Finite element modelling and damage detection of seam weld

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Abstract. Seam welds are widely used in assembled structures for connecting components. However, the dynamic effects of a seam weld are often difficult to characterise in numerical models for several reasons: (1) it is often not wise to build a fine mesh on the seam line which will add considerable computational cost for a structure with many welds, (2) the mechanical properties of weld materials are not well known; (3) sometimes some geometric information about welds is not known beforehand. In this work, the finite element model of a welding connection part is developed by employing CSEAM element in NASTRAN and its feasibility for representing a seam weld is investigated. Based on this result, a damage detection method by updating the properties of the built CSEAM elements is also proposed for welding quality assurance. The damage takes the form of a gap in the weld which causes a sharp change of model strain energy at the edges of the gap for certain vibration modes. Specifically, the model strain energy shape is used as the objective function. A Kriging model is introduced for efficiency and simulation of a T-shaped welded plate structure to demonstrate the effectiveness of this method.

Keywords: Seam Weld, Model Updating, Kriging Model, Model Strain Energy

1 Introduction

Assembled structures are widely used in civil and mechanical engineering. Different structural members are produced independently and then connected together by special joining techniques. The joint formed has an important influence on the overall dynamical characteristics of the structure. Thus the accurate representation of the joints in the Finite Element (FE) models has significant research value [1].

Welding is one of the most commonly used joining techniques, whose FE modelling has drawn great attention in the past decades [2]. At the early stage, special elements are designed for connection, like partly rigid beam element used in frame structures [3-

5]. Then, with the development of Computer-aided engineering software, many useful connectors are available in structural analysis software packages to represent welded joints in the FE model, like RBE2, ACM2 and CWELD [6]. The last one is specially designed for spot weld modelling in Nastran [7] and has been wildly used and studied by many researchers. References [8, 9] show that after reliable model updating, CWELD elements can successfully represent the laser spot weld joints in a top-hat structure. Further research by Abu Husain et al. about damage identification [10] and uncertainty analysis [11] on that kind of structure also benefits from the superiority of the CWELD element.

Unlike the mature application of spot weld modelling, a seam weld which creates a weld seam line to connect parts together has not attracted much research, even though it is also widely used. Most research about FE modelling of seam welded joints does not focus on reflecting the dynamical characteristics, but on their deformation [12], rotational stiffness [13], fatigue capacity [14, 15] and residual stresses [16]. In the field of structural dynamics, Zahari et al. [17, 18] studied the modelling of friction stir weld joints. But their model is simplistic and thus cannot be used generally. Chee [19] and Rahman [20] presented common finite element modelling of T-shaped structures connected by fillet welds. Their focus was on the impact of the types of elements used in plates while qualified welding is completed. The joint part modeled by a shell or a solid element is not flexible enough to represent the damage in a weld. In most cases, a seam weld provides a firm connection. But it may still suffer from cracks, underfill, burn through, incomplete fusion, long services, welders' faults and so on. Therefore, an effective modelling method of seam welded joints that can properly reflect the welding quality and can be further applied in damage identification of the welding part is required in modern industry.

Besides different joint modelling methods, model updating is applied in most of the publications mentioned above. In actual applications, geometric and material properties of the welding part are often assumed but they may slowly vary over time or their accurate values are not known beforehand. One remedy is to update these models by minimizing the differences between the predicted results and the measured experimental results by optimization methods. After that, the welding part is regarded as properly modelled. More details about model updating methods are given in [21].

Model updating is also an effective means for damage identification, especially when an initial model is available. Usually, the stiffness of the models before and after damage is compared and the reduction of the stiffness of the later indicates presence of damage. Vibration-based damage identification make use of dynamic responses such as frequency, mode shape, frequency response function, mode shape curvature and modal strain energy (MSE), any of which can be used as a damage index. Among them, MSE is adopted in this research as it was found to be more sensitive to damage by Alvandi and Cremona [22]. Several works about MSE-based damage identification are listed as follows:

Doebling et al. [23] used MSE as a mode selection criteria as a basis for other methods. Stubbs et al. [24] proposed a method using the decrease of MSE as an index for damage detection and successfully applied it on the I-40 bridge. Cornwell et al. [25]

generalized that method to plate-like structures. In these two papers, MSE was calculated by the curvature of the measured mode shapes. On the other hand, Shi et al. [26-28] developed a similar method in which the MSE calculation was based on the FE model. In addition to localization, he also explored a damage quantification method by updating the MSE of suspected damaged elements and derived the sensitivity formulas for optimization. Recently, some improvement of that method has been made by Li et al. [29] and Moradipour et al. [30, 31]. Artificial Intelligence algorithms were also introduced in this method by Seyedpoor [32] and Kaveh et al. [33, 34].

In this paper, the CSEAM element of Nastran is adopted to model the seam weld joint of a T-shape plate structure. The advantages and disadvantages of this element are discussed and its application in damage identification is shown through an FE simulation. In the simulation, the damage is in the form of an unwelded gap in the weld seam line and model updating method based on MSE is applied to detect the gap. The resulting optimization problem is solved by a genetic algorithm while Kriging model is added for computational efficiency.

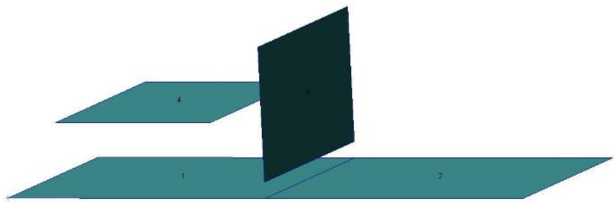
2 Seam welded joint modelling

The simplest way to join two plates (modelled by shell elements) of a T-shaped structure together is to delete extra overlapping points at the connection parts, as shown in Fig. 2a, Section 4. Thus, the two parts are assumed to be melt together representing the weld is firm enough. But in this case, the joint of the model is not adjustable and cannot be used for further calibration.

A smarter approach is to include extra shell or solid elements and delete overlapping points between them and the two plates. In this way, the quality of the weld can be changed through modifying the parameters of these elements. Model updating methods could be applied to make the model more accurate and reliable.

However, all of these methods mentioned above are limited by the mesh distribution of the two plates. They can hardly work with misalignment of the two meshes especially when the two element sizes are different. In order to solve more general problems, in this research the CSEAM element of Nastran is explored. This type of element is specially designed for modelling seam welds and can easily overcome the misalignment problem.

Demonstration about how a CSEAM element works is shown in Fig. 1. At first, two pairs of shell elements are built in Patran (Fig. 1a). Then, two hex elements are added between those two pairs and each node of the hex elements is connected to the four nodes of the corresponding shell elements by RBE3 elements (Fig. 1b). After this, the relative displacement between two connected shell elements is limited and a CSEAM element is created as such an element set.



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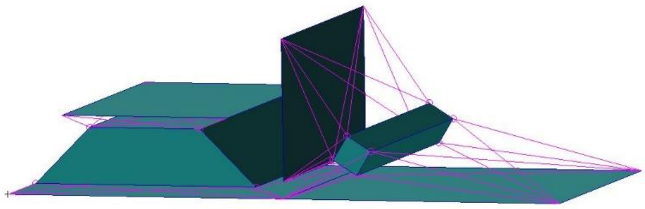

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Fig. 1. Demonstration about how CSEAM element is built in Nastran: (a) Semi-finished structure in Patran; (b) CSEAM elements rebuilt in Patran.

Two shell elements can easily be connected by a CSEAM element whether they are parallel or vertical. It is not necessary for the CSEAM element to take the same length as the shell elements. Thus it can easily join two plates with different meshes together. Obviously, this element can not only be used for a tee joint, but also a lap joint, corner joint and edge joint. But it is not included in any pre- or post-processing software packages, which makes it inaccessible.

3 Basic theory

3.1 Model updating method

When an FE model is built, the discrepancies between the measured and predicted responses, like natural frequencies and mode shapes, are unavoidable. And the model can be improved by systematically adjusting the structural parameters to minimise these discrepancies. Such a procedure is called model updating.

In seam weld joint modelling, even though the welds are well produced, model updating should still be applied to reflect the real firmness. Usually the stiffness of the whole seam element is selected as a design parameter and the minimisation is carried out via a residual-based objective function:

$$\min \sum_{i=1}^{m} W_i \left(\frac{\omega_i^{\mathrm{a}}}{\omega_i^{\mathrm{e}}} - 1 \right)^2 \tag{1}$$

where $\boldsymbol{\omega}_i^{\mathrm{a}}$ and $\boldsymbol{\omega}_i^{e}$ are the predicted and measured frequencies of the *i*th mode, respectively. $\mathbf{W}_i$ indicates the weighting coefficient of the residual of that mode.

Measured frequencies are enough for updating the intact model. However, in this research, the focus is the damage identification of the weld joint. As mentioned before, the damage is introduced as a gap within the weld seam representing missing connection. This time the mode shape must be used to localize the gap.

Raw mode shape data are not very sensitive to damage. Many methods took advantage of their derivatives, for example, mode shape curvature, flexibility matrix and MSE were used and compared in reference [22]. The MSE is adopted here, whose theory and improvement are shown as follows:

Energy stored in the *j*th element at mode *i* before and after the occurrence of damage is defined as:

$$MSE_{ij} = \boldsymbol{\phi}_i^{\mathrm{T}} \mathbf{K}_j \boldsymbol{\phi}_i, \qquad MSE_{ij}^{\mathrm{d}} = \boldsymbol{\phi}_i^{\mathrm{dT}} \mathbf{K}_j \boldsymbol{\phi}_i^{\mathrm{d}} \tag{2}$$

where $\boldsymbol{\phi}_i$ is the mode shape vector, $\mathbf{K}_j$ is the global stiffness matrix of the *j*th element. $(\ast)^{\mathrm{d}}$ represents the damaged states.

In traditional MSE methods, the elemental modal strain energy change ratio is used as a good indicator for damage localization and is defined as:

$$MSECR_{ij} = \frac{\left| MSE_{ij}^{\mathrm{d}} - MSE_{ij} \right|}{MSE_{ij}} \tag{3}$$

But in this case, damage exists in the weld part whose mode shapes cannot be measured directly. Thus the measured MSEs of joint elements are unavailable and equation (3) is unusable.

To solve this problem, in this paper a new MSE-based model updating method is proposed for seam weld joint damage identification. This method utilizes the MSE of the shell elements of the horizontal plate connected by the weld joint to localize the gap. Assuming the vertical plate is in bending, the MSE of the shell elements of the horizontal plate will increase and decrease sharply across the seam line at the edge of the gap. If the theoretical model predicted MSEs are closest to the experimental ones, it can be regarded as properly updated and the reduction of the design parameter shows the location of the gap. The objective function to maximise this MSE shape similarities is defined as:

$$\min f = -\frac{\mathbf{MSEa}^{\mathrm{T}} \cdot \mathbf{MSEe}}{\left|\mathbf{MSEa}\right| \cdot \left|\mathbf{MSEe}\right|} \tag{4}$$

where a '-' is added to turn the maximum problem into a minimum problem. **MSEa** and **MSEe** are theoretical and experimental MSE vectors of the shell elements along the weld seam line for a certain mode, respectively. One row of shell elements nearest and parallel to the weld is enough to localize a gap in the weld.

3.2 Kriging surrogate model

To solve the optimization problem in equation(4), it is not wise to use sensitivity based method as formulas will be complex and it is time-consuming to extract global stiffness and mass matrices from a structural analysis software. Instead, one of the famous evolutionary algorithms, genetic algorithm (GA), is applied.

GA is commonly used to generate a high-quality solution to optimization problems by using bio-inspired operators such as mutation, crossover and selection. It has a higher probability of identifying a global optimum solution than the gradient-based approach. Even if the process of computing gradient is impossible, it can still be applied.

Direct utilization of GA can easily solve the problem. However, GA usually requires many calculations of the objective function to get a result. Each of them needs to invoke Nastran to get the dynamic responses of the structure with the changed parameters. The whole process incurs a high computational workload.

To decrease the time taken for Nastran to calculate responses, a Kriging model is first established and then inserted into the GA process. A Kriging model is a surrogate model based on a stochastic process. It maps the input parameters to the corresponding responses mathematically which can be written as:

$$y(\mathbf{x}_i) = \mathbf{f}^{\mathbf{T}}(\mathbf{x}_i)\boldsymbol{\beta} + z(\mathbf{x}_i), \quad i = 1, 2, \cdots, n \tag{5}$$

where $\mathbf{f}(\mathbf{x})$ is a polynomial vector of the sample $\mathbf{x}$, $\boldsymbol{\beta}$ is the vector of the linear regression coefficients to be estimated and $z(\mathbf{x})$ represents errors and is assumed to be a stochastic process that follows a normal distribution of $N(0, \sigma^2)$ with a zero mean and standard deviation σ.

After the initial sampling of the design parameters (input) and the corresponding objective function values (output), a surrogate model is built for fast calculating output at random input points instead of invoking Nastran. In this way, much time saving is made when applying GA.

In the event that the initial sampled Kriging model is not precise enough to get proper results, new samples should be added to make the model more reliable especially in the area around the minimum. A Kriging model can predict the response of a new point and its mean square error. If the distance between the new point and the existing sample points is longer, this error is larger. A balance about the value and error must be reasonably considered to find the most likely minimum point. Such a point will be added to the samples and the Kriging model will be renewed until convergence.

This method is called efficient global optimization. For more details and formulas of this methods and Kriging model, the readers are referred to references [35, 36]. The procedure to implement this method to solve optimization problems is concluded as follows:

Step 1: Generate initial sample points of the updating parameters.

Step 2: Run the FE analysis program to calculate the objective function output vector of the sample points and construct the initial Kriging Model.

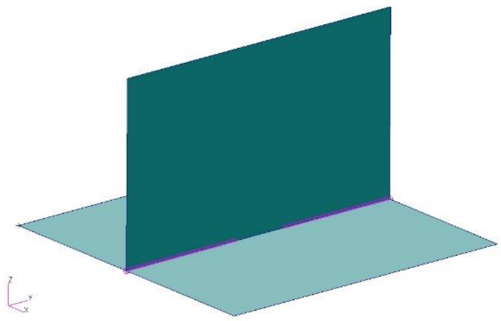
Step 3: Find the the most likely minimum point of the current Kriging model by GA and add it into the set of the sample points.

Step 4: Calculate the response output of the new sample point and reconstruct the Kriging model by the all sample points and response output.

Step 5: Check whether the procedure has converged. If so, then stop and the sample point leading to the minimum objective function value is the updating result. Otherwise, go back to step 3 and continue adding new sample points.

4 Numerical simulation

In this section, a finite element simulation is used to test the performance and robustness of the presented method. As shown in Fig. 2, two plates are connected together to simulate a tee weld joint configuration with free edge conditions.



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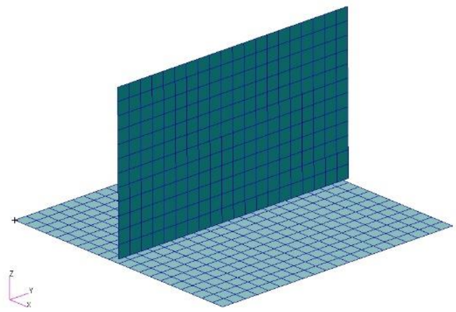

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Fig. 2. Illustrations of two T-shape models used in simulation by Patran: (a) Fine mesh with merged-node joint (b) General mesh with CSEAM element joint.

The sizes of the two plates are set to 200×200 mm (horizontal) and 200×120 mm (vertical) with 6 mm thickness. They share the same material properties: Young's modulus of 70 GPa, mass density of 2769 kg/m^3 and Poisson's ratio of 0.33. They are welded across the middle line of the horizontal plate. Two different models are constructed with shell elements.

In the 1st model shown in Fig. 2a, very fine mesh (element size is set to 1mm) is distributed onto both plates. Their connected parts, edge and middle line, are overlapped. Welding connection is constructed by merging the 2 nodes at the same location as mentioned in Section 2. The purple line shows the nodes that have been merged. Also, a gap is left in the middle of the weld seam representing the part of the welding failure (damage). The responses of this model are regarded as experimental responses.

In the 2nd model shown in Fig. 2b, the element size is set to 10mm for general use. There is a small distance between the two plates due to thickness of the horizontal plate. 20 CSEAM elements are created on both sides of the vertical plate to join the two plates together element by element, which cannot be seen in Patran. The material properties of these elements are set the same as the plates initially. Then their Young's modulus (E) is updated based on the first five frequencies of the 1st model without damage by equation (1). The updated E is 28 GPa and the improvement in frequencies is shown in Table 1.

Table 1. Natural frequency improvement after updating.

Mode	1st model (Hz)	Original 2nd model(Hz)	Updated 2nd model(Hz)
1	401.3	442.1	398.9
2	459.8	463.1	456.3
3	642.6	668.0	651.2
4	680.3	694.5	656.9
5	897.5	920.7	909.8
Average error		3.9%	1.5%

The proposed method is applied to identify the damage gap in the 1st model. The left edge point and length of the gap is chosen as updating parameters a and b, respectively. They both change from 0 to 20 with the constraint: $a+b \leq 20$ (unit: cm). As the gap represents the welding fault, the connection stiffness is 0 in the gap. During the updating process, E of each CSEAM element is multiplied by a factor which equals the proportion of its element length that is not covered by the gap represented by a and b.

In this simulation, the MSE of the 1st mode is used for damage identification. As shown in Fig. 3 and Fig. 4, in this mode the vertical plate is bending laterally across the seam line and the MSE values of the elements near the edges of the gap are extremely high, which indicates that the model updating procedure will get correct results.

All of the MSEs used are extracted from the output files of Nastran while 100 elements' MSEs of the 1st model are added to be used in equation (4) with the MSE of 1 element of the 2nd model in the same area. After the optimization, the updated a and b are compared with their actual values in Table 2. It is clear that the damage gap can be identified correctly with a tiny error by this method. This error is caused by inevitable modelling error which influences the updating result of the gap length more.

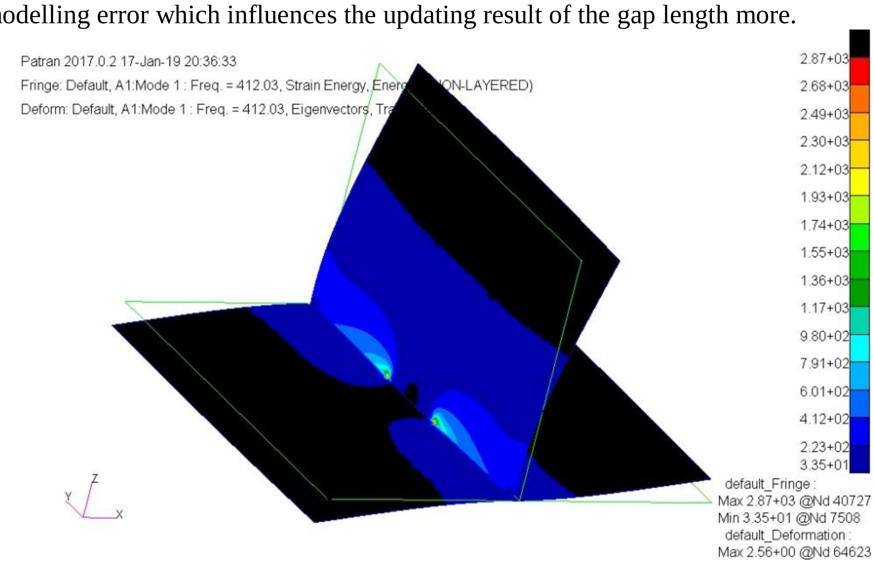

Fig. 3. Mode shape of Mode 1 of the 1st model and each element's MSE shown in color.

Table 2. Actual location and updating results of the gap.

Parameters	Actual value	Updating results without noise	Error	Updating results with noise	Error
a	7.9	8.06	2%	8.14	3%
b	4.5	4.2	6.7%	4.0	11%

To check the robustness of the proposed method, 30 percent white Gaussian noise is added to the experimental MSE of the 1st model. The contaminated MSE of the horizontal plate is shown in Fig. 4 and the updating result by these data is also listed in Table 2. This time the error of the result is larger caused by noise, but is still acceptable for damage localisation.

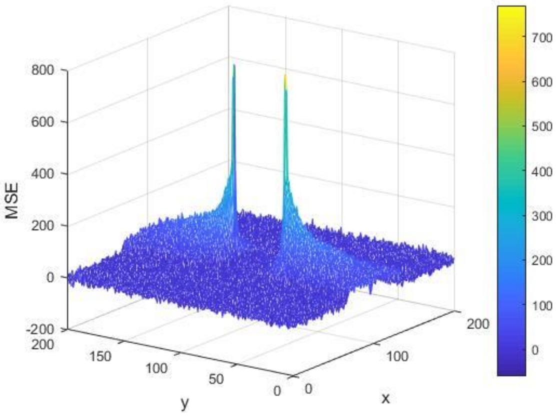

Fig. 4. Contaminated MSE of the horizontal plate of the 1st model in mode 1.

It should be noted that although considerable noise has been added to represent the real response, in practice the error of the measured MSE could be even larger. This is because the angular displacements are difficult to measure and play an important role in equation (2) especially for shell elements. So research about how to effectively measure and calculate MSE and the noise influence based on it should be further explored.

5 Conclusions

In this paper, the finite element modelling of seam weld joint based on CSEAM element is presented. This element performs well in connecting two shell element meshes together and can be conveniently used in model updating and damage identification. A Kriging surrogate model updating method using model strain energy is then proposed to detect and localise the damage caused by a welding fault in the form of a gap. The simulation results show that this method can localize the gap correctly and is robust when the experimental test results are fairly accurate.

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