some managerial insights.

The rest of the paper is organized as follows. Section 2 characterizes the dynamics of inventory level over time under item decay and random demand using Itō diffusion. Section 3 develops the cost model of the system. Section 4 carries out numerical experiments to illustrate the effectiveness of the proposed model. Finally, Section 5 draws conclusiongs.

**2. Characterize the dynamics of inventory level under item decay and random demand**

This paper studies a single-product, single-location inventory system subject to decay. The demand is uncertain and replenishments require a positive lead time. Shortages are allowed and backorders-lost sales mixtures are considered. A continuous review (*Q*, *r*) model is applied. The objective is to find the lot size and the reorder point that minimize the long-run expected total cost per time unit.

This section introduces the stochastic model used to describe the evolution of the inventory level during lead time, taking into account the effect of both decay and (random) demand. Being the basis to develop the cost formulation proposed in this paper, this model is presented in a separate, preliminary section. Although many relations are essentially known in literature, it is interesting to discuss how the deterministic model can be generalized to include randomness. Moreover, some properties of the stochastic model will be discussed and formalized in propositions whose proofs are part of the contributions provided with this work.

Consider a single-product, single-location inventory system. Let be an initial time instant such that the corresponding inventory level is positive. Suppose that each item in the stock at any is characterized by the same instantaneous decay rate , and that the instantaneous demand rate at time is . Then, the inventory level at any such that can be described by the ordinary differential equation

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| --- | --- |
|  | (1) |

with initial condition , . Equation (1) has been largely used in literature assuming a variety of expressions for and (see, e.g., Ai et al., 2017; Hemmati et al., 2017; Wu et al., 2017). A reasonably complete and quantitative derivation of (1), with the inclusion of reliability concepts that show the link with a perishable inventory system, seems to be not present in literature. Hence, in Appendix A we provide an intuitive demonstration about how (1) can be obtained.

To include randomness in the system caused by the presence of uncertain demand, we assume that the demand rate is perturbed by a white noise process amplified by a time-dependent factor , for all *t* (the practical meaning of will be evident in the next section). Equation (1) becomes:

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|  | (2) |

Note that this procedure to include randomness in a deterministic model by adding a stochastic noise term to an ordinary differential equation is frequently adopted in physics and other natural sciences (Gardiner, 2004).

It is possible to follow a number of steps (Øksendal, 2013) which allow us to rewrite (2) in a more consistent mathematical form, so as to give a precise meaning to its solution . This procedure leads to the following stochastic differential equation (SDE)

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|  | (3) |

where is the standard one-dimensional Brownian motion. The function is the local drift (i.e., velocity) coefficient and is called diffusion coefficient of the Itō diffusion described by (3).

Basically, (3) describes a decaying inventory with Brownian demand process. Note that such a demand process can be understood as the *netput* process, which captures the difference between the real demand and some input processes, such as customer returns (Yao, 2017).

In the inventory management literature, there exist a lot of examples where a SDE is used to model the evolution of inventory over time, such as Benkherouf (2007); Pinçe et al. (2008); Yao (2017); Lin (2017). Some of these contributions include the effect of decay (Benkherouf et al., 2003; Y. Li et al., 2017). However, none of them has taken into account a positive replenishment lead time. In addition, the typical optimization approach followed in these papers is based on variational methods whose application in practice may be difficult due to the complex optimization procedure that is needed to achieve the optimal solution.

A general result assures that the SDE (3) admits a unique solution , for (Øksendal, 2013). The following propositions establish some properties characterizing the stochastic process , which are useful to derive the cost model presented in the subsequent section for the inventory system under consideration.

**Proposition 1.** *For , the following stochastic process is the unique solution of the SDE/initial value problem (3):*

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*where* .

*Proof.* See Appendix B. □

**Proposition 2.** *For fixed , the random variable is Gaussian with mean and variance respectively given by*

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*and*

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*Proof.* See Appendix B. □

**3. The cost model**

The cost model concerns a single-product, single-location inventory system, which is managed according to a continuous review, lot size-reorder point (*Q*, *r*) policy. Inventory is subject to a constant decay rate *δ*, which means that each unit in the stock has an exponentially distributed lifetime with parameter *δ*. A positive replenishment lead time, *L*, is taken into account. Demands occurring in the stockout period are lost with a ratio *β*, while the fraction is backordered. In this section, we first provide some preliminary results and introduce additional notation; then develop the cost model.

***3.1. Preliminary results and additional notation***

The cumulative demand, , up to time *t* is given by

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where is the cumulated demand at time . Although a more general expression for could be easily included into the model, we will assume that the demand rate is constant over time, i.e., .

The cost model is developed following a similar approximate treatment to Hadley and Whitin (1963) for the standard (*Q*, *r*) model. This permits us to overcome the above-mentioned difficulties in modelling the inventory system under consideration (Nahmias and Wang, 1979). More precisely, we suppose that (*i*) there is never more than one order outstanding, and that (*ii*) the safety stock, i.e., the expected inventory level (the expected stock on hand minus backorders) just before the arrival of the order, is positive.

While these hypotheses may be considered questionable in some circumstances, in many others they are able to give a reasonably accurate description of the real system. For instance, in many practical cases, the proportion of time spent out of stock is small (Nahmias, 2013). In addition, it is likely in reality that shortages occur just prior to the arrival of an order (Nahmias, 1981). Last but not least, they permits to achieve analytically tractable expressions. Note that these assumptions have been largely endorsed by many authors, such as Hariga and Ben-Daya (1999); Chang et al. (2006); Eynan and Kropp (2007); Pinçe et al. (2008); Sazvar et al. (2014); Braglia et al. (2017). Nonetheless, the validity of the hypotheses introduced in the model will be numerically verified in a subsequent section.

According to the previously made assumptions, the long-run expected total cost per time unit can be evaluated by means of the renewal theoretic approach. Under the (*Q*, *r*) control policy, the stochastic process representing the inventory position repeats itself probabilistically at the epochs when the inventory position hits the reorder point *r* (Hadley and Whitin, 1963; Pinçe et al., 2008). Hence, the reordering instances constitute regeneration points and a *cycle* is defined as the time between two consecutive order placements. The long-run expected total cost per time unit is finally calculated exploiting the renewal reward theorem as the ratio of the expected cost per cycle to the expected cycle length. This approach to the cost model formulation is typically known as *cycle-based approximation* (Platt et al., 1997).

To obtain the long-run expected total cost per time unit, the expected cost per cycle must be evaluated. Hence, our calculations are restricted to observing the inventory behavior within a cycle. Considering this relatively limited time frame, the diffusion coefficient can be supposed to be constant over time, i.e., . Recalling that even the demand rate, , and the decay rate, , are assumed constant, the (positive) inventory level in any interval within the inventory cycle follows the dynamics

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|  | (4) |

with . If *u*, with , is a time instant such that for and if is sufficiently small, we can assume that (4) is able to give a sufficiently good description of the stock dynamics in . That is, if the portion of time within a cycle when the on-hand inventory is zero and backorders accumulates is small enough, we can hypothesize that (4) is able to describe the stock dynamics even in the stockout period.

To conclude this section, we provide the additional notation that will be used in the formulation of the cost model:

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| *x*+ | Maximum between 0 and *x* |
|  | Standard normal probability density function (p.d.f.) |
|  | Standard normal cumulative distribution function (c.d.f.) |
|  | Standard normal loss function |
|  | Expected value operator |
| *Q* | Order quantity (decision variable) [unit quantity] |
| *r* | Reorder point (decision variable) [unit quantity] |
| *A* | Ordering cost per order [$/order] |
| *v* | Stockholding cost rate per unit [$/unit quantity/unit time] |
| *c* | Penalty cost per decayed unit [$/unit quantity] |
| *p* | Penalty cost per unit shortage [$/unit quantity] |

***3.2. Model development***

In this subsection, we develop the heuristic cost model for the considered inventory system. Calculations are firstly carried out for a deterministic lead time. Then, we discuss the extension to the stochastic lead time case.

We begin with the computation of the safety stock, , which is the expected inventory level just before the arrival of the order. To this aim, we must consider the stochastic evolution of the inventory level as soon as the reorder point is hit. That is, if is the time instant in which the reorder point is reached, we determine the solution to the SDE

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for . Let be such solution evaluated at the end of lead time, i.e., just before the arrival of the order. From Section 2, we know that the p.d.f. of is Gaussian whose mean and variance are

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|  | (5) |

and

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|  | (6) |

respectively.

It is evident that, in case of full backordering, the inventory level just before the arrival of the order is . If all demand during the stockout period is lost, then the inventory level just before the arrival of the order is . If the fraction *β* of demand during the stockout period is lost and the quota is backordered, then the inventory level just before the arrival of the order, , is given by

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Hence, the safety stock can be determined as the expected value of :

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where is given by (5) and , with given by (6).

The number of units which go short in a cycle is 0 if , and otherwise. Hence, the expected shortage per cycle is

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We recall that *β* is the fraction of shortage that is lost. Hence, is the expected number of lost sales per cycle, and is the expected number of backorders per cycle.

Without loss of generality, we assume that the inventory cycle begins at time . Let *T* be the mean cycle length. For the mean value function becomes

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|  | (7) |

The expression for the mean cycle length can be found from (7) by imposing the boundary conditions and , where is the safety stock. Thus, we find

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which gives

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|  | (8) |

To evaluate the expected stockholding cost rate, it is necessary to calculate the expected on-hand inventory, , which can be readily obtained by averaging the mean-value function, , over :

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|  | (9) |

where *T* is given by (8).

It remains to evaluate the expected number of units decaying per unit time. The mean inventory level decreases by *Q* units due to the joint effect of demand and deterioration. Since is the average number of units demanded in a cycle, the average number of units decaying in a cycle is . Hence, the expected number of units decaying per unit time, , is given by

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where is given by (9).

All the terms in the expected average cost per unit time are determined; is

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|  | (10) |

where *T* is given by (8). It is relatively easy to show that

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|  | (11) |

If we let and be the mean and the standard deviation of the lead-time demand, respectively, we can note that (11) perfectly agrees with Hadley and Whitin (1963). From (11), it is possible to deduce that the parameter *σ* is able to represent the standard deviation of forecast errors per unit of time (Nahmias, 2013).

The objective is to find the values of *Q* and *r* that minimize (10). Although it is hard to verify analytically whether is convex or not, numerical tests have shown that, in general, it is not convex. However, it admits a unique stationary point, which consists in a minimum. Hence, the values and that minimize must satisfy the equations and . To obtain and , a standard numerical method can be used to solve the system of equations obtained from the first-order condition for optimality. For instance, the iterative method proposed by Hadley and Whitin (1963) can be adopted.

Note that (10) is formulated considering a deterministic lead time. If *L* is supposed to be a random variable with support and c.d.f. , then the procedure to determine is slightly different. In particular, it is required to evaluate (i.e., the safety stock) and (i.e., the expected shortage per cycle) taking into account the marginal distribution of the inventory level just before the arrival of the order. With stochastic lead time, and are given by

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and

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respectively.

To conclude this section, we would highlight some differences between the model of Nahmias and Wang (1979) and ours. We remind the reader that only the model of Nahmias and Wang (1979) is taken into reference in the comparison as we were not able to find additional models approaching the decay problem, in a continuous review system, with random demand and positive lead time. These models differ in how the safety stock and the expected shortage per cycle are obtained. In our case, these quantities are determined according to the procedure presented in this section. Instead, Nahmias and Wang (1979) evaluate the safety stock as the difference between the reorder point and the expected number of items consumed in lead time because of the joint effect of demand and decay. They estimate the expected shortage per cycle as the loss function associated to a given lead-time demand distribution, whose type and parameters are *a priori* established by assumption. A last difference consists in the fact that our model includes the mixture of backorders and lost sales, while the original model of Nahmias and Wang (1979) neglects this feature.

**4. Numerical experiments**

The objective of this section is threefold:

1. To evaluate the performance of the heuristic cost model proposed in this research;
2. To provide a comparison with the model of Nahmias and Wang (1979), which will be referred to as the NW model (or NW heuristic) henceforth in this section; and
3. To carry out a sensitivity analysis in order to draw some managerial insights.

These experiments are performed on a specifically developed numerical test platform within the MATLAB® R2017a environment.

We begin with the first two objectives of the experiments. This analysis is carried out by means of simulation. In particular, for a given set of parameter values, the optimal policy for our cost model and the optimal policy for the NW model are determined. The “real” cost related to these policies is evaluated by simulating the behavior of the inventory system. The simulated costs of the optimal policies for our model and for the NW model are then compared with the cost of the policy that is found to approximate the true optimum of the system.

The optimum of the system is estimated by performing a simulation-driven Lucas search, which is based on the use of Lucas numbers. A sequence of (*Q*, *r*) values is created by a “one at a time” Lucas search on each variable (more details about the Lucas search algorithm can be found in the work of Subasi et al. (2004)). For each (*Q*, *r*) pair, after a warmup period of 7 inventory cycles the system is simulated over 200 time periods to obtain an estimate of the cost per time unit. Each simulation run is repeated 20 times; in each run, the simulation is initialized with a different random seed. Hence, 20 independent observations for the cost per time unit are gathered. An estimate of the true optimal policy is the (*Q*, *r*) pair that gives the smallest average cost per unit time. A similar procedure was adopted to obtain an estimate of the simulated costs of the optimal policy for our model and for the NW model.

To simulate a nearly continuous path for the inventory level, each period is broken down into *N* = 20 steps with an average demand of per step, where *λ* is the average demand per period. If is the variance of the demand per period, then the variance of the demand per step is . Observations for step size demand are generated from a gamma distribution with parameters and .

Given the initial condition at time , the simulated inventory level at time step *t* is determined according to the law

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where *τ* is the size of each time step, i.e., , and is the observation for step size demand at time *t*. The relation between and is obtained by applying the backward Euler method. If the inventory level at time *t* is , the amount of decayed inventory in the time interval is given by . The rule is adopted that, if is the first time instant in a cycle such that , at the time the inventory level is raised up by a quantity of *Q* units.

We considered two cases for the NW model. In the first one, the density of the lead-time demand is a gamma with parameters and (we will refer to this model as the NWG model). In the other one, the lead-time demand follows a Gaussian distribution with mean and variance (we will refer to this model as the NWN model).

To consider positive values of *β*, it is necessary to extend the NW model to include lost sales (in its original version, lost sales are not taken into account). If we use the same notation as in Nahmias and Wang (1979), the safety stock *s*, in their model, is readily obtained as

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If *f* is a gamma density with parameters *k* and , it can be noted that

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where is the ordinary gamma function and is the upper incomplete gamma function.

The fixed parameter values considered in these experiments are *λ* = 10 units/period and *v* = $1/unit/period. We take into account different values of , *δ*, *L*, *p*, *c*, *β* and *A*. Experiments are performed on a PC with an Intel® Core™ i7 processor at 2.4GHz and 16GB RAM memory. Obtaining the results for a given combination of parameter values requires on average about 4 minutes exploiting parallel computing on 4 logical cores. The results of tests are given in Tables 2 and 3 for the cases *A* = $600/order and *A* = $800/order, respectively, with *c* = $3/unit and *β* = 0.2. Tables 4 and 5 show the results for *A* = $600/order and *A* = $800/order, respectively, with *c* = $12/unit and *β* = 0.2. Tables 6 and 7 concerns experiments with *c* = $3/unit and *c* = $12/unit, respectively, considering *A* = $800/order and *β* = 0.8.

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In these tables, column (1) shows the policy that minimizes (10), while columns (2) and (3) give the optimal policy for the NW model in the gamma distribution case (NWG model) and in the normal distribution case (NWN model), respectively. Column (4) represents the solution obtained from the simulation-driven Lucas search. Column (5) provides the 95% paired-t confidence interval (Law and Kelton, 2000) for the average cost corresponding to the solution obtained from the simulation-driven Lucas search. Columns (6)–(8) compare the cost performance of three policies in columns (1)–(3) using the mean absolute percentage error (MAPE) with respect to the cost achieved by policy in column (4). The MAPE is defined as

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where *N* is the number of instances, is the cost of the policy obtained with the simulation-driven Lucas search, and is the cost of any policy in columns (1)–(3).

It is possible to note that the MAPE of the new model is below 5% in

* 27 out of 36 test cases in Tables 2 and 4;
* 28 out of 36 test cases in Table 3;
* 30 out of 36 test cases in Tables 5, 6, and 7.

Hence, we may assert that the new model performs, in absolute terms, quite well.

Now, consider the relative performance of the new heuristic with respect to the NW models. In some cases, all heuristic models are substantially equivalent, i.e., the respective values of MAPE are close. However, in many other cases, the new model gives a solution that is more than 90% better in terms of MAPE than the NW heuristics. Note that when the models do not approximate the optimal solution from simulation well (i.e., MAPE > 5%), their performance is substantially similar. In a very limited number of test instances, the new model provides the worst solution (see, e.g., instance number 6 in Table 2 and instance number 28 in Table 5), and when this happens the difference of performance between all heuristics is practically negligible. However, the new model is able to give the best solution on average in more than 30 test cases out of 36.

In few cases, among all experiments, the MAPE achieved by the heuristic models is larger than 20% and, only in two circumstances, is above 40%. Such deviations cannot, however, be neglected. Hence, an in-depth analysis is necessary to motivate the observed error.

We have investigated the instances where MAPE > 20%, and it has been noted that, in these cases, the optimal policy for the heuristic models makes the fundamental assumptions behind the heuristics, which were stated in Section 3, violated. That is, the system simulated considering the optimal policy of any heuristic model shows a behavior in which these assumptions are not verified.

Note that such large deviations typically occur when the values of *L* or *δ* are high. A larger *L* or *δ* (with the other parameters fixed) leads to an increased probability of running out of stock during lead time, and hence to a larger expected shortage. It is known that the heuristic models based on the assumptions stated in Section 3 become less efficient as the expected shortage increases (Hadley and Whitin, 1963).

Clearly, the fact that, in a given instance, the assumptions are not respected in correspondence to a certain (*Q*, *r*) pair only depends upon the relative value of parameters. It is also true that, for a given combination of parameter values, it is possible to find (*Q*, *r*) pairs that make the assumptions satisfied and (*Q*, *r*) pairs for which the same assumptions are violated. Hence, given a combination of parameter values, the heuristic models are appropriate only over a sub-region of all possible (*Q*, *r*) pairs.

From results, it is possible to analyze how the performance of the new model changes as parameter values vary. We can observe that it improves for larger values of *A* and smaller values of *L*, *p*, and *β*. If we consider the effect of , the model becomes less efficient for larger values of , although in some instances the performance is not good even for small values of . A similar observation can be made for *δ*. With respect to the parameter *c*, it seems that no evident effect on the performance of the new model can be observed. We may justify this behavior, about how the performance changes as parameters vary, with the following observations:

* Since larger values of *A* reduce the optimal reorder point and increase the optimal order quantity, it is more likely that the characteristic of stochastic repeatability of cycles be satisfied (i.e., that the on-hand inventory is raised above the reorder point after a replenishment), and hence that all fundamental hypotheses for the heuristic models be verified as well.
* The larger the lead time *L* (for fixed demand and decay rates), the smaller the probability to avoid a stockout occurrence. Hence, for smaller *L*, we can expected that model (4) is able to give a better description of the inventory level dynamics. A similar observation can be made for the variance of the demand per period and for the decay rate, as larger or *δ* (with the other parameters fixed) give a higher stockout probability.
* With regard to the parameter *β* about the fraction of the demand during the stockout period that is lost, we recall that the heuristic treatment for the lost sales case is more complex, and the approximation is acceptable in more narrow conditions, than the case with backordered demand (see, e.g., Hadley and Whitin, 1963; Silver et al., 2017). This justifies the worse performance of the heuristic models for larger values of *β*.

It is interesting to note that the optimal reorder point for the new model and the optimal reorder point obtained from the simulation-driven Lucas search are very close (see Tables 8, 9, and 10). While there is not a large difference between lot sizes provided by the heuristic models, we can observe that, with the exception of very few cases, the optimal reorder point for the new model is surprisingly closer to the optimal reorder point obtained from simulation than the optimal reorder point obtained from the other heuristic models (i.e., NWG and NWN). This means that the new model should be able to provide a better estimate of the optimal safety stock than the NWG and NWN models.

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TABLE 8 HERE

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TABLE 9 HERE

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TABLE 10 HERE

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Given the results presented above, we can conclude that the new heuristic is substantially very efficient, and hence is valuable in a practical context. This observation can be made under two viewpoints:

1. It is efficient in absolute terms as its MAPE is below 5% in about 80% of all considered instances, and is above 10% in nearly 13% of all experiments, on average;
2. It is efficient in relative terms, with respect to the NW heuristics, as it provides the best solution (i.e., the smaller MAPE) in about 84% of all experiments, on average.

A sensitivity analysis is finally carried out in order to draw some managerial insights concerning the new heuristic model. The analysis is conducted by changing each of the considered parameters by -40%, -20%, +20%, and +40%, taking one parameter at a time and keeping the value of the remaining parameters unchanged. Results are shown in Table 11. Note that the results concerning parameters *c* and *h* are the same for the considered significant digits, while they differ at smaller order of magnitudes. The following observations can be made:

* The optimal batch size increases as parameters *A*, *λ*, *σ*, *L*, and *p* increase, while it decreases with an increment in *δ*, *β*, *c*, and *h*. The sensitivity of is high (total variation > 20%) for changes in *A*, *λ*, *c*, and *h*, moderate (total variation between 20% and 5%) for changes in *δ* and *σ*, and low (total variation below 5%) for changes in *β*, *L*, and *p*.
* The optimal reorder point increases as parameters *λ*, *δ*, *β*, *L*, and *p* increase, while it decreases with an increment in *A*, *σ*, *c*, *h*. Note that a similar behavior with respect to changes in *σ* can be observed even for the optimal reorder point obtained from the simulation-driven Lucas search (see, e.g., instances number 3 and 15 in Table 4), and only the new heuristic is able to capture this peculiarity. The sensitivity of is high for changes in *λ*, *L*, and *p*, moderate for changes in *A*, *δ*, and *β*, and low for changes in *σ*, *c*, and *h*.
* The optimal cost grows as any parameter increases. The sensitivity of is high for changes in *A*, *λ*, and *δ*, moderate for changes in *c* and *h*, and low for changes in *σ*, *β*, *L*, and *p*.

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TABLE 11 HERE

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**5. Conclusions and further remarks**

This paper considered a single-item inventory subject to uncertain demand and decay under a continuous review policy. It is known that optimizing this system when a positive lead time is taken into account is a very difficult problem. We approached this question by developing a heuristic, approximate lot size-reorder point (*Q*, *r*) inventory model, in which the inventory dynamics during lead time is captured by means of an Itō diffusion.

Numerical experiments were carried out to evaluate the efficiency of the proposed model. A comparison was made with respect to the solution found by a search procedure over a response surface approximating the “true” cost function and the solution found from a former heuristic model owed to Nahmias and Wang (1979). According to numerical results, the proposed model should provide a very good approximation of the optimal policy obtained from simulation over a reasonably wide range of parameter values. Moreover, it seems to provide better results than the former model in a substantial number of test cases. A sensitivity analysis was finally carried out in order to draw some managerial insights concerning the proposed model.

It was interesting to observe that our model is able to give a better estimate of the optimal reorder point from simulation than Nahmias and Wang (1979) in a very large number of cases (in some cases where the estimate is particularly not accurate, all heuristic models showed a bad performance). Moreover, the value provided of the reorder point is very close to the optimal one from simulation (absolute percentage error < 5%) in several test instances. All heuristic models give very similar estimates of the optimal order quantity obtained from simulation.

While in Nahmias and Wang (1979) the standard deviation of the demand per period is constant and a more general expression cannot be considered, our approach permits to include any sufficiently regular function of time . This feature is clearly applicable if a meaningful change in the standard deviation of the demand per period can be observed inside an inventory cycle.

Although this paper considered the case of constant and , more complex expressions may be used. For example, a stock-dependent demand rate as in Padmanabhan and Vrat (1995) or in Giri and Chaudhuri (1998) may be taken into account. When demand is random, many items may move from one inventory cycle to the next one. In this condition, if a non-constant decay rate is adopted, items in inventory have a different probability to fail in the next time interval. This issue should be carefully considered in the model. A future research may be devoted to investigating this aspect.

Since the optimal reorder point calculated with the model presented in this research seems substantially efficient over a wide range of parameter values, a question that deserves attention may be how to obtain a better estimate of the optimal order quantity. A further research topic that may be worth investigating is the development of a heuristic cost model for a periodic review inventory system with a not negligible deterministic or stochastic lead time.

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**Appendix A**

Consider a single item inventory that is subject to the effect of decay only (demand is temporarily neglected). Let be an initial time instant such that the corresponding inventory level is positive. Suppose that each item in the stock at any is characterized by the same instantaneous decay rate . According to basic reliability theory (Verma et al., 2016), this means that each item in the stock has a probability of of having survived between and , for any . Hence, if is the inventory level at time , the total number of items on hand at time is a binomial random variable with parameters and , and the expected inventory level at time is . Moreover, the expected number of decayed items in the time is .

Now, suppose items are also subject to an instantaneous demand rate at time *t*. The joint effect of decay and demand produces a negative variation, , in the inventory level between and which is given by

|  |  |
| --- | --- |
|  |  |

For a sufficiently small *h*, we can write:

|  |  |
| --- | --- |
|  |  |

As *h* approaches 0, we get the following ordinary differential equation which describes the inventory level at any such that :

|  |  |
| --- | --- |
|  | (A.1) |

with initial condition , .

Equation (A.1) provides a completely deterministic description of the stock level for an item that is subject to the joint effect of decay and demand. In fact, the decay process included into (A.1) arises as the mean value of a perishable inventory problem with random lifetimes. When is very large (for example, in the case of radioactive materials), the law of large numbers guarantees that (A.1) will be an accurate representation of the system for all realizations of the item lifetimes (Nahmias, 2013).

**Appendix B**

Let and be measurable functions on with values in . Let be a standard one-dimensional Brownian motion. Consider the following real-valued one-dimensional SDE/initial value problem:

|  |  |
| --- | --- |
|  | (B.1) |

**Assumption B.1.** *Given the SDE/initial value problem (B.1), b and σ satisfy Assumption B.1 if they are measurable in and if there exist constants and such that, for each and ,*

|  |  |
| --- | --- |
|  |  |

If we take and , where , and are continuous functions in and such that Assumption B.1 is satisfied, then (B.1) becomes

|  |  |
| --- | --- |
|  | (B.2) |

Moreover, a known general theorem guarantees that (B.2) admits a unique solution (Øksendal, 2013). The following result gives the solution to (B.2):

**Proposition B.1.** *For , the following stochastic process is the unique solution of the SDE/initial value problem (B.2):*

|  |  |
| --- | --- |
|  | (B.3) |

*where .*

*Proof.* We apply Itō’s lemma to the stochastic process given by (B.3) and rewrite it in the form

|  |  |
| --- | --- |
|  | (B.4) |

where

|  |  |
| --- | --- |
|  |  |

The differential of , and is respectively given by

|  |  |
| --- | --- |
|  |  |

Since (B.4) is structured as , whose differential is , then we have:

|  |  |
| --- | --- |
|  |  |

which proves the assertion. □

The following lemma is required later to derive the distribution of the random variable , for fixed *t*.

**Lemma B.1.** *For fixed , the random variable is Gaussian with mean 0 and variance given by*

|  |  |
| --- | --- |
|  | (B.5) |

*Proof*. By applying Itō’s lemma to the stochastic process , we readily obtain

|  |  |
| --- | --- |
| . |  |

Now, consider the characteristic function of the random variable (for fixed *t*). By definition, we have

|  |  |
| --- | --- |
|  |  |

If we let and use Itō’s lemma on , we find

|  |  |
| --- | --- |
|  |  |

Note that . Hence, we get

|  |  |
| --- | --- |
|  | (B.6) |

We recall that (Øksendal, 2013). Hence, the expected value of (B.6) gives:

|  |  |
| --- | --- |
|  | (B.7) |

By taking the first-order time derivative of (B.7), we obtain (note that is parameterized by both *t* and *u*):

|  |  |
| --- | --- |
|  | (B.8) |

It is possible to observe that . We can thus rewrite (B.8) as follows:

|  |  |
| --- | --- |
|  | (B.9) |

If we put , (B.9) becomes

|  |  |
| --- | --- |
|  |  |

To evaluate the characteristic function , it is therefore required to solve the following initial value problem:

|  |  |
| --- | --- |
|  | (B.10) |

To this aim, we adopt the method of characteristics. Let be the curve that brings the Cauchy data . can be parameterized by the following equations:

|  |  |
| --- | --- |
|  | (B.11) |

To determine the general solution to problem (B.10), it is enough to find the characteristics by solving the characteristic system of ordinary differential equations:

|  |  |
| --- | --- |
|  | (B.12) |

with initial conditions (B.11). It is relatively easy to obtain the solution of the system of ordinary differential equations (B.12), which gives

|  |  |
| --- | --- |
|  | (B.13) |

The solution to the initial value problem (B.10) is therefore given by (B.13):

|  |  |
| --- | --- |
|  |  |

This demonstrates that, for fixed , the random variable is Gaussian with mean 0 and variance given by (B.5).

□

**Proposition B.2.** *For fixed , the random variable defined in (B.3) is Gaussian with mean and variance given by*

|  |  |
| --- | --- |
|  | (B.14) |
|  | (B.15) |

*Proof*. For fixed , let be the characteristic function of the random variable . By definition of characteristic function, is given by

|  |  |
| --- | --- |
|  | (B.16) |

If we put , then (B.16) can be rewritten as follows:

|  |  |
| --- | --- |
|  |  |

where is the characteristic function of .

According to Lemma B.1, we know that is a Gaussian random variable with mean 0 and variance given by (B.5). The characteristic function of is thus expressed as follows:

|  |  |
| --- | --- |
|  |  |

Therefore, becomes

|  |  |
| --- | --- |
|  |  |

This proves that, for fixed , is a Gaussian random variable with mean and variance given by (B.14) and (B.15), respectively.

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**Table 5.** Results of experiments for *A* = $800/order, *c* = $12/unit, and *β* = 0.2.

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**Table 11.** Sensitivity analysis of new heuristic model.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Model features** | | | | |  |  |
|  | Demand | | Lead time | | Additional features |  |  |
| **Authors** | *Deterministic* | *Stochastic* | *Positive* | *Zero* |  | **Modelling method** | **Optimization approach** |
| Nahmias and Wang (1979) |  | ✓ | ✓  (Deterministic) |  | Backorders | * Ordinary differential equation for average inventory level * The lead-time demand follows a distribution imposed by assumption | Check of Kuhn-Tucker conditions |
| Benkherouf et al. (2003) |  | ✓ |  | ✓ | Backorders | Brownian demand process | Application of variational methods |
| Dey et al. (2008) | ✓ |  | ✓  (Deterministic) |  | * Time-dependent demand * Inflation and time value of money * Two-warehouse system * Backorders and lost sales | Ordinary differential equation for inventory level | Meta-heuristic algorithm |
| Rong et al. (2008) | ✓ |  | ✓  (Fuzzy) |  | * Two-warehouse system * Price-dependent demand * Distance-dependent unit holding cost rate * Backorders and lost sales |  | Check of Kuhn-Tucker conditions |
| Jha and Shanker (2009) |  | ✓ | ✓  (Deterministic) |  | * Single vendor-single buyer supply chain * Service level constraint * Controllable lead time | * Ordinary differential equation for average inventory level * The lead-time demand follows a distribution imposed by assumption |  |
| Skouri et al. (2011) | ✓ |  |  | ✓ | * Ramp-type demand rate * Permissible delay in payments * Backorders and lost sales | Ordinary differential equation for inventory level | Check of Kuhn-Tucker conditions |
| Sazvar et al. (2013) | ✓ |  | ✓  (Stochastic) |  | * Time-dependent unit holding cost rate * Backorders | Ordinary differential equation for average inventory level | Check of Kuhn-Tucker conditions |
| Maihami and Karimi (2014) |  | ✓ |  | ✓ | * Price-dependent demand * Promotional effort * Non-instantaneous decay * Backorders and lost sales | * Ordinary differential equation for average inventory level * A random noise is used to include randomness in the demand rate | Check of Kuhn-Tucker conditions |
| Wu et al. (2016) | ✓ |  |  | ✓ | * Trapezoidal-type demand rate * Time-dependent decay rate * Time value of money * Backorders and lost sales | Ordinary differential equation for inventory level | Check of Kuhn-Tucker conditions |
| Chen and Sarker (2017) | ✓ |  |  | ✓ | * Single manufacturer-multiple retailer supply chain * Price-dependent demand rate * Backorders | Ordinary differential equation for inventory level | Meta-heuristic algorithm |
| Li et al. (2017) |  | ✓ |  | ✓ | * Price- and time-dependent demand * Time-dependent price * Backorders | Brownian demand process | Application of variational methods |
| Pervin et al. (2018) | ✓ |  |  | ✓ | * Time-dependent demand rate * Time-dependent decay rate * Time-dependent unit holding cost rate * Backorders and lost sales | Ordinary differential equation for inventory level | Check of Kuhn-Tucker conditions |
| Lashgari et al. (2018) | ✓ |  | ✓  (Deterministic) |  | * Non-instantaneous decay * Hybrid partial payment * Partial trade credit * Backorders and lost sales | Ordinary differential equation for inventory level | Check of Kuhn-Tucker conditions |
| This paper |  | ✓ | ✓  (Deterministic  /stochastic) |  | Backorders and lost sales | Brownian demand process | Check of Kuhn-Tucker conditions |

**Table 1.** Overview of some key literature about decaying inventory.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Test No. | *σ*2 | *p* | *δ* | *L* | Optimal policy from new model | Optimal policy from the NWG model | Optimal policy from the NWN model | Optimal policy from simulation | 95% confidence interval for the cost of solution from simulation | Mean absolute percentage error achieved by policy in column (1) | Mean absolute percentage error achieved by policy in column (2) | Mean absolute percentage error achieved by policy in column (3) |
| 1 | 5 | 50 | 0.1 | 3 | (114.4, 37.7) | (114.8, 47.7) | (114.6, 47.8) | (120.1, 38.6) | [153.8, 154.2] | 0.30 | 5.30 | 5.48 |
| 2 |  |  |  | 6 | (115.4, 86.7) | (115.9, 95.3) | (115.7, 95.4) | (102.6, 88.8) | [155.7, 156.2] | 0.31 | 2.48 | 2.53 |
| 3 |  |  | 0.3 | 3 | (112.0, 50.6) | (112.9, 71.6) | (112.6, 71.7) | (121.7, 50.2) | [213.7, 214.2] | 0.34 | 11.21 | 11.19 |
| 4 |  |  |  | 6 | (112.4, 173.4) | (113.7, 144.9) | (113.5, 144,9) | (166.8, 167.3) | [225.1, 225.7] | 11.64 | 12.09 | 12.04 |
| 5 |  |  | 0.5 | 3 | (107.7, 70.1) | (109.2, 90.9) | (108.9, 91.1) | (115.1, 74.2) | [266.5, 266.8] | 0.22 | 6.00 | 5.97 |
| 6 |  |  |  | 6 | (107.9, 187.3) | (109.7, 184.9) | (109.5, 184.8) | (186.7, 186.7) | [305.9, 306.9] | 5.08 | 4.82 | 4.83 |
| 7 |  | 100 | 0.1 | 3 | (114.7, 39.9) | (115.2, 50.2) | (114.8, 50.1) | (112.8, 41.0) | [157.1, 157.6] | 0.67 | 6.20 | 6.05 |
| 8 |  |  |  | 6 | (115.8, 90.4) | (116.3, 98.9) | (116.0, 98.7) | (98.8, 89.9) | [157.7, 158.1] | 0.58 | 2.31 | 2.14 |
| 9 |  |  | 0.3 | 3 | (113.0, 54.3) | (114.0, 75.0) | (113.5, 74.9) | (110.6, 54.3) | [219.8, 220.3] | 0.98 | 11.22 | 11.28 |
| 10 |  |  |  | 6 | (113.5, 183.7) | (114.8, 150.0) | (114.4, 149.7) | (179.9, 311.5) | [232.8, 233.5] | 45.13 | 44.91 | 45.76 |
| 11 |  |  | 0.5 | 3 | (109.1, 76.5) | (110.6, 95.4) | (110.1, 95.2) | (106.2, 82.1) | [273.1, 273.7] | 0.40 | 5.64 | 5.63 |
| 12 |  |  |  | 6 | (109.2, 413.3) | (111.0, 191.4) | (110.6, 190.8) | (303.5, 302.0) | [375.2, 376.3] | 32.21 | 32.43 | 32.88 |
| 13 | 20 | 50 | 0.1 | 3 | (118.4, 40.3) | (119.8, 50.5) | (118.8, 51.0) | (123.3, 43.5) | [162.1, 163.1] | 0.70 | 3.09 | 3.39 |
| 14 |  |  |  | 6 | (120.4, 91.0) | (122.0, 101.1) | (120.9, 101.5) | (141.4, 87.2) | [164.4, 165.3] | 0.42 | 2.09 | 2.33 |
| 15 |  |  | 0.3 | 3 | (117.1, 52.3) | (119.3, 75.6) | (118.3, 76.3) | (113.7, 54.2) | [224.4, 225.3] | 0.47 | 8.68 | 9.14 |
| 16 |  |  |  | 6 | (117.9, 177.9) | (120.9, 155.7) | (119.9, 155.8) | (154.5, 202.5) | [232.6, 233.7] | 6.27 | 7.30 | 7.57 |
| 17 |  |  | 0.5 | 3 | (112.8, 69.7) | (116.0, 96.4) | (115.1, 97.3) | (140.6, 61.6) | [277.8, 278.9] | 0.52 | 5.32 | 5.70 |
| 18 |  |  |  | 6 | (113.1, 195.7) | (117.1, 200.2) | (116.1, 200.0) | (135.1, 186.2) | [310.3, 312.3] | 3.02 | 3.88 | 4.22 |
| 19 |  | 100 | 0.1 | 3 | (119.0, 44.8) | (120.6, 55.9) | (119.2, 55.7) | (122.8, 44.1) | [168.9, 169.9] | 1.40 | 5.15 | 5.01 |
| 20 |  |  |  | 6 | (121.1, 98.7) | (122.9, 108.9) | (121.5, 108.4) | (120.2, 97.1) | [174.8, 176.1] | 0.56 | 2.06 | 2.28 |
| 21 |  |  | 0.3 | 3 | (118.9, 60.3) | (121.6, 83.4) | (120.0, 82.9) | (141.8, 60.5) | [238.1, 239.4] | 0.63 | 8.32 | 7.96 |
| 22 |  |  |  | 6 | (119.8, 198.6) | (123.1, 167.0) | (121.6, 165.6) | (191.3, 197.2) | [253.8, 255.9] | 30.81 | 30.32 | 32.09 |
| 23 |  |  | 0.5 | 3 | (115.3, 83.7) | (119.0, 106.4) | (117.3, 105.8) | (118.9, 90.2) | [294.9, 296.5] | 1.12 | 5.25 | 4.78 |
| 24 |  |  |  | 6 | (115.7, 446.2) | (119.7, 214.2) | (118.2, 212.1) | (326.3, 426.1) | [382.9, 385.0] | 25.81 | 26.67 | 27.78 |
| 25 | 50 | 50 | 0.1 | 3 | (123.2, 43.2) | (126.3, 53.0) | (123.7, 54.6) | (144.0, 41.8) | [172.1, 173.6] | 1.36 | 3.32 | 4.25 |
| 26 |  |  |  | 6 | (126.4, 95.7) | (129.8, 107.2) | (127.2, 108.5) | (94.8, 102.8) | [179.9, 182.4] | 1.58 | 1.76 | 1.25 |
| 27 |  |  | 0.3 | 3 | (123.1, 53.9) | (127.4, 79.4) | (124.9, 81.7) | (131.8, 47.7) | [239.9, 241.4] | 1.56 | 8.66 | 9.64 |
| 28 |  |  |  | 6 | (124.4, 181.9) | (129.8, 167.9) | (127.3, 168.7) | (156.1, 249.5) | [244.3, 246.2] | 1.71 | 3.98 | 4.07 |
| 29 |  |  | 0.5 | 3 | (132.3, 86.3) | (124.6, 101.5) | (122.2, 104.4) | (94.8, 73.2) | [299.2, 301.1] | 3.12 | 5.00 | 5.74 |
| 30 |  |  |  | 6 | (119.4, 370.2) | (126.1, 218.0) | (123.7, 218.1) | (139.0, 570.3) | [316.8, 319.8] | 0.86 | 1.17 | 1.21 |
| 31 |  | 100 | 0.1 | 3 | (123.9, 50.5) | (127.9, 62.3) | (124.3, 62.1) | (142.2, 52.7) | [184.5, 186.0] | 3.04 | 4.61 | 4.38 |
| 32 |  |  |  | 6 | (127.2, 108.2) | (131.4, 120.7) | (127.9, 119.6) | (129.0, 118.0) | [191.7, 194.5] | 1.49 | 1.88 | 1.42 |
| 33 |  |  | 0.3 | 3 | (125.7, 66.9) | (131.4, 93.2) | (127.3, 92.5) | (108.2, 66.4) | [260.5, 262.3] | 0.82 | 7.12 | 6.68 |
| 34 |  |  |  | 6 | (127.2, 216.8) | (133.5, 187.7) | (129.8, 184.4) | (194.9, 263.5) | [270.1, 274.2] | 16.61 | 18.34 | 22.50 |
| 35 |  |  | 0.5 | 3 | (122.5, 92.3) | (129.6, 119.7) | (125.4, 118.3) | (136.5, 94.7) | [326.0, 329.3] | 0.89 | 4.45 | 4.03 |
| 36 |  |  |  | 6 | (123.0, 486.0) | (130.3, 242.4) | (126.7, 237.2) | (291.1, 701.3) | [402.5, 406.3] | 18.31 | 19.96 | 20.68 |

**Table 2.** Results of experiments for *A* = $600/order, *c* = $3/unit, and *β* = 0.2.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Test No. | *σ*2 | *p* | *δ* | *L* | Optimal policy for new model | Optimal policy for NWG model | Optimal policy for NWN model | Optimal policy from Lucas search | 95% confidence interval for the cost of solution from simulation | Mean absolute percentage error achieved by policy in column (1) | Mean absolute percentage error achieved by policy in column (2) | Mean absolute percentage error achieved by policy in column (3) |
| 1 | 5 | 50 | 0.1 | 3 | (134.1, 37.1) | (134.6, 49.4) | (134.3, 49.5) | (147.3, 39.3) | [179.5, 179.8] | 0.46 | 6.96 | 6.95 |
| 2 |  |  |  | 6 | (135.2, 85.7) | (135.7, 99.0) | (135.5, 99.1) | (136.2, 86.9) | [179.9, 180.3] | 0.15 | 4.43 | 4.36 |
| 3 |  |  | 0.3 | 3 | (133.2, 49.4) | (134.1, 76.7) | (133.9, 76.9) | (137.6, 51.1) | [253.6, 254.0] | 0.45 | 14.62 | 14.82 |
| 4 |  |  |  | 6 | (133.6, 170.3) | (135.1, 155.8) | (134.9, 155.9) | (163.0, 190.8) | [258.1, 258.9] | 3.32 | 4.00 | 4.00 |
| 5 |  |  | 0.5 | 3 | (145.7, 81.9) | (130.6, 99.0) | (130.4, 99.2) | (124.5, 64.0) | [317.0, 317.8] | 10.41 | 10.38 | 10.46 |
| 6 |  |  |  | 6 | (129.0, 226.0) | (131.3, 202.1) | (131.1, 202.2) | (171.0, 232.5) | [339.6, 340.4] | 1.78 | 2.10 | 1.98 |
| 7 |  | 100 | 0.1 | 3 | (134.5, 39.4) | (135.0, 52.0) | (134.7, 52.0) | (111.1, 41.3) | [183.3, 183.8] | 0.22 | 5.63 | 5.57 |
| 8 |  |  |  | 6 | (135.6, 89.7) | (136.2, 102.8) | (135.9, 102.7) | (144.3, 89.6) | [182.7, 183.5] | 0.60 | 3.76 | 3.88 |
| 9 |  |  | 0.3 | 3 | (134.4, 53.5) | (135.5, 80.5) | (135.0, 80.5) | (139.0, 54.8) | [258.9, 259.6] | 0.25 | 14.06 | 14.10 |
| 10 |  |  |  | 6 | (134.9, 181.1) | (136.4, 161.6) | (136.0, 161.3) | (177.2, 218.5) | [265.2, 265.8] | 18.31 | 19.91 | 20.23 |
| 11 |  |  | 0.5 | 3 | (130.6, 75.1) | (132.3, 104.1) | (131.9, 104.0) | (125.2, 76.9) | [325.5, 326.1] | 0.89 | 9.96 | 9.85 |
| 12 |  |  |  | 6 | (130.8, 406.7) | (132.9, 209.4) | (132.5, 209.0) | (315.7, 364.5) | [406.4, 407.9] | 20.92 | 21.42 | 21.49 |
| 13 | 20 | 50 | 0.1 | 3 | (138.4, 39.1) | (139.7, 51.4) | (138.8, 52.1) | (146.2, 37.7) | [188.2, 189.4] | 0.65 | 4.75 | 5.27 |
| 14 |  |  |  | 6 | (140.5, 89.0) | (142.1, 103.8) | (141.1, 104.4) | (136.7, 88.6) | [188.6, 189.5] | 0.84 | 4.38 | 4.49 |
| 15 |  |  | 0.3 | 3 | (138.5, 49.8) | (140.8, 79.6) | (139.9, 80.6) | (135.9, 44.8) | [262.6, 263.7] | 0.61 | 12.16 | 12.89 |
| 16 |  |  |  | 6 | (139.3, 171.1) | (142.7, 165.3) | (141.8, 165.8) | (161.8, 199.0) | [265.7, 266.8] | 0.78 | 1.26 | 1.43 |
| 17 |  |  | 0.5 | 3 | (145.7, 85.3) | (137.7, 102.9) | (136.9, 104.1) | (120.4, 70.0) | [327.5, 328.9] | 9.32 | 9.29 | 9.96 |
| 18 |  |  |  | 6 | (134.4, 355.7) | (139.2, 215.9) | (138.3, 216.2) | (153.9, 325.8) | [341.7, 343.2] | 0.91 | 1.55 | 1.48 |
| 19 |  | 100 | 0.1 | 3 | (139.1, 43.9) | (140.8, 57.1) | (139.4, 57.1) | (136.1, 42.9) | [195.6, 196.8] | 1.73 | 6.00 | 5.85 |
| 20 |  |  |  | 6 | (141.3, 97.1) | (143.2, 112.1) | (141.9, 111.8) | (159.3, 97.0) | [197.0, 198.2] | 0.68 | 3.03 | 2.87 |
| 21 |  |  | 0.3 | 3 | (140.7, 58.4) | (143.7, 88.2) | (142.0, 88.0) | (161.2, 56.1) | [278.5, 279.9] | 0.54 | 11.73 | 11.62 |
| 22 |  |  |  | 6 | (141.8, 194.1) | (145.4, 177.8) | (144.0, 176.7) | (197.9, 191.0) | [284.3, 286.5] | 12.51 | 12.01 | 12.56 |
| 23 |  |  | 0.5 | 3 | (137.3, 80.7) | (141.4, 114.3) | (139.8, 114.0) | (126.7, 87.8) | [345.8, 347.8] | 0.32 | 7.72 | 7.40 |
| 24 |  |  |  | 6 | (137.7, 432.4) | (142.4, 231.7) | (140.9, 230.0) | (315.9, 318.4) | [415.7, 418.3] | 17.10 | 17.17 | 17.95 |
| 25 | 50 | 50 | 0.1 | 3 | (143.4, 41.2) | (146.4, 53.0) | (144.0, 54.9) | (166.3, 34.7) | [197.8, 199.3] | 1.00 | 3.41 | 4.30 |
| 26 |  |  |  | 6 | (146.8, 92.5) | (150.2, 108.6) | (147.8, 110.3) | (171.5, 80.1) | [201.9, 203.4] | 0.79 | 1.20 | 1.83 |
| 27 |  |  | 0.3 | 3 | (144.8, 49.5) | (149.2, 81.7) | (147.0, 84.6) | (152.7, 42.3) | [277.4, 279.5] | 1.75 | 11.24 | 12.58 |
| 28 |  |  |  | 6 | (146.2, 170.1) | (152.1, 175.6) | (149.8, 177.3) | (153.2, 338.6) | [282.6, 284.5] | 0.63 | 0.69 | 0.60 |
| 29 |  |  | 0.5 | 3 | (145.8, 67.3) | (146.5, 105.3) | (144.6, 109.5) | (145.4, 39.5) | [340.5, 342.2] | 6.97 | 8.67 | 10.15 |
| 30 |  |  |  | 6 | (139.1, 215.3) | (148.8, 231.5) | (146.5, 232.9) | (165.7, 210.0) | [352.8, 354.8] | 2.19 | 2.36 | 2.33 |
| 31 |  | 100 | 0.1 | 3 | (144.4, 49.0) | (148.4, 62.3) | (144.8, 63.1) | (157.5, 44.5) | [210.9, 213.9] | 1.02 | 3.72 | 3.82 |
| 32 |  |  |  | 6 | (148.0, 105.7) | (152.2, 123.0) | (148.7, 122.4) | (184.4, 103.9) | [213.8, 215.9] | 0.85 | 2.42 | 2.12 |
| 33 |  |  | 0.3 | 3 | (148.1, 64.2) | (154.0, 97.1) | (150.0, 97.0) | (159.7, 56.0) | [298.3, 300.9] | 1.22 | 10.02 | 10.13 |
| 34 |  |  |  | 6 | (149.7, 209.6) | (156.6, 197.4) | (152.9, 195.0) | (175.7, 219.7) | [305.6, 309.1] | 7.56 | 7.48 | 8.25 |
| 35 |  |  | 0.5 | 3 | (145.1, 87.4) | (152.7, 126.4) | (148.7, 126.0) | (149.2, 74.6) | [376.6, 379.3] | 0.85 | 5.93 | 6.14 |
| 36 |  |  |  | 6 | (145.6, 463.3) | (153.9, 258.9) | (150.3, 254.8) | (301.2, 302.6) | [435.4, 439.6] | 13.02 | 12.93 | 14.56 |

**Table 3.** Results of experiments for *A* = $800/order, *c* = $3/unit, and *β* = 0.2.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Test No. | *σ*2 | *p* | *δ* | *L* | Optimal policy for new model | Optimal policy for NWG model | Optimal policy for NWN model | Optimal policy from Lucas search | 95% confidence interval for the cost of solution from simulation | Mean absolute percentage error achieved by policy in column (1) | Mean absolute percentage error achieved by policy in column (2) | Mean absolute percentage error achieved by policy in column (3) |
| 1 | 5 | 50 | 0.1 | 3 | (85.9, 36.8) | (86.3, 43.3) | (86.1, 43.4) | (85.7, 38.8) | [193.4, 193.8] | 0.30 | 3.13 | 3.19 |
| 2 |  |  |  | 6 | (86.8, 85.2) | (87.3, 86.9) | (87.0, 87.0) | (88.2, 89.5) | [195.0, 195.5] | 0.33 | 0.22 | 0.38 |
| 3 |  |  | 0.3 | 3 | (68.2, 47.6) | (67.1, 55.3) | (66.9, 55.6) | (69.6, 49.5) | [301.6, 302.1] | 1.21 | 2.17 | 2.26 |
| 4 |  |  |  | 6 | (66.8, 166.2) | (67.5, 113.5) | (67.4, 113.7) | (105.5, 106.2) | [330.7, 331.4] | 8.50 | 8.70 | 8.88 |
| 5 |  |  | 0.5 | 3 | (64.7, 69.3) | (58.0, 64.0) | (57.9, 64.3) | (61.0, 78.1) | [382.8, 383.6] | 0.64 | 1.16 | 1.17 |
| 6 |  |  |  | 6 | (60.8, 32.5) | (58.3, 132.5) | (58.1, 132.7) | (97.2, 54.5) | [387.3, 388.3] | 8.20 | 8.00 | 8.13 |
| 7 |  | 100 | 0.1 | 3 | (86.1, 39.2) | (86.5, 45.9) | (86.2, 45.9) | (93.6, 39.4) | [198.7, 199.6] | 0.53 | 3.20 | 3.25 |
| 8 |  |  |  | 6 | (87.0, 59.3) | (87.4, 90.8) | (87.1, 90.7) | (98.7, 89.5) | [201.2, 201.7] | 1.54 | 1.21 | 1.64 |
| 9 |  |  | 0.3 | 3 | (67.0, 52.5) | (67.6, 59.2) | (67.3, 59.2) | (64.9, 54.4) | [312.3, 313.3] | 0.44 | 2.01 | 2.08 |
| 10 |  |  |  | 6 | (67.3, 178.5) | (68.1, 119.0) | (67.9, 118.8) | (151.7, 163.1) | [392.5, 393.6] | 48.27 | 47.41 | 47.67 |
| 11 |  |  | 0.5 | 3 | (87.8, 79.8) | (58.7, 68.8) | (58.5, 68.8) | (75.8, 76.1) | [412.7, 413.5] | 2.17 | 8.68 | 8.80 |
| 12 |  |  |  | 6 | (58.0, 397.8) | (59.0, 139.0) | (58.7, 138.8) | (130.1, 229.1) | [572.9, 574.9] | 22.91 | 24.65 | 24.71 |
| 13 | 20 | 50 | 0.1 | 3 | (89.5, 38.3) | (90.8, 44.7) | (89.9, 45.5) | (97.3, 39.3) | [204.5, 205.5] | 0.47 | 1.85 | 2.23 |
| 14 |  |  |  | 6 | (91.3, 87.6) | (92.7, 90.8) | (91.8, 91.4) | (91.3, 85.9) | [206.8, 208.0] | 0.58 | 0.74 | 0.89 |
| 15 |  |  | 0.3 | 3 | (79.2, 40.4) | (71.8, 56.0) | (71.1, 57.1) | (74.5, 44.7) | [313.2, 314.4] | 1.41 | 2.60 | 3.22 |
| 16 |  |  |  | 6 | (71.1, 160.6) | (72.7, 119.3) | (72.0, 120.1) | (86.7, 101.5) | [338.9, 341.2] | 4.74 | 6.02 | 6.21 |
| 17 |  |  | 0.5 | 3 | (64.7, 54.6) | (62.1, 65.3) | (62.1, 65.5) | (64.4, 57.6) | [391.9, 393.7] | 0.71 | 1.48 | 1.65 |
| 18 |  |  |  | 6 | (60.8, 78.6) | (62.9, 140.3) | (62.3, 141.2) | (76.9, 41.2) | [393.2, 395.7] | 5.44 | 6.04 | 6.28 |
| 19 |  | 100 | 0.1 | 3 | (89.7, 43.3) | (91.2, 50.6) | (89.9, 50.7) | (69.7, 44.3) | [219.0, 220.7] | 1.33 | 1.64 | 1.70 |
| 20 |  |  |  | 6 | (91.5, 96.2) | (93.0, 99.2) | (91.8, 99.0) | (91.4, 93.3) | [221.7, 224.6] | 0.81 | 0.98 | 0.85 |
| 21 |  |  | 0.3 | 3 | (71.1, 56.2) | (72.9, 64.8) | (71.7, 65.0) | (83.0, 54.3) | [341.0, 342.8] | 0.40 | 2.16 | 2.18 |
| 22 |  |  |  | 6 | (71.7, 188.2) | (73.8, 131.3) | (72.8, 130.9) | (153.9, 149.2) | [408.4, 411.2] | 34.39 | 36.43 | 38.41 |
| 23 |  |  | 0.5 | 3 | (67.8, 74.1) | (63.9, 75.5) | (62.9, 75.8) | (74.0, 74.4) | [439.8, 441.5] | 1.78 | 2.78 | 3.16 |
| 24 |  |  |  | 6 | (62.0, 162.4) | (64.2, 154.6) | (63.4, 153.9) | (134.1, 172.7) | [577.6, 581.4] | 19.21 | 19.35 | 20.50 |
| 25 | 50 | 50 | 0.1 | 3 | (98.9, 39.8) | (96.7, 45.3) | (94.5, 47.5) | (126.7, 35.3) | [222.1, 224.1] | 1.31 | 1.59 | 1.75 |
| 26 |  |  |  | 6 | (96.8, 89.8) | (99.8, 93.9) | (97.6, 96.0) | (102.3, 78.5) | [223.0, 225.5] | 0.86 | 1.12 | 1.01 |
| 27 |  |  | 0.3 | 3 | (89.2, 33.2) | (76.8, 56.9) | (76.6, 57.7) | (88.2, 27.9) | [334.2, 337.2] | 1.21 | 3.29 | 3.73 |
| 28 |  |  |  | 6 | (77.0, 144.5) | (79.3, 123.9) | (77.5, 126.9) | (104.1, 174.5) | [344.2, 347.9] | 4.78 | 4.37 | 5.59 |
| 29 |  |  | 0.5 | 3 | (64.8, 41.1) | (66.4, 67.1) | (66.6, 67.2) | (63.6, 47.0) | [406.0, 409.6] | 1.22 | 2.69 | 2.72 |
| 30 |  |  |  | 6 | (72.8, 9.6) | (68.9, 147.0) | (67.3, 150.5) | (92.0, 15.2) | [402.0, 405.3] | 3.41 | 4.26 | 5.02 |
| 31 |  | 100 | 0.1 | 3 | (94.0, 48.0) | (97.5, 55.5) | (94.3, 56.1) | (94.0, 48.4) | [234.9, 237.2] | 1.23 | 2.36 | 2.64 |
| 32 |  |  |  | 6 | (98.9, 104.0) | (100.4, 108.6) | (97.4, 108.6) | (115.5, 105.8) | [244.8, 246.8] | 1.46 | 2.05 | 2.45 |
| 33 |  |  | 0.3 | 3 | (75.9, 60.2) | (79.8, 70.6) | (76.9, 71.7) | (95.7, 58.6) | [376.3, 379.5] | 0.64 | 1.38 | 1.83 |
| 34 |  |  |  | 6 | (77.0, 198.9) | (81.0, 145.6) | (78.4, 145.0) | (155.5, 209.1) | [432.6, 436.3] | 24.29 | 23.70 | 25.17 |
| 35 |  |  | 0.5 | 3 | (87.9, 69.3) | (70.7, 82.4) | (68.0, 83.9) | (89.4, 72.0) | [482.9, 486.7] | 0.30 | 0.84 | 1.22 |
| 36 |  |  |  | 6 | (66.9, 180.2) | (70.9, 173.1) | (68.6, 171.7) | (134.7, 177.9) | [592.9, 598.1] | 14.04 | 15.17 | 17.24 |

**Table 4.** Results of experiments for *A* = $600/order, *c* = $12/unit, and *β* = 0.2.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Test No. | *σ*2 | *p* | *δ* | *L* | Optimal policy for new model | Optimal policy for NWG model | Optimal policy for NWN model | Optimal policy from Lucas search | 95% confidence interval for the cost of solution from simulation | Mean absolute percentage error achieved by policy in column (1) | Mean absolute percentage error achieved by policy in column (2) | Mean absolute percentage error achieved by policy in column (3) |
| 1 | 5 | 50 | 0.1 | 3 | (100.3, 36.1) | (100.8, 44.3) | (100.6, 44.5) | (102.1, 36.4) | [224.2, 224.7] | 0.48 | 4.68 | 4.91 |
| 2 |  |  |  | 6 | (101.3, 84.1) | (101.7, 89.4) | (101.5, 89.6) | (103.5, 81.9) | [224.3, 224.8] | 0.92 | 1.86 | 2.13 |
| 3 |  |  | 0.3 | 3 | (82.3, 53.2) | (78.8, 58.1) | (78.8, 58.1) | (79.7, 44.7) | [350.2, 350.9] | 3.67 | 5.72 | 5.71 |
| 4 |  |  |  | 6 | (94.2, 91.5) | (79.4, 119.6) | (79.3, 119.8) | (97.8, 87.8) | [363.4, 364.1] | 2.65 | 2.78 | 2.71 |
| 5 |  |  | 0.5 | 3 | (70.9, 10.9) | (68.2, 68.6) | (68.2, 68.6) | (74.4, 11.5) | [430.7, 431.7] | 0.59 | 5.14 | 5.10 |
| 6 |  |  |  | 6 | (76.2, 89.1) | (68.9, 141.2) | (68.8, 141.4) | (89.2, 99.6) | [422.8, 423.8] | 1.82 | 2.00 | 1.90 |
| 7 |  | 100 | 0.1 | 3 | (100.5, 38.7) | (100.9, 47.2) | (100.6, 47.2) | (96.2, 40.0) | [230.1, 230.9] | 0.73 | 4.72 | 4.67 |
| 8 |  |  |  | 6 | (101.5, 88.4) | (102.0, 93.6) | (101.7, 93.5) | (97.7, 90.3) | [231.0, 232.0] | 0.52 | 1.78 | 1.49 |
| 9 |  |  | 0.3 | 3 | (78.9, 51.6) | (79.6, 62.3) | (79.3, 62.3) | (82.8, 54.3) | [365.3, 366.2] | 0.37 | 4.32 | 4.29 |
| 10 |  |  |  | 6 | (79.2, 176.1) | (80.1, 125.6) | (79.9, 125.5) | (162.6, 163.9) | [426.2, 427.3] | 28.55 | 28.64 | 28.51 |
| 11 |  |  | 0.5 | 3 | (71.4, 72.5) | (69.4, 73.2) | (69.2, 73.3) | (77.8, 72.8) | [472.9, 474.0] | 0.51 | 0.97 | 1.14 |
| 12 |  |  |  | 6 | (68.5, 390.0) | (69.7, 148.5) | (69.5, 148.3) | (129.4, 218.5) | [605.3, 608.0] | 15.20 | 14.44 | 14.61 |
| 13 | 20 | 50 | 0.1 | 3 | (104.1, 37.0) | (105.4, 45.0) | (104.6, 45.8) | (104.6, 39.1) | [235.1, 236.2] | 0.53 | 3.25 | 3.67 |
| 14 |  |  |  | 6 | (106.0, 85.4) | (107.4, 92.1) | (106.6, 92.9) | (109.1, 80.1) | [236.1, 237.1] | 1.67 | 3.24 | 3.51 |
| 15 |  |  | 0.3 | 3 | (96.3, 35.7) | (82.8, 59.1) | (82.9, 59.1) | (69.2, 40.3) | [363.5, 365.0] | 4.53 | 5.48 | 5.44 |
| 16 |  |  |  | 6 | (83.2, 149.7) | (84.8, 123.5) | (84.2, 124.6) | (102.8, 131.6) | [368.0, 369.5] | 1.92 | 2.10 | 2.12 |
| 17 |  |  | 0.5 | 3 | (70.9, 7.5) | (71.8, 70.1) | (71.9, 70.1) | (69.5, 3.8) | [436.4, 439.2] | 1.51 | 8.62 | 8.54 |
| 18 |  |  |  | 6 | (66.2, 110.6) | (73.7, 146.8) | (73.2, 148.0) | (89.0, 42.4) | [424.7, 426.5] | 2.05 | 2.41 | 2.21 |
| 19 |  | 100 | 0.1 | 3 | (104.4, 42.3) | (105.8, 51.2) | (104.6, 51.5) | (124.3, 39.4) | [248.9, 250.9] | 0.55 | 1.79 | 1.77 |
| 20 |  |  |  | 6 | (106.3, 94.5) | (107.9, 101.2) | (106.7, 101.2) | (112.6, 90.1) | [250.6, 252.7] | 0.59 | 1.25 | 1.27 |
| 21 |  |  | 0.3 | 3 | (83.2, 54.3) | (85.1, 67.0) | (84.0, 67.5) | (84.5, 52.0) | [390.9, 393.1] | 0.60 | 4.30 | 4.72 |
| 22 |  |  |  | 6 | (83.9, 183.2) | (86.2, 137.0) | (85.2, 136.8) | (146.3, 169.3) | [439.4, 442.3] | 22.49 | 21.34 | 22.12 |
| 23 |  |  | 0.5 | 3 | (79.4, 71.6) | (74.9, 78.9) | (74.0, 79.5) | (76.8, 73.0) | [503.9, 506.2] | 1.01 | 1.40 | 1.18 |
| 24 |  |  |  | 6 | (72.8, 394.4) | (75.4, 163.0) | (74.5, 162.7) | (141.4, 142.7) | [611.9, 615.0] | 10.30 | 9.65 | 9.96 |
| 25 | 50 | 50 | 0.1 | 3 | (108.7, 37.7) | (111.4, 44.5) | (109.5, 46.9) | (97.5, 42.4) | [249.0, 251.0] | 0.66 | 1.82 | 2.58 |
| 26 |  |  |  | 6 | (111.8, 86.1) | (114.8, 93.8) | (112.7, 96.2) | (112.4, 84.6) | [250.0, 252.0] | 0.58 | 1.89 | 1.81 |
| 27 |  |  | 0.3 | 3 | (96.3, 38.7) | (87.3, 60.1) | (87.5, 60.2) | (91.3, 33.8) | [372.3, 375.3] | 3.05 | 7.01 | 7.11 |
| 28 |  |  |  | 6 | (94.2, 110.9) | (91.8, 125.4) | (90.3, 129.2) | (115.7, 79.7) | [377.7, 381.0] | 1.79 | 1.73 | 1.76 |
| 29 |  |  | 0.5 | 3 | (71.0, 12.2) | (75.9, 71.6) | (76.0, 71.7) | (78.9, 9.8) | [444.1, 447.3] | 1.15 | 11.14 | 10.94 |
| 30 |  |  |  | 6 | (86.2, 120.1) | (80.0, 149.7) | (78.7, 154.4) | (99.6, 9.5) | [427.2, 431.6] | 2.76 | 2.88 | 2.91 |
| 31 |  | 100 | 0.1 | 3 | (108.9, 46.5) | (112.4, 55.4) | (109.3, 56.3) | (129.7, 47.4) | [267.0, 269.6] | 0.89 | 1.97 | 2.50 |
| 32 |  |  |  | 6 | (112.0, 101.4) | (115.6, 109.6) | (112.6, 110.0) | (117.0, 99.1) | [274.7, 276.9] | 1.61 | 2.77 | 2.29 |
| 33 |  |  | 0.3 | 3 | (88.4, 57.1) | (92.3, 71.7) | (89.6, 73.4) | (92.0, 57.4) | [420.1, 424.0] | 1.00 | 3.89 | 4.40 |
| 34 |  |  |  | 6 | (89.5, 190.5) | (93.9, 150.0) | (91.4, 150.1) | (154.3, 253.0) | [467.6, 471.1] | 13.60 | 13.54 | 15.30 |
| 35 |  |  | 0.5 | 3 | (84.5, 81.1) | (81.9, 84.3) | (79.5, 86.6) | (83.7, 81.5) | [544.4, 548.8] | 0.42 | 0.87 | 0.79 |
| 36 |  |  |  | 6 | (78.0, 392.3) | (82.5, 180.0) | (80.3, 179.7) | (139.1, 309.7) | [628.8, 635.3] | 5.23 | 7.29 | 7.65 |

**Table 5.** Results of experiments for *A* = $800/order, *c* = $12/unit, and *β* = 0.2.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Test No. | *σ*2 | *p* | *δ* | *L* | Optimal policy for new model | Optimal policy for NWG model | Optimal policy for NWN model | Optimal policy from Lucas search | 95% confidence interval for the cost of solution from simulation | Mean absolute percentage error achieved by policy in column (1) | Mean absolute percentage error achieved by policy in column (2) | Mean absolute percentage error achieved by policy in column (3) |
| 1 | 5 | 50 | 0.1 | 3 | (133.9, 37.8) | (134.6, 48.9) | (134.3, 49.1) | (140.8, 36.5) | [175.1, 175.6] | 0.23 | 6.51 | 6.67 |
| 2 |  |  |  | 6 | (134.9, 86.9) | (135.7, 98.4) | (135.5, 98.6) | (128.8, 86.3) | [176.5, 177.1] | 0.41 | 4.18 | 4.28 |
| 3 |  |  | 0.3 | 3 | (133.2, 51.2) | (134.1, 76.1) | (133.9, 76.3) | (152.1, 49.4) | [250.1, 250.6] | 0.33 | 14.12 | 14.35 |
| 4 |  |  |  | 6 | (133.6, 175.0) | (135.1, 155.3) | (134.9, 155.4) | (165.2, 170.6) | [253.2, 254.1] | 5.98 | 6.16 | 6.48 |
| 5 |  |  | 0.5 | 3 | (129.3, 71.8) | (130.6, 98.4) | (130.4, 98.6) | (131.4, 66.1) | [312.6, 313.5] | 0.30 | 10.31 | 10.43 |
| 6 |  |  |  | 6 | (129.4, 350.0) | (131.3, 201.6) | (131.1, 201.7) | (166.7, 293.2) | [358.1, 359.7] | 3.15 | 3.84 | 3.73 |
| 7 |  | 100 | 0.1 | 3 | (134.4, 39.7) | (135.0, 51.8) | (134.7, 51.8) | (120.3, 40.1) | [178.6, 179.2] | 0.36 | 7.00 | 6.97 |
| 8 |  |  |  | 6 | (135.5, 90.2) | (136.2, 102.6) | (135.9, 102.5) | (151.9, 90.0) | [180.9, 181.6] | 0.54 | 4.78 | 4.70 |
| 9 |  |  | 0.3 | 3 | (134.4, 54.1) | (135.5, 80.3) | (135.0, 80.3) | (139.3, 56.8) | [255.4, 256.2] | 0.68 | 14.78 | 14.78 |
| 10 |  |  |  | 6 | (134.9, 182.8) | (136.4, 161.3) | (136.0, 161.1) | (184.5, 175.6) | [262.5, 263.7] | 22.94 | 23.49 | 24.17 |
| 11 |  |  | 0.5 | 3 | (130.8, 76.5) | (132.3, 103.8) | (131.9, 103.8) | (129.2, 77.0) | [321.4, 322.8] | 0.47 | 10.16 | 10.12 |
| 12 |  |  |  | 6 | (131.0, 413.1) | (132.9, 209.2) | (132.5, 208.8) | (330.2, 955.3) | [403.6, 405.5] | 33.98 | 33.50 | 33.42 |
| 13 | 20 | 50 | 0.1 | 3 | (137.9, 40.6) | (139.7, 50.4) | (138.8, 51.2) | (144.6, 42.5) | [182.7, 183.5] | 0.55 | 4.04 | 4.67 |
| 14 |  |  |  | 6 | (139.8, 91.6) | (142.1, 102.6) | (141.1, 103.3) | (138.6, 86.3) | [184.8, 186.4] | 2.28 | 5.34 | 5.53 |
| 15 |  |  | 0.3 | 3 | (138.5, 53.7) | (140.8, 78.4) | (139.9, 79.5) | (130.5, 46.1) | [256.8, 258.8] | 1.31 | 12.88 | 13.64 |
| 16 |  |  |  | 6 | (139.3, 181.8) | (142.7, 164.2) | (141.8, 164.8) | (166.7, 226.9) | [260.5, 261.9] | 2.22 | 2.51 | 2.28 |
| 17 |  |  | 0.5 | 3 | (134.7, 74.0) | (137.7, 101.5) | (136.9, 102.9) | (130.4, 59.0) | [319.8, 321.4] | 1.15 | 9.88 | 10.41 |
| 18 |  |  |  | 6 | (135.0, 402.0) | (139.2, 214.8) | (138.3, 215.3) | (218.5, 268.3) | [353.3, 356.7] | 4.89 | 4.70 | 4.66 |
| 19 |  | 100 | 0.1 | 3 | (138.9, 44.5) | (140.8, 56.7) | (139.4, 56.7) | (148.5, 42.1) | [188.9, 190.2] | 0.98 | 5.64 | 5.64 |
| 20 |  |  |  | 6 | (141.0, 98.1) | (143.2, 111.6) | (141.9, 111.4) | (167.6, 93.4) | [194.2, 196.0] | 0.76 | 3.45 | 3.32 |
| 21 |  |  | 0.3 | 3 | (140.8, 59.7) | (143.7, 87.7) | (142.0, 87.6) | (145.3, 58.6) | [268.0, 270.9] | 0.80 | 13.49 | 13.57 |
| 22 |  |  |  | 6 | (151.8, 176.7) | (145.4, 177.3) | (144.0, 176.4) | (175.9, 173.6) | [276.7, 280.3] | 11.25 | 13.12 | 13.00 |
| 23 |  |  | 0.5 | 3 | (137.6, 83.7) | (141.4, 113.8) | (139.8, 113.6) | (135.3, 71.8) | [339.6, 342.8] | 1.17 | 10.02 | 9.96 |
| 24 |  |  |  | 6 | (137.9, 446.2) | (142.4, 232.2) | (140.9, 229.6) | (300.8, 599.9) | [406.7, 411.2] | 29.16 | 28.63 | 27.37 |
| 25 | 50 | 50 | 0.1 | 3 | (142.5, 43.8) | (146.4, 51.1) | (144.0, 53.5) | (136.8, 43.7) | [189.7, 191.2] | 0.81 | 3.31 | 4.10 |
| 26 |  |  |  | 6 | (145.6, 101.8) | (150.2, 106.4) | (147.8, 108.5) | (121.7, 93.1) | [194.9, 197.1] | 2.07 | 4.55 | 5.37 |
| 27 |  |  | 0.3 | 3 | (144.6, 56.7) | (149.2, 79.4) | (147.0, 82.9) | (158.0, 43.9) | [265.4, 268.0] | 2.67 | 12.26 | 13.86 |
| 28 |  |  |  | 6 | (151.9, 160.2) | (152.1, 173.6) | (149.8, 175.7) | (154.5, 148.0) | [266.5, 270.0] | 0.96 | 1.08 | 1.35 |
| 29 |  |  | 0.5 | 3 | (141.0, 76.8) | (146.5, 102.8) | (144.6, 107.6) | (120.2, 51.8) | [324.5, 328.3] | 2.73 | 10.91 | 12.41 |
| 30 |  |  |  | 6 | (161.4, 414.5) | (148.8, 229.6) | (146.5, 231.5) | (235.8, 788.3) | [351.5, 355.2] | 2.14 | 2.46 | 2.36 |
| 31 |  | 100 | 0.1 | 3 | (144.0, 50.0) | (148.4, 62.1) | (144.8, 62.5) | (135.2, 47.6) | [199.9, 203.0] | 1.17 | 3.66 | 3.79 |
| 32 |  |  |  | 6 | (147.5, 107.4) | (152.2, 122.1) | (148.7, 121.7) | (139.4, 105.6) | [204.6, 208.0] | 3.65 | 7.61 | 7.19 |
| 33 |  |  | 0.3 | 3 | (148.0, 66.5) | (154.0, 96.2) | (150.0, 96.4) | (158.0, 56.9) | [284.9, 288.3] | 1.25 | 12.16 | 12.36 |
| 34 |  |  |  | 6 | (159.6, 202.1) | (156.6, 196.6) | (152.9, 194.4) | (193.8, 156.8) | [304.7, 309.3] | 2.34 | 2.60 | 3.49 |
| 35 |  |  | 0.5 | 3 | (145.4, 92.5) | (152.7, 125.5) | (148.7, 125.4) | (139.3, 89.8) | [354.5, 359.5] | 2.03 | 10.45 | 10.48 |
| 36 |  |  |  | 6 | (145.9, 486.9) | (153.9, 258.2) | (150.3, 254.3) | (246.5, 309.3) | [420.4, 429.5] | 19.95 | 17.43 | 17.87 |

**Table 6.** Results of experiments for *A* = $800/order, *c* = $3/unit, and *β* = 0.8.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Test No. | *σ*2 | *p* | *δ* | *L* | Optimal policy for new model | Optimal policy for NWG model | Optimal policy for NWN model | Optimal policy from Lucas search | 95% confidence interval for the cost of solution from simulation | Mean absolute percentage error achieved by policy in column (1) | Mean absolute percentage error achieved by policy in column (2) | Mean absolute percentage error achieved by policy in column (3) |
| 1 | 5 | 50 | 0.1 | 3 | (99.9, 37.1) | (100.8, 43.6) | (100.6, 43.9) | (96.9, 37.2) | [220.5, 221.3] | 0.55 | 4.33 | 4.50 |
| 2 |  |  |  | 6 | (100.7, 85.8) | (101.7, 88.6) | (101.5, 88.8) | (102.1, 81.8) | [222.5, 223.4] | 0.27 | 0.64 | 0.64 |
| 3 |  |  | 0.3 | 3 | (78.0, 49.5) | (78.8, 57.2) | (78.8, 57.2) | (79.3, 48.4) | [347.5, 348.7] | 1.03 | 4.84 | 4.95 |
| 4 |  |  |  | 6 | (78.2, 170.6) | (79.4, 118.7) | (79.3, 118.9) | (117.5, 180.3) | [380.7, 382.0] | 3.56 | 4.24 | 4.11 |
| 5 |  |  | 0.5 | 3 | (67.5, 63.9) | (68.2, 67.7) | (68.2, 67.7) | (65.5, 57.9) | [444.6, 446.7] | 0.77 | 0.84 | 0.97 |
| 6 |  |  |  | 6 | (67.6, 220.1) | (68.9, 140.3) | (68.8, 140.6) | (94.2, 184.4) | [472.1, 473.9] | 0.44 | 0.46 | 0.47 |
| 7 |  | 100 | 0.1 | 3 | (100.4, 39.1) | (100.9, 46.9) | (100.6, 47.0) | (108.6, 38.4) | [225.8, 226.7] | 0.72 | 5.11 | 5.17 |
| 8 |  |  |  | 6 | (101.3, 89.1) | (102.0, 93.2) | (101.7, 93.2) | (97.8, 88.0) | [228.7, 230.0] | 0.49 | 0.97 | 1.18 |
| 9 |  |  | 0.3 | 3 | (78.7, 52.7) | (79.6, 61.9) | (79.3, 62.0) | (72.7, 54.1) | [361.6, 363.4] | 0.35 | 4.60 | 4.74 |
| 10 |  |  |  | 6 | (79.1, 179.0) | (80.1, 125.2) | (79.9, 125.2) | (162.8, 221.5) | [421.0, 423.2] | 34.91 | 35.09 | 35.18 |
| 11 |  |  | 0.5 | 3 | (68.4, 73.9) | (69.4, 72.8) | (69.2, 72.9) | (76.4, 109.9) | [469.7, 471.5] | 0.48 | 0.62 | 0.84 |
| 12 |  |  |  | 6 | (68.5, 401.4) | (69.7, 148.1) | (69.5, 148.0) | (147.6, 638.6) | [660.4, 663.7] | 12.28 | 11.99 | 11.75 |
| 13 | 20 | 50 | 0.1 | 3 | (103.2, 39.1) | (105.4, 43.5) | (104.6, 44.5) | (92.9, 35.4) | [229.6, 231.6] | 0.74 | 2.48 | 3.00 |
| 14 |  |  |  | 6 | (104.9, 89.1) | (107.4, 90.4) | (106.6, 91.4) | (100.7, 83.6) | [232.8, 234.9] | 0.83 | 1.17 | 1.61 |
| 15 |  |  | 0.3 | 3 | (81.4, 50.3) | (82.8, 57.3) | (82.9, 57.3) | (87.4, 43.7) | [357.3, 360.0] | 1.15 | 4.48 | 4.60 |
| 16 |  |  |  | 6 | (82.0, 119.5) | (84.8, 128.5) | (84.2, 129.2) | (118.7, 121.8) | [377.8, 381.2] | 2.62 | 2.98 | 3.25 |
| 17 |  |  | 0.5 | 3 | (70.6, 99.6) | (71.8, 68.2) | (71.8, 68.2) | (56.1, 136.6) | [448.6, 453.3] | 2.41 | 2.88 | 2.77 |
| 18 |  |  |  | 6 | (70.8, 373.8) | (73.7, 144.7) | (73.2, 146.3) | (83.0, 446.8) | [464.8, 468.4] | 1.36 | 1.83 | 1.95 |
| 19 |  | 100 | 0.1 | 3 | (104.0, 43.2) | (105.8, 50.6) | (104.6, 51.0) | (97.3, 45.3) | [239.8, 241.5] | 0.96 | 2.07 | 2.82 |
| 20 |  |  |  | 6 | (105.8, 95.9) | (107.9, 100.5) | (106.7, 100.6) | (89.6, 189.8) | [250.3, 253.3] | 2.13 | 1.14 | 1.20 |
| 21 |  |  | 0.3 | 3 | (82.9, 56.7) | (85.1, 66.2) | (84.0, 66.8) | (96.9, 59.3) | [388.4, 390.6] | 0.91 | 3.49 | 3.88 |
| 22 |  |  |  | 6 | (83.6, 189.7) | (86.2, 136.2) | (85.2, 136.2) | (149.1, 229.3) | [431.4, 435.0] | 27.24 | 25.22 | 25.81 |
| 23 |  |  | 0.5 | 3 | (72.4, 89.3) | (74.9, 78.0) | (74.0, 78.8) | (71.4, 98.0) | [489.3, 494.9] | 1.64 | 2.25 | 1.98 |
| 24 |  |  |  | 6 | (72.6, 421.4) | (75.3, 162.3) | (74.5, 162.1) | (135.0, 610.7) | [654.2, 662.7] | 10.12 | 11.09 | 11.60 |
| 25 | 50 | 50 | 0.1 | 3 | (107.1, 41.4) | (111.4, 41.8) | (109.5, 44.8) | (94.4, 35.0) | [237.4, 240.4] | 2.09 | 2.46 | 2.80 |
| 26 |  |  |  | 6 | (109.7, 92.9) | (114.8, 90.7) | (112.7, 93.6) | (91.3, 85.4) | [245.6, 249.0] | 1.22 | 1.42 | 1.12 |
| 27 |  |  | 0.3 | 3 | (85.4, 51.0) | (87.3, 57.2) | (87.5, 57.2) | (87.4, 39.3) | [359.6, 363.1] | 4.58 | 7.82 | 7.88 |
| 28 |  |  |  | 6 | (86.3, 125.3) | (91.8, 121.6) | (90.3, 130.1) | (103.4, 125.9) | [373.6, 379.0] | 2.94 | 3.09 | 3.34 |
| 29 |  |  | 0.5 | 3 | (74.3, 97.9) | (75.9, 68.7) | (76.0, 68.8) | (53.8, 118.5) | [452.8, 460.1] | 4.15 | 5.49 | 5.46 |
| 30 |  |  |  | 6 | (75.5, 366.8) | (80.0, 145.8) | (78.7, 151.4) | (112.4, 557.0) | [470.7, 477.7] | 1.34 | 1.94 | 1.46 |
| 31 |  | 100 | 0.1 | 3 | (108.3, 47.9) | (112.4, 54.3) | (109.3, 55.6) | (121.4, 43.2) | [256.4, 259.1] | 1.49 | 3.45 | 3.97 |
| 32 |  |  |  | 6 | (111.2, 103.8) | (115.6, 108.4) | (112.6, 109.1) | (98.9, 100.0) | [266.6, 271.1] | 2.74 | 4.35 | 4.26 |
| 33 |  |  | 0.3 | 3 | (87.8, 61.4) | (92.3, 70.2) | (89.6, 72.4) | (73.3, 51.6) | [401.5, 407.1] | 4.31 | 7.69 | 8.69 |
| 34 |  |  |  | 6 | (88.8, 202.2) | (93.9, 148.7) | (91.4, 149.1) | (125.5, 150.1) | [453.2, 459.9] | 18.59 | 15.39 | 18.05 |
| 35 |  |  | 0.5 | 3 | (78.2, 59.1) | (81.9, 82.6) | (79.5, 85.5) | (79.0, 57.5) | [516.9, 524.4] | 2.33 | 3.28 | 2.75 |
| 36 |  |  |  | 6 | (77.3, 445.0) | (82.5, 178.8) | (80.3, 178.7) | (134.5, 152.1) | [644.7, 654.1] | 11.38 | 11.10 | 10.80 |

**Table 7.** Results of experiments for *A* = $800/order, *c* = $12/unit, and *β* = 0.8.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | ***A* = $600/order** | | | ***A* = $800/order** | | |
| Test No. | *σ*2 | *p* | *δ* | *L* | Difference between *r* from Lucas search and *r* for new model | Difference between *r* from Lucas search and *r* for NWG model | Difference between *r* from Lucas search and *r* for NWN model | Difference between *r* from Lucas search and *r* for new model | Difference between *r* from Lucas search and *r* for NWG model | Difference between *r* from Lucas search and *r* for NWN model |
| 1 | 5 | 50 | 0.1 | 3 | 0.9 | -9.2 | -9.3 | 2.2 | -10.0 | -10.2 |
| 2 |  |  |  | 6 | 2.0 | -6.5 | -6.6 | 1.1 | -12.1 | -12.2 |
| 3 |  |  | 0.3 | 3 | -0.4 | -21.3 | -21.5 | 1.7 | -25.6 | -25.7 |
| 4 |  |  |  | 6 | -6.1 | 22.5 | 22.5 | 20.5 | 35.0 | 34.9 |
| 5 |  |  | 0.5 | 3 | 4.1 | -16.7 | -16.9 | 17.9 | -35.0 | -35.3 |
| 6 |  |  |  | 6 | -0.6 | 1.8 | 1.9 | 6.5 | 30.3 | 30.3 |
| 7 |  | 100 | 0.1 | 3 | 1.2 | -9.1 | -9.0 | 1.9 | -10.7 | -10.6 |
| 8 |  |  |  | 6 | -0.6 | -9.0 | -8.9 | -0.1 | -13.3 | -13.2 |
| 9 |  |  | 0.3 | 3 | ≈ 0 | -20.8 | -20.6 | 1.3 | -25.8 | -25.7 |
| 10 |  |  |  | 6 | 128.2 | 161.4 | 161.8 | 37.5 | 57.0 | 57.3 |
| 11 |  |  | 0.5 | 3 | 5.6 | -13.3 | -13.1 | 1.9 | -27.2 | -27.0 |
| 12 |  |  |  | 6 | -111.3 | 110.7 | 111.2 | -42.2 | 155.1 | 155.5 |
| 13 | 20 | 50 | 0.1 | 3 | 3.3 | -7.0 | -7.5 | -1.4 | -13.7 | -14.4 |
| 14 |  |  |  | 6 | -3.8 | -13.9 | -14.3 | -0.4 | -15.2 | -15.7 |
| 15 |  |  | 0.3 | 3 | 1.9 | -21.5 | -22.2 | -5.0 | -34.8 | -35.7 |
| 16 |  |  |  | 6 | 24.6 | 46.8 | 46.7 | 27.9 | 33.6 | 33.2 |
| 17 |  |  | 0.5 | 3 | -8.1 | -34.8 | -35.7 | -15.3 | -32.9 | -34.1 |
| 18 |  |  |  | 6 | -9.5 | -14.0 | -13.9 | -29.8 | 110.0 | 109.6 |
| 19 |  | 100 | 0.1 | 3 | -0.7 | -11.8 | -11.6 | -0.9 | -14.2 | -14.2 |
| 20 |  |  |  | 6 | -1.6 | -11.9 | -11.3 | -0.1 | -15.2 | -14.8 |
| 21 |  |  | 0.3 | 3 | 0.5 | -22.8 | -22.4 | -2.3 | -32.1 | -32.0 |
| 22 |  |  |  | 6 | -1.5 | 30.1 | 31.5 | -3.1 | 13.2 | 14.3 |
| 23 |  |  | 0.5 | 3 | 6.5 | -16.3 | -15.6 | 7.1 | -26.6 | -26.3 |
| 24 |  |  |  | 6 | -20.1 | 211.9 | 214.0 | -114.0 | 86.7 | 88.4 |
| 25 | 50 | 50 | 0.1 | 3 | -1.3 | -11.1 | -12.8 | -6.5 | -18.3 | -20.3 |
| 26 |  |  |  | 6 | 7.1 | -4.4 | -5.7 | -12.3 | -28.5 | -30.1 |
| 27 |  |  | 0.3 | 3 | -6.2 | -31.6 | -33.9 | -7.3 | -39.4 | -42.4 |
| 28 |  |  |  | 6 | 67.6 | 81.6 | 80.8 | 168.5 | 162.9 | 161.3 |
| 29 |  |  | 0.5 | 3 | -13.1 | -28.3 | -31.2 | -27.8 | -65.8 | -70.0 |
| 30 |  |  |  | 6 | 200.1 | 352.3 | 352.2 | -5.3 | -21.4 | -22.9 |
| 31 |  | 100 | 0.1 | 3 | 2.2 | -9.7 | -9.4 | -4.5 | -18.3 | -18.5 |
| 32 |  |  |  | 6 | 9.8 | -2.7 | -1.7 | -1.8 | -19.0 | -18.4 |
| 33 |  |  | 0.3 | 3 | -0.6 | -26.8 | -26.0 | -8.2 | -41.1 | -41.0 |
| 34 |  |  |  | 6 | 46.7 | 75.8 | 79.1 | 10.1 | 22.3 | 24.7 |
| 35 |  |  | 0.5 | 3 | 2.3 | -25.0 | -23.6 | -12.8 | -51.8 | -51.4 |
| 36 |  |  |  | 6 | 215.3 | 459.0 | 464.1 | -160.7 | 43.7 | 47.8 |

**Table 8.** Difference between reorder point from Lucas search and reorder points for heuristic models for *c* = $3/unit and *β* = 0.2.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | ***A* = $600/order** | | | ***A* = $800/order** | | |
| Test No. | *σ*2 | *p* | *δ* | *L* | Difference between *r* from Lucas search and *r* for new model | Difference between *r* from Lucas search and *r* for NWG model | Difference between *r* from Lucas search and *r* for NWN model | Difference between *r* from Lucas search and *r* for new model | Difference between *r* from Lucas search and *r* for NWG model | Difference between *r* from Lucas search and *r* for NWN model |
| 1 | 5 | 50 | 0.1 | 3 | 2.0 | -4.5 | -4.7 | 0.3 | -7.9 | -8.1 |
| 2 |  |  |  | 6 | 4.3 | 2.6 | 2.5 | -2.1 | -7.4 | -7.6 |
| 3 |  |  | 0.3 | 3 | 1.9 | -5.8 | -6.0 | -8.5 | -13.4 | -13.4 |
| 4 |  |  |  | 6 | -60.0 | -7.3 | -7.5 | -3.7 | -31.8 | -32.0 |
| 5 |  |  | 0.5 | 3 | 8.8 | 14.1 | 13.8 | 0.6 | -57.1 | -57.1 |
| 6 |  |  |  | 6 | 22.0 | -78.0 | -78.2 | 10.5 | -41.6 | -41.9 |
| 7 |  | 100 | 0.1 | 3 | 0.2 | -6.6 | -6.5 | 1.3 | -7.2 | -7.2 |
| 8 |  |  |  | 6 | 0.3 | -1.2 | -1.2 | 1.8 | -3.3 | -3.3 |
| 9 |  |  | 0.3 | 3 | 1.9 | -4.8 | -4.8 | 2.7 | -8.0 | -8.1 |
| 10 |  |  |  | 6 | -15.4 | 44.2 | 44.3 | -12.1 | 38.4 | 38.4 |
| 11 |  |  | 0.5 | 3 | -3.7 | 7.2 | 7.1 | 0.3 | -0.4 | -0.5 |
| 12 |  |  |  | 6 | -168.7 | 90.1 | 90.3 | -171.4 | 70.1 | 70.2 |
| 13 | 20 | 50 | 0.1 | 3 | 1.0 | -5.4 | -6.2 | 2.1 | -5.9 | -6.7 |
| 14 |  |  |  | 6 | -1.7 | -4.8 | -5.5 | -4.5 | -11.3 | -12.1 |
| 15 |  |  | 0.3 | 3 | 4.3 | -11.3 | -12.4 | 4.6 | -18.8 | -18.8 |
| 16 |  |  |  | 6 | -59.1 | -17.8 | -18.6 | -18.1 | 8.1 | 7.0 |
| 17 |  |  | 0.5 | 3 | 3.0 | -7.8 | -7.9 | -3.7 | -66.3 | -66.3 |
| 18 |  |  |  | 6 | -37.4 | -99.1 | -100.0 | -68.2 | -104.3 | -105.6 |
| 19 |  | 100 | 0.1 | 3 | 1.0 | -6.3 | -6.4 | -3.0 | -11.8 | -12.1 |
| 20 |  |  |  | 6 | -2.9 | -5.9 | -5.7 | -4.4 | -11.1 | -11.1 |
| 21 |  |  | 0.3 | 3 | -1.8 | -10.4 | -10.7 | -2.1 | -14.8 | -15.3 |
| 22 |  |  |  | 6 | -39.0 | 17.9 | 18.3 | -13.8 | 32.4 | 32.5 |
| 23 |  |  | 0.5 | 3 | 0.3 | -1.1 | -1.4 | 1.4 | -5.9 | -6.5 |
| 24 |  |  |  | 6 | 10.3 | 18.1 | 18.8 | -251.7 | -20.3 | -20.0 |
| 25 | 50 | 50 | 0.1 | 3 | -4.5 | -10.0 | -12.2 | 4.7 | -2.1 | -4.5 |
| 26 |  |  |  | 6 | -11.4 | -15.4 | -17.6 | -1.4 | -9.2 | -11.6 |
| 27 |  |  | 0.3 | 3 | -5.3 | -28.9 | -29.8 | -4.9 | -26.3 | -26.4 |
| 28 |  |  |  | 6 | 30.1 | 50.6 | 47.6 | -31.2 | -45.7 | -49.4 |
| 29 |  |  | 0.5 | 3 | 5.9 | -20.1 | -20.2 | -2.4 | -61.8 | -61.9 |
| 30 |  |  |  | 6 | 5.6 | -131.8 | -135.2 | -110.6 | -140.2 | -145.0 |
| 31 |  | 100 | 0.1 | 3 | 0.3 | -7.1 | -7.7 | 0.9 | -8.0 | -8.9 |
| 32 |  |  |  | 6 | 1.9 | -2.8 | -2.8 | -2.3 | -10.6 | -11.0 |
| 33 |  |  | 0.3 | 3 | -1.6 | -11.9 | -13.1 | 0.3 | -14.3 | -16.0 |
| 34 |  |  |  | 6 | 10.2 | 63.5 | 64.1 | 62.5 | 103.0 | 102.9 |
| 35 |  |  | 0.5 | 3 | 2.7 | -10.5 | -12.0 | 0.4 | -2.8 | -5.1 |
| 36 |  |  |  | 6 | -2.3 | 4.8 | 6.2 | -82.6 | 129.7 | 130.1 |

**Table 9.** Difference between reorder point from Lucas search and reorder points for heuristic models for *c* = $12/unit and *β* = 0.2.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | ***c* = $3/unit** | | | ***c* = $12/unit** | | |
| Test No. | *σ*2 | *p* | *δ* | *L* | Difference between *r* from Lucas search and *r* for new model | Difference between *r* from Lucas search and *r* for NWG model | Difference between *r* from Lucas search and *r* for NWN model | Difference between *r* from Lucas search and *r* for new model | Difference between *r* from Lucas search and *r* for NWG model | Difference between *r* from Lucas search and *r* for NWN model |
| 1 | 5 | 50 | 0.1 | 3 | 1.3 | 12.4 | 12.6 | -0.1 | 6.5 | 6.7 |
| 2 |  |  |  | 6 | 0.7 | 12.2 | 12.3 | 3.9 | 6.8 | 7.0 |
| 3 |  |  | 0.3 | 3 | 1.8 | 26.7 | 26.9 | 1.1 | 8.8 | 8.8 |
| 4 |  |  |  | 6 | 4.4 | -15.3 | -15.2 | -9.7 | -61.6 | -61.3 |
| 5 |  |  | 0.5 | 3 | 5.7 | 32.2 | 32.5 | 6.0 | 9.8 | 9.8 |
| 6 |  |  |  | 6 | 56.8 | -91.6 | -91.5 | 35.7 | -44.1 | -43.8 |
| 7 |  | 100 | 0.1 | 3 | -0.4 | 11.7 | 11.7 | 0.7 | 8.5 | 8.5 |
| 8 |  |  |  | 6 | 0.2 | 12.6 | 12.5 | 1.1 | 5.2 | 5.2 |
| 9 |  |  | 0.3 | 3 | -2.7 | 23.5 | 23.5 | -1.5 | 7.8 | 7.9 |
| 10 |  |  |  | 6 | 7.3 | -14.3 | -14.5 | -42.5 | -96.2 | -96.3 |
| 11 |  |  | 0.5 | 3 | -0.5 | 26.9 | 26.8 | -35.9 | -37.1 | -36.9 |
| 12 |  |  |  | 6 | -542.2 | -746.1 | -746.5 | -237.2 | -490.5 | -490.6 |
| 13 | 20 | 50 | 0.1 | 3 | -1.9 | 7.9 | 8.7 | 3.7 | 8.0 | 9.1 |
| 14 |  |  |  | 6 | 5.3 | 16.3 | 17.0 | 5.6 | 6.8 | 7.8 |
| 15 |  |  | 0.3 | 3 | 7.6 | 32.2 | 33.4 | 6.6 | 13.6 | 13.6 |
| 16 |  |  |  | 6 | -45.1 | -62.7 | -62.1 | -2.3 | 6.7 | 7.4 |
| 17 |  |  | 0.5 | 3 | 15.0 | 42.5 | 43.9 | -37.1 | -68.4 | -68.4 |
| 18 |  |  |  | 6 | 133.7 | -53.5 | -53.0 | -72.9 | -302.0 | -300.5 |
| 19 |  | 100 | 0.1 | 3 | 2.3 | 14.6 | 14.6 | -2.2 | 5.3 | 5.7 |
| 20 |  |  |  | 6 | 4.7 | 18.3 | 18.0 | -93.9 | -89.3 | -89.2 |
| 21 |  |  | 0.3 | 3 | 1.1 | 29.1 | 29.0 | -2.6 | 6.9 | 7.6 |
| 22 |  |  |  | 6 | 3.1 | 3.7 | 2.8 | -39.6 | -93.1 | -93.1 |
| 23 |  |  | 0.5 | 3 | 11.8 | 42.0 | 41.8 | -8.7 | -20.1 | -19.3 |
| 24 |  |  |  | 6 | -153.7 | -368.7 | -370.3 | -189.3 | -448.4 | -448.6 |
| 25 | 50 | 50 | 0.1 | 3 | 0.1 | 7.4 | 9.8 | 6.4 | 6.8 | 9.8 |
| 26 |  |  |  | 6 | 8.7 | 13.3 | 15.4 | 7.5 | 5.3 | 8.3 |
| 27 |  |  | 0.3 | 3 | 12.9 | 35.5 | 39.0 | 11.6 | 17.9 | 17.9 |
| 28 |  |  |  | 6 | 12.2 | 25.7 | 27.8 | -0.8 | -4.3 | 4.2 |
| 29 |  |  | 0.5 | 3 | 24.9 | 50.9 | 55.8 | -20.6 | -49.7 | -49.7 |
| 30 |  |  |  | 6 | -373.8 | -558.7 | -556.8 | -190.2 | -411.2 | -405.6 |
| 31 |  | 100 | 0.1 | 3 | 2.4 | 14.5 | 14.9 | 4.7 | 11.1 | 12.4 |
| 32 |  |  |  | 6 | 1.9 | 16.5 | 16.1 | 3.8 | 8.4 | 9.1 |
| 33 |  |  | 0.3 | 3 | 9.6 | 39.3 | 39.5 | 9.8 | 18.6 | 20.8 |
| 34 |  |  |  | 6 | 45.3 | 39.8 | 37.6 | 52.1 | -1.4 | -1.0 |
| 35 |  |  | 0.5 | 3 | 2.7 | 35.7 | 35.6 | 1.6 | 25.1 | 28.0 |
| 36 |  |  |  | 6 | 177.6 | -51.1 | -55.0 | 292.8 | 26.7 | 26.6 |

**Table 10.** Difference between reorder point from Lucas search and reorder points for heuristic models for *A* = $800/order and *β* = 0.8.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | **% change in the solution** | | |
| **Parameter** | **% change** |  |  |  |
| *A* | -40% | -25.7109 | 6.5884 | -24.9217 |
|  | -20% | -12.2834 | 3.0268 | -11.9241 |
|  | +20% | 11.4646 | -2.6868 | 11.1511 |
|  | +40% | 22.3044 | -5.1438 | 21.7091 |
| *λ* | -40% | -16.3794 | -49.3695 | -17.1161 |
|  | -20% | -7.6167 | -23.6769 | -7.9077 |
|  | +20% | 6.8335 | 22.7128 | 7.0475 |
|  | +40% | 13.0874 | 44.8600 | 13.4689 |
| *δ* | -40% | 2.3624 | -8.4075 | -17.7034 |
|  | -20% | 1.2661 | -4.3937 | -8.6533 |
|  | +20% | -1.3360 | 4.7643 | 8.3169 |
|  | +40% | -2.6874 | 9.8943 | 16.3430 |
| *σ* | -40% | -1.9607 | 0.4104 | -2.1486 |
|  | -20% | -0.9822 | 0.2353 | -1.0733 |
|  | +20% | 0.9861 | -0.2982 | 1.0713 |
|  | +40% | 1.9764 | -0.6620 | 2.1404 |
| *β* | -40% | 0.0541 | -4.0926 | -0.6292 |
|  | -20% | 0.0149 | -1.8774 | -0.2973 |
|  | +20% | 0.0094 | 1.6317 | 0.2696 |
|  | +40% | 0.0011 | 3.0759 | 0.5164 |
| *L* | -40% | -0.7026 | -47.3086 | -0.7672 |
|  | -20% | -0.2964 | -25.0760 | -0.3232 |
|  | +20% | 0.2214 | 28.2812 | 0.2411 |
|  | +40% | 0.3892 | 60.1755 | 0.4236 |
| *c* | -40% | 14.7157 | 2.0069 | -8.1360 |
|  | -20% | 6.6698 | 0.9654 | -3.9462 |
|  | +20% | -5.6365 | -0.9029 | 3.7390 |
|  | +40% | -10.4751 | -1.7535 | 7.2990 |
| *p* | -40% | -0.9756 | -14.6183 | -2.2499 |
|  | -20% | -0.4280 | -5.6266 | -0.9444 |
|  | +20% | 0.3417 | 4.0814 | 0.7376 |
|  | +40% | 0.6230 | 7.2655 | 1.3403 |
| *h* | -40% | 14.7157 | 2.0069 | -8.1360 |
|  | -20% | 6.6698 | 0.9654 | -3.9462 |
|  | +20% | -5.6365 | -0.9029 | 3.7390 |
|  | +40% | -10.4751 | -1.7535 | 7.2990 |

**Table 11.** Sensitivity analysis of new heuristic model.