Reliability Analysis of an Axial Compressor Based on One-Dimensional Flow Modeling and Survival Signature

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This paper presents a procedure for the reliability analysis of a multi-stage axial compressor regarding blade-specific roughness effects, based on the survival signature approach. As a result, a time-dependent evolution of the system reliability is obtained along with a prioritization technique for monitoring and regeneration of the rough blade rows by capturing the most critical system components. For this purpose, a one-dimensional flow model is developed and utilized to evaluate the aerodynamic influences of the blade-specific roughness on the system performance parameters, namely the overall pressure ratio and the isentropic efficiency. In order to achieve transparency and high numerical efficiency for time-dependent analyses in practice, the physics-based compressor model is translated into an illustrative, function-based system model. This system model is established by conducting a Monte Carlo simulation along with a variance-based global sensitivity analysis, with the input variables being the row-specific blade roughness. Based on the system model, the roughness impact in different blade-rows is ranked by the relative importance index, and the corresponding time-dependent reliability of the compressor system in terms of pressure ratio and efficiency is estimated through its survival function. Furthermore, uncertainties in the roughness-induced failure rates of the components are modeled using imprecise probabilities. Consequently, bounds on the reliability function and the importance indices for the blade-surface roughness in each blade row are captured, which enhances the decision-making process for maintenance activities under uncertainty.

1 Introduction

Motivation: Gas turbines are widely utilized in propulsion and power generation industries, where for safety, economic, and environmental reasons their degradation must be appropriately addressed to maintain reliable and efficient operation with minimal costs. For instance, 90% of the complete life cycle cost of a gas turbine are due to maintenance and fuel expenditures compared to 10% due to acquisition, installation and operation [1]. Additionally, in the aviation sector, the regeneration of jet engines due to deterioration of engine parts alone constitutes 8% of airplane operating cost [2]. Therefore, studying deterioration mechanisms and their impacts on the performance of the engine remain an

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attractive research topic besides reliability estimates and optimal maintenance measures. This paper considers the blade-roughness due to erosion as a deterioration mechanism in a multi-state axial compressor, which constitutes a substantial part of the gas turbine as well as the jet engine, is considered. The impact of the blade roughness on the performance of the compressor is studied, and a tool for reliability estimation and regeneration prioritization is suggested.

During the operating life of axial compressors, the blade surfaces naturally deteriorate over time due to erosion and corrosion effects which result from a wide range of operational and environmental factors. The most common sources are the ingested aerosols such as salt spray from marine applications [3, 4], sand [5, 6], and even volcanic ash [7]. The material particles and liquid droplets in the ingested air erode or deposit on the blade surface, and thus modify the airfoil shape and lead to an increased surface roughness [8–10]. Consequently, many facets of the compressor performance such as efficiency, pressure ratio, and stall margin downgrade from their design specifications [11]. This in turn results in increased specific fuel consumption and even in a drastically declined safety in the operation of the complete gas turbine in case of loss of compressor stall margin due to excessive roughness (deposition buildup) [12,13]. More recent insights into the the evaluation and analysis of performance deterioration of axial compressors due to roughness effects can be found in [14–18].

Similar to all mechanical systems, when the operational efficiency of an axial compressor drops below a specific threshold, appropriate maintenance and regeneration measures must be applied to bring the system back to its best possible function. The regeneration activities, nevertheless, should be organized in such a manner that the corrective actions take place at the right time to prevent any serious consequences from the system failure, while simultaneously keeping the interruption time to a minimum and avoiding unnecessary maintenance actions. This issue is of paramount importance with the increasing competitiveness in the gas turbine market, and in mechanical systems in general where the high reliability and availability of the systems is crucial. Therefore, a tool that (i) diagnoses the current health state of a machine, (ii) predicts the time to future failure taking uncertainties into consideration, and (iii) prioritizes the maintenance actions will be vital for the reliability and availability of the system [19]. Such a tool enables the operator to undertake the appropriate action before any critical malfunction takes place, to schedule the maintenance procedure in advance, and to proactively deploy replacement parts [20–22]

Scope of the paper: The present paper introduces an integrated methodology for the time-dependent reliability estimation of an axial compressor due to roughness effects, using a system representation approach. For this purpose, first, a one-dimensional aerodynamic flow-model is developed, and used to evaluate the effect of blade-specific surface roughness on the performance of a multi-stage axial compressor, with the overall pressure ratio and isentropic efficiency being the performance parameters. In addition to the meridional flow-path geometry, the developed model considers variable geometric and aerodynamic input parameters in order to anticipate their influences on individual stages, as well as on the overall compressor performance. In the next step, a functional-based system representation of the axial compressor is constructed based on stochastic and global sensitivity analyses of the one-dimensional flow-model. With these analyses, the blade rows in the stage-levels are classified into different component types of the representative system model according to the impact of their surface-roughness on performance parameters. In the third step, the time-dependent evolution of the system reliability against roughness effects is estimated using a survival signature approach. In this approach, the relative importance index is utilized to rank the importance of the system components to the reliability of the system. By doing so, a regeneration prioritization is achieved such that only the blades whose roughness reach a specific value must be maintained or regenerated. The epistemic uncertainties involved in failure rates of the components due to roughness are modeled using imprecise probabilities, which allows the drawing of lower and upper bounds on the survival function of the system, as well as on the relative importance indices of its components.

In the context of this paper, the term *failure* means an unacceptable loss of aerodynamic performance. Accordingly, a turbomachine *fails*, when it does not meet its aerodynamic design parameters because of deteriorated blade aerodynamics. Although the described methodology is capable of analyzing various other failure modes like *vibration failure* due to rotor unbalance or *integrity failure* due to component fracture, this paper focuses on the concept of *performance failure* due to aerodynamic blade deterioration.

The complete methodology serves as a very convenient procedure for supporting the decision making process in the regeneration and maintenance of any mechanical system according to a physics-based evaluation of possible deterioration mechanisms on the performance and, consequently, reliability of that system. The reliability analysis part of the methodology, in particular, is a superior technique in the context of regeneration and maintenance due to its ability to completely separate the structure of the mechanical system from the random failure of its components. Therefore, when updating the random failure rates of the regenerated system components, the system structure remains unchanged and, consequently, the time-dependent system reliability is recomputed and updated very efficiently.

2 Survival Signature Approach for System Reliability

Survival signature [23] is a recently developed methodology for estimating the time-dependent reliability of networked systems of multi-type components. It basically relies on two concepts: first, the concept of system survival analysis [24] which has applications in diverse fields such as biology, economics, and reliability engineering. In the later context, survival analysis is referred to as reliability analysis which quantifies the survival probability of the considered system at a certain point in time through the survival (or reliability) function. The second concept is the system signature [25] which estimates the reliability of systems consisting of exchangeable components with the capability of a complete separation between the system structure and the probabilistic description of its components' random failure. The major limitation of system signature is that it only fits systems with components of the same type, which is not the case in most real-world problems. This issue is overcome in the survival signature approach by the ability to also consider independent and identically distributed component characteristics. Further advancements to the approach are achieved in [26] and [27] for deriving the survival signature of a system from the signatures of two subsystems, and for considering Bayesian perspective, respectively.

In [28] the survival signature has been extended to incorporate imprecision into the probabilistic description of component characteristics. This is of substantial importance when reflecting the incomplete information or knowledge about the random failure rates of the components. The reliability function in this case will be encompassed by lower and upper bounds representing its extreme values due to indeterminacy in components' random failure rates. Moreover, a measure for ranking the importance of each component to the reliability of the system, named as *relative importance* (*RI*) index, is also presented in [28] based on the survival signature. Through this index, the time-dependent importance evolution of a component is estimated for both precise and imprecise component characteristics cases.

A theoretical minimum of the survival signature methodology is summarized in the following subsections, in order to illuminate its appropriateness as a supportive tool for the regeneration of mechanical systems in general and to familiarize the reader with its basic concepts. Detailed information can be found in [23, 26], and [28].

2.1 System Structure Function

Let us suppose a system with m components of the same type. The state vector of the components is defined as $\underline{x} = (x_1, x_2, \ldots, x_m) \in \{0,1\}^m$, where $x_i = 1$ if component i is in a functioning state and 0 if not. The function $\phi = \phi(\underline{x}) : \{0,1\}^m \rightarrow \{0,1\}$ is called the structure function, and it describes the system state based on the state vector. This means, for a given state vector $\underline{x}$, ϕ equals 1 if the system functions and 0 if not. In case of systems with $K \geq 2$ types of m components, the total number of the components is $\sum_{k=1}^{K} m_k = m$ with m_k being the number of components of each type. The state vector for this situation is given as $\underline{x} = (\underline{x}^1, \underline{x}^2, \ldots, \underline{x}^K)$, where $\underline{x}^k = (x_1^k, x_2^k, \ldots, x_{m_k}^k)$ being the state vector of type k. This is, of course, possible under the assumption that the components' failure times of each type are independent and identically distributed (*iid*).

2.2 Survival Signature and Survival Function

The survival signature of a multi-type system is denoted by $\Phi(l_1, l_2, \ldots, l_k)$, with $l = 0, 1, \ldots, m_k$ for $k = 1, 2, \ldots, K$, and defined as the functioning probability of the system given that l_k out of its m_k components of type k are in a functioning state, for each $k \in \{1, 2, \ldots, K\}$. This means the value of Φ equals 0 (minimum value) when non of the components work, and it reaches 1 (maximum value) when all the components of all types work. The set of all state vectors with $l_1, l_2, \ldots, l_K$ working components of types $k = 1, 2, \ldots, K$ and for which the system functions is defined as $S_{l_1, l_2, \ldots, l_K}$. Under the assumption that the random failure time is independent of components of different types, the survival signature of the system is given by

$$\Phi(l_1, l_2, \ldots, l_K) = \left[\prod_{k=1}^{K} \binom{m_k}{l_k}^{-1} \right] \times \sum_{\underline{x} \in S_{l_1, l_2, \ldots, l_K}} \phi(\underline{x}). \quad (1)$$

The survival signature of the system (equation 1) depends only on the structure of the system, i.e. the way in which the components are connected, and is fully independent of the random failure time of its components.

Now, suppose $F_k(t)$ to be the cumulative distribution function (CDF) describing the random failure time of the components of type k, i.e., components of the same type are exchangeable, and $C_k(t) \in \{0, 1, \ldots, m_k\}$ to be the number of working components of type k at time t. The probability that $C_k(t) = l_k$ for $k = 1, 2, \ldots, K$ will be

$$\begin{aligned} P(\cap_{k=1}^{K} \{C_k(t) = l_k\}) &= \prod_{k=1}^{K} P(C_k(t) = l_k) \\ &= \prod_{k=1}^{K} \binom{m_k}{l_k} \left[F_k(t)\right]^{m_k - l_k} \left[1 - F_k(t)\right]^{l_k}. \end{aligned} \quad (2)$$

This equation represents the probabilistic structure of the system, i.e., it describes the way the components fail regardless of the system structure. The survival function, which quantifies the time-dependent system reliability, of the system with K types of components at time T_s is defined by

$$P(T_s > t) = \sum_{l_1=0}^{m_1} \cdots \sum_{l_k=0}^{m_k} \Phi(l_1, l_2, \ldots, l_K) P(\cap_{k=1}^{K} \{C_k(t) = l_k\}). \quad (3)$$

The explicit and complete separation between the structure of the system and its probabilistic structure (or model) has substantial merit in the context of this paper. As will be highlighted in the discussion, the system structure will be developed based on a physics-based analysis and then the probabilistic structure can be updated at any time, for example in response to regeneration and maintenance activities.

2.3 Relative Importance Index

Identifying the components most critical to the failure of the system is a fundamental objective in most reliability analysis studies. Such identification enables the appropriate management of uncertainties in the system, and it provides a powerful frame for developing optimal monitoring, inspection and maintenance strategies for the system. The relative importance index proposed in [28] is a powerful measure for the influence of each component type on the reliability or survivability, of the system. This index is naturally based on the survival signature principle, and it is defined for a specific component i as the difference between the probability that the system survives when the component i is working and the same probability when that component is not working. Therefore, the index is also a function of time:

$$RI_i(t) = P(T_s > t|T_i > t) - P(T_s > t|T_i \leq t), \quad (4)$$

where $P(T_s > t|T_i > t)$ is the probability that the system works given that component i is working, and $P(T_s > t|T_i \leq t)$ is the probability that system works given that component i is not working. At a certain point in Time T_s, the larger the value of RI_i for component i compared to other indices is, the larger is the influence of the uncertainties of component i on the reliability of the system.

2.4 Imprecision in The Probabilistic Structure

In many real-world situations, a rigorous description of the random failure time of system components with a cumulative distribution function $F_k(t)$ is not possible. This might be a result of incomplete information, lack of data, or even lack of understanding the failure mechanisms of the components. More generalized probabilistic methods such as imprecise probabilities [29–32] have been utilized to overcome this problem. These methods encapsulate all possible $F_k(t)$s, which might be candidates for describing the random failure time of a certain component, with lower and upper bounds, $\underline{F}_k(t)$ and $\overline{F}_k(t)$, respectively. The obtained circumscription $[\underline{F}_k(t), \overline{F}_k(t)]$ is named a "probability box" or "p-box" [33–35]. These bounds are constructed based on the extreme values assigned to the parameters of the distribution $F_k(t)$.

In [28] the imprecise probability concept is integrated to the survival signature approach, and numerical procedures are introduced for estimating the bounds of the survival function as well as for the bounds of RI indices of the components. These procedures are also followed in this paper.

3 Aerodynamic Compressor Model

3.1 Model Purpose

In the light of todays capabilities in high-fidelity turbulent flow simulations, namely Reynolds-Averaged Navier-Stokes (RANS) turbulence models, Large Eddy Simulations (LES), and Direct Numerical Simulations (DNS), an attempt to simplified compressor-flow analysis is only acceptable if computational speed and model robustness are needed for a first step into the interdisciplinary field of reliability analysis of multi-stage axial turbomachines. It must be clearly stated, that the model applied in this study suits below investigations, which focus on methodological aspects rather than aerodynamic accuracy. In this paper, the authors want to demonstrate their approach applying the methods of reliability analysis to a turbomachinery application. Once the multi-disciplinary framework between the methods of stability analysis and turbomachinery aerodynamics is established, it will be straightforward to overcome the remaining drawbacks on the aerodynamic side in future studies by including higher-order models for two- and three-dimensional flow phenomena, if accuracy should become more important than computational speed. An existing two-dimensional code for meridional flow-field analyses will be the next step towards high-fidelity reliability analysis in turbomachinery applications.

3.2 Fundamental Equations and Assumptions

The current compressor-flow model, which provides the state function for the reliability analysis, is a one-dimensional flow model of multi-stage axial compressor. The main flow path in the meridional plane is bounded by the hub and shroud contours, and the effective channel cross section is reduced from the geometric annulus area to account for blockage effects along the side walls (see Figure 2). In this one-dimensional model, the primary flow quantities, namely

- absolute total pressure,
- absolute total temperature,
- absolute flow angle, and
- absolute flow Mach number

are predicted by the turbomachinery specific formulation of the fundamental fluid dynamics equations in the absolute (stationary) frame of reference for steady-state conditions. These are the following:

- Continuity equation

$$\dot{m}_{i+1} = \dot{m}_i \quad \text{for} \quad i = 1, ..., N_{stations} - 1 \quad (5)$$

$$\text{with} \quad \dot{m}_i = \rho_i \cdot c_{mi} \cdot A_i \quad \text{for} \quad i = 1, ..., N_{stations} \quad (6)$$

where i indicates each of the N axial stations along the flow path, at which each flow quantitiy is evaluated, $\dot{m}$ denotes the mass-flow rate, ρ the density, c_m the meridional flow velocity, and A the flow-path cross section,

- Euler's turbine equation (balance of angular momentum)

$$\Delta h_{t,j} = u_{out,j} \cdot c_{\theta,out,j} - u_{in,j} \cdot c_{\theta,in,j} \text{for} j = 1, ..., N_{rotors}, \quad (7)$$

where j indicates the rotor of each stage with an inlet in and an outlet out. Under the assumption of an adiabatic process, $\Delta h_{t,j}$ is the change in specific total enthalpy across each rotor row j, u is the spanwise averaged rotational speed of the rotor blade, and c_θ is the circumferential or swirl component of the flow at the inlet or outlet of this rotor,

- first and second law of thermodynamics,

$$\Delta h_{t,j} = h_{out,j} + \frac{c^2_{out,j}}{2} - h_{in,j} + \frac{c^2_{in,j}}{2} \text{for} j = 1, ..., N_{stages}, \quad (8)$$

  where the first law of thermodynamics (equation 8) balances specific thermal energy in terms of the static enthalpy h and specific kinetic energy against the change in total enthalpy h_t over each stage j for adiabatic conditions. The second law of thermodynamics is incorporated in the set of equations by the definition of a friction-based loss coefficient in total pressure ω (equation 13), the definition of the isentropic efficiency η (equation 11), and the following two requirements

$$0 \leq \eta \leq 1 \quad \text{and} \quad 0 \leq \omega \leq 1. \quad (9)$$

This set of equations is evaluated at each axial position between two adjacent blade rows of the compressor. More details on the implementation of these equations in a similar one-dimensional compressor model can be found in reference [36]. Flow variables, especially the velocity components and geometric information of the blade shapes and flow channel, are linked by kinematic relations e.g. the velocity triangles. In the meridional plane, the compressor main flow path is bounded by the hub and shroud contours. Dry air is described as an ideal gas with temperature-dependent caloric properties [37]. The performance of a multi-stage axial compressor is evaluated in terms of the following quantities:

- Total-to-total pressure ratio π between the absolute total outlet pressure p_{t2} and the absolute total inlet pressure p_{t1}

$$\pi = \frac{p_{t2}}{p_{t1}}. \quad (10)$$

- Total-to-total isentropic efficiency η as the ratio between isentropic total enthalpy difference Δh_{ts} to the total enthalpy change Δh_t

$$\eta = \frac{\Delta h_{ts}}{\Delta h_t}. \quad (11)$$

3.3 Blade-Element Model

A multi-stage compressor combines multiple stages in a series of connected stages through which the gas flow proceeds in the stream-wise direction. In the case of an axial compressor, this direction of the main flow is basically oriented axially which means parallel to the center-line of compressors rotating shaft. Depending on the local curvature of the meridional flow channel and the resulting shift in the radial position of the mean streamline, a radial velocity component is added to the axial flow component. Additionally, each rotor blade row increases the absolute flow swirl, whereas each stator blade row transforms it from dynamic into static pressure.

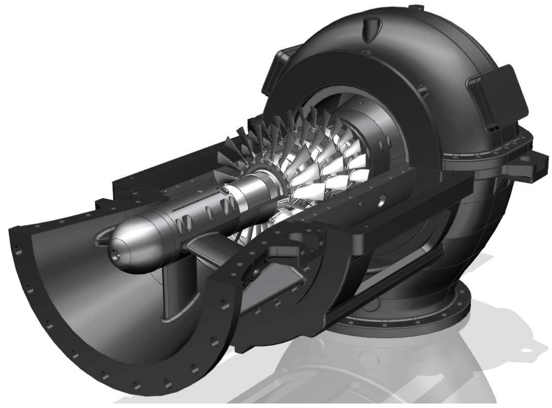

Fig. 1. Multi-stage axial compressor.

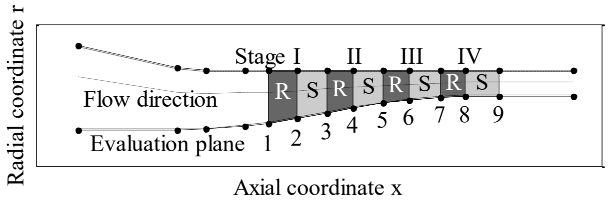

Fig. 2. Illustration of the meridional compressor contour.

Thus, the flow turning by each blade is crucial for the aerodynamic compressor performance. In order to account for this flow turning effect between the inlet and the outlet plane, each blade row in the one-dimensional model is characterized by its flow angles at the blade leading and trailing edge. Depending on the operating point, the inlet flow angle α_1 may deviate from the leading edge angle β_{LE} by an incidence angle i

$$i = \alpha_1 - \beta_{LE}. \quad (12)$$

In the case of a misalignment between the incoming flow and the orientation of the blade leading edge, the incidence differs from zero and the flow turning of the blade row increases. Depending on the magnitude and direction of the flow incidence, as well as the blade surface condition, the absolute total pressure loss and the deviation angle will change.

In order to account for operational and blade-surface conditions e.g. roughness in the compressor performance analysis, the aerodynamic blade performance parameters total-pressure loss coefficient ω (equation 13) and deviation angle δ (equation 14) are prescribed as blade-individual profile-loss for each blade row.

The loss coefficient ω describes the drop in total pressure p_t over the blade row (inlet: 1, outlet: 2) divided by the available dynamic inlet pressure $p_{ti} - p_1$

$$\omega = \frac{p_{t2} - p_{t1}}{p_{t1} - p_1}. \quad (13)$$

The difference angle between the blade trailing edge angle β_{TE} and the outlet flow angle α_2 is the deviation angle δ

$$\delta = \beta_{TE} - \alpha_2. \tag{14}$$

3.4 Roughness Model

In order to account for surface roughness effects as one possible origin of compressor performance deterioration, the roughness Reynolds number is introduced in [38] as a criterion for hydraulically smooth compressor blade surfaces

$$Re_k = \frac{w_1 k_s}{\nu} \leq 90, \tag{15}$$

where w_1 is the relative inlet velocity, k_s is the equivalent sand roughness, and ν is the kinematic viscosity. In [38] it is proposed to scale the arithmetical averaged roughness of a blade surface k_{CLA} of reasonable magnitude;

$$k_s \approx 6.2 k_{CLA}. \tag{16}$$

In cases of high chord-based Reynolds numbers Re_c above $2.5 \cdot 10^5$, the roughness effect on blade performance strongly depends on the ratio of blade surface roughness to chord length k_s/l [38]

$$Re_c = \frac{w_1 l}{\nu}. \tag{17}$$

For the investigated four-stage axial compressor (Table 1), the averaged equivalent sand roughness above which an aerodynamic impact on the flow can be expected is about 6.5 μm. Below this representative roughness limit, the compressor blade surfaces can be considered to be hydraulically smooth. The actual roughness of the machined blades lies above this limit.

The roughness limit is evaluated for each blade row individually in such a way, that if the prescribed surface roughness exceeds the critical surface roughness, the aerodynamic blade performance in terms of the loss coefficient ω is scaled by an increasing factor ϕ_ω and the deviation is increased by an angle $\Delta\delta$

$$\phi_\omega = \frac{\omega(k_s)}{\omega_0} \geq 1 \,\mathrm{for} k_s \geq k_{s,crit}, \tag{18}$$

$$\Delta\delta = \delta(k_s) - \delta_0 \geq 0 \,\mathrm{for} k_s \geq k_{s,crit}, \tag{19}$$

where the index 0 denotes the properties of the blade in a hydraulically smooth reference state. For predicting the roughness-induced deterioration in blade performance for a given blade roughness k_s appropriate correlations for the loss coefficient $\omega(k_s)$ and the deviation angle $\delta(k_s)$ are needed. In the current one-dimensional formulation, the approach introduced in [39] provides an experimentally validated correlation for the loss coefficient ω of a rough compressor blade with a chord length l and a NACA 65-A506 profile shape at high Reynolds numbers Re above $4 \cdot 10^5$ in a two-dimensional cascade

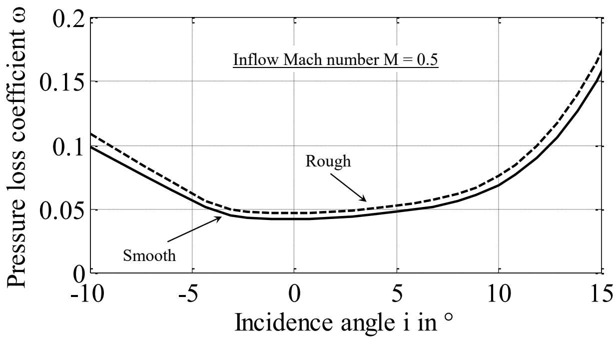

Fig. 3. Example for profile-loss curve (stator 2).

$$\omega = 0.0439 \cdot \left(\frac{k_s}{l}^3\right)^{0.034} \mathrm{for\ NACA65-A506}. \tag{20}$$

At this point, it must be stressed, that the above equation was derived from compressor blades, which are not of the same profile type as the Controlled-Diffusion Airfoils (CDA) of the baseline compressor. The roughness-induced change of the deviation angle δ is derived using graphical data from the experiments by [1]. Although it is preferable to include a similar roughness correlation for CDA rather than NACA profiles, a lack of more suitable experimental data prevents this increase in aerodynamic model accuracy.

3.5 Model Capacities and Limits

In its current version, the one-dimensional model is capable of analyzing axial compressors of an arbitrary shape and stage number in terms of pressure ratio and efficiency, while accounting for blockage effects and blade roughness under changing operating conditions. Rotor and stator blades are characterized by the axial positions of their respective inlet and outlet planes, as well as the angles at the blade leading and trailing edge at mid-span.

In order to account for the aerodynamic effects of a blade row, each blade row is reduced to its blade-individual profile-loss and deviation curves (see Figure 3). The total flow state, flow direction, and mass-flow rate at the first rotor inlet, as well as the rotor speed define the compressor operating conditions. Each combination of these parameters is evaluated to obtain the compressor performance in terms of the total-to-total pressure ratio and the total-to-total isentropic efficiency for individual stages, as well as for the overall compressors.

For example, flow boundary layers will develop along the channel sidewalls. A velocity gradient between the flow velocity in the idealized inviscid free-stream region and the velocity at the no-slip wall exists within these boundary layers. This velocity gradient between the hub and shroud is oriented perpendicular to the main flow direction. A similar gradient between free-stream and wall-bounded velocity exists between the pressure and suction side of the adjacent blades in a blade row. Since by definition the one-dimensional flow model of this study is not capable of resolving these flow gradients, a high-fidelity flow resolution requires empirical correlations in order to account for changes in the discontinuous flow evolution over several blades rows in the stream-wise direction. Since flow conditions in a turbomachine are highly three-dimensional and unsteady, the simplicity of the chosen modeling approach suffers from the risk of lacking reliability in terms of the predicted flow phenomena and their technological impact, if no corrections for these flow phenomena are applied. To overcome this disadvantage, additional terms e.g. for the loss, the sidewall blockage, flow deviation etc. are included in the set of equations in order to increase the model validity and the reliability of its results.

The main challenges in accounting for these higher-order effects are the identification of the aerodynamically most suitable and sensitive mathematical terms as well as a valid calibration of the results. With the objective of an interdisciplinary combination of reliability analysis and multi-stage compressor dynamics, the tedious work of calibrating a low-order model to either higher-order numerical or experimental data seems favorable for improving the quality of the results. On the other hand, this calibration step does not contribute to the methodology concept of this work. Thus, model simplicity is given priority over result accuracy in order to achieve a robust and easily understandable aerodynamic state function. On this basis, the compressor model of the current study does not claim to give the most accurate results in terms of aerodynamic performance prediction, but rather to serve as an extremely stable and time-efficient prediction tool for extensive parameter variations. Beside its numerical robustness under multi-dimensional parameter variation, the simplicity of the model supports engineering judgment on the results based upon the underlying aerodynamics.

3.6 Baseline Axial Compressor

The four-stage high-speed axial compressor of the Institute of Turbomachinery and Fluid Dynamics at Leibniz Universität Hannover is the baseline compressor of this study. Compressor performance data at the aerodynamic design point are listed in Table 1 and given in more detail by [40,41] and [42].

Table 1. Performance data of the reference multi-stage axial compressor.

Parameter desciption	Notation	Value	Unit
Design speed	N	18000	rpm
Mass flow rate	$\dot{m}$	2.93	kg/s
Overall total-total pressure ratio	π	2.72	
Overall isentropic efficiency	η	0.89	
Averaged Reynolds number	Re_c	$5 \cdot 10^5$	

4 System Representation of the Axial Compressor

The core idea of this study relies on developing a functionally based system representation of the axial compressor based on the one-dimensional flow model introduced in the previous section. This is due to the fact that the relationship between blade-specific roughness in each row in the axial compressor and its performance parameters is governed by aerodynamic laws, not by explicit networked components. Therefore, in order to develop a decision-making platform based on the survival signature approach, which best suits this purpose, a stochastic analysis using the aerodynamical flow model is conducted beforehand. This analysis can be summarized in the following steps:

1. The blade roughness, represented in the equivalent sand-grain coefficient k_s of the flow model, in each row of rotors and stators of the compressor is regarded as a random variable with a uniform probability distribution function $\mathcal{U}(2,17)$. This parameter can vary between 2 and 17 μm in order to significantly deteriorate the overall compressor efficiency with respect to the baseline design. Below the limit of 2 μm, the roughness does not have any aerodynamic influence. Of course, the distribution might take other forms than uniform, which is regarded here as a general case. Additionally, it should be mentioned that the roughness is regarded to be uniformly positioned on the blade surface, although some studies evaluated the non-uniform placement of roughness on the compressor performance and showed that roughness-position distribution in blade height direction has a minimal effect on the entire compressor performance [17]. This issue can be smoothly considered in case numerical models evaluating the distribution direction on the compressor performance are available.
2. A Monte Carlo simulation is conducted on the computationally inexpensive flow model to estimate variation of the performance parameters of the model, i.e., overall pressure ratio and overall isentropic efficiency, represented in the coefficient of variation COV as maps of the variations in the stage-specific blade roughness.
3. A global sensitivity analysis is conducted using Sobol′ indices [43] to decompose the computed variation in the performance parameters into the contributions of the eight input variables, which refer here to the blade-roughness in each row of rotors and stators of the four stages.

In Figures 4 and 5 the results of the Monte Carlo simulation and the sensitivity analysis for the case of the isentropic efficiency are provided, respectively. Because the sensitivity measures for the overall pressure ratio were very close

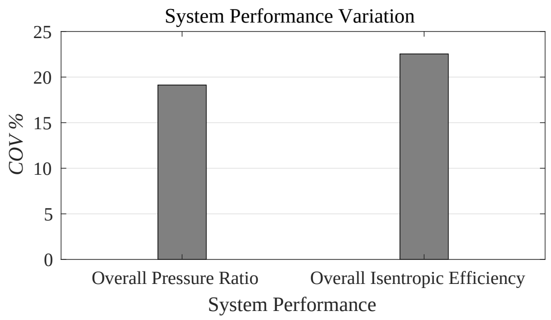

Fig. 4. The coefficients of variation COV of the performance parameters due to mapping of inputs variation.

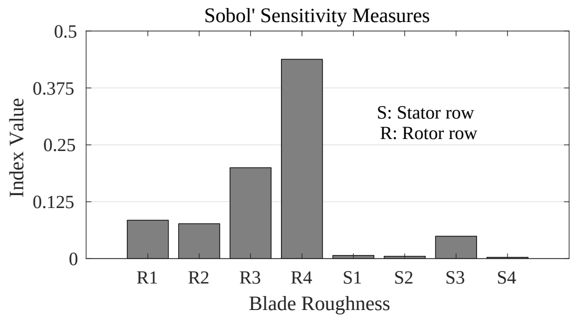

Fig. 5. Sobol$'$ sensitivity indices of the roughness of stator and rotor blade-rows to the isentropic efficiency.

to those of the overall isentropic efficiency, they are not included in Figure 5. According to the sensitivity results, the roughness in the rotors has much greater influence on the performance parameters than in the stators, and this influence increases starting from the front stage towards the rear one. This behavior is plausible because the Reynolds number and thus the sensitivity with respect to an absolute value of surface roughness increases, because the non-dimensionalized roughness increases. The non-dimensionalized roughness scales the aerodynamic importance of surface roughness in such a way, that for fixed absolute roughness dimensions, the relative aerodynamic impact on the compressor flow increases with an increasing Reynolds number. According to [44], the studies conducted by [45] and [46] using in-service data showed that the stators are more prone to roughness than rotors. This means that even though the stator blades deteriorate faster than those of the rotors due to roughness effects, this deterioration has a minor influence on the compressor performance. This fact will be reflected in the system representation of the compressor.

In the second stage, a threshold is defined for the admissible variation in the performance parameters, overall pressure ratio and overall isentropic efficiency, which is caused by the roughness of the blades. This threshold is regarded to be 25% and assigned based on expert knowledge. This means if 25% of the total variation of one output variable (system performance parameter) estimated in Figure 4 using Monte Carlo simulation, is reached due to roughness effects, the system will be regarded as being in a non-functioning state. Here, the variation of the isentropic efficiency is taken

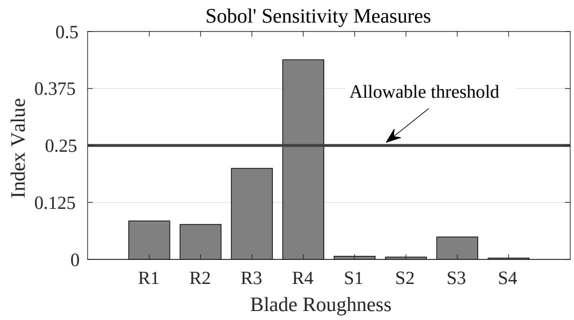

Fig. 6. Demonstration of the allowable threshold of system performance variation along with the sensitivity measures which are used in constructing the representative system model.

into consideration as it is slightly more sensitive than the pressure ratio. It is known that the values of Sobol$'$ indices reflect the contribution of the corresponding input variables to the total variance of the output parameter. Therefore, the combination of the 25% threshold and the sensitivity indices illustrated in Figure 6, is used to construct the system model demonstrated in Figure 7. In this system representation, the blade rows in the stages are classified into four component types. The first and second rows of rotor blades are regarded to be of the same component type t1, i.e., two components of type t1, because they have very similar contribution to the output variation (i.e., similar sensitivity measures). The rotor blades in the third and fourth rows are regarded as different component types t2 and t3, respectively, due to their distinct sensitivity measures. Finally, the four rows of stator blades are regarded to represent four components of the same type t4. Subsequently, the arrangement of the connections between the eight components, representing the eight rows of blades, are defined based on their sensitivity measures in combination with the assigned threshold. That is, when both components of type t1 fail due to roughness effects, i.e., the roughness reaches some extreme value that can be defined as failure by an expert assessment, the complete system will still work as long as the roughness in the other components is small enough not to result in an accumulated effect of 25% of performance variation. This is the case for component t2 as well. Therefore, the components of type t1 and t2 are arranged in the way illustrated in Figure 7. Component t3, however, is connected in series with the other components because it has a major effect on system performance and even a moderate variation in its roughness already results in a 25% of performance variation. All the components of type 4 are grouped in parallel and connected in series with the other components because even though the roughness reaches a severe value in all of them, the resulting variation in system performance is still much smaller than the defined threshold. Nevertheless, to avoid any potential danger to the mechanical and structural stability of stator blades, if the roughness in all the stator rows reaches an extreme value, the system is considered to have failed.

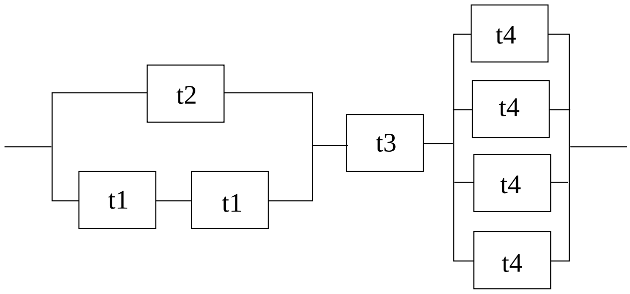

Fig. 7. Representation of the axial compressor as a system whose components being the roughened rotor and stator blade-rows.

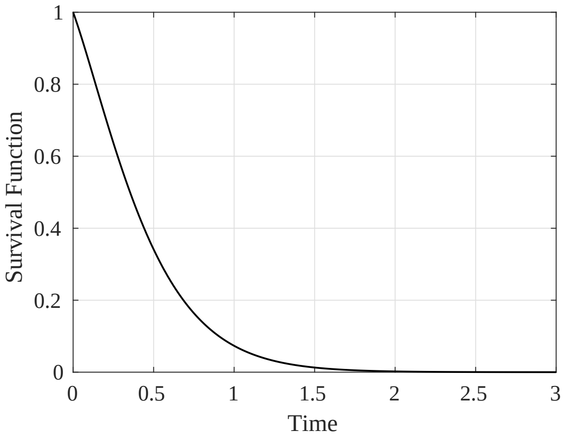

Fig. 8. The survival function of the axial compressor based on the representative system model and using exponential distribution function for the random time failure for all components.

5 Reliability Analysis and Regeneration Prioritization

After accomplishing a system representation of the relationship between row-specific blade roughness and compressor performance, the time-dependent reliability of this system is estimated in terms of its survival function. To achieve this, first the values of the survival signature of the system are computed only once based on the system structure and according to equation 1. Second, the probabilistic model including the failure time-rate of the components is constructed. The probability distribution function describing the failure time-rate must normally depend on available operational data in real-world problems. However, for the purpose of proof of concept and applicability, an exponential function with a parameter $\lambda = 0.8$ is considered for all components. Generally, different distributions can be utilized for different component types, and the distributions can be naturally updated in case a specific component has been maintained or replaced with a new one. This flexibility in the probabilistic model and its complete separation from the system structure reflects the suitability of the methodology for the time-dependent reliability analysis of continuously maintained and/or partially renewed, i.e. regenerated, systems in general. The survival function of the system is computed and illustrated in Figure 8, which demonstrates the deterioration of system survivability with time due the due to evolving roughness effects as a deterioration mechanism.

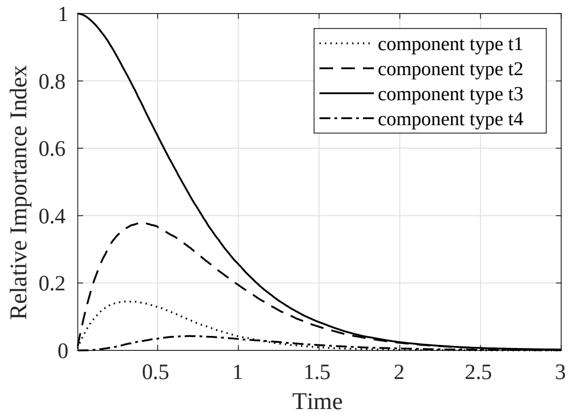

Fig. 9. The time-dependent relative importance indices of the system components.

For the sake of regeneration prioritization, the relative importance index is utilized. This index, as already mentioned, ranks the evolving significance of the components to the reliability of the system as a function of time. Therefore, it can be used as a decision metric for prioritizing the maintenance, monitoring, and inspection plans such that all this activities be optimally conducted to keep the efficiency of the system under best possible form regarding economical and environmental conditions, as well as meeting the safely requirements of the system. Figure 9 depicts the *RI* indices of the components for the representative system model of the axial compressor, which clearly demonstrates the importance of component t3 to the reliability of the system. Hence, this component must have priority in monitoring and regeneration plans. It is worth mentioning here that, understandably, the *RI* indices in this example are compatible with the system structure as well as with Sobol′ indices, which are used to develop the system structure, and it can be argued why it is needed to repeat the sensitivity measures twice. The authors emphasize, however, that the time dependent character of *RI* indices, which originates from the probabilistic model, makes a substantial difference to Sobol′ indices. Therefore, when renewing or maintaining a specific component in the system, its failure time-rate will be updated accordingly, and its importance compared to other components will also be updated. This might significantly change the importance of the component based on the new roughness level. Moreover, when developing a system representation of a complex mechanical system which might consist of many physics-based subsystems, the importance to the sub-system performance estimated by Sobol′ indices can be completely different from the importance to the overall system, which will be best captured by *RI* indices.

The uncertainties in describing the random failure time of system components are also considered in this study. As already stated, in most real-world cases having a probabilistic model that fully characterizes the random failure time of the components is not achievable. This might be due to scarce data, statistical errors, or incomplete understanding of the deterioration mechanism. Therefore, in this exam-

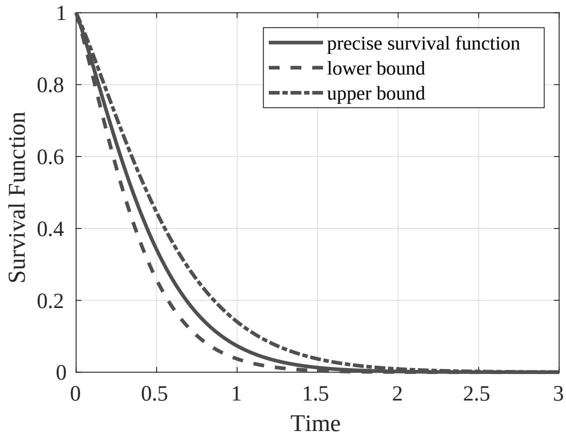

Fig. 10. The survival function of the axial compressor system with imprecise distribution parameters of the components random failure time.

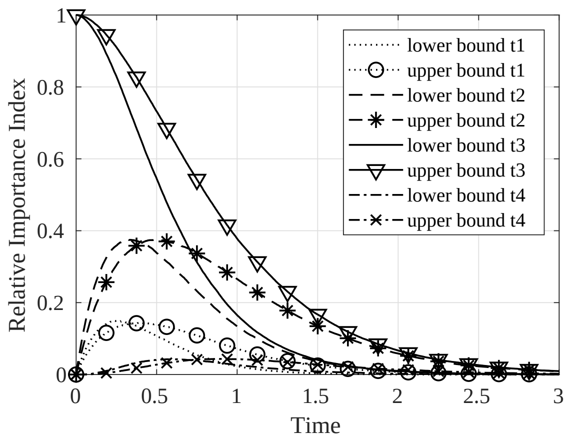

Fig. 11. The time-dependent relative importance indices of the system components with imprecise distribution parameters of the components random failure time.

ple, lower and upper bounds are introduced for parameter $\lambda \in [0.6, 1.0]$ of the exponential distribution utilized to model the random failure time. Subsequently, the survival function along with the *RI* indices are computed and depicted in Figures 10 and 11, respectively. As can be seen in these figures, the introduced bounds for parameter λ result in extreme values for the estimated survival function and *RI* indices. This constitutes a substantial and dependable basis for decision-making under uncertainty, including epistemic uncertainty, regarding inspection, monitoring, and regeneration of the axial compressor due to roughness effects.

6 Summary and Conclusions

This paper presents a comprehensive approach for studying the time-dependent reliability of a multi-stage axial compressor performance in vision of the effects of roughness evolution of its rotor and stator blades. A one-dimensional aerodynamic flow model is developed, validated and utilized as a physics-based description of the relationship between blade-roughness in different stages and compressor performance criteria. With the help of this model, a function-based system representation of the aforementioned relationship is constructed and further analyzed using the survival signature approach to estimate the reliability of the system as a function of time and for ranking the importance of the components to its functionality and performance.

By combining the knowledge obtained from the survival function of the system and the *RI* indices, the maximum allowable roughness in each blade-row of the compressor can be determined with respect to system performance. For instance, the maximum allowable roughness in rotors of the fourth stage in this example should be considerably less than that of all the other rotor and stator blade rows. Therefore, the failure time-rate due to roughness should be defined accordingly. This will be reflected in the parameter of the corresponding probability distribution function.

This study is significant in that it is valid even when applied to a variety of mechanical systems where a direct description in the form of components and connections is not available. In such cases, the physics-based analysis of the system can be translated to an illustrative function-based system model which enables a transparent and highly efficient numerical estimation of the time-dependent system reliability based on the survival signature method. Moreover, for complex systems consisting of several physics-based subsystems, the presented methodology is suitable for enabling reliability studies of the overall system performance due to any deterioration mechanism or failure effects of its subsystem components. Even different deterioration mechanisms can be regarded in different sub-systems. A great example is a complete gas turbine or a complete turbojet engine failing due to any deterioration mechanism such as fouling, geometry alterations, or erosion and corrosion in all or some of the sub-systems (compressor, turbine, fan, or combustion chamber). Additionally, the substantial contribution of the relative importance indices in the priority management of different maintenance actions provides a valuable decision-making tool with respect to minimizing the regeneration activities while complying with safety and reliability criteria. This issue is of paramount importance with regards to sustainability requirements and the environmentally friendly operation of mechanical systems.

In case of complex systems, the physics-based model for evaluating the influence of different deterioration mechanisms on system performance can be computationally very demanding as it may rely on finite volume methods or computational fluid dynamics simulations. Therefore, using these models in the stochastic and the sensitivity analyses for constructing a functional-based system representation can be prohibitive. In these cases, however, efficient surrogate techniques can be trained and used to substitute the original models.

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