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METHODS FOR PREDICTING VERTICAL VELOCITY DISTRIBUTIONS IN OPEN CHANNEL FLOWS WITH SUBMERGED RIGID VEGETATION

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ABSTRACT

This paper evaluates the four analytical models of Klopstra, Defina, Baptist, and Nepf model for predicting vertical velocity distributions of flow in an open channel with submerged rigid vegetation, in order to examine their sensibility to vegetation density and submergence ratios (H/h), where H is flow depth and h is vegetation height. Fourteen experimental datasets used cover the ratios of H/h =1.25 ~ 3.33, and various vegetation density of a =1.1~ 18.5 m^{-1} (a defined as the frontal areas of the vegetation per unit volume). For submerged vegetated flow, the vertical velocity distribution can often be described by two layers, the vegetation layer in the lower part and the non-vegetation layer in the upper part. The vegetation retards flow by exerting drag force on the flow, thus resulting in different velocities between the vegetation layer and the upper surface layer. Based on an eddy mixing-layer analogy, different analytical models have been proposed for predicting vertical velocity distribution in the two layers, and four models were chosen to examine their sensibility when tested against different independent datasets from those used in their original papers. Our studies show that the four models can predict reasonably well under a certain range of vegetation density and submergence, and that the prediction of all the models are less sensitive in the lower region. The studies also reveal that under the same submergence, the predictions of the four models in the upper layer are less sensitive to low vegetation density (e.g. ah <1) than to high vegetation density (ah >1), but that for the same vegetation density, all four models are much sensitive to the vegetation submergence ratio (H/h). It was also found that the Defina and Klopstra models are almost the same despite some simplification made in the Klopstra model, if the same mixing length scale of eddies (λ) is used. Finally, close examination on λ in the analytical models, and it was found that the predictions of the models (the Baptist, Defina and Klopstra models) are less sensitive to the vegetation density and submergence when λ is evaluated by $0.02\sqrt{Hh}$.

Keywords: Vertical velocity profile, vegetated flow, open channel flow, rigid vegetation, analytical model

1. INTRODUCTION

Vegetation exists in many rivers and wetlands. The presence of vegetation affects the flow field, increases flow resistance and changes the vertical velocity profile. The impact of vegetation on the vertical velocity profile depends on whether the vegetation is rigid or flexible and whether the vegetation is submerged or emergent. As a prerequisite for analysis of flow resistance, pollutant mixing process, and so on, the velocity profile in vegetated flows has drawn the attention of many researchers (e.g. Tsujimoto & Kitamur 1990, Shimizu & Tsujimoto, 1994; Klopstra et al., 1997; Meijer & Van Velzen, 1999; Nepf & Koch, 1999; Nepf & Vivoni, 2000; Lopez & Carcia, 2001; Gahisalberti & Nepf, 2004; Defina & Bixio, 2005; Baptist et al., 2007; Kubrak, et al. 2008; Huai, et al. 2009; Yang & Choi, 2010; Dimitris & Panayotis, 2011; Nepf, 2012; Nguyen , 2012; Tang & Ali, 2013; Hao et al. 2014). However, due to differences in vegetation types and flow conditions, various methods have been proposed to predict the velocity profile, based on 1-D models of the momentum equation. Currently, the most common method is a two-layer approach used for the flow with submerged vegetation, in which different analytical models are applied in the lower vegetation layer and the upper surface layer, based on an eddy mixing-layer analogy. (Klopstra et al., 1997; Meijer & Van Velzen, 1999; Gahisalberti & Nepf, 2004; Defina & Bixio, 2005; Baptist, et al., 2007; Huai, et al. 2009; Yang & Choi, 2010; Nepf, 2012). For example, Klopstra et al. (1997) proposed a two-layer model for predicting the velocity profile through and above submerged rigid vegetation by dividing the flow field into two layers, one with vegetation and one above is called the surface layer. In the vegetation layer, the turbulent eddy viscosity of turbulent shear stress is described by the product of a characteristic length (λ) and velocity, where λ was found to be related to H/h (H is the flow depth and h is the height of vegetation) via an empirical equation fitted to experimental data. Afterward, Meijer & Van Velzen (1999) further studied this model and recommended that λ could be approximated as $0.0144\sqrt{Hh}$ after examining more data. Similarly, Defina & Bixion (2005) and Baptist et al. (2007) established their analytical solutions of velocity in the vegetation layer in different forms. However, Nepf (2012) proposed a model in a different equation with corresponding empirical parameters.

In the present paper, its aim is to compare the four approaches by Klopstra et al. (1997), Defina & Bixio (2005), Baptist et al. (2007) and Nepf (2012) for predicting the velocity profiles in rigid submerged vegetation against a wide range of separate datasets, in order to evaluate the capability of each model. The 14 datasets used in our study include different submergence (H/h) ranging from 1.25 to 3.33, various vegetation densities (defined by a, the frontal area of the vegetation per unit volume) (a =1.1~18.5 m^{-1}) and bed slopes S_o (4x10^{-4} ~4x10^{-3}).

2. ANALYTICAL MODELS

The analytical models proposed by Klopstra et al. (1997) and Defina & Bixio (2005) are based on the momentum equation under steady uniform flow. For the channel flow with vegetation, the bed and wall boundary stress are both assumed to be negligible compared with the drag force on the vegetation (Nepf & Vivoni, 2000; Stone & Shen, 2002). Therefore, the governing equation for fully-developed 1-D vegetated flow may be described by (Klopstra et al., 1997; Ghisalberti & Nepf, 2004; Defina & Bixio, 2005 and Kubrak et al., 2008) as.

$$\frac{1}{\rho}\frac{\partial\ (z)}{\partial} = F_v - gS_o \tag{1}$$

where τ is the shear stress, g the gravity, z the vertical coordinate above the bed, S_o the bed slope and F_v is the drag force per unit mass generated by the vegetation (see Figure 1). The drag force F_v is given by

$$F_v = \begin{cases} \frac{1}{2} C_D a\, u^2, & z \le h \\ 0, & z > h \end{cases} \quad ; \quad a = mA_v \tag{2}$$

where u is the average velocity, h the height of vegetation, C_D the drag coefficient, a the density of vegetation, i.e. the frontal area of vegetation (A_v) per unit volume, and m is the number of vegetation per unit area.

The shear stress (τ) is described by Boussinesq hypothesis through a mixing length concept as:

$$\tau(z) = \rho\nu_T \frac{\partial}{\partial} = \rho\ \ u\frac{\partial}{\partial} \tag{3}$$

where ν_T is the total eddy viscosity of vegetated flow, and λ is a mixing length.

Under steady flow condition, inserting Eqs. (2) & (3) into Eq. (1) gives:

$$u\frac{\partial^2 u}{\partial\ ^2} + \lambda\left(\frac{\partial}{\partial}\right)^2 = \frac{1}{2}C_D a\, u^2 - gS_o \tag{4}$$

or

$$\lambda\frac{\partial^2(u^2)}{\partial\ ^2} - C_D a\, u^2 + 2gS_o = 0 \tag{5}$$

For given vegetation density (a) and drag coefficient (C_D), an analytical solution for u^2 in Eq. (5) can be obtained under appropriate boundary conditions (e.g. Klopstra et al., 1997; Defina & Bixio, 2005; Baptist el al. 2007). The boundary conditions are considered as follows:

At the bed (z = 0), where the bed shear stress is neglected, the local equilibrium between gravity force and vegetation drag gives:

$$u_o = u|_{z=0} = \sqrt{\frac{2gS_o}{aC_D}} \tag{6}$$

At the top of the vegetation ($z = h$), the boundary shear stress is given by:

$$\tau(z = h) = \rho \ (H - h)S_o \tag{7}$$

2.1 Model by Klopstra et al. (1997)

Following Eq. (5) with boundary conditions (6) & (7), the analytical solutions for the velocity profiles are:

For the vegetation layer:

$$u_z = \sqrt{C_1 e^{-\sqrt{2A}} - C_1 e^{\sqrt{2A}} + {u_0}^2} \tag{8}$$

in which

$$C_1 = \frac{-2gS_o(H-h)}{\lambda\sqrt{2A}(e^{\sqrt{2A}} + e^{-\sqrt{2A}})} \tag{9}$$

$$A = \frac{aC_D}{2\lambda} \tag{10}$$

$$\lambda = 0.0793h\ l\imath\ \left(\frac{H}{h}\right) - \ 0.0009 \qquad \text{when} \quad \lambda \geq 0.001 \tag{11}$$

For the surface layer, the velocity is described by the well-known logarithmic profile:

$$u = \frac{u_*}{\kappa}\ \ln\left(\frac{z-z_m}{z_o}\right) = \frac{u_*}{\kappa}\ \ln\left[\frac{z-(h-h_s)}{z_o}\right] \tag{12}$$

where κ is von Karman's constant, z_m (= $h - h_s$) is the zero-plane displacement of the logarithmic profile, h_s is the distance between the top of vegetation and the virtual bed of the surface layer (see Figure 1), z_o is the equivalent bed roughness height, and u_* is the shear velocity, given by:

$$u_* = \sqrt{gS_o(H-h)} \tag{13}$$

$$h_s = g\,\frac{1+\sqrt{1+\frac{4E^2\kappa^2(H-h)}{g}}}{2E^2\kappa^2} \tag{14}$$

$$z_o = h_s e^{-F} \tag{15}$$

with

$$E = \frac{-C_1\sqrt{2A/S_o}\ e^{h\sqrt{2A}}}{2\sqrt{{u_0}^2 - C_1 e^{h\sqrt{2A}}}} \tag{16}$$

$$F = \frac{\kappa\sqrt{{u_0}^2 - C_1 e^{h\sqrt{2A}}}}{\sqrt{gS_o[H-(h-h_s)]}} \tag{17}$$

2.2 Model by Defina and Bixio (2005)

Based on Eq. (5), with similar boundaries to Eqs. (6) & (7), and using the mixing length concept, Defina & Bixio (2005) proposed another form of velocity profile by introducing a parameter β, which is related to λ/h, as described as follows:

For the vegetation layer:

$$u_z = \sqrt{\frac{2gS_o}{\beta(\lambda/h^2)}\left\{\left(\frac{H}{h} - 1\right)\frac{\mathrm{s}}{\mathrm{c}}\ \frac{\left[\beta\left(\frac{z}{h}\right)\right]}{(\beta)} + \frac{1}{\beta}\right\}} \quad ; \qquad \beta = \sqrt{\frac{C_D a h}{(\lambda/h)}} \tag{18}$$

where λ is recommended as $0.0144\sqrt{Hh}$.

For the surface layer, the velocity is in the same form as Eq. (12) where h_s and z_o are expressed respectively as:

$$h_s = \frac{gS_c + \sqrt{(gS_c)^2 + 4(\kappa \quad /\partial \ |_{z=h})^2 gS_c(H-h)}}{2(\kappa \quad /\partial \ |_{z=h})^2} \tag{19}$$

$$z_o = h_s e^{-\kappa(\frac{u|_{z=h}}{u_*})} \tag{20}$$

2.3 Model by Baptist et al. (2007)

In the vegetation layer, based on Eq. (5), Baptist et al. (2007) related the mixing length (λ) to the length scale of vegetation (l) with a coefficient c_p of turbulence intensity, given by

$$\lambda = c_p \, l \tag{21}$$

If the length scale of vegetation l and the coefficient c_p are assumed constant over the vegetation, the analytical solutions of Eq. (5) can be obtained.
For the vegetation layer:

$$u_z = \sqrt{u_0{}^2 + a_v \exp(\frac{z}{L})} \tag{22}$$

where u_o is given by Eq.(6), and the constants of L and a_v are given by,

$$L = \sqrt{\frac{\lambda}{a\,C_D}} \tag{23a}$$

$$a_v = \frac{2gS_c(H-h)L}{\lambda \mathrm{e} \quad (h/L)} \tag{23b}$$

For the surface layer, the velocity takes the same form as Eq.(12), where h_s and z_o are expressed respectively by:

$$h_s = L(1 - \exp(-h/L)) \tag{24}$$

$$z_o = h_s e^{-\kappa\sqrt{\frac{2L}{\lambda}\left(1+\frac{L}{H-h}\right)}} \tag{25}$$

Baptist et al. (2007) validated their method using the data by Nepf and Vivoni (2000), and recommended λ = (H-h)/20.

2.4 Model by Nepf (2012)

Nepf (2012) divides the velocity profile of the submerged vegetation into three layers. Although the model is supposed to be applicable to both flexible and rigid vegetation, most of the parameters are only proposed for rigid vegetation. Therefore, only the parameters for the rigid vegetation are given here. For the upper non-vegetation layer, a logarithmic profile is assumed, like Eq. (12), where z_m and z_0 are given by:

$$z_m = h - \frac{0.1}{a \quad _D} \tag{26}$$

$$z_o = (0.04 \pm 0.02)a^{-1} \tag{27}$$

where z_o in Eq. (27) is recommended for $ah > 0.1$; otherwise, z_o is estimated to be $0.1h$ (Nepf and Ghisalberti, 2008).
The vegetation layer is divided into two zones depending on the penetration of the turbulence stresses from the surface layer into the vegetation layer. An empirical equation is proposed for estimating the depth of turbulent shear penetration (□) into the vegetation layer for (ah) values of (0.2-3) as:

$$\delta = \frac{0.2 \ \pm 0.6}{a \ \ _D} \tag{28}$$

Therefore, in the upper region of the vegetation layer (h-δ < z < h), the velocity profile is assumed to be exponential, where the flow is driven by gravity and turbulent stresses and balanced by vegetation drag. The turbulent stresses are modelled using a mixing length concept and the following is obtained:

$$u = u_2 + (u_h - u_2)\exp[-k_u(h - z)] \tag{29}$$

$$k_u = (8.7 \pm 1.4)a \ \ _D \tag{30}$$

where $u_h = u$ at the top of the vegetation, and u_2 = velocity in the lower region of vegetation ($z < h$-δ), given by Eq. (6), as the momentum equation is assumed to be balanced between gravitational and vegetation drag forces, where the bed drag and viscous and turbulent stresses are neglected.

3. DATA USED FOR STUDY

To compare the four models in Section 2, we used a wide range of different experimental data for submerged rigid vegetation from the literature. A total of 14 datasets used cover various relative depths of submergence (H/h), which are 1.25 ~ 3.33, the vegetation densities a =1.1~ 18.5 m^{-1}, and bed slopes S_o ranged from 4×10^{-4} to 4×10^{-3}. The details of the datasets are given in Table 1, where the data of Nguyen (2012) cover a wide range of vegetation density and various diameter sizes of vegetation. The values of C_D in the dataset of emergent vegetation were estimated for all datasets. It should be mentioned that all the tests were carried out in laboratory flumes.

4. RESULTS AND DISCUSSIONS

4.1 Comparison of results between the models

For convenience of comparison, the models by Klopstra et al. (1997), Defina & Bixio (2005), Baptist et al. (2007) and Nepf (2012) are denoted as Klopstra, Defina, Baptist and Nepf models respectively in the subsequent figures and context. The comparisons between the four models are given in Figures 2-4, which show that all the models can predict profiles reasonably well in the vegetation layer, particularly close the channel bed, except in the cases of shallow submergence (H/h < 1.5 defined by Nepf (2012)), e.g. Hao et al. data. For the surface layer, the Defina model over-estimates the velocity, the Klopstra model under-estimates, whereas the Nepf and Baptist models lie between them and agree with the data reasonably well, see the data by Dunn et al. (1996), Huai et al. (2009) and Lopez & Garcia (2001) in Figures 2 and 3, and in Figure 4 by Nguyen (2012) when the density of vegetation (a) is not very high (e.g. a < 7.1 m^{-1}). However, if the density of vegetation is very high, e.g. a > 7.1 m^{-1}, it appears that in the surface layer, the velocity predicted by the Nepf model is larger than those given by the other three models where the results by the Defina and Baptist models are close, and the Klopstra model gives the lowest velocity.

Close examination of the measured velocity profiles in the vegetation layer shows that the velocity starts to increase exponentially or linearly from distances very close to the channel bed, because in such a region the flow is driven by the turbulent stress and pressure gradient with the balancing force of the vegetation drag, and thus this may be evidence of deep penetration by the turbulent stresses. In this case, the dispersive stress might have become more important because the flow field in the vegetation zone is expected to become relatively more turbulent due to a deep penetration of the turbulent stresses.

Although the Klopstra, Defina and Baptist models have similar assumptions, the Baptist model performs better than any of the Defina and Klopstra models in most cases. This may be due to their different λ models used. It was found that if the same λ model is used, the predicted velocity profiles by the Klopstra and Defina models are almost the same, but slightly different from that by the Baptist model, because z_0 and z_m proposed by three models are not the same, as clearly shown in Eqs. (8), (18) and (22) respectively.

The Nepf model does not perform well in certain cases, e.g. in the cases of very high density of vegetation (A30, B30 and C30 of Nguyen's data). This is mainly due to the limitation of Eq. (29) for predicting the velocity profile in the upper layer of vegetation, which indicates that velocity in the upper region of vegetation depends on two velocity predictions; one at the top of vegetation and the other in the lower part of the vegetation. Therefore, if these two values deviate from the measured ones significantly, this will then result in the prediction of velocity. Because the equation proposed for the lower part of vegetation (i.e. u_2) neglects the dispersive, turbulent and viscous stresses and, as mentioned before, in the experiments where the vegetation is deeply penetrated by the turbulent stress, the dispersive stress might become significant. Bearing in mind that this model is based on the empirical equations for estimation of h_s and z_o, which depend on C_D and the vegetation density (a), further study is needed.

4.2 Further analysis of Klopstra, Defina & Baptist models

Through close examination of the Klopstra, Defina and Baptist models, it was found that their differences of velocity prediction between the Klopstra and Defina are almost the same, but they are less than the prediction by the Baptist model, if the same λ assumption is used, see an example in Figure 5, where $\lambda = 0.05(H\text{-}h)$. Because both z_0 and h_s are important to the velocity profile in the surface layer, they are also closely related to the λ assumption in the models, where λ is the eddy mixing length of vegetated flow. Since both flow depth (H) and vegetation height (h) will have impact on λ, and it was found that λ can be approximated by $k\sqrt{Hh}$. The predicted velocity by all three models decreases with increasing value of constant k. Close examination of all the data studied here found that k=0.02 in all models gives the best agreement with the data.

Table 1. The dataset used for evaluating the models of submerged rigid vegetation

Author	Run	Flow depth H (m)	Vegetation height h (m)	H/h	Frontal area a (m^{-1})	C_D	S_o	ah
Dunn et al. (1996)	8	0.391	0.1175	3.33	2.46	1.13	0.0036	0.289
	9	0.214	0.1175	1.82	2.46	1.13	0.0036	0.289
Lopez & Garcia	01	0.335	0.12	2.79	1.09	1.13	0.0036	0.131
(2001)	09	0.214	0.12	1.78	2.49	1.13	0.0036	0.299
Huai et al. (2009)	R1	0.291	0.19	1.53	1.20	1.0	0.0004	0.228
	R2	0.383	0.19	2.02	1.20	1.0	0.0004	0.228
Nguyen (2012)	A60-15	0.15	0.10	1.5	1.78	1.13	0.004	0.178
	A30-15	0.15	0.10	1.5	7.11	1.13	0.004	0.711
	B60-15	0.15	0.10	1.5	3.67	1.13	0.004	0.367
	B30-15	0.15	0.10	1.5	14.67	1.13	0.004	1.467
	C60-15	0.15	0.10	1.5	4.61	1.13	0.004	0.461
	C30-15	0.15	0.10	1.5	18.44	1.13	0.004	1.844
Hao et al. (2014)	Test 1	0.10	0.08	1.25	1.355	1.13	0.0035	0.108
	Test 2	0.11	0.08	1.38	1.355	1.13	0.004	0.108

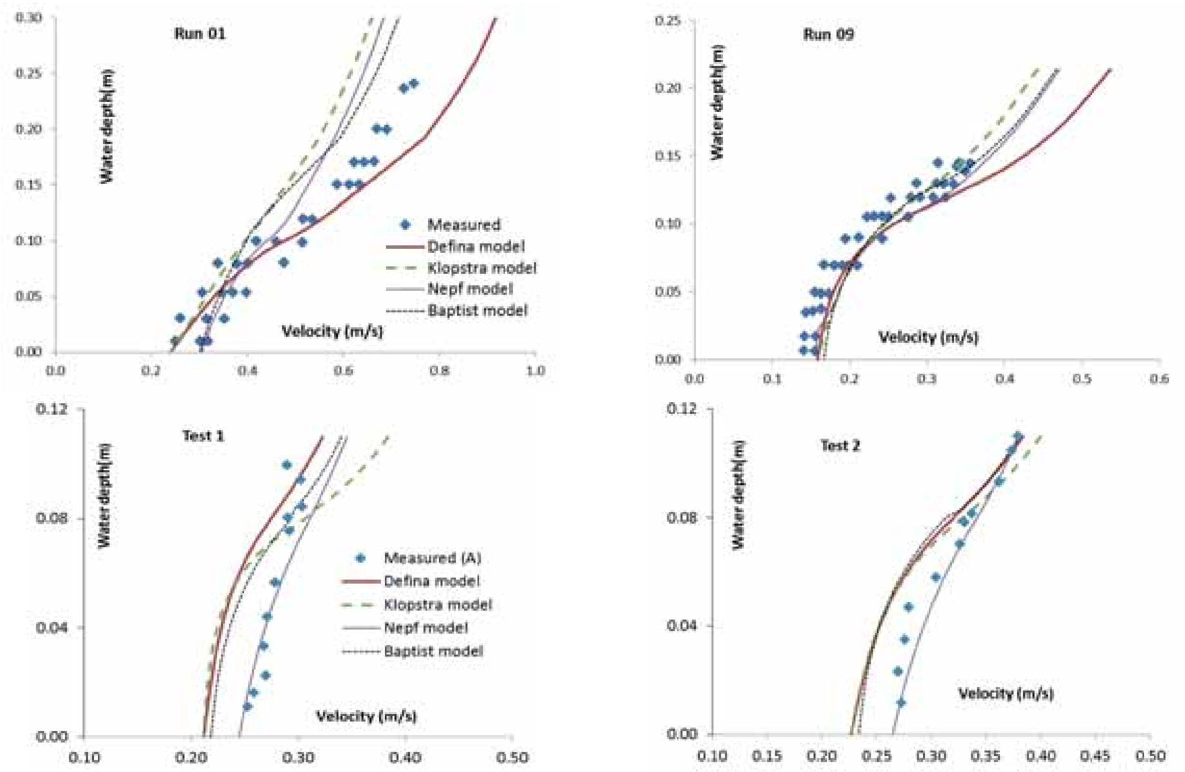


Figure 2. Prediction of velocity profile for the data by Lapez & Garcia (2001): 01 & 09 and Hao et al. (2014): 1 & 2

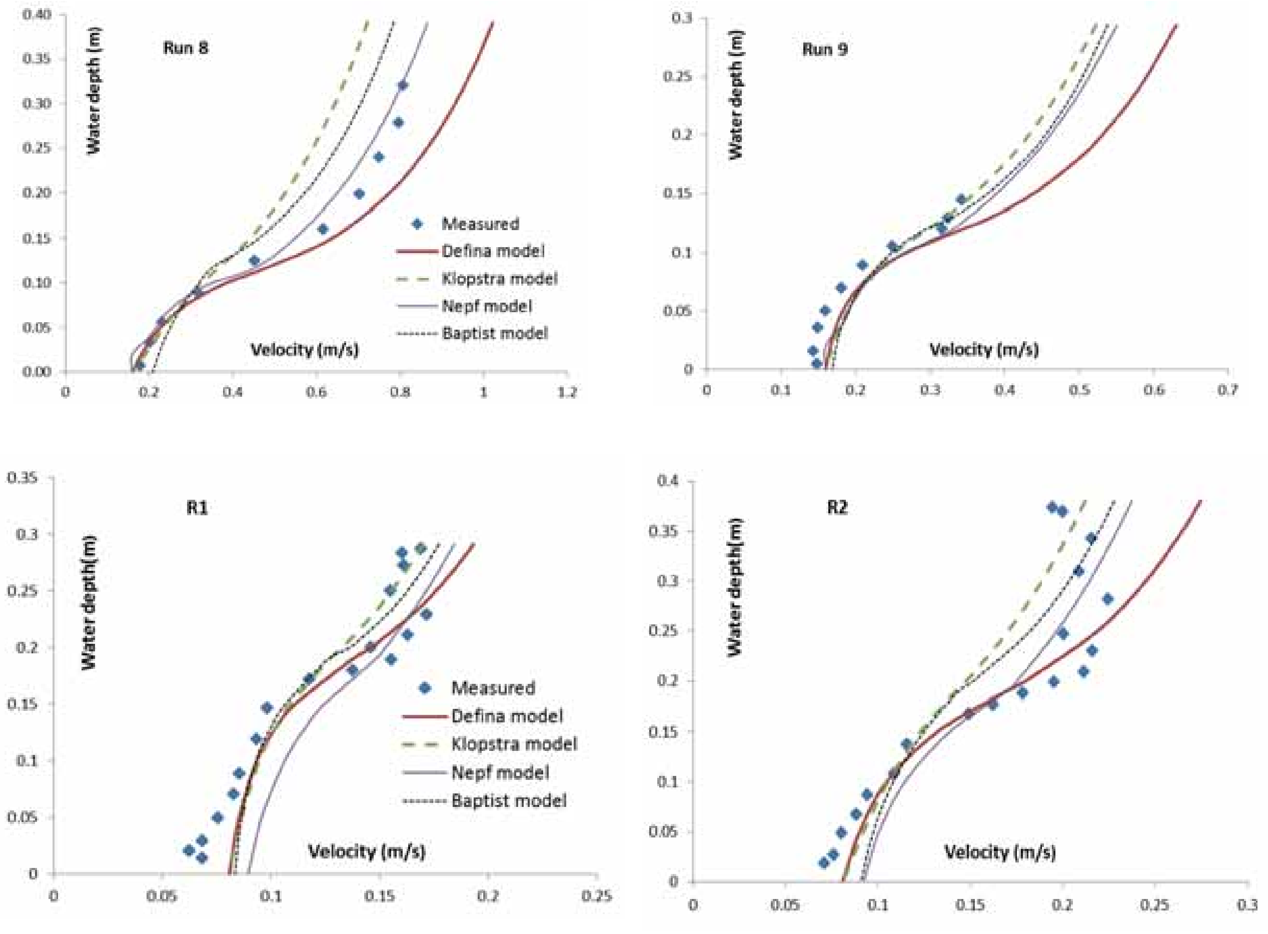
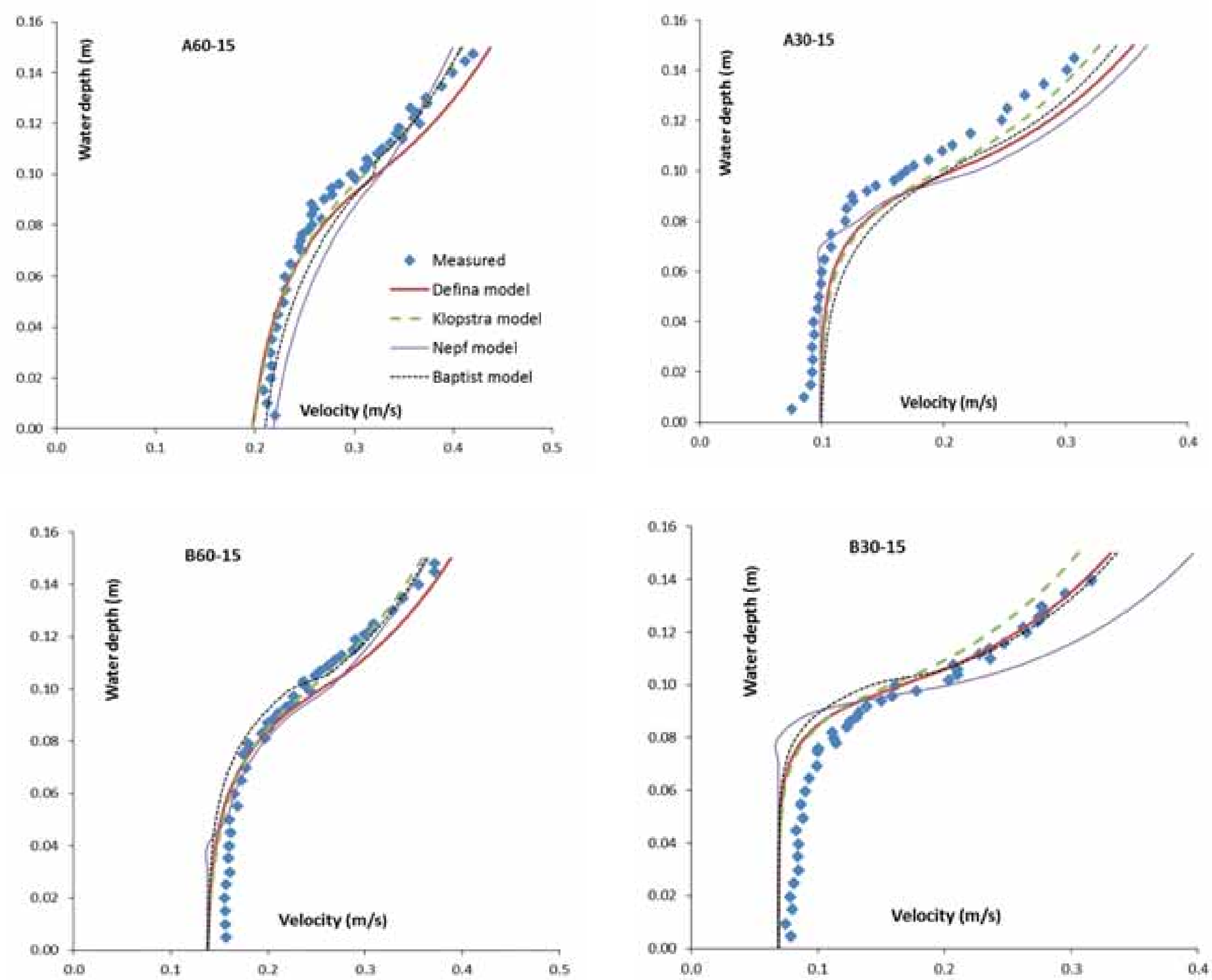


Figure 3. Prediction of velocity profile for the data by Dunn et al. (1996): 8 & 9 and Huai et al. (2009): R1 & 2.

5. CONCLUSIONS

Vegetation significantly affects the mean velocity profiles. Within submerged vegetation, the vertical velocity profile in the vegetation region is significantly different from the non-vegetation surface region. Through comparison of four commonly used two-layer models by Klopstra et al. (1997), Defina & Bixio (2005), Baptist et al. (2010) and Nepf (2012), it was found that all the models can predict the velocity reasonably well in the vegetation layer close to bed if *ah* >1.25 or *a* < 7. In the surface layer, all the models are capable of predicting velocities well to a certain degree over a range of vegetation densities using their proposed values for the parameters when *ah* >1.25 or *a* < 7. Generally, the Defina model significantly over-estimates the velocity, whereas the Klopstra model under-estimates the velocity, and between them are the Baptist and Nepf models, which are close in most cases. Although the Nepf model can predict reasonably well for some cases with lower density of vegetation (up to a limit of *ah* about 0.35), the model will significantly over-estimate velocity when *ah* is large (> 0.7). The Baptist model has shown overall best prediction for a relatively wide range of *ah*.

Close examination of the Baptist, Defina and Klopstra models shows that the velocity predictions by the Baptist and Defina models are very close if the same parameter λ (mixing length) is used, and the prediction by Baptist is slightly larger than the latter two. It was also found that when λ can be approximated by $k\sqrt{Hh}$ with *k* having an optimum value of 0.02, then all three models can predict the velocity profiles reasonably well for a wide range of vegetated flows tested in this study. More data are needed to evaluate the new recommended value of *k*.



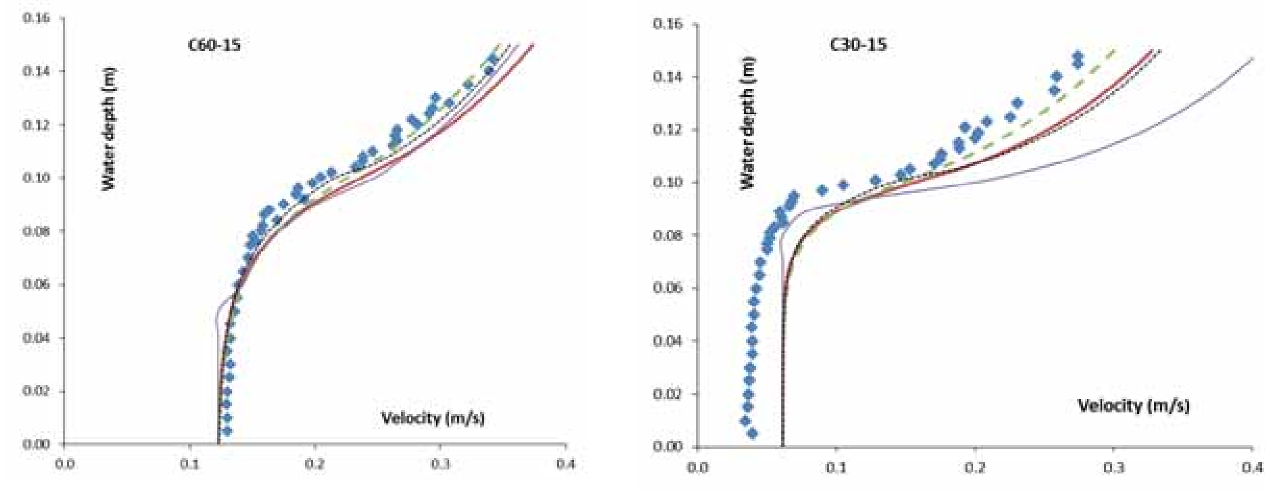


Figure 4. Prediction of velocity profile for the data by Nguyen (2012).

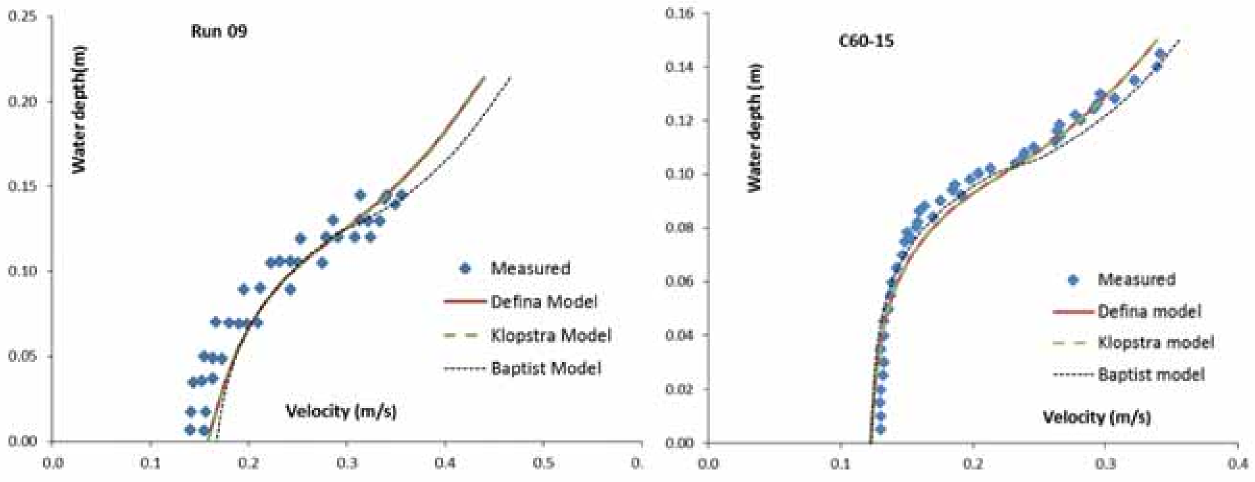


Figure 5. Prediction of velocity profile for the data by Dunn et al. (1996): 8 and Nguyen (2012):C60-15.

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