A NEW APPROACH FOR PREDICTING DISCHARGE IN STRAIGHT, SYMMETRIC HOMOGENEOUS COMPOUND CHANNELS

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Accurate estimation of flow discharge in a compound river channel becomes increasingly important in river flood risk management and eco-environment design. In this paper, a new approach is proposed to improve the prediction of flow based on the concept of the apparent shear stress at the diagonal interface between main channel and floodplain. The new approach is compared with a wide range of our experimental data and the data available in the literature. The 22 datasets used include homogenously symmetric channels, which have various aspect ratios [channel total width (B) at bankfull / main channel bottom (b) =1.5 ~ 15.8] and bed slopes (S_o = 3.0x10^{-4} ~2.024x10^{-3}). It was found that the new approach has significantly improved the accuracy of flow prediction compared with the conventional DCM (divided channel method) against all the datasets, particularly for relatively low flow depths of floodplain where the flow discharge is most difficult to predict correctly. The new approach predicts the total discharge well for homogeneously symmetric channels, within a relative error of 5% on average. Furthermore, the new approach can have a better prediction of zonal discharge percentage compared with the DCM.

Keywords: Overbank flow, compound channel, symmetric compound channel, open channel, momentum exchange

1. INTRODUCTION

Many natural rivers have a deep main channel adjoined with one or two shallow floodplains, which becomes a compound channel, or called a two-stage channel. In certain cases, e.g. in urban river restoration and environmental design, compound channels are deliberately constructed in order to increase channel flow capacity in times of floods, or create environmental friendly space in the floodplain. The existence of floodplain increases the dimension of river cross-section, thus increasing the transport capacity of flow; meanwhile, the wide and shallow floodplain can provide wealthy nutrients that contribute to the reproduction and diversity of species. Compound channels have drawn much attention from river and environmental engineers. The accurate prediction of flow in a compound channel is a prerequisite to the flood risk and environmental management of river in order to eliminate or mitigate environmental impact, economic or human losses.

Conventional one-dimensional (1-D) channel divisional methods, namely the Divided Channel Method (DCM), and the Single Channel Method (SCM), still are widely used in practice because of their simplicity. However, it is well-known that these methods either over-estimate or under-estimate channel discharge, particularly for zonal discharge (i.e. discharge in main channel and its floodplain) (Wormleaton et al. [26]; Knight & Hamed [11]; Tang & Knight [21]; Yang et al. [27]; Tang [19]). When a floodplain is inundated, the velocity differences between the main channel and floodplain result in a mixing shear layer due to lateral momentum exchange. Early research (Wormleaton et al.[26]; Knight & Demetriou [10]; Knight & Hamed [11]; Prinos & Townsend [17]; Christodoulou [4]) indicated the importance of considering the main channel / floodplain interaction effects. Most recently, Hamidifar et al. [7] compared SCM and various DCMs with their experimental data, and concluded that these methods are less accurate compared with the Coherence Method (COHM) by Ackers [1] and quasi-2D analytical method (called SKM) by Shiono and Knight [18].

Despite the availability of quasi-2D method, e.g. SKM by Shiono and Knight [18] and its modified methods by Tang & Knight ([22]-[24]) and Knight & Tang [12], and 3-D approaches that take into account the interaction between the main channel and floodplain, e.g. Krishnappan & Lau [13], Cater & Williams [3] and Marjang & Merkley [14], these methods are usually complex and require more information and turbulence parameters, which are usually not available. Therefore, 1-D method has still been developing ever since because of its simplicity and practical significance.

In the river eco-environmental design and flood risk management, it is required precisely to predict not only the overall discharge but also zonal discharge (the discharge in the main channel and floodplain, respectively) in a compound river channel. Recently, some new 1-D methods have been proposed, for example, the Interacting Divided Channel Method (IDCM) by Huthoff et al. [8], the Apparent Shear Stress Method (ASSM) (Wormleaton et al. [26]; Christodoulou [4]; Moreta and Martin-Vide [16]), the Momentum-Transfer Divided Channel Method (MTDCM) by Yang et al. [27], the Modified Divided Channel Method (MDCM) by Khatua et al. [9], Mohanty & Khatua [15] and Devi et al. [5], the Energy Concept Method (ECM) by Yang et al. [28], and the modified energy-based method by Tang [20]. These methods have taken into account the effect of the lateral interaction of momentum in different forms, but they were developed and validated based on their own limit data. Fernandes et al. [6] assessed various other methods for predicting discharge of compound channels and concluded that the methods that count for the momentum exchange between the main channel and floodplain show much improved results than the conventional methods. Most recently, Tang [19] compared some of the above methods against a large set of data in homogenous compound channels and concluded that the methods can predict the total discharge reasonably well within an averaged error of 8% for symmetric compound channels, but do not for asymmetric channels, particularly in the prediction of zonal discharge. However, none of these methods can predict discharge well for relatively lower flow depth of floodplain, which is most difficult to predict with relatively high precision.

In the present paper, we propose a new method for predicting the total discharge and zonal discharge of flow in a compound channel, based on the force balance within each zone, which is separated along a diagonal line between the main channel and floodplain. The apparent shear stress along the diagonal interface is evaluated by the velocity difference between the adjoined zones. The proposed new method was compared with the DCM against a wide range of 22 datasets of homogeneously symmetric compound channel, and it was found that the new method has greatly improved the accuracy of flow prediction compared with the DCM and also has a better prediction of zonal discharge percentage.

2 METHOD

For the convenience of reference in the subsequent context, the sketched cross-section of a symmetric compound channel is shown in Figure 1, where H, h and h_f are the flow depth of main channel, bankfull, and floodplain (subscript f), respectively. b and b_f are the widths of the main channel bottom and floodplain, respectively; S_c and S_f are the side slopes of the main channel and floodplain, respectively.

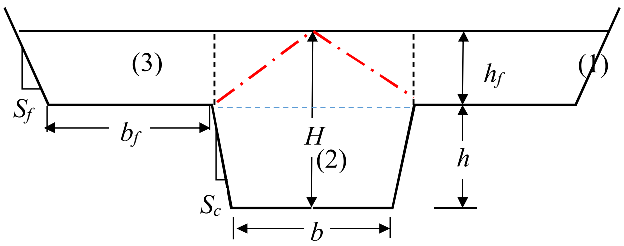


Figure 1. The sketched cross-section of symmetric compound channel

In this study, the proposed new method is to take into account the momentum transfer of flow between the main channel and the floodplain along the diagonal line as illustrated by the dot-line in Figure 1, and is described as follows:

Based on the force balance of each part of channels per unit length separately by the vertical line (i.e. the dash fine between the main channel and floodplain), it follows,

$$\rho g A_c S_o = \rho f_c U_c^2 P_c + N_f \tau_a h_f \tag{1}$$

$$\rho g A_f S_o = \rho f_f U_f^2 P_f - \tau_a h_f \tag{2}$$

where U is the cross-sectional velocity, A is the cross-sectional area, ρ is the density of fluid, S_o is the bed slope of channel, h_f is the depth of flow at the interface (i.e. the flow depth of floodplain), P is the wetted perimeter, f is the frictional factor, N_f is the number of floodplain, and the subscripts c & f denote the main channel and floodplain, respectively. The apparent shear stress (τ_a) at the interface between main channel and its floodplain can be evaluated by the zonal velocities at each region, as recommended by Christodoulou [4], Huthoff et al. [8], Moreta and Martin-Vide [16], and Yang et al. [27].

$$\tau_a = \frac{1}{2}\rho \alpha_m (U_c^2 - U_f^2) \tag{3}$$

where α_m is an interface constant.

Similarly, if the diagonal line is imaged to split the channel cross-section to three zones, as denoted as 1, 2 3 in Figure 1, the apparent shear stress (τ_a) along the diagonal interface (i.e. the plane on the diagonal line) is then evaluated by a similar equation (3), where the velocity differences are between in zone 2 and in zone 1 or 3. In a symmetric homogenous compound channel, the apparent shear stress on each diagonal interface is then equal and is given by

$$\tau_{a12} = \frac{1}{2}\rho\alpha_{12}(U_2^2 - U_1^2)\ ; \quad \tau_{a23} = \frac{1}{2}\rho\alpha_{23}(U_2^2 - U_3^2) \tag{4}$$

where $\tau_{\alpha 12}$ and $\tau_{\alpha 23}$ are the apparent shear stress at the diagonal interfaces on the left and right, respectively, and α_{12} and α_{23} are their corresponding coefficients of moment transfer. Due to the symmetric channel, $\tau_{\alpha 12}$ and $\tau_{\alpha 23}$ are equal whereas α_{12} and α_{23} are the same, let them as α_m. U is the average velocity of sub-section, and subscripts (1, 2, 3) denote the sub-sections as shown in Figure 1.

Based on the force balance of each sub-section (1, 2 and 3), we can obtain the averaged velocity of each sub-section. For the symmetric compound channel, we take the right half of the cross-section as analysis, and it fellows that

$$U_1^2 = U_{1,0}^2 + \frac{\varepsilon_f}{1+\varepsilon_f+2\varepsilon_c}\left(U_{2,0}^2 - U_{1,0}^2\right) \tag{5}$$

$$U_2^2 = U_{2,0}^2 - \frac{2\varepsilon_c}{1+\varepsilon_f+2\varepsilon_c}\left(U_{2,0}^2 - U_{1,0}^2\right) \tag{6}$$

with the coefficients:

$$\varepsilon_c = \frac{1}{2}\alpha_m h'/f_c P_c \quad ; \quad \varepsilon_f = \frac{1}{2}\alpha_m h'/f_f P_f \ ; \ h' = \sqrt{h_f^2 + (B_c/2)^2} \tag{7}$$

where U is the cross-sectional velocity, B_c is the width of main channel at bankfull, h_f is the depth of flow at the interface (i.e. the flow depth of floodplain), P is the wetted perimeter, f is the frictional factor, the subscripts 1, 2 & 3 denote sub-sections, and the subscript (,0) denotes the values based on the DCM with vertical interface excluded.

Then, the zonal velocity of both main channel and floodplain can be obtained by:

$$U_f = U_1 \ ; \ U_c = U_2 + (U_1 - U_2)h_f B_c/2A_c \tag{8}$$

where A is the cross-sectional area.

3 DATA USED FOR STUDY

To test the new method in Section 2, we used a wide range of different experimental datasets in symmetric, homogeneous compound channels. These data are from our data (in www.flowdata.bham.ac.uk) and other available data in the literature. A total of 22 datasets used covers different channel cross-sections, in which the aspect ratio (B/b) is 1.5 ~ 15.8 and the bed slope $S_o = 3.0\text{x}10^{-4} \sim 2.024\text{x}10^{-3}$. The details are given in Table 1, where N_f is the number of floodplain, N is the number of experiment runs, Dr , = $(H-h)/H$, is the relative depth of floodplain, n is the Manning coefficient, and other notations see Figure 1.

Table 1. Details of experimental datasets of symmetric compound channels used

Series	N_f	N	n	h (m)	b_f(m)	b (m)	B/b	S_c	S_f	Q_t (m^3/s)	D_r
FCF data (1992), S_o = 0.001027											
FCF1	2	8	0.01	0.15	4.10	1.50	6.67	1	0	0.2082-1.0145	0.056-0.400
FCF2	2	10	0.01	0.15	2.25	1.50	4.20	1	1	0.2123-1.1142	0.041-0.479
FCF3	2	10	0.01	0.15	0.75	1.50	2.20	1	1	0.2251-0.8349	0.051-0.500
FCF8	2	8	0.01	0.15	2.25	1.50	4.00	0	1	0.1858-1.1034	0.050-0.499
FCF10	2	9	0.01	0.15	2.25	1.50	4.40	2	1	0.2368-1.0939	0.051-0.464
University of Birmingham (2001), S_o=0.002024											
BU-S	2	11	0.0091	0.05	0.4073	0.398	3.05	0	0	0.0154-0.0552	0.162-0.475
Knight and Demetriou (1983), S_o=0.000966											

KD83A	2	6	0.0090	0.076	0.076	0.152	2.00	0	0	0.0052-0.0171	0.108-0.409
KD83B	2	5	0.0093	0.076	0.152	0.152	3.00	0	0	0.0050-0.0234	0.131-0.491
KD83C	2	6	0.0097	0.076	0.229	0.152	4.00	0	0	0.0049-0.0294	0.106-0.506
Bahram (2006), S_o=0.002003											
BD01	2	10	0.0091	0.05	0.10	0.398	1.50	0	0	0.0120-0.0451	0.053-0.536
BD02	2	11	0.0091	0.05	0.20	0.398	2.01	0	0	0.0120-0.0500	0.051-0.522
BD03	2	11	0.0091	0.05	0.30	0.398	2.51	0	0	0.0120-0.0501	0.062-0.487
BD04	2	9	0.0091	0.05	0.40	0.398	3.01	0	0	0.0121-0.0501	0.086-0.468
Khatua et al. (2012), So=0.0019											
KH01	2	5	0.0108	0.120	0.16	0.12	3.67	0	0	0.0087-0.0391	0.118-0.461
Mohanty and Khatua (2014), S_o=0.0011											
KH02	2	6	0.01	0.065	1.745	0.330	11.97	1	0	0.0150-0.1062	0.073-0.115
Patra et al. (2012), S_o=0.000311											
Patra	2	7	0.0098	0.08	0.805	0.12	15.75	1	0	0.0494-0.0958	0.111-0.136
Prinos and Townsend (1984), S_o=0.0003											
PT01	2	5	0.011	0.102	0.381	0.203	5.26	0.5	0	0.0066-0.0205	0.089-0.328
PT02	2	5	0.011	0.102	0.381	0.305	3.83	0.5	0	0.0106-0.026	0.089-0.328
Noutsopoulos and Hadjipanos (1983), S_o=0.0015											
NH-A1	2	5	0.01	0.075	0.425	0.15	6.67	0	0	0.0090-0.0450	0.186-0.459
NH-A3	2	6	0.01	0.075	0.325	0.15	5.33	0	0	0.0150-0.0370	0.285-0.468
NH-A4	3	5	0.01	0.075	0.225	0.15	4.00	0	1	0.0100-0.0300	0.248-0.479
Wormleaton et al. (1982), S_o=0.00043											
W-A	2	7	0.01	0.12	0.460	0.29	4.17	0	0	0.0134-0.043	0.115-0.367

4 RESULTS AND DISCUSSION

To evaluate the errors of the new method against the experimental data, the error percentage of predicted discharge was used as a criterion for the purpose of method evaluation. The percentage of error in predicted discharge of each flow depth is calculated by,

$$\%E_{Q,i} = \frac{|Q_{cal,i} - Q_{exp,i}|}{Q_{exp,i}} \times 100\% \quad (9)$$

where $\%E_{Q,i}$ is the error percentage of predicted discharge, $Q_{cal,i}$ and $Q_{exp,i}$ are the predicted and observed discharge at *i*th flow depth, respectively. Therefore, the average error of each method for an experiment is obtained by

$$\%E_Q = \frac{1}{N}\sum_{i=1}^{N}(\%E_{Q,i}) \quad (10)$$

where N is the total number of runs in an experiment.

In subsequent figures, subscripts (t, c, f) denote the values for the total channel, main channel and floodplain, respectively. Figure 2 shows the average percentage errors of predicted discharge by the new method for all 22 datasets, along with the DCM method as a comparison.

As described in Equation (3), the interface coefficient (α_m) is introduced for the evaluation of the apparent shear stress (τ_a) along the diagonal interface in the proposed new method, and the value of α_m is expected to small. After examining all the data, we found that the new model agree the data well when α_m = 0.001, as shown in Figure 2, where the predictions by the DCM are included for comparison. Figure 2 shows that the new method, which takes into account the momentum transfer of flow between the main channel and floodplain, gives an overall improved prediction of total discharge (Q_t) within the combined average percentage of error less than 5% (also see Figure 3), when compared with the DCM. Further analysis in Figure 3 shows that the averaged error of prediction by both the new method and the DCM increases slightly as increasing B/b.

The analysis of zonal discharge prediction shows that the new method has also improved the prediction of zonal discharge percentage (both Q_c/Q_t and Q_f/Q_t) compared with the DCM, as demonstrated in Figure 4, which includes two cases: one has a large B/b (FCF1) and the other has a smaller B/b (KD83A).

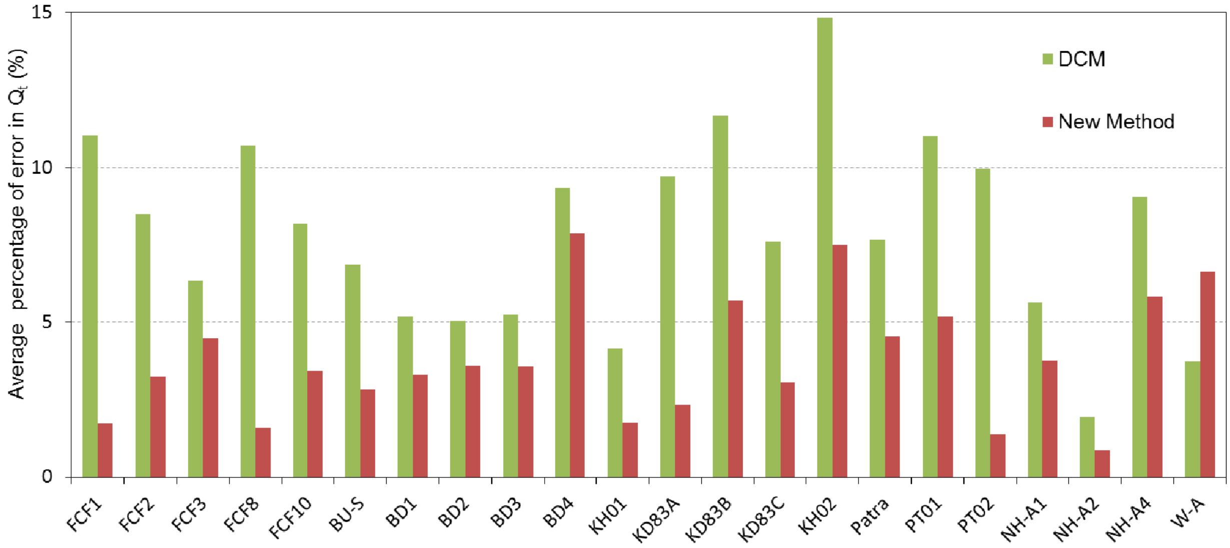


Figure 2. Averaged error percentage of total discharge (%Q_t).

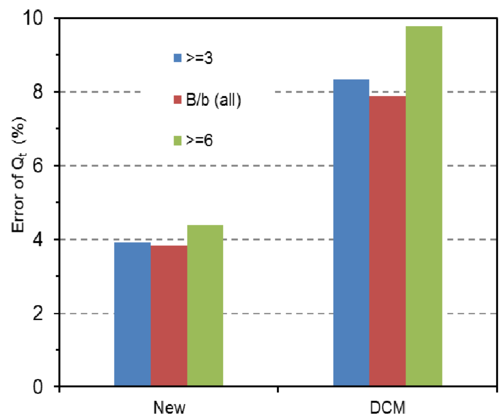


Figure 3. Averaged error percentage of Q_t.

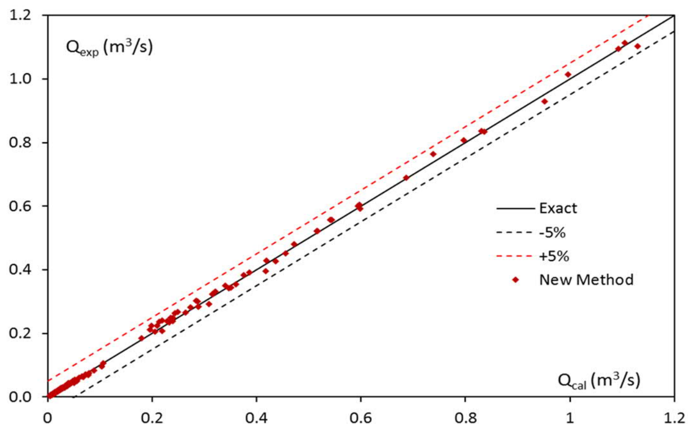
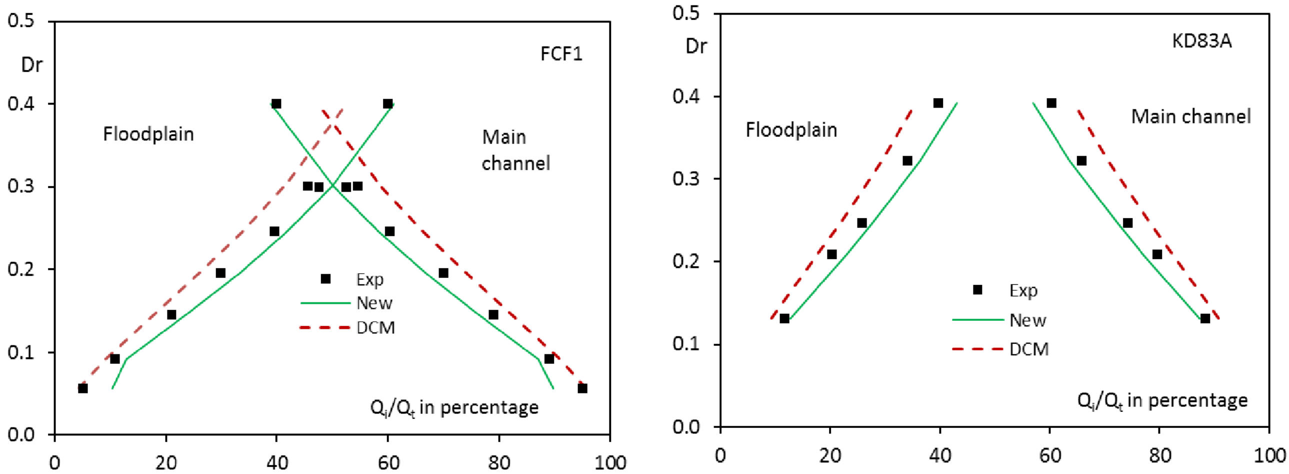


Figure 6. Comparison of the discharge prediction for all datasets.



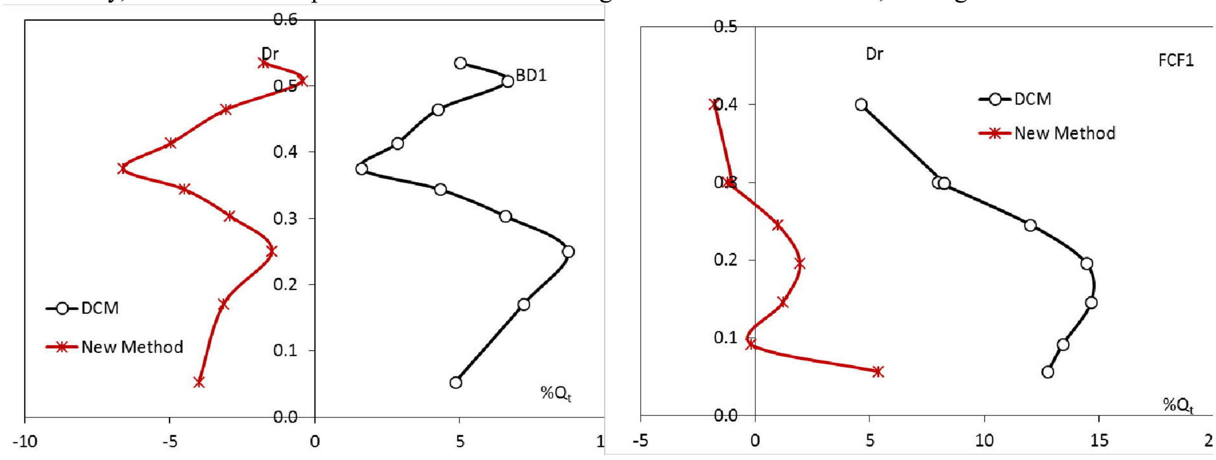
(a) FCF1: B/b = 6.67

(b) KD83A: B/b = 2

Figure 4. Comparison of zonal discharge percentage for FCF1 and KB83A.

Figure 5 indicates that the errors of predicted discharge by the new method are smaller within 6% over different depths compared with the DCM, which has a large error percentage up to 15% in small depths of flow. Figure 5 also shows that the prediction errors of Q_t by the new method are relatively less sensitive to depth of flow than those by the DCM. Moreover, the errors of discharge prediction by the two methods appear to decrease as increasing B/b.

Finally, the new method predicts the overall discharge well for all the datasets, see Figure 6.



(a) BD01: B/b = 2.01 (b) FCF1: B/b = 6.67

Figure 5. Variation of percentage errors of Q_t with relative depth D_r for symmetric channels.

5 CONCLUSIONS

Through the test against a wide range of data in symmetric, homogeneous compound channels, the proposed new method shows that:

- Compared with the DCM, the new method improves the prediction of total discharge (Q_t) within the error 5% on average, and the method also shows good prediction of zonal discharge percentage (Q_c/Q_t and Q_f/Q_t).
- The prediction errors of Q_t over different depths by the new method are smaller than those by the DCM, and the prediction errors of the new method also vary relatively small with depth of flow.
- It appears that the prediction errors by the two methods increases as increasing B/b for symmetric homogeneous compound channels. Owing to limited data analyzed, this finding may need further study using more datasets in the future.

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