PHYSICAL REVIEW D **92,** 045006 (2015)

Geometric constructions of two-dimensional (0,2) SUSY theories

Radu Tatar[*]

Division of Theoretical Physics, Department of Mathematical Sciences, The University of Liverpool, Liverpool L69 3BX, United Kingdom

(Received 1 July 2015; published 5 August 2015)

We consider the field theories on multiple stacks of D5-branes wrapped on four cycles of resolved/deformed conifold geometries fibered over a two-torus. The central charges of the D5-branes are slightly misaligned when the branes are wrapped on various rigid holomorphic two-cycles or when they have different charges with respect to a magnetic flux turned on the two-torus. The wrapped D5-branes preserve (0, 2) supersymmetry in two dimensions if the Kahler moduli and the magnetic flux are related. Our geometries are T-dual to the brane configurations considered by Kutasov-Lin, and we provide a geometric interpretation for their equality between the field theory D-terms and the magnetic fluxes. We also consider the geometric transitions for rigid holomorphic two-cycles fibered over a two-torus with magnetic flux and discuss the partial breaking of supersymmetry after the geometric transition.

DOI: 10.1103/PhysRevD.92.045006 PACS numbers: 11.10.Kk, 11.25.Tq

I. INTRODUCTION

The supersymmetric field theories enjoy some elegant descriptions in string theory compactifications. One successful direction of research studies geometric transitions which map wrapped brane setups into flux configurations, as proposed in [1] and extended in [2]. The transition can also be understood by studying matrix models which allow perturbative insights into nonperturbative physics [3]. A configuration with wrapped antibranes can also provide supersymmetric configurations before and after the transition [4]. A natural generalization to hybrid system of branes and antibranes was considered in [5] to tackle the problem of D-term supersymmetry breaking.

Soon after the geometric transition was described by studying D5-branes wrapped on two-cycles, a T-dual picture was proposed where the wrapped D-branes are mapped into D-branes suspended between various types of NS branes [6]. The brane picture allows a lift to M theory and the use of the MQCD approach to obtain details about the geometric transitions. The configuration of D4 and NS branes is lifted as a unique M5 brane which splits into a collection of simpler M5 branes after the geometric transition [6].

Recently there has been an increasing interest in using branes and geometry to study two-dimensional field theories. A class of interesting theories are the chiral (0,2) SUSY theories in two dimensions. The first brane construction was proposed some time ago and involved three sets of orthogonal NS branes [7]. More recently, two-dimensional (0,2) theories emerged from compactifications of six-dimensional theories on four-manifolds with a partial topological twist [8]. This led to the realization of some interesting two-dimensional triality as an IR equivalence between three different theories [9]. Other developments include twisted compactifications of the four-dimensional Leigh-Strassler fixed point on closed hyperbolic Riemann surfaces [10] and a Pfaffian description [11].

An alternative approach was proposed in [12,13] utilizing brane configurations with color D4-branes and flavor D6-branes suspended between orthogonal NS branes. The corresponding four-dimensional $\mathcal{N}=1$ supersymmetric field theories were further compactified on a two-torus to yield (2,2) SUSY two-dimensional theories. A D-term for the field theory on the D4-branes (either color or flavor groups) and a magnetic flux on the two-torus were added as extra ingredients representing rotations and displacements of various D4-branes and NS branes. This leads generically to SUSY breaking but a fine tuning for the D-term and the magnetic flux can conspire to partially preserve some supersymmetry, in particular (0,2) SUSY in two dimensions.

In this work, our goal is to study the geometric picture arising from T-dualizing the brane configuration of [12]. The T-duality leads to multiple stacks of D5-branes wrapped on $\mathbf{P}^1$ cycles or noncompact holomorphic cycles. To obtain a two-dimensional theory, we fiber the resolved conifold geometries over a two-torus and reinterpret the setup as wrapped D5-branes on $\mathbf{P}^1$ fibers over the T^2. After turning on a D-term on the $\mathbf{P}^1$ fiber (making $\mathbf{P}^1$ cycle rigid), the central charges of the branes become misaligned [5] which potentially leads to supersymmetry breaking. On the other hand, turning on a magnetic flux through the two-torus could also break supersymmetry. We discuss how these two types of SUSY breaking can compensate each other and partially preserve the SUSY for branes wrapped on four-cycles inside $SU(4)$ structure manifolds (when an extra NS flux is present). We consider the SUSY condition

[*]rtatar@liverpool.ac.uk

for wrapped D5-branes on two-cycles and four-cycles of $SU(3)$ and $SU(4)$ holonomy manifolds derived in [14] and replace the Kahler two-form J with its complexified version. The SUSY condition becomes an equality between the Kahler form and the magnetic flux through the two-torus base, which represents a geometric interpretation of the equality between the D-term and the magnetic flux proposed in [12,13].

In Sec. II, we start by reviewing the geometric D-term SUSY breaking considered in [2,5]. For a single stack of D5-branes, a SUSY configuration can be obtained even in the presence of D-terms/rigid cycles but this is not true for D5-branes wrapped on arbitrary rigid $\mathbf{P}^1$ cycles or noncompact two-cycles. We also consider the boost supergravity solution described in [15] and discuss the gauge coupling constant on wrapped D5-branes. In Sec. III, we review the proposal of [16] to build Calabi-Yau fourfolds as resolved/deformed conifolds fibered over a genus g base and we restrict to the case $g = 1$.

In Sec. IV, we consider the unbroken supersymmetry condition for D5-branes wrapped on rigid $\mathbf{P}^1$ cycles fibered over T^2 with magnetic flux. Our main claim is that the condition of SUSY preservation is satisfied when D5-branes wrap Kahler calibrations and the Kahler moduli and the magnetic field are related, reproducing the condition derived in [12]. In Sec. V we consider the geometric transition inside the SU(4) structure manifolds, in the presence of nonzero D-terms and magnetic fluxes. After the transition, the color D5-branes are replaced by fluxes through various $S^3 \times S^1$ cycles and the gluino condensates are equal to the integrals of the holomorphic four-form on such cycles. The flavor degrees of freedom lie on D5-branes wrapped on noncompact two-cycles, which remain unchanged during the geometric transition. The cancellation between the global symmetry D-term and the magnetic field remains valid during the transition and assures SUSY preservation.

II. D-TERMS FOR WRAPPED D5-BRANES

A. The geometry of D-terms

We start by reviewing the geometric interpretation of the D-terms for $\mathcal{N} = 1, d = 4$ field theories. Consider a resolved conifold and wrap some D5-branes on the nonrigid $\mathbf{P}^1$ cycle. The gauge coupling is

$$\frac{4\pi}{g_{\rm YM}^2} = \frac{b_{\rm NS}}{g_s} \tag{1}$$

where $b_{\rm NS}$ is the integral over $\mathbf{P}^1$ of the NS two-form field on the D5-branes.

In addition, we can turn a small nonzero Fayet-Iliopoulos parameter ξ for the $U(1)$ center of the gauge group which contributes to the Lagrangian with a term $\sqrt{2}\xi{\rm TrD}$. Its geometric interpretation was provided in [5], where it was associated to turning on the real part j of the complexified Kahler class of the $\mathbf{P}^1$ cycle. The central charge for wrapped D5-branes is the integral of the complexified Kahler form

$$Z = \int_{S^2} (J + iB_{\rm NS}) = j + ib_{\rm NS}, \tag{2}$$

where j is related to ξ by

$$\xi = \frac{j}{4\pi g_s}. \tag{3}$$

For $j \neq 0$, the phase of the central charge is modified and the supersymmetry appears to be broken due to the presence of the Fayet-Iliopoulos term. Nevertheless, this is not necessarily true for any $j \neq 0$ [2]. For a single set of wrapped branes on a $\mathbf{P}^1$, the theory has an alternative SUSY description with a bare coupling constant related to the quantum volume of the resolution $\mathbf{P}^1$ cycle as

$$\frac{4\pi}{g_{\rm YM}^2} = \frac{\sqrt{b_{\rm NS}^2 + j^2}}{g_s}. \tag{4}$$

For a product group obtained on several stacks of D5-branes wrapped on different $\mathbf{P}^1$ cycles, we have the freedom to turn different values for j on each of the $\mathbf{P}^1$ cycles. For two stacks of branes wrapped on $\mathbf{P}^1$ cycles with $j_1 \neq j_2$, the central charges have different phases, they cannot align and the supersymmetry is broken [5].

B. The supergravity interpolating solution

The variation of the parameters J and $B_{\rm NS}$ for the wrapped D5-branes was studied in supergravity by many authors [15,17–19]. [17,18] considered a flow between a Maldacena-Nunez solution [20] and a Klebanov-Strassler solution [21]. The Maldacena-Nunez solution corresponds to large values for J and zero $B_{\rm NS}$ whereas the Klebanov-Strassler solution is valid for zero J (fractional branes) and nonzero $B_{\rm NS}$. The solution involves a reduction of ten-dimensional spinors $\epsilon_i, i = 1, 2$ to six-dimensional spinors $\eta_+^i, i = 1, 2$ which are related to the SU(3) invariant spinors η_+ as [17]

$$\eta_+^1 = \frac{1}{2}(\alpha + \chi)\eta_+; \qquad \eta_+^2 = \frac{1}{2i}(\alpha - \chi)\eta_+. \tag{5}$$

The choice $\alpha = 0$ (or $\chi = 0$) corresponds to the Maldacena-Nunez solutions and $\alpha = \pm i\chi$ corresponds to the Klebanov-Strassler solution. The interpolating solution is parametrized by a phase ω related to χ and α as $\chi = i\sin(\omega/2); \alpha = \cos(\omega/2)$. The relation (5) becomes

$$\eta_+^1 = ie^{i\omega}\eta_+^2. \tag{6}$$

We now compare (6) with the supersymmetry condition obtain for D5-branes wrapped on a two-cycle of an $SU(3)$ structure manifold (when NS flux is present). The corresponding relation between η^i_+ was considered in [14] for $SU(3)$ holonomy and extended in [22] to $SU(3)$ structure manifolds as

$$\eta^1_+ = -e^{-i\rho}\eta^2_+, \tag{7}$$

where ρ is a geometric parameter. We see that the supergravity parameter ω and ρ are related as $\omega = \pi/2 - \rho$.

The flow of [17] was reinterpreted in [15] as starting with D5-branes with no NS flux and performing a boost which provides some NS flux, after a series of S and T dualities. This approach was subsequently used by [19,23] to describe wrapped D5-branes on a resolved conifold. It was argued that the SUSY preservation implies that the D5-branes should wrap a cycle inside a non-Kahler deformation of the resolved conifold. When the dilaton is constant, the IIB configuration of [15,19] implies the following form for the RR and NS three-forms:

$$H_{\rm RR} = \cosh\beta *_6 dJ, \qquad H_{\rm NS} = -\sinh\beta dJ, \tag{8}$$

where the Hodge star is with respect to the non-Kahler metric on the resolved conifold. The supersymmetry is preserved if the $G_3 = H_{\rm RR} - ie^{\phi}H_{\rm NS}$ flux is of the (2,1) form. For a complex internal manifold, the dilaton is constant $\phi = \phi_0$ and the complex structure is provided by

$$\gamma = e^{\phi_0}\mathrm{cotanh}\beta, \tag{9}$$

where γ was introduced in [19] in the definition of the complex forms needed to separate the (2,1) and (1,2) pieces of the fluxes. We repeat the steps of [15] in the case of two stacks of D5-branes. We start with two sets of D5-branes wrapped on two $\mathbf{P}^1$ cycles and compactify three extra coordinates of the D5-branes into a three-torus and T-dualize along them to obtain two sets of D2-branes wrapped on $\mathbf{P}^1$ cycles. We lift this configuration to M theory and get two stacks of M2-branes. The configuration is compactified on a seven-dimensional manifold whose base is the resolved geometry with two $\mathbf{P}^1$ cycles. We now perform the boost of [15]:

$$t \to \cosh\beta t - \sin\beta x_{11}, \qquad x_{11} \to -\sinh\beta t - \cos\beta x_{11}. \tag{10}$$

After reducing back to type IIA and reversing the three T-dualities, we reach a type IIB solution with two stacks of D5-branes wrapped on $\mathbf{P}^1$ cycles. The calibration condition becomes

$$B_{\rm NS} = \sinh\beta \quad e^{-2\phi}J. \tag{11}$$

When integrating (11) over the two $\mathbf{P}^1$ cycles for a constant dilaton, the supersymmetry condition implies that $b_i = \int_{\mathbf{P}^1_i} B_{\rm NS}$ and $j_i = \int_{\mathbf{P}^1_i} J$ are related as

$$b_i = \sinh\beta \quad e^{-2\phi_0}j_i \to \frac{b_1}{j_1} = \frac{b_2}{j_2}. \tag{12}$$

On the other hand, the central charges on the two stacks of D5-branes are $j_i + ib_i$ so the equality (12) implies that the phases of the central charges are equal. For generic values of j_i, b_i, the condition (12) is not satisfied and the supersymmetry is broken.

C. Gauge coupling constant on the wrapped D5-branes

The interpolating supergravity solution (5), (6) constructed in [17] involves a parameter ω which is related to the boosting parameter β as

$$\cos\omega = -\tanh\beta \quad e^{\phi}. \tag{13}$$

We now consider the gauge coupling constant on D5-branes wrapped on $\mathbf{P}^1$ cycles. In [15] this was related to ω by looking at the superpotential after the geometric transition. Here we discuss the case before the geometric transition. If we replace J by its complexified version $J + iB$, reconsider the compactification manifold as an $SU(3)$ structure manifold and use (7), we get the second condition to preserve SUSY for a D5-brane wrapped on the $\mathbf{P}^1$ cycle [14]

$$(J + iB_{\rm NS})e^{-i\rho} = \mathrm{vol}_{\mathbf{P}^1}. \tag{14}$$

The angle ρ is a parameter, as defined in equation (7). In order to obtain a real left-hand side in equation (14), $J + iB_{\rm NS}$ should have a phase equal to ρ so we can identify the parameter as $J\tan(\rho) = B_{\rm NS}$.

After integrating over $\mathbf{P}^1$, the left-hand side of (14) becomes

$$j\cos\rho + b\sin\rho = \sqrt{j^2 + b^2}, \tag{15}$$

which is the inverse of the gauge coupling constant for D5-branes wrapped on a rigid $\mathbf{P}^1$.

D. T-dual picture

What happens when we perform a T-duality along the angular direction of the $\mathbf{P}^1$ cycle? The singular lines inside the resolved conifold are replaced by two orthogonal NS branes and the value of the D-term maps into some extra separation between the NS branes and a rotation of the D4-branes. If the NS branes extend in the directions (012345) and (012389), $j = 0$ corresponds to NS branes being separated only in the x^6 direction. $j \neq 0$ adds an extra displacement of the NS branes in the direction x^7 and a rotation of the D4-branes in the (x^6, x^7) plane.

When starting with two stacks of D5-branes wrapped on two rigid $\mathbf{P}^1$ cycles, the T-dual configurations contains three NS branes separated in the (x^6, x^7) plane with two stacks of D4-branes between them. For different values of j_1, j_2, the two stacks of D4-branes are rotated by different angles, which signals SUSY breaking. When one of the $\mathbf{P}^1$ cycles is replaced with a noncompact holomorphic two-cycle, we get a gauge group with flavors. In the T-dual picture, flavor groups are represented by semi-infinite D4-branes, rotated in the (x^6, x^7) plane when $j \neq 0$. Turning on a Fayet-Iliopoulos term implies a rotation of the flavor (semi-infinite) D4-branes with respect to the color (finite) D4-branes in the (x^6, x^7) plane and the supersymmetry is generically broken. The configuration with rotated semi-infinite D4-brane and unrotated finite D4-branes maps into the D6-brane picture of [12] via a Hanany-Witten brane creation effect [24].

Supersymmetry is broken when the relation (12) is not satisfied, as it happens for arbitrary j_i, b_i. To remove the requirement (12), we uplift the resolved conifold geometry to a resolved conifold fibration over a two-torus T^2. The D5-branes wrap a $\mathbf{P}^1$ fibration over T^2 and we add some magnetic flux on the T^2 base which combines with the rigidity parameter j to provide a SUSY configuration. Such constructions (without magnetic flux and D-terms) were introduced in [16] and we consider these geometries in the next section.

III. D5-BRANES WRAPPED ON A FOUR-CYCLE INSIDE CY FOURFOLDS

We briefly review the setup of resolved/deformed conifold fibrations over T^2 proposed in [16]. We consider D5-branes wrapped on four-cycles inside Calabi-Yau four-folds described as fibrations over genus g surfaces. We start with a conifold represented by the equation

$$x_1 x_2 - x_3 x_4 = 0, \tag{16}$$

in terms of the complex coordinates $x_i, i = 1, \ldots, 4$. When the coordinates x_i are line bundles $\mathcal{L}_i$ over a curve C, the eight-dimensional manifold becomes a Calabi-Yau fourfold X_s in the five-dimensional complex variety $\mathcal{L}_1 \oplus \mathcal{L}_2 \oplus \mathcal{L}_3 \oplus \mathcal{L}_4 \to C$.

The singular Calabi-Yau fourfold X_s can be made smooth by either a small resolution (a fourfold denoted X_r) or by a deformation along the curve C (a fourfold denoted X_d).

A. Resolved conifold side

Consider X_r an $O(-1) \oplus O(-1) \to \mathbf{P}^1$ fibration over a genus g curve. The $\mathbf{P}^1$ fibration over C gives rise to a compact two complex dimensional surface S with Euler characteristic $\chi(S) = 4 - 4g$. If the line bundle $\mathcal{L}_1 \otimes \mathcal{L}_4^{-1}$ has degree n, the volume of S is

$$\mathrm{Vol}(S) = \frac{n}{2}(J^F)^2 + J^F J^C, \tag{17}$$

where J^F and J^C measure the volumes of the $\mathbf{P}^1$ fiber and of the curve C. If we wrap D5-branes on S, we get a two-dimensional field theory with reduced supersymmetry.

There are some other two-cycles in the $O(-1) \oplus O(-1) \to \mathbf{P}^1$ fiber. The small resolution is covered by two copies of $\mathbf{C}^3$ with coordinates Z, X, Y and Z', X', Y' respectively. One can define two types of noncompact holomorphic cycles $\tilde{B}_1 : Y = 0, X = m$ or $\tilde{B}_2 : Y' = 0, X' = M$. If we wrap D5-branes on the noncompact holomorphic cycles, they will correspond to massive flavor (if wrapped on $\tilde{B}_1$) or flavor with expectation value (if wrapped on $\tilde{B}_2$). The $\tilde{B}_1$ fibration over C is a noncompact two complex dimensional surface B_1 and the $\tilde{B}_2$ fibration over C is a noncompact two complex dimensional surface B_2. If we wrap D5-branes on B_1 or B_2, we obtain two-dimensional flavor fields.

B. Deformed conifold side

After a geometric transition on the fiber, a Calabi-Yau fourfold X_d is obtained by deforming the singular fiber of X_s into a deformed conifold with a deformation parameter ϵ:

$$xy - uv = \epsilon. \tag{18}$$

The singular point is replaced by a three-cycle S^3 and its Poincaré dual P. There are several types of nontrivial four-cycles inside X_d:

(i) $2g$ four-cycles $D_n, n = 1, \ldots, 2g$ of topology $S^1 \times S^3$ generated by transporting the S^3 fibers along the nontrivial $2g$ cycles of the base.

(ii) $2g - 3$ four-cycles of topology S^4.

C. Our case: Two-torus (g = 1)

In this work we consider a two-torus base and choose the value of n is (17) to be zero. This simplifies the problem because

(i) the volume of the surface S in the resolved conifold is $J^F J^C$

(ii) there is no cycle of topology S^4.

For each deformation S^3 cycle, we get two $S^1 \times S^3$ cycles which we denote by D_1, D_2 and their Poincaré duals $\tilde{D}_1, \tilde{D}_2$.

The four-cycles B_1 and B_2 which correspond to massive flavors or flavors with expectation values also exist in the deformed geometry.

IV. SUPERSYMMETRIC CONFIGURATIONS WITH D-TERMS AND MAGNETIC FLUXES THROUGH T^2

Consider D5-branes wrapped on various $\mathbf{P}^1$ or $\tilde{B}_i$ fibrations over a two-torus. We are interested to have a $U(N_c) \times U(N_f)$ theory with $U(N_c)$ a gauge group and $U(N_f)$ a global flavor group. In this work we only consider the case of a D-term for a $U(1)$ subgroup of $U(N_f)$. In the absence of magnetic flux through the two-torus, the resulting eight-dimensional manifold inherits the $SU(3)$ holonomy from the resolved conifold. The theory on each D5-brane is two-dimensional with a (2,2) supersymmetry. The four-cycle $T^2 \times \mathbf{P}^1$ is holomorphic and the coupling constant of the two-dimensional field theory obtained on each D5-branes wrapped on $T^2 \times \mathbf{P}^1$ is

$$\frac{1}{g_{(2,2)}} = \frac{b_{\mathrm{NS}} A_{T^2}}{g_s}, \tag{19}$$

where b_{NS} is the integral of B_{NS} through the nonrigid $\mathbf{P}^1$ cycle and A_{T^2} is the area of the two-torus. Due to the presence of the NS flux, the four-cycle $\mathbf{P}^1 \times T^2$ is a generalized holomorphic cycle embedded in a $SU(3)$ structure manifold.

In the absence of magnetic fluxes and in the limit $\omega = 0$ for the four-dimensional $N = 1$ SUSY theory, the relation (6) becomes

$$\eta_+^1 = i\eta_+^2, \tag{20}$$

which is just the limit $\theta = 0$ of the unbroken supersymmetry condition for D-branes wrapped on a four-cycle [14]. We conclude that the $\theta = \omega = 0$ solution (19) fits the SUSY condition for D5-branes wrapped on four-cycles inside $SU(3)$ structure manifolds. We now vary θ and ω. When θ is arbitrary for $\omega = 0$, the supersymmetry is preserved and remains (2,2) in two dimensions. For arbitrary θ and ω, the supersymmetry is generically broken.

A. SUSY and magnetic fluxes

We first consider the case of a constant $\omega = 0$ and an arbitrary θ. This correspond to having $J = 0$ (infinite boost) and a vector potential $A_2 = Mx^1$ on the two-torus. If the D5-branes are only wrapped on a two-torus with magnetic flux, the SUSY conditions imply a relation between the spinors η_+^i containing a parameter θ: $\eta_+^1 = -e^{-i\theta}\eta_+^2$ and one involving two-forms,

$$\mathrm{vol}_{T^2} + iM = e^{i\theta}\frac{\sqrt{|g+M|}}{\sqrt{|g|}}\mathrm{vol}_2, \tag{21}$$

where vol_{T^2} is the volume form of the two-torus and M is a two-form.

We now consider wrapping the D5-branes on a trivial $\mathbf{P}^1$ fiber over T^2, with B_{NS} on $\mathbf{P}^1$. The trivial $\mathbf{P}^1$ fiber over T^2 is a four-cycle. For $\omega = 0$ (infinite boost), we have $J = 0, B_{\mathrm{NS}} \neq 0$. The right-hand side of (21) becomes the volume of a four-cycle, whereas the left-hand side of (21) is multiplied by iB_{NS}. We insert the factor i in the relation between the η_+^i, which becomes $\eta_+^1 = ie^{-i\theta}\eta_+^2$. This is exactly the unbroken supersymmetry condition for D-branes wrapped on four-cycles of SU(3) holonomy manifolds [14] (modified to SU(3) structure in the presence of B_{NS}). The supersymmetry condition (21) becomes

$$B_{\mathrm{NS}} \wedge (\mathrm{vol}_{T^2} + iM) = e^{i\theta}\frac{\sqrt{|g+M|}}{\sqrt{|g|}}\mathrm{vol}_4, \tag{22}$$

where vol_4 is the volume form of the $\mathbf{P}^1$ fibration over T^2.

We introduce a phase σ as a function of the magnetic flux M:

$$M = \tan(\sigma)\mathrm{vol}_{T^2}. \tag{23}$$

We integrate (22) over the $\mathbf{P}^1$ fibration over the T^2 and denote

$$b_{\mathrm{NS}} = \int_{P^1} B_{\mathrm{NS}}, \qquad I_4 = \int \frac{\sqrt{|g+M|}}{\sqrt{|g|}}\mathrm{vol}_4,$$
$$A_{T^2} = \int_{T^2} \mathrm{vol}_{T^2}. \tag{24}$$

As M is a constant flux on the two-torus, $\int_{T^2} M = M A_{T^2}$, where M is now a number. The result of integrating (22) is then

$$b_{\mathrm{NS}} A_{T^2}\sqrt{1+M^2}e^{i\sigma} = e^{i\theta} I_4. \tag{25}$$

This relation is satisfied if $\sigma = \theta$ and

$$b_{\mathrm{NS}} A_{T^2}\sqrt{1+M^2} = I_4. \tag{26}$$

This enables us to identify the SUSY parameter θ with σ of (23) and the gauge coupling constant with the inverse of $b_{\mathrm{NS}} A_{T^2}\sqrt{1+M^2}$. An argument for this value of the coupling constant was provided in [25] where the coupling constant for the gauge theory on a D-brane wrapped on a torus with area A_{T^2} and in the presence of a magnetic flux M was shown to be $A_{T^2}\sqrt{1+M^2}$. This is exactly the interpretation of (26) after a further compactification on a nonrigid $\mathbf{P}^1$. (26) therefore provides the (2,2) two-dimensional coupling constant on D5-branes wrapped on the direct product $T^2 \times \mathbf{P}^1$, with magnetic flux M:

$$\frac{1}{g_{(2,2)}} = \frac{b_{\mathrm{NS}} A_{T^2}\sqrt{1+M^2}}{g_s}. \tag{27}$$

The change in the coupling constant determined by the magnetic flux can also be understood in a T-dual picture with D-branes, after the T-duality is taken along one of the directions of T^2. The D4-branes are replaced by D3-branes with a tilting in the (1,2) plane due to the magnetic field. The two-dimensional coupling constant is inverse proportional to the length of D3-brane i.e. proportional to $1/\sqrt{1+M^2}$.

B. SUSY configurations with magnetic flux and rigid cycles

We consider the general solution for arbitrary θ and an arbitrary boosting parameter ω. This corresponds to allowing some arbitrary J and M. When starting with a group $U(N_c) \times U(N_f)$, the magnetic flux is chosen such that the gauge group is broken to $U(N_c) \times SU(N_f/2)^2 \times U(1)$. We want zero entries for the $N_c \times N_c$ block as the $U(N_c)$ fields are not charged under the magnetic flux.

The resolved conifold is now nontrivially fibered over T^2 and we deal with a Calabi-Yau fourfold. We have various types of stable four-cycles inside Calabi-Yau fourfolds. One type of stable four-cycles are the Kahler calibrations which are calibrated by J^2 and are complex submanifolds. The second type of four-cycles are the Lagrangian submanifolds L which are calibrated by $\alpha_\psi = \mathrm{Re}(e^{i\psi}\Omega)$ where Ω is the holomorphic (4,0) form and ψ is a phase. The most general calibration is the Cayley calibration when the four-cycles are calibrated by $J^2 + \mathrm{Re}(e^{i\psi}\Omega)$. The Cayley calibrations for wrapped D-branes were first used in [26,27].

In this work we consider S is a nontrivial $\mathbf{P}^1$ or $\tilde{B}$ fibration over T^2. It is a compact complex submanifold, as introduced in [16]. This implies that we deal with Kahler calibrations on which $\mathrm{Re}(f^*(e^{i\psi}\bar{\Omega}))$ is zero. The conditions for unbroken SUSY on D5-branes wrapped on Kahler calibrations are [14]

$$\eta_+^1 = ie^{-i(\theta+\phi)}\eta_+^2 \tag{28}$$

and

$$(J + iB_{\mathrm{NS}})(\mathrm{vol}_{T^2} + iM) = e^{i(\theta+\phi)}\frac{\sqrt{|g+M|}}{\sqrt{|g|}}\mathrm{vol}_4. \tag{29}$$

The conditions (28) and (29) for D5-branes wrapped on a four-cycle of an $SU(4)$ structure manifold have two angular parameters θ and ϕ which we want to relate to the parameters

$$B_{\mathrm{NS}} = \tan(\rho)J, \qquad M = \tan(\sigma)\mathrm{vol}_{T^2}. \tag{30}$$

To do this, we start with D5-branes wrapped on a two-torus with magnetic flux which requires $\eta_+^1 = -e^{-i\theta}\eta_+^2$ and fix the θ parameter to be equal to σ such that $M = \tan(\theta)\mathrm{vol}_{T^2}$. We then wrap the D5-branes on an extra $\mathbf{P}^1$ cycle such that the $\mathbf{P}^1$ fiber over the two-torus is a Kahler calibration inside an $SU(4)$ holonomy manifold, as in [16].

In case of extra wrapping on a rigid four-cycle, we multiply $\mathrm{vol}_{T^2} + iM$ by $J + iB_{\mathrm{NS}}$ and the relation between $\eta_+^i, i = 1, 2$ changes from $\eta_+^1 = -e^{-i\theta}\eta_+^2$ (D5-branes on two-cycle) to $e^{i\rho}\eta_+^1 = -e^{-i\theta}\eta_+^2$ (D5-branes on four-cycle). The factor $e^{i\rho}$ is the phase of $J + iB_{\mathrm{NS}}$ and is extracted once we consider that the quantum volume of the P^1 cycle is $\sqrt{j^2+b^2}$. The relation between η_+^i can be rewritten as

$$\eta_+^1 = ie^{-i(\theta+\rho)+i\pi/2}\eta_+^2. \tag{31}$$

When comparing (28) and (31) we see that, besides $\theta = \sigma$, we can also identify $\phi = \rho - \pi/2$. The reality condition $\theta + \phi = 0$ becomes $\sigma = \pi/2 - \rho$, which implies

$$\tan\rho = \mathrm{cotan}\sigma \to B_{\mathrm{NS}} \wedge M = J \wedge \mathrm{vol}_{T^2}, \tag{32}$$

which is the geometric version of the equality between the D-term and the magnetic flux considered in [12].

The supersymmetric condition also requires

$$A_{T^2}\sqrt{j^2 + b_{\mathrm{NS}}^2}\sqrt{1+M^2} = I_4, \tag{33}$$

which implies that the two-dimensional gauge coupling for the (0,2) gauge theory is

$$\frac{1}{g_{(0,2)}} = A_{T^2}\frac{\sqrt{j^2+b_{\mathrm{NS}}^2}\sqrt{1+M^2}}{g_s}. \tag{34}$$

In the above discussion, we have considered that the four-cycle wrapped by the D5-branes is a $\mathbf{P}^1$ fiber over T^2. We can also take the limit when the cycle $\mathbf{P}^1$ is replaced by a noncompact holomorphic cycle. In this case we get flavor D5-branes wrapped on noncompact cycles. The SUSY compatibility between wrapped D5-branes remains the same as in (32).

V. GEOMETRIC TRANSITION WITH D-TERMS AND MAGNETIC FLUXES

We now consider the geometric transition [1] in the presence of rigid two-cycles and magnetic fluxes. The geometric transition between resolved and deformed geometries for pure gauge theories starts with N_c D5-branes wrapped on a resolved conifold, continues with shrinking the $\mathbf{P}^1$ cycle and replacing it with an S^3 cycle with a size equal to the field theory gluino condensate,

$$S = \int_{S^3}\Omega_3, \tag{35}$$

where Ω_3 is the holomorphic three-form on the deformed conifold. The color D5-branes disappear and are replaced by N units of Ramond-Ramond flux through the S^3 cycle,

$$\int_{S^3} H^{\mathrm{RR}} = N. \tag{36}$$

There are others quantities which map from the resolved conifold side to the deformed conifold side, involving P, the Poincaré dual to S^3. The bare gauge coupling map is

$$\int_{\mathbf{P}^1} B_{\mathrm{NS}} \leftrightarrow \int_P H_{\mathrm{NS}}, \tag{37}$$

and the Fayet-Iliopoulos D-term map is [5]

$$\int_{\mathbf{P}^1} J \leftrightarrow \int_P dJ. \tag{38}$$

A nonzero value of dJ in the deformed geometry implies the existence of some nonzero torsion classes, a setup studied in detail in [28]. On the other hand, if fundamental flavors are present, they live on D5-branes wrapped on noncompact holomorphic two-cycles which survive the geometric transition. In this case $J \neq 0$ on the noncompact two-cycle before the geometric transition maps into $J \neq 0$ on the noncompact two-cycle after the geometric transition. There is no $dJ \neq 0$ contribution from the surviving noncompact holomorphic two-cycles.

A. Breaking SUSY with fluxes and noncompact cycles

We saw in the resolved conifold geometry that the supersymmetry is broken if D5-branes wrap two-cycles with different values for j. In particular, the SUSY is broken when the N_c color D5-branes wrap a nonrigid $\mathbf{P}^1$ cycle and the N_f D5-branes wrap a rigid noncompact holomorphic two-cycle. How do we translate this statement into the deformed conifold side?

Consider the deformed conifold configuration with N_c units of RR flux through the S^3 cycle (coming from the N_c D5-branes wrapping a $\mathbf{P}^1$ cycle) and a noncompact two-cycle with N_f D5-branes wrapped on it. This represents the strong coupling limit of the $SU(N_c)$ field theory with N_f fundamental flavors. To discuss the SUSY breaking, we consider this configuration as a limit of a geometry describing the strongly coupled $SU(N_c) \times SU(N_f)$ gauge theories when the gauge coupling of $SU(N_f)$ goes to zero. If S_1, S_2 are the gluino condensates for $SU(N_c) \times SU(N_f)$, this limit implies that $S_2 \to 0$. For two S^3 cycles with sizes S_1, S_2, the prepotential is

$$2\pi i F_0 = \frac{1}{2} S_1^2 \log\left(\frac{S_1}{\Lambda_0^2} - \frac{3}{2}\right) + \frac{1}{2} S_2^2 \log\left(\frac{S_2}{\Lambda_0^2} - \frac{3}{2}\right) - S_1 S_2 \log\frac{a}{\Lambda_0}, \tag{39}$$

whose derivatives are the two B-periods Π_1, Π_2 of the geometry. In the limit $S_2 \to 0$, the contribution of Π_1 to the effective superpotential is the usual one for a decoupled $SU(N_c)$ gauge theory,

$$N_c(3S_1 \log\Lambda_0 + S(1 - \log S_1)), \tag{40}$$

whereas the contribution of Π_2 becomes

$$S_2 \log\left(\frac{a}{\Lambda_0}\right). \tag{41}$$

We recognize the quantity (41) as the additional superpotential coming from the contribution of the D5-branes wrapped on noncompact two-cycles in the deformed geometry [2,6]. a is either the mass of the flavors [2] or their expectation value [6]. The geometry with nonzero S_1, S_2 continuously deforms into its $S_2 \to 0$ limit with one S^3 cycle and a noncompact two-cycle.

We can take a similar $S_2 \to 0$ limit when we start with a geometry with two S^3 cycles of nonvanishing sizes with $dJ \neq 0$ on their Poincaré duals. As considered in [5], in this case the critical points of the tree-level effective superpotential correspond to values for S_1, S_2 containing the factors

$$\left(\frac{a}{\Lambda}\right)^{N_i \cos(\theta_{12})}, \quad i = 1, 2, \tag{42}$$

where θ_{12} is the relative phase between the central charges $Z_i, i = 1, 2$ of the SU groups. The relative phase originates from the terms

$$\int_{P_i} dJ/g_s = j_i/g_s; \qquad j_1 \neq j_2. \tag{43}$$

In the limit $S_2 \to 0$, the cycle S_2^3 is replaced by a holomorphic noncompact two-cycle and a nonzero value of dJ on P_2 maps into a nonzero value of J on the noncompact two-cycle $\tilde{B}_2$. θ_{12} remains the relative phase between the central charges but now originates from the terms

$$\int_{P_1} dJ/g_s = j_1/g_s; \qquad \int_{\tilde{B}_2} J/g_s = j_2/g_s. \tag{44}$$

The three-cycle P_1 is the Poincaré dual to S_1^3 and should not be confused with the two-cycle $\mathbf{P}^1$. If $j_1 \neq j_2$ in (44), the supersymmetry is broken. The particular case we are interested is when $j_1 = 0$ and $j_2 \neq 0$ which occurs when a D-term is turned only for the flavor group. We see that SUSY is broken in this particular case.

B. Reduction on a two-torus and partial supersymmetry restoration

We saw in the previous subsection that the deformed geometry with a noncompact rigid two-cycle generically breaks SUSY when $j_1 \neq j_2$ in (44). We now argue that the procedure employed in the resolved conifold geometry (extra compactification on T^2 with a magnetic flux on the torus) to preserve SUSY can also be applied after the geometric transition. The geometry becomes a deformed conifold with an extra noncompact two-cycle, fibered over a two-torus. The deformation cycle S^3, its Poincaré dual P and the noncompact two-cycle are all uplifted to four-cycles.

In the fourfold language, the identification (35) becomes

$$S = \int_{D_1} \Omega_4, \tag{45}$$

where Ω_4 is the holomorphic four-form on the Calabi-Yau fourfold and D_1 is a four-cycle $S^3 \times S_i^1$ where $S_i^1, i = 1, 2$ are the one-cycles of the two-torus. The flavor degrees of freedom live on a noncompact four-cycle B_i which is an holomorphic noncompact two-cycle fibered over T^2.

To get the flux contribution to the effective superpotential, we use the results of [29] for the superpotential in case of a compactification on a Calabi-Yau fourfold with a nonzero four-form flux:

$$W = \int_Y \Omega \wedge G, \tag{46}$$

where Ω is a holomorphic four-form on Y and G is an integral four-form. This can be reduced to Calabi-Yau threefolds by considering the Calabi-Yau fourfold as an elliptic fiber over a base B and expanding G as in [29]

$$G = q + p \wedge \chi + \sum_i H_i \wedge \theta^i, \tag{47}$$

where q, p and H_i are forms of degree 4, 2 and 3 on the base B, $\theta^i, i = 1, 2$ form a basis of integral one-forms on the fiber and χ is an integral two-form generating the two-dimensional cohomology of the elliptic fiber.

In our case, the Calabi-Yau fourfold is a deformed conifold fibered over a two-torus. How do we incorporate the FI terms into the effective superpotential on the fourfolds? When compactifying on a Calabi-Yau threefold, [5] has argued that the FI terms transform as a vector (E_1, E_2, E_3) under an $SU(2)_R$ R-symmetry and, when considered as entries of a 2×2 matrix, the action appears as

$$\frac{1}{4\pi} \mathrm{Re}(\mathrm{Tr} X \bar{E}). \tag{48}$$

X is a 2×2 matrix depending on j, and b_{NS} and E_i are integrals over the P cycle:

$$E_1 = \int_P \frac{H_{\mathrm{NS}}}{g_s}, \qquad E_2 = \int_P H_{\mathrm{RR}}, \qquad E_3 = \int_P dJ/g_s. \tag{49}$$

We know from [5] that the relevant supersymmetry variations of the $SU(2)_R$ doublets of fermions (ψ, λ) are given by

$$\delta \Psi^i = X^{ij} \epsilon_j \quad i, j = 1, 2. \tag{50}$$

In case of several gauge groups with different values for j, there are several matrices X_a with zero eigenvalues but the supersymmetry is broken for a configuration with arbitrary values for j_a.

To partially preserve supersymmetry, we consider the deformed conifold fibered over the two-torus. We want to uplift the quantities E_i to the Calabi-Yau fourfold. The term $H_i \wedge \theta^i$ in (47) contains θ^i, the basis for one-forms on the torus

$$dz = dx_1 + \tau dx_2, \qquad d\bar{z} = dx_1 + \bar{\tau} dx_2. \tag{51}$$

We consider a vector potential $A_2 = M x^1$. The noncompact three-cycle P becomes a collection of two $S_i^1 \times P, i = 1, 2$ four-cycles denoted as $\tilde{D}_1, \tilde{D}_2$. The uplift to the fourfold is

$$\text{gauge coupling constant:} \int_B H_{\mathrm{NS}} \leftrightarrow \int_{\tilde{D}_1} H_{\mathrm{NS}} \wedge dx^1, \tag{52}$$

$$E_1 = \int_B \frac{H_{\mathrm{NS}}}{g_s} \leftrightarrow \tilde{E}_1 = \int_{\tilde{D}_1} H_{\mathrm{NS}} \wedge dx^1, \tag{53}$$

$$E_2 = \int_B H_{\mathrm{RR}} \leftrightarrow \tilde{E}_2 = \int_{\tilde{D}_1} H_{\mathrm{RR}} \wedge dx^1, \tag{54}$$

$$E_3 = \int_B dJ/g_s \leftrightarrow \tilde{E}_3 = \int_{\tilde{D}_1} dJ/g_s \wedge dx^1. \tag{55}$$

On the other hand, we also have contributions from $\tilde{D}_2$ as

$$\int_{\tilde{D}_2} H_{\mathrm{NS}} \wedge A_2 dx^2, \qquad \int_{\tilde{D}_2} dJ \wedge A_2 dx^2, \qquad \int_{\tilde{D}_2} H_{\mathrm{RR}} \wedge A_2 dx^2. \tag{56}$$

More involved is the calculation of the matrices X_a for the deformed conifold fibration over the two-torus. To do this, we need to split the four-dimensional fermions into right (left) moving fermions on $R^{1,1}$. The definition of X would contain both j and the magnetic flux M. As mentioned before, in this work we restrict to the case when only flavor branes are charged under the magnetic flux and there is no $dJ \neq 0$ on the $\tilde{D}_i$ cycles. All the information about the magnetic flux and nonzero D-terms is encoded in the noncompact two-cycle. We plan to develop a general discussion for arbitrary X and E in a future publication.

We now return to our concrete example in this work and deal with the mismatch between $j_1 = 0$ and $j_2 \neq 0$ in (44). When lifted to the $SU(4)$ structure manifold, the phase introduced by the integral of J over $\tilde{B}_2$ is matched by the phase introduced by M on T^2. When integrated over the B_2, the $\tilde{B}_2$ fiber over T^2, the two phases cancel each other if the relation (32) is valid. For flavor branes, the geometric transition provides a set of cycles $D_i, \tilde{D}_i, B_2$. The integral of dJ over $\tilde{D}_i$ is zero when no D-term is considered for the gauge group. For magnetic flux on the two-torus that only the flavors are charged under, the SUSY condition for D5-branes wrapped on a noncompact four-cycle in the deformed conifold side is identical to the one in the resolved conifold side and requires the condition (32) to be true. We conclude that the condition (32) ensures that the SUSY is preserved during the geometric transition.

As the geometries discussed here have $SU(4)$ structure in the presence of NS flux on Calabi-Yau fourfolds, it would be interesting to consider the approach of [30] involving manifolds with SU(4) structure. This would allow us to consider more involved assignments of flavor and colour charges under the magnetic flux.

VI. CONCLUSIONS

In this work, our goal was to provide a geometric picture for a partial supersymmetry breaking yielding (0,2) two-dimensional theories. Our setup is T-dual to the brane configurations of [12]. We start with D5-branes wrapped on various compact or noncompact two-cycles of resolved conifold geometries which are further fibered over a two-torus. Consequently, the D5-branes wrap four-cycles S which are (compact or noncompact) two-cycles fibered over the two-torus. The supersymmetry is partially preserved for rigid two-cycles and when a magnetic flux is considered on the two-torus, if the magnetic flux and the rigidity parameter j are related as in (32). This reproduces the equality between the D-terms and the magnetic fluxes proposed in [12]. We also consider the supersymmetry preservation after a geometric transition. For the case discussed in this paper, the supersymmetry condition involves noncompact two-cycles wrapped by flavor branes which are fibered over the two-torus with magnetic flux. The relative phase between the central charges of various stacks of branes is zero when the relation (32) is obeyed.

ACKNOWLEDGMENTS

This work was supported in part by STFC under Contract No. ST/L000431/1. We would like to thank Keshav Dasgupta for important comments on the draft.

Note added.—Recently a paper [31] appeared which considers 2d (0,2) quiver gauge theories on the world volume of D1-branes probing singular toric Calabi-Yau fourfolds. Our approach uses D5-branes wrapped on four-cycles inside Calabi-Yau fourfolds and $SU(4)$ structure manifolds.

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