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Lateral Shear Layer and Its Velocity Distribution of Flow in Rectangular Open Channels

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Abstract

The lateral velocity distribution of flow in the shear layer of open channel is required to many problems in river and eco-environment engineering, e.g. distribution of pollutant dispersion, sediment transport and bank erosion, and aquatic habitat. It is not well understood about how the velocity varies laterally in the wall boundary layer. This paper gives an analytical solution of lateral velocity distribution in a rectangular open channel based on the depth-averaged momentum equation proposed by Shiono & Knight. The obtained lateral velocity distributions in the wall shear layer are related to the two hydraulic parameters of lateral eddy viscosity (λ) and depth-averaged secondary flow (Γ) for given roughened channels. Preliminary relationships between the above two parameters and the aspect ratio of channel, B/H, are obtained from two sets of experimental data. The lateral width (δ) of the shear layer was investigated and found to relate to the λ and the bed friction factor (f), as described by Equation (26). This study indicates that the lateral shear layer near the wall can be very wide ($\delta/H = 14.6$) for the extreme case ($\lambda = 0.6$ and $f = 0.01$).

Keywords

Lateral Shear Layer, Velocity Distribution, Analytical Model, Lateral Width, Open Channel Flow, Secondary Flow

1. Introduction

In open channel flow, like natural rivers and canals, flows are of highly three-dimensional characteristics owing to the influence of boundaries, varying roughness and non-uniform shapes. Even modeling flow in straight prismatic channels becomes surprisingly difficult, as shown by Chiu & Chiou [1] and Nezu & Nakagawa [2]. This is partly due to the nature of the anisotropic turbulence, which is generated by various factors, such as variations in channel geometry and roughness, the consequent formation of secondary flow cells, and the level of turbulence closure.

Due to the complexity of three-dimensional flow, under certain assumptions, one- or two-dimensional analyses are often used in engineering practice. Lots of research has been studied on velocity vertical distribution in open channel flows [2] [3] [4] [5], which could not meet the requirement of lateral velocity distribution in engineering application. The lateral velocity distribution is pre-requisite in solving certain problems in open channel flows, such as lateral distribution of pollutant, sediment transport and aquatic habitat [6]. However, there is not an analytical formula for predicating the lateral velocity distribution of flow in a rectangular channel.

This paper provides an analytical solution of lateral velocity distribution in the shear layer of rectangular channels, which have various aspect ratios, based on the depth-averaged flow model proposed by Shiono & Knight [7] [8]. Shiono and Knight's model has a wide range of application for curved channels [9], compound weirs [10] [11] and compound channels [12] [13]. The obtained analytical solution of lateral velocity distribution is related to three parameters: the eddy viscosity parameter (λ), secondary flow term (Γ) and the bed friction factor (f). Through analysis on the analytical velocity distribution, it was found that the lateral extent of the shear layer is related to the eddy viscosity parameter (λ) and the bed friction factor (f). An empirical relationship was also established by analyzing two sets of experimental data from Knight *et al.* [14] and Atabay [15], also see www.flowdata.bham.ac.uk. The results of the lateral width of the shear layer indicate that the shear layer near the wall could be very wide for natural rivers when the λ is very large.

In the following sections, Section 2 provides theoretical background with the assumption of model and its analytical solution, followed by experimental data used for model parameter evaluation in Section 3. Section 4 shows the detailed comparison between the analytical model and experiment, along with the discussion on the lateral extent of shear layer. Some conclusions are given in Section 5.

2. Theoretical Consideration

2.1. Introduction

For steady flow in a prismatic open channel, the governing momentum equation in the stream wise direction may be combined with the continuity equation to give [8] [16]:

$$\rho\left[\frac{\partial}{\partial y}(UV)+\frac{\partial}{\partial z}(UW)\right]=\rho g S_o+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z} \tag{1}$$

where U, V, W are the time-averaged velocity components in the x (streamwise), y (lateral) and z (normal to bed) directions respectively (see **Figure 1**), S_o is the

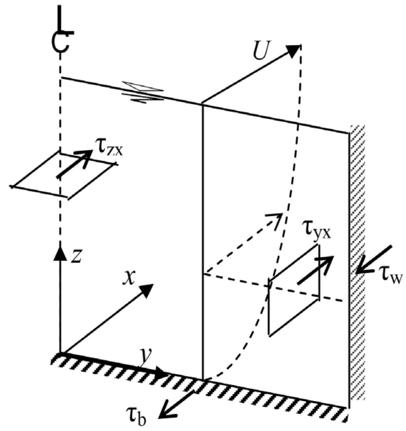

Figure 1. Streamwise Reynolds stress in an open channel.

bed slope, ρ is the density of water, and g is the gravitational acceleration. τ_{yx} and τ_{zx} denote the turbulent shear stresses on the planes perpendicular to the y and z directions, respectively. The left terms of Equation (1) thus signify the secondary flow per unit weight, which is balanced by the weight component and both the lateral and vertical Reynolds stresses in x direction.

For steady flow in a rectangular channel, consider that $W\big|_{z=H} = W\big|_{z=0} = 0$ and by integrating Equation (1) over the water depth (H), the depth-averaged momentum equation can be obtained as (Shiono & Knight [8]):

$$\frac{\partial}{\partial y}\left[H\left(\rho UV\right)_d\right] = \rho gHS_o + \frac{\partial\left(H\overline{\tau}_{yx}\right)}{\partial y} - \tau_b \tag{2}$$

where τ_b is the bed shear stress, the overbar or subscript d denotes the depth-averaged value, and

$$\left(\rho UV\right)_d = \frac{1}{H}\int_0^H \left(\rho UV\right)\mathrm{d}z \tag{3}$$

$$\overline{\tau}_{yx} = \frac{1}{H}\int_0^H \left(-\rho uv\right)\mathrm{d}z \tag{4}$$

in which u and v are the fluctuating velocity component in x and y direction respectively. Through an eddy viscosity concept, the Reynolds stress, $\overline{\tau}_{yx}$, can be described as:

$$\overline{\tau}_{yx} = \rho\overline{\varepsilon}_{yx}\frac{\partial U_d}{\partial y} \tag{5}$$

$$\overline{\varepsilon}_{yx} = \lambda U_* H \tag{6}$$

where λ is the dimensionless eddy viscosity coefficient and U_* ($=\sqrt{\tau_b/\rho}$) is the local shear velocity. The bed shear stress, τ_b, may be related to the depth-averaged velocity, U_d, using the Darcy-Weisbach friction coefficient (f), which is

$$\tau_b = \rho\left(\frac{f}{8}\right)U_d^2 \tag{7}$$

or

$$U_* = \sqrt{\frac{f}{8}} U_d \tag{8}$$

Then, inserting Equations (5) and (7) into (2) gives:

$$\frac{\partial}{\partial y}\left[H\left(\rho UV\right)_d\right] = \rho g H S_o - \rho\left(\frac{f}{8}\right)U_d^2 + \frac{\partial}{\partial y}\left[\frac{\rho\lambda H^2}{2}\sqrt{\frac{f}{8}}\frac{\partial U_d^2}{\partial y}\right] \tag{9}$$

Experimental results by Shiono & Knight [8] demonstrate that in overbank flow the term of secondary flow, $H(\rho UV)_d$, varies almost linearly with y in the main and floodplain regions of a channel. Therefore, the lateral gradient of the secondary flow force may be approximated as

$$\frac{\partial}{\partial y}\left[H\left(\rho UV\right)_d\right] = \Gamma \tag{10}$$

where Γ is a dimensionless constant of secondary flow, which varies in different flow regions (Knight *et al.* [17]). Thus Equation (9) can be expressed by:

$$\rho g H S_o - \rho\left(\frac{f}{8}\right)U_d^2 + \frac{\partial}{\partial y}\left[\frac{\rho\lambda H^2}{2}\sqrt{\frac{f}{8}}\frac{\partial U_d^2}{\partial y}\right] = \Gamma \tag{11}$$

which is the governing equation of depth-averaged flow in the streamwise direction.

2.2. Analytical Solution for U_d^2

For given channels, when f, λ and Γ are fixed values, Equation (11) becomes a standard ordinary differential equation in terms of variable U_d^2. An analytical solution to Equation (11) can be obtained under appropriate boundary conditions. The detailed solution process is given by Knight *et al.* [17]. The solutions for the channel with linearly varying bed can also refer to Knight & Shiono [18] and Tang & Knight [13] [19] for further details.

In a rectangular channel, which has the constant depth of flow (*i.e.* H = constant), the analytical solution of U_d to Equation (11) has the following form:

$$U_d = \left(A_1 e^{\gamma y} + A_2 e^{-\gamma y} + k\right)^{1/2} \tag{12}$$

where

$$k = \frac{8 g H S_o}{f}(1-\beta) \tag{13}$$

$$\gamma = \frac{1}{H}\sqrt{\frac{2}{\lambda}}\left(\frac{f}{8}\right)^{1/4} \tag{14}$$

$$\beta = \frac{\Gamma}{\rho g H S_o} \tag{15}$$

where A_1 and A_2 are two integral constants, which can be obtained by applying the boundary conditions of flow continuity and the no-slip condition at the wall, as shown by Knight *et al.* [20] [21]. For a simple rectangular channel of semi-width, b, see **Figure 2**, a single panel is used to represent half of the cross-section, and then the two A coefficients are obtained as:

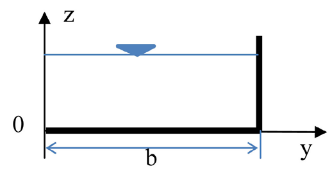

Figure 2. Half rectangular channel cross-section.

$$A_1 = A_2 = -\frac{k}{2\cosh(\gamma b)} \tag{16}$$

Thus for a rectangular channel, Equation (12) becomes

$$U_d = \sqrt{C\cosh(\gamma y) + k} \tag{17}$$

where

$$C = -k/\cosh(\gamma b) \tag{18}$$

3. Experimental Data for Parameter Evaluation

3.1. Calibration of Coefficients and Modelling Philosophy

Strictly speaking, Equation (15) suits for the flow region of constant depth where a constant Γ value exits. If Γ varies laterally, the cross section of a channel can be split into a discrete number of panels if necessary. Generally, panel junctions coincide with the place where the roughness of channel bed or flow depth is discontinuous. Thus, for a symmetric uniform rectangular channel, only half the channel needs to be modeled because of symmetry of the channel. If complex secondary flow cells present, then further division of the channel into more panels may be required [21].

The friction coefficient is generally assumed to be constant in each panel, and it can be back calculated from $f = 8\tau_b/(\rho U_d^2)$ based on experimental data. Note that the variation of λ has great effect for open-channel flows due to lateral shear near wall [22]. Hence, both the secondary flow parameter (Γ) and lateral eddy viscosity coefficient (λ) require calibration in modeling the lateral distribution of depth-averaged streamwise velocity.

3.2. Experimental Data in the Study

The data of lateral flow velocity used herein were obtained from Knight *et al.* [14] and Atabay [15]. These experiments have covered a wide range of aspect ratios (B/H), which vary from 1.0 to 16.0. The earlier experiments by Knight *et al.* [14] were conducted in a non-tilting 15 m long flume with a rectangular cross-section of width B =152 mm and a bed slope S_o of 9.66 × 10^{-4}. The more recent experiments by Atabay [15] were undertaken in a 22 m long flume with a channel width of 50 mm and a bed slope of 2.024 × 10^{-3}. These data are also accessible from www.flowdata.bham.ac.uk.

In the present study, only half the channel was modeled due to the symmetry of flow, as shown in **Figure 3**, where only one panel is used for Equation (17). The parameters adopted in the modeling are given in **Table 1**, where DWK and

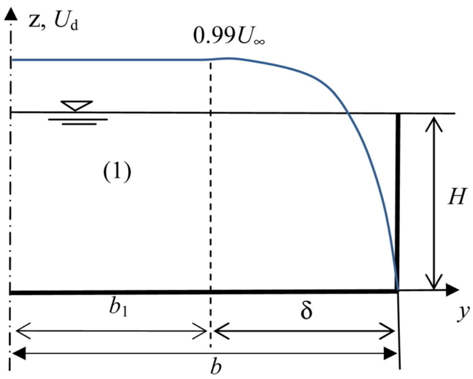

Figure 3. Sketch of half rectangular channel.

Table 1. Parameters adopted in the application of Equation (17).

Experiment	B/H	f	λ	Γ	MAPE (%)
DWK	0.99	0.0205	0.0003	0.093	1.34
	1.38	0.0205	0.0005	0.54	0.98
	1.77	0.0205	0.001	0.42	0.73
	4.20	0.016	0.015	0.25	5.70
	4.79	0.016	0.02	0.15	4.97
AS	6.60	0.019	0.025	0.1	3.72
	9.30	0.02	0.038	0.02	2.36
	11.86	0.023	0.048	0.008	3.49
	15.18	0.026	0.055	0.006	2.93

AS denote the experimental runs by Knight *et al.* [14] and Atabay [15], respectively. In the modelling, $\rho = 1000$ kg/m^3 and $g = 9.807$ m/s^2.

4. Results

4.1. Prediction of Lateral Distributions of U_d

For given parameters of f, λ and Γ, Equation (17) can predict the lateral variation of depth-averaged velocity. Figure 4 and Figure 5 show the lateral distributions of U_d between the prediction and experimental data. The results show that the prediction by Equation (17) agrees well with the experimental data for flows in rectangular open channels, which have a wide range of aspect ratios (B/H).

To evaluate the robustness of Equation (17) against the experimental data, error analyses were carried out. MAPE (Mean Absolute Percentage Error) is used as a measure of the error percentage for predicted values of model against the measured values. The individual differences are called as residuals for the data sample that is used for estimation, and the residuals are known as estimation errors for the sample [23] [24] [25].

The percentage of error in predicted discharge of each flow depth is calculated by,

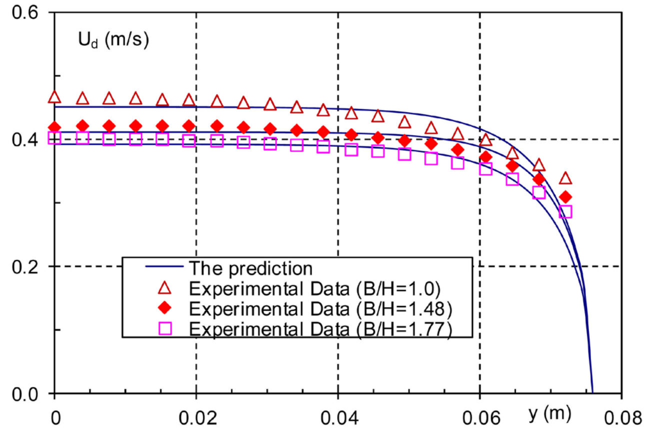

Figure 4. U_d distributions for the experiments by Knight *et al.* [14].

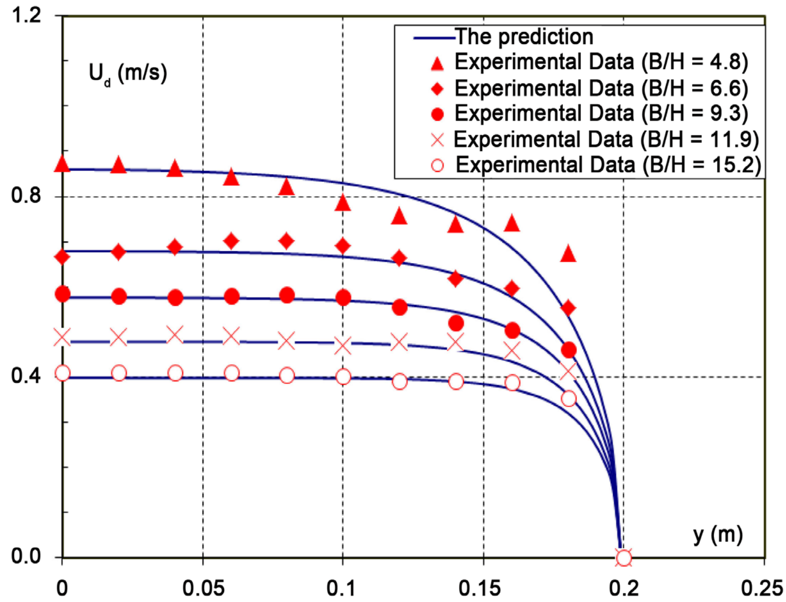

Figure 5. U_d distributions for the experiments by Atabay [15].

$$E_{u,i} = \frac{\left|u_{a,i} - u_{e,i}\right|}{u_{e,i}} \tag{19}$$

where $E_{u,i}$ is the error percentage of predicted velocity; $u_{a,i}$ and $u_{e,i}$ are the predicted and observed velocity at ith measured point (y), respectively. Therefore, the average error percentage of model for an experiment can be computed by

$$E_u = \frac{1}{N}\sum_{i=1}^{N}\left(E_{u,i}\right) \tag{20}$$

where N is the total number of obervation in an experiment.

The values (E_u) of MAPE for all experiments are shown in Table 1, which shows that the prediction by Equation (17) has the averaged percentage error of 1% to 6%.

In this study, the optimum values of λ and Γ in are found to be well related to the aspect ratios of channel, as expressed by:

$$\lambda = \exp\left[-2.36 - 8.22/\left(B/H\right)\right], R^2 = 0.993 \tag{21}$$

$$\Gamma = -0.09 + \left\{0.174\ln\left(B/H\right) + 0.924\right\}/\left(B/H\right), R^2 = 0.992 \tag{22}$$

where B is the width of channel, H is the flow depth, and R is the correlation coefficient.

4.2. Lateral Extent of Shear Layer

From the analytical solution of U_d, given by Equation (17), it reveals that the depth-averaged velocity has little change in the major central part of channel, but it decreases rapidly toward zero at the wall, as shown in **Figure 4** & **Figure 5**. With increasing the aspect ratio (B/H) of channel, the extent of almost constant flow velocity is getting wider in the central channel, which can be treated as one-dimensional uniform flow.

When a channel is shallow and very wide, *i.e.* the ratio of B/H is very large, the value of $\cosh(\gamma b)$ in Equation (18) can be very large (∞), so that the C valve in Equation (17) becomes a very small value (close to zero). Note that the secondary flow is weak in such a shallow channel, thus Γ is negligible (Knight *et al.* [21]). Therefore Equation (17) becomes:

$$U_d = \sqrt{k} = \sqrt{\frac{8g}{f}}\sqrt{HS_o} \tag{23}$$

The obtained Equation (23) is the well-known velocity formula for one-dimensional (1-D) uniform flow, *i.e.* Darcy-Weisbach equation.

In open channel flow, a lateral shear layer exists near the wall. The extent of lateral shear layer can be defined by a lateral distance, δ, away from the wall, for which $U_d\big|_{y=b-\delta} = 0.99U_\infty$. U_∞ is the velocity of 1-D flow, given by Equation (23), as illustrated in **Figure 3**. The boundary distance δ in the shear layer can be obtained as follows:

Applying Equation (17) for $U_d\left(\delta\right) = 0.99U_\infty = \sqrt{k}$ when $y = b - \delta$ gives:

$$0.99\sqrt{k} = \sqrt{C\cosh\left[\gamma\left(b-\delta\right)\right] + k} \tag{24a}$$

or

$$0.9801k = C\cosh\left[\gamma\left(b-\delta\right)\right] + k \tag{24b}$$

Inserting Equation (18) into (24b) yields

$$\cosh\left(\gamma\delta\right) - \tanh\left(\gamma b\right)\times\sinh\left(\gamma\delta\right) = 0.0199 \tag{25}$$

Since the parameter (γ), given by Equation (14), is a function of depth, Equation (25) is implicit for δ. However, δ can be obtained numerically by solving Equation (25), as graphically shown in **Figure 6**. This implies that when γb is about 5 the value of U_d at $y = b - \delta$ is approximate to $0.99U_\infty$. In other words, in the region away from the wall, where $\gamma\delta = 5$, the flow can be treated as uniform flow.

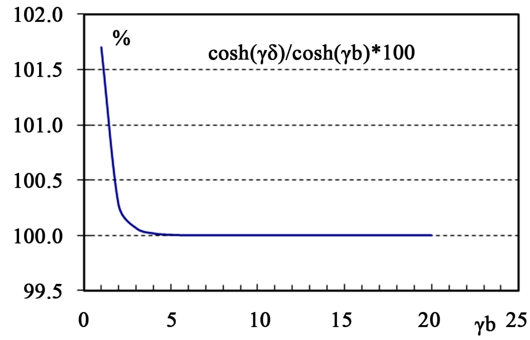

Figure 6. The boundary distance (δ) for $U_d = 0.99\,U_\infty$.

Therefore, replacing γ in Equation (14) for $\gamma\delta = 5$ gives:

$$\frac{\delta}{H} = 5\sqrt{\frac{\lambda}{2}}\left(\frac{8}{f}\right)^{1/4} = 3.54\sqrt{\lambda}\left(\frac{8}{f}\right)^{1/4} \tag{26}$$

Samuels [26] obtained the similar boundary distance (δ) to Equation (26) with a coefficient 3.4 rather than 3.54 here.

In order to estimate the shear layer width (δ) for a natural river, it needs to estimate the eddy viscosity coefficient (λ). When bed generated turbulence dominates the lateral momentum transfer, Cunge *et al.* [27] suggested that the typical value of λ is 0.5, which represents the value for the cross-sectional exchange coefficient in the range of 0.23 to 0.6. By taking the extreme case where $\lambda = 0.6$ and $f = 0.01$, Equation (26) gives $\delta/H \approx 14.6$.

In practical applications for open channel flows, within the boundary width ($\gamma\delta = 5$) Equation (17) can be used to predict the U_d distribution in the lateral shear layer near the wall, whereas the velocity is calculated by Equation (23) in the central region of channel beyond the shear layer. In fact, the flow in the central part is uniform flow if the channel is very shallow (*i.e.* large aspect ratio B/H).

By taking the values of f and λ from **Table 1**, Equation (26) will establish a relationship between δ/H and B/H, as shown in **Figure 7**. The relationship can be described as:

$$\delta/H = 1\Big/\left[0.272 + 3.982\big/(B/H)^2\right], \quad R^2 = 0.994 \tag{27}$$

5. Conclusions

The depth-averaged velocity distribution in lateral shear layer in a rectangular open channel can be described by Equation (17), which is analytically obtained based on the depth-averaged momentum equation proposed by Shiono & Knight. The analytical solution of Equation (17) agrees well with the experimental data with the aspect ratios ranging from 1 to 16. The percentage error in MAPE is 1% to 6%.

Based on Equation (17), the lateral width of the shear layer is found to be related to the dimensionless eddy viscosity coefficient (λ) and the bed friction (f), as given by Equation (26). The shear layer width (δ) near wall varies with the

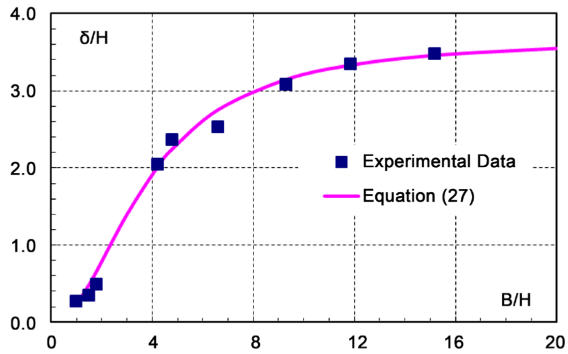

Figure 7. Plot of δ/H based on Equation (27).

aspect ratios of channel. It reveals that the boundary width can be very large: e.g. $\delta/H = 14.6$ for the extreme case ($\lambda = 0.6$ and $f = 0.01$) in natural rivers.

The obtained Equation (17) for the lateral velocity distribution was obtained for a rectangular open channel with uniform boundary, but it can be extended to use for velocity analysis in an open channel with non-uniform boundary, where the channel can be split to serial panels of having similar roughness to apply for.

Future work may need to establish the relationships (21) & (22) for estimations of λ and Γ in channels with heterogeneous roughness, along with the model application on field data if available.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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