

How Does the Stock Market View Bank Regulatory Capital Forbearance Policies?*

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Abstract

During the subprime crisis, the FDIC has shown, once again, laxity in resolving and closing insolvent institutions. [Romn and Verma \(1986\)](#) call the tolerance level below which a bank closure is triggered the regulatory policy parameter. We derive a model in which we make this parameter stochastic and bank-specific to infer the stock market view of the regulatory capital forbearance value. For 565 U.S. listed banks during 1990 to 2012, the countercyclical forbearance fraction in capital, most substantial in recessions, could represent 17%, on average, of the market valuation of bank equity and could go as high as 100%.

Keywords: Bank regulatory closure rules or policy parameter, bank insolvency, regulatory forbearance, market-based closure rules, financial crises

JEL classification: G17, G21, G28

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1 Introduction

Using market information and discipline to improve the design and implementation of bank regulatory policies, bank risk management, and deposit insurance has long been an important topic in the banking literature. Notwithstanding the efficient market hypothesis not necessarily holding and the emergence of endogenous risk (see, e.g., [Danielsson et al., 2012](#)) especially during financial crises, [Flannery \(1998\)](#), [Flannery \(2001\)](#), [Gunther et al. \(2001\)](#), [Krainer and Lopez \(2004\)](#), and others, do confirm that market information is useful for ranking banks and provides incremental information for bank regulators' supervisory monitoring and assessment.

Taking advantage of the relatively higher liquidity and efficiency of listed bank equity prices, our study focuses on the use of stock market information to infer the market perception of the regulatory closure rules, known following [Ronn and Verma \(1986\)](#) as the regulatory policy parameter. This parameter is most often driven at least in part by politics, especially in the case of systemically important financial institutions (SIFI) which will be subject to enhanced capital requirements according to the current Basel 3 regulatory framework. This parameter represents a (hypothetical, conjectural, or even real) limit, expressed as a percentage ρ ($0 \leq \rho \leq 1$) of the total debt value D of the bank at the time of supervisory audit, beyond which the dissolution of assets by regulatory bodies would be a reasonable alternative. If the value of the bank falls between ρD and D , the insuring agency forbears (e.g., the Savings and Loans crisis in the 1980s), and in the case of extreme market turmoil, the government intervenes, as witnessed during crises such as the 2007-2009 subprime crisis via the Troubled Asset Relief Program (TARP, see [Veronesi and Zingales, 2010](#)), infusing up to $(1 - \rho)D$ and making it equal to D . If the bank value falls below ρD , the insuring agency steps in to dissolve the assets of the bank. A compelling reason for not closing an insolvent bank is the loss of significant franchise value

(stemming from core deposits, customer relationships, and valued personnel) that occurs after an FDIC seizure. If the market insolvency closure rule is strictly followed, the policy parameter is equal to one, and there is regulatory capital forbearance when the policy parameter is below one. Along with other assumptions, [Ronn and Verma \(1986\)](#) fix ρ at a constant 0.97 to yield an aggregate weighted average premium of $1/12$ percent for the US deposit insurance premium. Since, in [Ronn and Verma \(1986\)](#)'s model, the value of forbearance is assumed to be the value of the capital assistance, many authors call this practice, regulatory capital forbearance.

Under the Basel 3 countercyclical capital buffer framework, bank regulators would try to ensure that banks build up capital levels during good times so that they can run it down in bad times (see, e.g., [Drehmann et al., 2010](#); [Hanson et al., 2011](#)). In effect, to conduct countercyclical capital requirements policies, regulators are compelled to adopt a time-varying policy parameter. To the best of our knowledge, only [Lai \(1996\)](#) treats the policy parameter as a stochastic process to reflect the uncertainty of the bank closure rules.¹

In this paper, we extend [Lai \(1996\)](#)'s framework by modeling ρ as a more realistic stochastic process that is mean-reverting and bound by zero and one. To justify this, we argue that the regulatory forbearance policy can be treated as "reduced form" and described by a state variable. Further, the policy is constrained by economic, legal, political, regulatory competition, and bureaucratic considerations that are mean-reverting throughout their respective cycles. Two main sources of uncertainty regarding the variation of bank regulatory forbearance may be posited: (1) asymmetric information and (2) stochastic state variables. In the first case, because of confidential information obtained from on-site examinations, private information may exist that is known only to the regulator.² For actively stock-listed banks in efficient markets with increased disclosure, such a scenario may be considered not as a major source of uncertainty. For closely-held banks, it is more plausible but we discard this possibility by only studying banks with available equity prices. In

¹[Kane \(1986\)](#) treats safety-net guarantees as a two-part option: a taxpayer put and a knock-in, stop-loss call on the firm's assets; while [Allen and Saunders \(1993\)](#) model forbearance as forfeiture by the deposit insurer of the value of its call component of the deposit insurance option.

²[DeYoung et al. \(2001\)](#) empirical results indicate that on-site examinations do produce value-relevant information about the future safety and soundness of banks not reflected immediately in their debenture prices whereas we focus on their stock prices.

the second situation, we suppose that while the policy of the regulator is known to all (e.g., the Too Big to Fail (TBTF) doctrine), it is a function of some external, hardly predictable state variables, structural changes, or unforeseeable events. While asymmetric information and stochastic state variables are two possible sources of regulatory uncertainty, only the latter is explicitly modeled in this paper.³ With this justification, we develop an enhanced [Ronn and Verma \(1986\)](#) model to infer the market-based, bank-specific, and more flexible policy parameter from the market value of bank equity.⁴ Then, based on the calibration of our model with U.S. listed banks, by gauging the size of the capital forbearance value and contributing to the ongoing debate on adequate bank capital, we seek answers to the following two research questions: 1) How does the time-varying capital forbearance portion embedded in bank equity depend on various banks' own risk and business cycle variables? and 2) How do banks' market-assessed intrinsic (i.e., devoid of the forbearance subsidy) capital ratios (or inverse leverage ratios) react to various business cycles and the banks' own risk variables?

Toward this end, we first develop a two-factor model, in which we model ρ as an exponential of a negative Cox-Ingersoll-Ross (CIR) process ([Cox et al., 1985](#)) and the value of the bank as a log normal process with a stochastic drift term. We derive a closed-form solution for the equity represented by market capitalization (market cap), which is viewed as a call option on the value of the bank following the structural approach of [Merton \(1974\)](#). The model is then calibrated to 565 U.S. banks' market capitalization and total debt data from 1990-2012 by means of the Unscented Kalman Filter in conjunction with the Quasi-Maximum Likelihood Estimate (QMLE) to obtain series of ρ s and implied asset values. Further, we derive the *Forbearance Fraction of Capital*, which is defined as one minus the ratio of the market value of intrinsic equity to total equity. We estimate

³Not postulating forbearance as the outcome of a random process, political-economy theorists treat it as a function of insurer resources and workloads and of insolvent firms' size, complexity, and political clout. When firms are truly too big or too many to fail, some of the insurer's deposit insurance calls are never or almost never going to be exercised. This allows these firms a real option to expand their balance sheets and "gamble for resurrection".

⁴To compare the current market capital-asset ratio to the current regulatory capital-asset ratio for a U.S. sample of publicly traded bank holding companies and savings and loans associations in 1990, [Cordell and King \(1995\)](#) present an approach for extending the forbearance factor to be bank and time-specific. However, [Cordell and King \(1995\)](#) arbitrarily assign values for the critical value of the policy parameter and use an interrelated approximation of what they call the conditional value of forbearance.

bank intrinsic equity, *Intrinsic Market Cap*, indicative of economic capital, as a hypothetical and counter-factual equity assuming zero capital forbearance as if the FDIC adhered to the market closure rule. We also compute the effective policy parameter, which weighs, not only the likelihood of a bank being insolvent, but also the likelihood of the bank not requiring forbearance. These two novel metrics provide us relatively cleaner measures of the market perceptions of the time-varying regulatory agencies' implementation of the bank closure rule.

Using Wells Fargo as a case study, we show that our model effectively captures the market view about the regulatory capital forbearance practice. From our aggregate results, during the financial crisis period, we find that the total estimated forbearance value surges over three years earlier than the massive liquidity provision by two crisis facilities, namely the Term Auction Facility (TAF) and the Term Asset-Backed Securities Loan Facility (TALF). Also, throughout the financial crisis, our estimated value of capital forbearance follows closely these facilities' changes in the liquidity provision measured by the changes in their outstandings.

To address our two research questions, we estimate a system of two equations using the "Two-step System Generalized Method of Moments (2SGMM)" (Blundell and Bond, 1998), which controls endogeneity between regression variables. In the first equation, the left-hand-side (LHS) variable is the Forbearance Fraction of Capital computed from the filtered policy parameters, and the right-hand-side (RHS) variables include a one-quarter-lagged LHS variable, the business cycle proxies (GDP Growth, GDP Output Gap, and S&P 500 index), banks' risk factors (Idiosyncratic Volatility, Asset Volatility, Systematic Beta, and Systemic Risk measures), and other control variables (log Total Assets, Intrinsic Market Cap to Implied Asset Value Ratio, and Implied Asset Value to Book Total Assets Ratio). Simply, $\text{Intrinsic Market Cap} / \text{Implied Asset Value}$ is a measure of the equity capital ratio devoid of the regulatory forbearance value, whereas the Implied Asset Value to Book Total Assets Ratio is a proxy for Keeley (1990)'s bank charter value. In the second equation, the LHS variable is the Intrinsic Market Cap to Implied Asset Value ratio. The RHS variables are the same as those in the first equation except that the Intrinsic Market Cap-to-Implied Asset Value ratio replaces the Forbearance Fraction in Capital.

Our regression results show that the market believes that larger banks benefit more from capital forbearance, which suggests that the “Too Big To Fail” (TBTF) doctrine is prevalent. Naturally, the market expects banks with strong performance and higher market-assessed levels of owner-contributed capital to receive less forbearance. The market also expects a bank with high market power to be given less forbearance, consistent with the competition-stability paradigm, and an institution with high franchise value to cost less to rescue. The stock market holds the view that, *ceteris paribus*, banks with higher idiosyncratic risk and systemic risk will benefit from more capital forbearance, thereby leading to higher bailout costs to taxpayers. The market expects banks to receive, in a countercyclical fashion, increased forbearance in bad times and less forbearance in boom times. More forbearance lowers the market intrinsic capital ratio (or bank owner’s contributed capital with the forbearance subsidy removed). For our period of study and our bank sample, the estimated annual capital forbearance subsidy amounts to 7.6 billion USD or 13.5% of the FDIC aggregate cost of explicit deposit insurance. With an embedded mean capital forbearance portion amounting to 17% of the market value equity, the largest banks exhibit a mean market capital to implied asset value ratio of 16.4% and a mean book equity to total assets ratio of 8.9%. It appears that our framework may be useful for market assessments and tracking of regulatory capital forbearance and may serve as an additional red flag tool for supervisory bodies and policymakers.

The remainder of the article is organized as follows. [Section 2](#) sets up the two-factor model to extend the framework of [Ronn and Verma \(1986\)](#) and [Lai \(1996\)](#) and derive a closed-form solution for the model used to extract the market view and the cost of bank regulatory forbearance policies. [Section 3](#) describes the procedure for model calibration and data in detail. [Section 4](#) contains a study case. Sections [5](#) and [6](#) present the empirical results and discuss the findings. [Section 7](#) concludes the paper. The Online Appendices contain technical details and supplemental results.

2 A model of the bank regulatory policy parameter

2.1 Pay-off function of a bank equity holder

We follow [Ronn and Verma \(1986\)](#)'s seminal model, as it is relatively simple and will be amenable to econometric implementation later. In the model, the equity of a firm is a European call option on the value of the firm, V , with the strike price being the debt face value, D . To model the FDIC closure rules, [Ronn and Verma \(1986\)](#) modify the model by simply adjusting the strike price, i.e., multiplying the debt level by the regulatory policy parameter ρ . At supervisory examination time, if $\rho D \leq V \leq D$, the equity holder's pay-off is $V - \rho D$. In their model, the equity holder's pay-off, when $V > D$, is still $V - \rho D$. A rational investor would not include $(1 - \rho)D$ as part of her pay-off when the bank is solvent ($V > D$). Since in this case there is no forbearance, the pay-off, when $V > D$, is assumed to be $V - D$.⁵ The [Ronn and Verma \(1986\)](#) option pricing framework for analyzing bank equity valuation is parsimonious and useful, it has been employed in numerous empirical studies by academics and practitioners alike.⁶ Nevertheless, we reckon that the [Ronn and Verma \(1986\)](#) framework is a one-period model. To mimic a dynamic framework, we assume agents keep on refreshing the expectation on the limited term capital forbearance cash (real or conjectural) injection as new information arrives (which is how we empirically calibrate our model). This inevitably carries some bias due to the unmeasured continuation option value as shown in [Pennacchi \(1987\)](#).⁷ To gauge the bias, we conduct a simulation exercise described in Online Appendix A and find that the bias

⁵Admittedly, if one considers the sale of the bank's equity to a third party during a period in which the bank is not in distress and the FDIC is not stepping in to prop up the sale, the value should still reflect the forbearance which could occur as a function of future outcomes. Alternatively, if the pay-off function simply represents the fact that the FDIC is willing to facilitate a make-whole buyout along the lines of Wells Fargo's acquisition of Wachovia examined later, in our case study, it is not clear that the equity holders actually do receive the difference between the true value and the debt. If these were the cases, we underestimate the forbearance value.

⁶A large literature follows [Ronn and Verma \(1986\)](#) and employs a constant regulatory policy parameter applied to all banks to estimate the two unobservable values of the bank assets and asset volatility (e.g., [Flannery and Sorescu, 1996](#); [Cordell and King, 1995](#), among numerous others). These two inputs have been used for many ends, for instance, to estimate the value of historical government guarantees as in [Hovakimian and Kane \(2000\)](#) and [Flannery \(2014\)](#). A growing literature uses this methodology with constant ρ , to estimate banks' systemic risk; see, e.g., [Lehar \(2005\)](#), [Hovakimian et al. \(2012\)](#). There has also been a great deal of research on the impact of this policy parameter on deposit insurance pricing; e.g., [Lai \(1996\)](#) and [Hwang et al. \(2009\)](#) among others.

⁷We thank the referee for pointing out this feature with the [Pennacchi \(1987\)](#) reference.

is likely to be small. Furthermore, the alternative specification presented in Online Appendix A, which models the forbearance value directly as a put option, has implications on the interpretation of the aggregate forbearance fraction of capital in relation with some of Fed's emergency lending programs during the 2008 financial crisis. These are discussed in [Section 5.3](#).

2.2 Model setup

Let us define a 2×1 column vector of state variables

$$X_t = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

which, under the risk-neutral measure \mathbb{Q} , follows the stochastic differential equations (SDE)

$$dX_t = d \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \left[\begin{pmatrix} \kappa\theta \\ \mu \end{pmatrix} + \begin{pmatrix} -\kappa & 0 \\ \varphi & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \right] dt + \begin{pmatrix} \sigma_1 \sqrt{x_1(t)} & 0 \\ 0 & \sigma_2 \end{pmatrix} d \begin{pmatrix} w_1(t) \\ w_2(t) \end{pmatrix}. \quad (1)$$

x_1 follows a CIR process, and x_2 follows a Wiener process with a stochastic drift term. θ is the reversion level of x_1 , κ is the reversion speed of x_1 , σ_1 is the volatility scaled by the square root of x_1 , μ is the mean of x_2 and σ_2 is the volatility of x_2 . $w_1(t)$ and $w_2(t)$ are independent Wiener processes under the measure \mathbb{Q} . The correlation between x_1 and x_2 is captured by φ . Note that we model exogenously the stochastic policy parameter as $\rho_t = e^{-x_1(t)}$, so that ρ_t lies between zero and one as x_1 is non-negative, and ρ_t is mean reverting. Since we model the value of a bank as $V_t = e^{x_2(t)}$, the dynamic of V_t is given by,

$$dV_t = de^{x_2(t)} = \mu_V V_t dt + \sigma_2 V_t dw_2(t)$$

where

$$\mu_V = \mu + \varphi x_1(t) + \frac{\sigma_2^2}{2}.$$

For the sake of tractability, we model ρ_t and V_t in a reduced-form manner.⁸ We impose economic structure ex-post in later empirical sections to see how ρ_t and V_t correlate with various other variables (both bank-specific and macro variables).

At time t , the equity value of a bank represented by the market cap, E_t , which is a call option on V_t with maturity T , has the following pay-offs at time T :

$$E_T = \begin{cases} V_T - D & \text{if } V_T > D \\ V_T - \rho_T D & \text{if } \rho_T D \leq V_T \leq D \\ 0 & \text{if } V_T < \rho_T D. \end{cases} \quad (2)$$

The stock market assessment of the supervisory forbearance is captured by this pay-off function with a random strike price. Then, with $\mathbb{E}_t^Q [\cdot]$ denoting the expectation, under the measure Q conditional on the information up to t , we have⁹

$$\begin{aligned} E_t &= B_t(T) \mathbb{E}_t^Q (E_T) \\ &= B_t(T) \mathbb{E}_t^Q \left[(V_T - D) \mathbb{1}_{\{V_T > D\}} + (V_T - \rho_T D) \mathbb{1}_{\{\rho_T D \leq V_T \leq D\}} \right] \\ &= B_t(T) \left\{ \mathbb{E}_t^Q \left[(V_T - \rho_T D) \mathbb{1}_{\{V_T > \rho_T D\}} \right] - D \mathbb{E}_t^Q \left[(1 - \rho_T) \mathbb{1}_{\{V_T > D\}} \right] \right\} \\ &= B_t(T) \left\{ \mathbb{E}_t^Q \left[\left(e^{x_2(T)} - e^{-x_1(T)} D \right)^+ \right] \right. \\ &\quad \left. - D \mathbb{E}_t^Q \left[\left(1 - e^{-x_1(T)} \right) \mathbb{1}_{\{x_2(T) > \log D\}} \right] \right\}, \end{aligned} \quad (3)$$

where $B_t(T) = e^{-r(T-t)}$ is the discount factor, r is the assumed-constant interest rate, and $\mathbb{1}_{\{\cdot\}}$ is an indicator function equal to one when $\{\cdot\}$ holds, and zero otherwise. In the implementation, we set $D = F/B_t(T)$ where F is the book value of the debt level at time t .¹⁰

⁸Endogenizing ρ within the model would of course yield a richer framework which might offer interesting results, but we defer this challenge to a further research.

⁹The risk-free interest rate is assumed to be constant. As in [Ronn and Verma \(1986\)](#), it is also assumed that the effects of interest rate changes are captured in the assets value and associated volatility, i.e., the present value of assets are brought about by anticipated changes in both the investment opportunity set and the interest rate. Unanticipated changes in interest rates are accounted for in the asset risk.

¹⁰An implicit assumption behind $D = F/B_t(T)$ is that all debt is issued at the risk-free interest rate. As mentioned in footnote 7 of [Ronn and Verma \(1986\)](#), this is no doubt valid for the insured deposits, which for most banks represent a substantial portion of their total debt. Another implication of $D = F/B_t(T)$ is that V_t denotes the value of the assets net of the deposit insurance fees levied. In other words, V_t , on top of other value drivers, includes the net value of deposit insurance provided by the FDIC (i.e., benefits minus premiums paid).

In sum, we offer two enhancements: (a) we employ a modified pay-off function (see (2)) for the call option while [Ronn and Verma \(1986\)](#) and [Lai \(1996\)](#) use a pay-off that overrates the value of forbearance (supportive evidence is provided in [Section 5.5](#)),¹¹ and (b) in [Ronn and Verma \(1986\)](#), ρ is a constant over time, whereas we model ρ as a mean reverting stochastic process.

2.3 Market evaluation of the capital forbearance

To apply the model developed above, we propose two novel measures of the stock market evaluation of the forbearance: the *Forbearance Fraction of Capital* denoted by FFC, and the *Effective Policy Parameter* denoted by ρ^* . These are defined as

$$\text{FFC}_t = \frac{E_t - E_t^{\rho=1}}{E_t} \quad \text{and} \quad \rho_t^* = \mathbb{E}_t^P \left[\mathbb{1}_{\{V_T > D\}} + \rho_T \mathbb{1}_{\{V_T \leq D\}} \right],$$

where $E_t^{\rho=1} = B_t(T) \mathbb{E}_t^Q \left[(V_T - D)^+ \right]$ which corresponds to the value of the equity when assuming $\rho_t = 1$, and \mathbb{E}_t^P is the expectation under the physical measure P. $E_t^{\rho=1}$ represents a counter-factual valuation of the bank equity if the FDIC adheres strictly to the market closure rule. Whether ρ is constant or stochastic, we assume the same value for V_t .¹²

By construction, FFC represents how forbearance is valued in terms of total equity, and can be used to decompose the market cap into the forbearance subsidy and intrinsic value; whereas ρ^* represents an effective value of the policy parameter. ρ^* is better than ρ as a proxy for the market view of the policy parameter. This is because ρ is only relevant when the probability of $\{V_T \leq D\}$ is significant, while ρ^* also takes into account the case of no forbearance.

¹¹The pay-off function in [Ronn and Verma \(1986\)](#) and [Lai \(1996\)](#) adds an extra forbearance value to the bank while in fact it should not when the bank is in healthy status. Therefore, the forbearance implied from a calibrated [Ronn and Verma \(1986\)](#) model is in general higher than that resulting from a more reasonable and non-linear pay-off function.

¹²Albeit subject to the Lucas critique, this assumption is intuitively sensible, V_t can be roughly represented as $\rho_t F + E_t(\rho = \rho_t)$, it is reasonable to assume $(1 - \rho_t)F = E_t(\text{with } \rho \text{ stochastic when } \rho = \rho_t) - E_t(\text{with } \rho \text{ nonstochastic and } \rho = 1)$. Here F is the total debt book value at time t .

2.4 Closed-form solutions of E_t , FFC, and ρ^*

Within the purview of the affine framework, we employ the techniques developed in [Duffie et al. \(2000\)](#) to derive the closed-form solutions for E_t and ρ^* . By PROPOSITIONS 1 and 2 in [Duffie et al. \(2000\)](#), we have

$$\begin{aligned} \mathbb{E}_t^Q \left[\left(e^{x_2(T)} - e^{-x_1(T)} D \right)^+ \right] &= \mathbb{E}_t^Q \left[e^{[-1,0]X_T} \left(e^{[1,1]X_T} - D \right)^+ \right] \\ &= G_{[0,1],[-1,1]}(D; X_t, T-t) - DG_{[-1,0],[-1,1]}(D; X_t, T-t), \end{aligned} \quad (4)$$

$$\begin{aligned} \mathbb{E}_t^Q \left[\left(1 - e^{-x_1(T)} \right) \mathbb{1}_{\{x_2(T) > \log D\}} \right] &= \mathbb{E}_t^Q \left[\mathbb{1}_{\{[0,1]X_T > \log D\}} \right] - \mathbb{E}_t^Q \left[e^{[-1,0]X_T} \mathbb{1}_{\{[0,1]X_T > \log D\}} \right] \\ &= G_{[0,0],[0,-1]}(D; X_t, T-t) - G_{[-1,0],[0,-1]}(D; X_t, T-t), \end{aligned} \quad (5)$$

$$\begin{aligned} \mathbb{E}_t^P \left[\mathbb{1}_{\{V_T > D\}} + \rho_T \mathbb{1}_{\{V_T \leq D\}} \right] &= \mathbb{E}_t^P \left[\mathbb{1}_{\{[0,1]X_T > \log D\}} \right] + \mathbb{E}_t^P \left[e^{[-1,0]X_T} \mathbb{1}_{\{[0,1]X_T \leq \log D\}} \right] \\ &= G_{[0,0],[0,-1]}^P(D; X_t, T-t) + G_{[-1,0],[0,1]}^P(D^{-1}; X_t, T-t), \end{aligned} \quad (6)$$

where $[\cdot, \cdot]$ denotes a 1×2 row vector. [Eq. \(4\)](#) and [Eq. \(5\)](#) (multiplied by D) represent the first and second elements in the curly brackets of [Eq. \(3\)](#) used to obtain E_t , respectively. [Eq. \(6\)](#) is for ρ^* . $G_{b,\delta}(z; X_t, \tau)$ in these equations is defined in Online Appendix B.

Using the results from [Eq. \(4\)](#) to [Eq. \(6\)](#), we obtain the closed-form solutions for E_t , FFC, and ρ^* . Note that $E_t^{\rho=1}$, which is required for the calculation of FFC, is simply given by the [Black and Scholes \(1973\)](#) formula, since with $\rho = 1$, V_t is a geometric Brownian motion.¹³ To calibrate the model, [Duffee \(2002\)](#)'s specification for the market price of risk is employed to derive the dynamic of X_t under the physical measure P. That is, the relation between the Wiener processes under the two measures is given by

$$\begin{aligned} w_1(t) &= w_1^P(t) + \lambda_1 \int_0^t \sqrt{x_1(s)} ds \\ w_2(t) &= w_2^P(t) + \lambda_2 t, \end{aligned}$$

¹³When applying [Black and Scholes \(1973\)](#) formula to $E_t^{\rho=1}$, the volatility of V , σ_2 , is set at its estimate from the full sample calibration of E_t ; the value of V at time t is the filtered value of V_t from the calibration; the strike price is the debt level at time t ; and the interest rate is the same as the one used to compute E_t .

where $\lambda_1 = \frac{\kappa - \kappa^P}{\sigma_1}$ and $\lambda_2 = \frac{\mu^P - \mu}{\sigma_2}$. The moment conditions for X_t under the measure P used to calibrate the model are presented in Online Appendix C. The time to the next supervisory examination, $T - t$, is assumed to be one year in our empirical implementation. Note that in our setup, although the volatility of V_t , σ_2 , is constant, the volatility of E_t is stochastic by construction and is closely linked to $\sqrt{x_1(t)}$. As pointed out by [Duan \(1994\)](#), the widely employed [Ronn and Verma \(1986\)](#) estimation method that assumes constant equity volatilities is incompatible with the [Merton \(1974\)](#) model. We formally address the estimation problem in assuming a constant volatility of V_t in Online Appendix D. Note also that we allow for time-varying risk premia but we do not account for liquidity in our model. This is less critical for our equity model since stock markets are relatively much more liquid than debt and credit default swap markets considered for instance by [Acharya et al. \(2015\)](#) and [Kelly et al. \(2015\)](#) among others. Further, since we account for measurement errors in the observed market cap, the transformed-data maximum likelihood estimation (MLE) method developed in [Duan \(1994\)](#) is improper. Given the nonlinearity of the model and the latency of the factors, to calibrate the model, we employ the QMLE in conjunction with the UKF.¹⁴ We describe the methodology in [Section 3.2](#) and Online Appendix E.

3 Model calibration

3.1 Data description

From the Bloomberg database, we build a sample of 23 years (from 1990 to 2012) of both market and financial statements accounting data for 706 banks (most of these are bank holding companies, for short, banks hereafter) as well as macroeconomic variables. Because many banks merged or went out of business or do not have complete data during this period, banks are not necessarily included in every year of the sample, giving an un-

¹⁴In a nutshell, since our state variables are unobservable, and the observable series of bank market caps have a nonlinear dependence on the latent state variables, we obtain the QMLE of our model parameters using the UKF procedure. The UKF deals with nonlinearities in the measurement equation, works through deterministic sampling of points (sigma points) in the distribution of the innovations to the state variables, and captures the conditional mean and variance-covariance matrix of the state variables with adequate accuracy.

balanced panel.¹⁵ Our resulting sample consists of 706 banking firms that appear in any one of the 23 years. Based on their total assets, after a thorough analysis of bank size percentiles, we categorize, for statistical purpose, the banks into four groups as the following: Large Banks (\geq 90th percentile), Big Banks (75th - 89th percentile), Medium Banks (20th - 74th percentile), Small Banks (\leq 19th percentile). Following the empirical evidence on the TBTF implication on diversification and market discipline (see, e.g., [Filson and Olfati, 2014](#); [Acharya et al., 2015](#)), we exclude Small Banks group in our analysis to focus on sizable banks which are commonly believed to receive forbearance benefits, and replace it with All but Small Banks, thereafter All Banks (20th - 100th percentile). This leaves us with 565 banking firms. The average total assets of these four groups are: 149 billion, six billion, one billion, and 18 billion, respectively.¹⁶

To filter the regulatory forbearance parameter and implied asset values, in the model calibration we use one-year U.S. Treasury rates (used to calculate the discount factor $B_t(T)$ in [Eq. \(3\)](#)), banks' total liabilities, and market caps. The market data are monthly (month end observations from daily data), and the total liabilities data are quarterly. The total liabilities data are fitted to the monthly frequency using cubic smoothing splines to match the frequency of the market data.

3.2 State space, Quasi-Maximum Likelihood Estimates (QMLE), and Unscented Kalman Filter (UKF)

Basically, we set a two-factor model for the market cap of banks. The first factor is the unobservable regulatory policy parameter, and the second is the unobservable bank asset value. The model is cast into a state space, and then parameters are calibrated to

¹⁵We do not exclude merger and acquisition events (M&A) in our sample. A huge part of our data contains M&A, if we stick to an M&A-free sample, two third of our sample data will be thrown away. However, our model is robust to M&A news, for instance, information about the intention to seek for the forbearance from the TBTF status as noted by [Bijlsma and Mocking \(2013\)](#) and [Brewer and Jagtiani \(2013\)](#).

¹⁶These averages are calculated over the past two decades and across all individual banks within these groups. The cross-sectional averages in USD at the end of 2012 for the groups of Large, Big, Medium, and All Banks are 393 billion, 12 billion, two billion, and 45 billion, respectively. We are aware that the Dodd-Frank Act (DFA) enacted in 2010 has legally defined size thresholds and the DFA might impose differential regulatory requirements for banks above specified asset size thresholds, see e.g., [Bouwman et al. \(2018\)](#). However, these thresholds are defined based on the current bank size distribution and do not apply to studies looking at historical size distributions. Therefore, we believe our percentile-based size categorization is more robust in the sense that the number of banks in each category is stable over the different time periods.

the observed market cap using QMLE together with the UKF method. More details of the UKF are presented in Online Appendix E.

Our reduced-form model is similar to the term structure models in the interest rate and credit derivatives literature. The fact that we do not have term structure data as we rely exclusively on equity data makes certain parameters hard to pin down, for example, θ and κ^P . We therefore set them to be conventional values during the model calibration. In pretesting results, we also find that setting parameter φ to zero has little impact on the filtered state variables, but ensures more robust model calibration by avoiding dealing with trigonometric functions and imaginary numbers in the numerical procedure. Therefore, in our empirical analysis, we assume that φ is zero. More implementation details are listed in Online Appendix F.

To calibrate the model, we need the dynamics of the state variables under the real measure P. We employ the essentially affine specification for the market prices of risks (Duffee, 2002), which allows for time-varying risk premiums, to derive the dynamics under measure P from the dynamics under the risk-neutral measure Q in Eq. (1). That is, the transition equation in the state space is given by

$$dX_t = \left[\begin{pmatrix} \kappa\theta \\ \mu^P \end{pmatrix} + \begin{pmatrix} -\kappa^P & 0 \\ 0 & 0 \end{pmatrix} X_t \right] dt + \begin{pmatrix} \sigma_1 \sqrt{x_1(t)} & 0 \\ 0 & \sigma_2 \end{pmatrix} dW_t^P,$$

where W_t^P consists of two independent Wiener processes under the measure P. The change of measure reflects the risk premia carried by the state variables.¹⁷ Although the transition density of the latent factor is non-Gaussian, Duan and Simonato (1999) show that the first two moments of the latent factors can approximate the distribution of the CIR process very well. Therefore, when employing the UKF, we assume a Gaussian transition density and only consider the first two moments of the transition density.

Since a bank's market cap enters into the measurement equation in our state-space setup, we assume that the observed market caps denoted by y_t are contaminated by mea-

¹⁷The specification of the market price of risk is required to model non-diversifiable risks. The risks of our state variables are indeed non-diversifiable, here, especially the risk of the non-tradable variable ρ .

surement errors and the noise is iid and normally distributed as expressed by

$$y_t = E_t + \zeta_t \equiv \mathcal{G}(X_t; \Phi, D) + \zeta_t,$$

where Φ is the parameter set, $E_t \equiv \mathcal{G}(X_t; \Phi, D)$ is the nonlinear function of X_t given by Eq. (3) to Eq. (5), and

$$\zeta_t \sim \text{IID } N(0, \omega^2),$$

where ω is a free parameter. The parameters are estimated by maximizing the sum of the following log transition density over the whole sample:

$$ll_{t|t-\Delta t} \propto -\frac{\log(P_{y_t|t-\Delta t})}{2} - \frac{(y_{t|t-\Delta t} - y_t)^2}{2P_{y_t|t-\Delta t}},$$

where $P_{y_t|t-\Delta t}$ and $y_{t|t-\Delta t}$ are estimates of the variance and mean of the measurement at time $t - \Delta t$. These are outputs from the UKF procedure outlined in Online Appendix E.

To sum up, given a set of market caps, total liabilities, and model parameters, the UKF (with QMLE) procedure outputs a series of filtered forbearance factors ρ_s , implied asset values V_s and a likelihood value specific to a bank. Once the model is calibrated, we can readily compute FFC and ρ^* from ρ_s and V_s .¹⁸

4 Case study: Wells Fargo

In this section, we take Wells Fargo as an example to depict the dynamics of the stock market view of the regulatory forbearance factor manifest in a given bank.

Figure 1 illustrates the time series of the (fitted) market cap, implied firm value, smoothed total liabilities, ρ^* , and FFC of Wells Fargo (on a monthly basis) from early 1998 to late 2012.¹⁹ Before the financial crisis, FFC was relatively lower and stable. Wells Fargo was

¹⁸We only run in-sample calibration once for all, and do not use rolling windows to update model parameters. The rolling window calibration seemingly offers time-varying property for the model parameters, e.g., σ_1 and σ_2 . However, this time-varying property is theoretically inconsistent with the assumption that these parameters are constant in our framework.

¹⁹Wells Fargo in its present form is the result of a merger between San Francisco-based Wells Fargo & Company and Minneapolis-based Norwest Corporation in 1998, so the financial and market data for Wells Fargo in our database only starts in early 1998.

one of the banks to receive funds from the first round of government bailout money (the TARP) in the fall of 2008. The acquisition of Wachovia at the end of 2008 significantly increased Wells Fargo's total liabilities in 2009. In early 2009, when there were negative allegations against Wells Fargo in the media, Wells Fargo's market cap dropped significantly. Although it rebounded quickly, within a month, the total liabilities remained double what they had been. FFC surged from below 10% to about 70% within a quarter, then took almost four years to return to 10%. The drop in the market cap and the increase in the total liabilities in 2009 induced a significant decrease in both the Market Cap / Implied Asset Value (from 20% to 7%) and the Intrinsic Market Cap / Implied Asset Value (from 17% to 3%). The drop in Intrinsic Market Cap / Implied Asset Value was even more dramatic. The market was not optimistic about Wells Fargo's future despite the huge capital injection (with equity warrants/preferred stock holdings) from the government. In contrast, the Book Equity / Total Asset did not change much in 2009, meaning the book ratio were almost indifferent to the market turmoil.

Our model fits the equity data very well and there is barely any difference between the fitted Market Cap (the line with dots) and the actual Market Cap (the line with circles) (see the upper panel of Figure 1). The great fit of the model implies that the dynamic of our filtered implied asset values is congruent with the distribution of the observed equity values.

[Insert **Figure 1** about here]

5 Empirical analysis I: Aggregate results

5.1 FFC

The dynamics of the cross-sectional distribution of FFC over time, on a monthly basis, are shown in **Figure 2**. Note that the light gray area (75th-90th percentile) covers the majority of the distribution, meaning that the distribution of FFC is highly skewed to the left and has a heavy right tail. This pattern is more significant for larger-sized banks. While not necessarily taking these values literally, we find that, in large banks, capital forbear-

ance takes up on average 17% of the market value of bank equity, and can go as high as 100%. As depicted in the first panel in [Figure 2](#), even though the medians of FFC for all four groups started to rise around the same time during the 2008-2009 financial crisis, the FFCs of a few mega banks show a much earlier upward trend than others. This indicates that the stock market perceived large banks' troubles long before the crisis became widespread. Another interesting observation is that during 1998 the FFCs of most banks were actually relatively low. Only a few large banks had very high FFCs during this period. These large banks probably had more international exposure than the others, and would therefore have been more affected by the 1998 Asian and Russian financial crisis. This observation might also be related to the consequences of the Gramm-Leach-Bliley Act of 1999 that led to deregulation toward universal banking, allowing banks to consolidate and offer one-stop services. The aggregate pattern of the FFC very much resembles the time series of "Government Support" shown in Fig. 1 of [Correa et al. \(2014\)](#).²⁰

[Insert Figures 2 about here]

Since we assume a constant volatility for the bank asset value, one might be concerned that the estimated ρ_t could pick up the time varying dynamics of the volatility of the bank value when it is indeed stochastic.²¹ To assure that this is not the case, we conduct a simulation exercise described in Online Appendix D where we show that the estimated ρ is little affected by the stochastic volatility of the bank asset value V_t . We do find that under the stochastic volatility case, the estimated constant volatility σ_2 for the bank asset value, overstates the level of the actual volatility. This is similar to the observation that the implied volatility is usually higher than the realized volatility due to the premium priced in the option price for bearing stochastic volatility risk. Since the estimated σ_2 is not used in our empirical study, inherent bank value stochastic volatility does not alter our results.

²⁰It may be interesting to implement this model on nonbank firms where regulatory forbearance should not be an issue. However, since it is not straightforward to apply the [Ronn and Verma \(1986\)](#) framework tailored to banks which are special and heavily regulated, to the Merton-KMV class of models used in the pricing of corporate securities issued by the much less leveraged non-financial firms with complex capital structure but devoid of deposits, bank services and government safety nets, we defer this experiment to a future project.

²¹We thank the referee for raising this issue.

5.2 FC

To gauge the magnitude of the market-based forbearance fraction in capital in terms of observed market caps, we plot in [Figure 3](#), the dynamics of the cross-sectional distribution of the Forbearance (in units) of Capital (FC), which is defined as the product of the FFC and the market cap. If there were no capital forbearance, each year the aggregate owner-contributed capital of the publicly traded banks in our sample would have been 7.6 billion USD higher. In other words, from 1990 through to 2012, the stock market estimated that these banks saved over 100 billion USD of capital due to benefiting from the forbearance subsidization resulting from FDIC non-adherence to the market closure rule.²² In [Figure 3](#), we see much cross-sectional variation in the forbearance capital. This is, however, not surprising, given the fact that forbearance capital depends very much on the individual equity value. It is worth noting that much cross-sectional variation in forbearance capital does not necessary imply a very different effective policy parameter ρ^* (see the definition in [Section 2.3](#)), which measures the market-based forbearance treatment, cross individual banks. This can be seen from [Figure 4](#) showing that ρ^* has much less cross-sectional variation over time.

[Insert Figures 3 and 4 about here]

5.3 FC and crisis facilities liquidity provision

In Online Appendix A, we show that FFC from our analytical model quantitatively resembles that of a put option with unlimited term payoffs capturing the potential capital forbearance that allows the bank to survive. This representation associates the capital provided in emergency lending programs during the financial crisis with our metric, i.e., the forbearance in units of capital, FC via an option pricing framework. These are effectively the put option's potential payoffs and FC is the value of the put option. Since our focus is the regulatory forbearance enjoyed by depository institutions, according to [Bai et al. \(2018, Internet Appendix, Table IAIV\)](#), two facilities, namely the Term Auction Facil-

²²The total FC at the beginning of 1990 was 5.2 billion USD, and by the end of 2012 it was 105.9 billion USD. On average, the total FC increased annually by $(105.9 - 5.2)/23 = 7.6$ billion USD.

ity (TAF) and the Term Asset-Backed Securities Loan Facility (TALF) are most relevant to our study. Time series of the aggregate IFC of all banks are plotted against TAF and TALF capitals in Panel (a) of [Figure 5](#).²³ One important observation from [Figure 5](#) is that IFC starts surging significantly over three years earlier than the massive liquidity provided by these facilities. This implies that if we have the right tool to dissect market information, the stock market could be used as an effective early-warning system.

[Insert [Figure 5](#) about here]

It is also interesting to examine how changes in the provision of these lending facilities contemporaneously affect the market view of future capital forbearance. To this end, we regress the monthly changes of IFC with the monthly changes in the TAF + TALF outstandings from July 2007 to July 2011. The two time series are plotted together in Panel (b) of [Figure 5](#). Specifically, we run the following regression:

$$\Delta \text{IFC}_t = C_0 + C_1 \Delta(\text{TAF} + \text{TALF})_t + \epsilon_t,$$

where ΔIFC_t is the monthly change of IFC_t and $\Delta(\text{TAF} + \text{TALF})_t$ is the monthly change of $(\text{TAF} + \text{TALF})_t$. The OLS estimate of C_1 is 0.1 with Newey-West standard error of 0.03, meaning the estimate is highly significant (P-value < 0.06%). R^2 of the regression is 9%. Therefore, the regression results confirm that our estimated IFC is highly correlated with these two facilities emergency liquidity provision during the financial crisis period, which further indicates that our model is sensibly useful for inferring the market perception about the regulatory capital forbearance.

5.4 ICR

Following [Flannery \(2014\)](#), we also compute the Market Cap / Implied Asset Value ratio, which measures the market equity capital ratio, i.e., the market-based counterpart of the book equity ratio defined as Book Equity / Total Assets. The lower is this ratio,

²³The data on TAF and TALF capitals are calculated based on the lending outstandings data used to produce [Figure 9](#) in [Hamilton \(2014\)](#). Specifically, the TAF and TALF capitals are the positive differences between their outstandings in the current month and last month.

the lower is the owner-contributed equity ratio, and the higher is the leverage level. By deducting FFC from Market Cap / Implied Asset Value, we obtain the Intrinsic Capital Ratio and denote it by IICR.²⁴ IICR gives us a cleaner measure of the market-based equity ratio and a more accurate measure of the leverage risk since it removes the market view of the FFC from the bank stock market price and leaves in only the intrinsic value. **Figure 6** compares the dynamics of the quarterly medians of Market Cap / Implied Asset Value, IICR, and Book Equity / Total Assets. Notably, that of the market-based equity ratio exhibits much more systematic variation than that of the book equity ratio, which tracks the reverse of the non-risk-weighted leverage ratio. The latter contains important information about how the market view of bank capital structure and economic capital changes over time. During the sub-prime crisis, unsurprisingly, the book equity ratio exceeded the market-valued equity ratios. A similar pattern is also found in Fig. 3 of [Flannery \(2014\)](#).

[Insert **Figure 6** about here]

We find that, given the forbearance policy, IICR is much more heterogeneous than its partially observable counterpart Market Cap / Implied Asset Value and its recorded value Book Equity / Total Assets. The dynamics of the Book Equity to Total Assets ratio surely reflect the banks' book keeping and window dressing, carried out to meet regulatory capital requirements.

5.5 Comparing our model results with those obtained from the [Ronn and Verma \(1986\)](#) model

To show that our results are economically different than those from the Ronn-Verma framework, we compare our FFC values with those obtained using Ronn-Verma model. First, we provide supportive evidence that the Ronn-Verma framework overrates (under-rates) the value of forbearance during normal periods (crisis periods). Then, we show that our (in-sample) model provides better probability of default estimates than those derived from the Ronn-Verma model.

²⁴We thank the referee for suggesting this label.

We follow the same estimation procedure outlined in [Section 3](#) and calibrate Ronn-Verma’s model with our bank dataset. Using the definition in [Section 2.3](#), we calculate FFC based on the calibrated Ronn-Verma model. Time series of the cross-sectional average difference between Ronn-Verma FFC and ours at each month is presented in [Figure 7](#). The overall average of the difference is 3.1% with zero t-test p-value. In other words, this confirms that relative to our model, the Ronn-Verma model overstates the value of forbearance on average by 3.1% with high statistical significance. It is also worth noting that, as highlighted in [Figure 7](#), Ronn-Verma derived FFC drop sharply to a level lower (nearly 4% lower) than ours during the 2008 financial crisis period while reaching a record high relative to ours (about 10% higher) right before the crisis. This observation indicates that with a static (constant) ρ , Ronn-Verma’s model tends to overstate (understate) forbearance during normal periods (crisis periods) while our model is more flexible in matching the time-varying nature of the capital forbearance dynamics.

[Insert [Figure 7](#) about here]

Given its more flexible structure, we also expect our model to perform better than the Ronn-Verma one, on some other dimensions, for instance, bankruptcy prediction. To provide supportive evidence, from the FDIC’s failed bank list since October 2000 at <https://www.fdic.gov/bank/individual/failed/banklist.html>, we identify 31 defaults in our data. We use $\mathbb{E}_t^P \left[\mathbb{1}_{\{V_{t+1} < \rho_{t+1} D\}} \right]$ ($\rho_{t+1} = 0.97$ in Ronn-Verma’s case) to compute the one-year probabilities of default one year before the default dates for the 31 defaulted banks as well as another 31 randomly selected active banks. Then, we plot the Receiver Operating Characteristic (ROC) curves and calculate the Area Under the Curve (AUC) to compare the in-sample bankruptcy predictive performance.²⁵ The ROCs are plotted in [Figure 8](#). The higher the AUC the better is the bankruptcy predictive power. The AUCs are 85.2% and 72.8% for our model and Ronn-Verma, respectively. We also note that our ROC is higher than the one generated by Ronn-Verma model in [Figure 8](#). This evidence clearly reinforces that relative to the Ronn-Verma alternative, our stochastic ρ framework is capable to provide better bankruptcy predictions, at least from an in-sample perspective.

²⁵The ROC is the same as the “power curve” used in [Duffie et al. \(2007\)](#). For a perfect predictive model, AUC = 100%; for a model with no power at all, AUC = 50%.

[Insert Figure 8 about here]

6 Empirical analysis II: Regression results

6.1 Variables

We use the $\mathbb{F}\mathbb{F}\mathbb{C}$, the $\mathbb{I}\mathbb{C}\mathbb{R}$, and V obtained from Section 5, and other focal and control variables described below to run regressions to address our research questions.²⁶

The LHS variables in the regressions are $\mathbb{F}\mathbb{F}\mathbb{C}$ and $\mathbb{I}\mathbb{C}\mathbb{R}$, respectively, the latter defined as $(1 - \mathbb{F}\mathbb{F}\mathbb{C}) * \text{Market Cap} / \text{Implied Asset Value}$. Recall that $\mathbb{F}\mathbb{F}\mathbb{C}$ denotes the forbearance Fraction of Capital. Hence, the intrinsic capital ratio is not only market-assessed but also devoid of the value of regulatory forbearance.

On the RHS, we consider the business cycle, bank risks, and some other control variables. For business cycle proxies, we use the U.S. *GDP Growth*, *GDP Output Gap*, and *S&P 500 Index Returns*. The latter two are used for robustness checks. For bank risks, we compute the following variables. We obtain *Idiosyncratic Volatility*, following Shumway (2001) and Duan et al. (2012), by first regressing the daily returns of the firm's market cap on the daily returns of the S&P 500 index, within a quarter, then taking the standard deviation of the residuals of this regression. For *Asset Volatility*, we follow Duan et al. (2012)'s approach to estimate *Distance-To-Default (DTD)* for financial firms, and the *Asset Volatility* is then a byproduct of *DTD* (see the appendix in Duan et al., 2012). Thirdly, *Beta* is the coefficient of the return of the S&P 500 index in the regression for *idiosyncratic volatility*. *Beta* captures the extent to which a firm's stock returns are sensitive to systematic risk. Investors holding diversified stock portfolios care about systematic risk, while large shareholders, bank managers, and regulators pay attention to *idiosyncratic risk*. The *Asset Volatility* and *Beta* are used for a robustness check. For discussions on these risk metrics in the banking literature, see, for instance, Acharya (2009), DeYoung et al. (2013).

Following Moore and Zhou (2013), Engle et al. (2014), and many others, we also include the following control variables commonly considered in the extant banking literature:

²⁶Given that ρ^* exhibits much less variation, and mostly clusters around one, we use $\mathbb{F}\mathbb{F}\mathbb{C}$, which has more variation, in the regression analysis.

- *Implied Asset Value / Total Assets*, or Market to Book Asset Value ratio: a measure of the charter value, which is related to banks' risk taking. It also proxies the degree of competition or market power à la [Keeley \(1990\)](#).
- *Log Total Assets*: the natural logarithm of total assets captures the bank size.
- *Total Deposits / Total Liabilities*: a proxy for a bank's funding structure.
- *(Short Term Borrowing + Other Short Term Liabilities) / Total Assets*: a proxy for a bank's funding liquidity;
- *Net Income / Total Assets*: the Return on Assets (ROA).
- *Total Loans / Total Assets*: a proxy for the extent of a bank traditional activities.

Table 1 presents these variables summary statistics (exclusive of GDP Growth and the one-year Treasury rate). By just looking at these statistics, we do not see any significant size effect among these variables. All four groups have similar distributions for all the variables, except for log Total Assets which is used to discriminate between the different groups. However, we find a significant size effect when we conduct panel data regressions, as discussed later.

[Insert **Table 1** about here]

6.2 Testable hypotheses

In addressing our two research questions: 1) How does the time-varying capital forbearance portion embedded in bank equity depend on various banks' own risk and business cycle variables? and 2) How do banks' market-assessed intrinsic (i.e., devoid of the forbearance subsidy) capital ratios (or inverse leverage ratios) react to various business cycles and the banks' own risk variables?, we also postulate the following hypotheses. It is natural to expect that FFC is correlated with bank risk. The higher the risk, the bigger is FFC, therefore, we hypothesize that there is a positive relation between FFC and bank risk. In light of numerous studies on TBTF, we expect FFC to be positively related

to the size of a banking firm. Banks with relatively more deposits are costlier to save, as bank depositors are indemnified by the FDIC. Therefore, we hypothesize that there is a positive relation between FFC and bank reliance on deposit funding represented by the relative size of bank deposits. Economic intuition leads us to expect FFC to be related to the business cycle and we hypothesize that FFC is bigger in troubled times.

Intuitively, when the charter value increases, bank intrinsic market equity and implied asset value increase by the same amount.²⁷ Therefore, we expect there is a positive relation between the ICR and the charter value. Strong capital typically reduces bank systemic risk, hence, we hypothesize that the ICR is negatively associated with systemic risk.

6.3 Regression analysis

6.3.1 A system of two equations and the Generalized Method of Moments (GMM)

FFC has a positive relation with the degree of supervisory leniency, hence, the bigger is FFC, the higher is the expected regulatory capital forbearance. As stated earlier, ICR gives us a clean measure of the market-based equity ratio or the leverage ratio that excludes the forbearance subsidy. Recall that Implied Asset Value / Total Assets is our enhanced proxy for both Tobin's Q and the charter value.

Endogeneity exists between FFC and ICR, and the other variables, especially Implied Asset Value / Total Assets and Idiosyncratic Volatility since the latter are obtained from stock market data. To take into account the endogeneity between the variables, we use

²⁷Without loss of generality, we assume that the total assets (TA) and the total liabilities (TL) remain constant. The implied asset value (IA) has two components: market value of debt ($TL * \rho^*$) and market value of equity (MC). The MC can be decomposed into three components: non-charter (nonC), charter (C), and forbearance capital (FC). The intrinsic market cap (IMC) consists of nonC and C. When C increases (by ΔC), IMC and IA will both increase by ΔC , and therefore ICR will increase.

$$IA = TL * \rho^* + MC, MC = IMC + FC, IMC = nonC + C$$

$$C \uparrow_{\Delta C} \Rightarrow \left\{ \begin{array}{l} IMC \uparrow_{\Delta C} \\ MC \uparrow_{\Delta C} \Rightarrow IA \uparrow_{\Delta C} \end{array} \right\} \Rightarrow ICR \uparrow.$$

system GMM to estimate the following system of two dynamic panel equations:

$$\mathbb{F}FC_{j,t} = f_1 \left(\begin{array}{c} \mathbb{F}FC_{j,t-\frac{1}{4}}, \mathbb{I}CR_{j,t}, \\ \text{Idios. Volatility}_{j,t}, \text{GDP Growth}_t, \\ \text{Control Variables} \end{array} \right) + \epsilon_{j,t}, \quad (7a)$$

$$\mathbb{I}CR_{j,t} = f_2 \left(\begin{array}{c} \mathbb{I}CR_{j,t-\frac{1}{4}}, \mathbb{F}FC_{j,t}, \\ \text{Idios. Volatility}_{j,t}, \text{GDP Growth}_t, \\ \text{Control Variables} \end{array} \right) + \epsilon_{j,t}, \quad (7b)$$

where $f_1(\cdot)$ and $f_2(\cdot)$ are linear functions, $\epsilon_{j,t}$ and $\epsilon_{j,t}$ are residuals, and subscripts j and t indicate that the value is for the j th firm at time t , and the Control Variables include $\mathbb{I}CR_{j,t}$, $\log \text{Total Assets}_{j,t}$, and $\frac{\text{Total Deposits}_{j,t}}{\text{Total Liabilities}_{j,t}}$. One-quarter-lagged LHS variables are included on the RHS to capture the system dynamic structure. We use quarterly $\mathbb{F}FC$, Implied Asset Value, and $\mathbb{I}CR$, i.e., these variables values are taken from the last month of each quarter, to match the frequency of the other variables. The correlation coefficients of the variables are presented in [Table 2](#). Consistent with intuition, $\mathbb{F}FC$ is strongly negatively correlated with $\mathbb{I}CR$ (-0.72) and Implied Asset Value / Total Assets (-0.70) whereas these two latter variables are positively correlated with each other (0.78).

[Insert [Table 2](#) about here]

To avoid the “biases in dynamic models with fixed effects” pointed out in [Nickell \(1981\)](#),²⁸ we estimate [Eq. \(7a\)](#) to [Eq. \(7b\)](#) using the 2SGMM developed by [Blundell and Bond \(1998\)](#). Since individual $\mathbb{F}FC$ and $\mathbb{I}CR$ are unlikely to directly affect GDP Growth, we assume that GDP Growth is a strictly exogenous regressor. Naturally, $\mathbb{F}FC$, $\mathbb{I}CR$, Idiosyncratic Volatility, and Implied Asset Value / Total Assets are endogenous variables, as they are all determined endogenously in our model. Although $\log \text{Total Assets}$ and $\text{Total Deposits} / \text{Total Liabilities}$ are not explicit in our model, their values are likely to be affected by the market view of forbearance. Therefore, these two variables are also potentially endogenous to $\mathbb{F}FC$ and $\mathbb{I}CR$.

²⁸Since the LHS variables, $\mathbb{F}FC$ and $\mathbb{I}CR$, are both persistent processes, this bias is considered to be significant if we use the standard fixed-effect regression.

In the GMM estimation, GDP Growth is used to instrument itself.²⁹ Other than GDP Growth, all the RHS variables in Eq. (7a) to Eq. (7b) are considered to be predetermined (lagged LHS) or endogenous. Therefore, two-quarter and longer lags of these variables are used as GMM instruments. Since the time length of our panel data is not too short (about 36 quarters on average), we cap the lags at 9 and 20 quarters for the Large Banks and Big Banks groups, respectively. To further limit the number of instruments at a reasonable level relative to the number of observations, these GMM instruments are “collapsed” à la Beck and Levine (2004) and Roodman (2009). We compute standard errors with the Windmeijer (2005) correction. The system GMM approach handles firm fixed effects, and we include year dummies in the regression to account for the time fixed effects.³⁰

6.3.2 Results and discussion

We report the estimation results from Eq. (7a) in Table 3. The coefficients of the lagged $\mathbb{F}\mathbb{F}\mathbb{C}$ are all significantly positive for all four groups, which confirms that $\mathbb{F}\mathbb{F}\mathbb{C}$ is a persistent process. The coefficients of $\mathbb{I}\mathbb{C}\mathbb{R}$ for the All Banks and Medium Banks groups are significantly negative, consistent with the belief that banks with higher intrinsic equity ratios receive less capital forbearance. One standard deviation increase in the $\mathbb{I}\mathbb{C}\mathbb{R}$ reduces the capital forbearance by 0.02 for all banks and 0.03 for medium banks. This increase is 11% of the sample capital forbearance mean. Since $\mathbb{I}\mathbb{C}\mathbb{R}$ is the reverse of the (market-based) leverage risk, the significantly negative coefficients also reflect a positive relation between $\mathbb{F}\mathbb{F}\mathbb{C}$ and bank leveraging. This confirms our hypothesis that $\mathbb{F}\mathbb{F}\mathbb{C}$ is positively related to the leverage risk. However, the estimated coefficients for the Large and Big banks are statistically zero. This may be interpreted that for these mega banks $\mathbb{F}\mathbb{F}\mathbb{C}$ is not associated with the intrinsic capital fraction since it is tiny compared to the scale of forbearance and bailout contemplated by the market.

[Insert Table 3 about here]

²⁹Results are robust even with GDP Growth as an endogenous variable. However, to limit the number of instruments, we use GDP Growth as an exogenous variable.

³⁰We do not include quarter dummies because we have a quarterly macro variable GDP Growth in our regression, which makes coefficients of quarter dummies unidentifiable. Year dummies are, nevertheless, identifiable.

The significantly positive coefficient of Idiosyncratic Volatility indicates that the market believes, *ceteris paribus*, that banks with higher non-diversifiable risk will be given more forbearance. We find that one standard deviation in Idiosyncratic Volatility leads to 0.022 increase in FFC (13.75% the sample average capital forbearance). This effect is higher for big banks in absolute terms. This result is robust to the two other bank specific risk measures: Asset Volatility and Beta. This is in line with the hypothesis that regulatory forbearance rises with higher idiosyncratic risk.

Although we do not observe a significant size effect on FFC from the summary statistics presented in [Table 1](#), we do find formal confirmation of a significant size effect in [Table 3](#). This is confirmed by the significantly positive coefficients of log Total Assets for the All but Small Banks group. This positive size effect on FFC indicates that the market believes that larger banks benefit from more forbearance and cost more to rescue. Since we use log Total Assets to proxy for bank size, we also capture to some degree the nonlinearity in the relation between FFC and Total Assets. Judging from the coefficient estimates in [Table 3](#), the nonlinearity implies that the size effect is unnoticeable when banks become large. This feature explains why the coefficients for the Large Banks and Big Banks groups are insignificant. This is consistent with the hypothesis about the positive size effect on the forbearance value.

A high ratio of Total Deposits / Total Liabilities means banks draw mainly from deposits and rely much less on wholesale funding. The significantly positive relation between FFC and Total Deposits / Total Liabilities in [Table 3](#) implies that it is costlier for the government to forbear in the case of banks that depend on deposits. This is again in line with our hypothesis that since deposits are insured, the FDIC has a heavier obligation to fully payoff depositors of banks having relatively more deposits.

The coefficients of GDP Growth reveal the relation between FFC and the business cycle. The coefficients for All but Small Banks, Large Banks and Big Banks are significantly negative. Obviously, the market expects banks to benefit from more forbearance or more likely to be rescued in bad times, and to be less prone to requiring financial assistance during booms. FFC is countercyclical as we hypothesize. The results are robust to the other two proxies for the business cycle: GDP Output Gap and S&P Index. To save space,

their results are not presented. In Online Appendix G, we show that the countercyclical nature of forbearance is also confirmed in time-series analysis. [Lai et al. \(2018\)](#) also show that the market views on the banking industry are useful for forecasting economic growth.

The estimation results from [Eq. \(7b\)](#) are presented in [Table 4](#). Again, we find that IICR, our proxy for the inverse of the bank's leverage ratio (market-based), is a persistent process as the coefficients of the lagged value of IICR are all significantly positive. The significantly negative coefficients of FFC confirm the hypothesized positive relation between FFC and the leverage risk. Given the coefficient estimates of FFC for All but Small Banks group, we can see that a 10 basis point increase in FFC contemporaneously decreases IICR by 0.23 basis points. Further, the marginal contribution of forbearance (FFC) to the intrinsic bank net worth ratio (IICR) appears almost identical in all three size-based subsamples of banks. This interesting result suggests that, for the sake of valuing bank adequate capital, the marginal impact of FFC on the intrinsic capital ratio is the same regardless of the bank size. The significantly negative coefficients of Idiosyncratic Volatility indicate a positive relation between bank idiosyncratic volatility and the leverage ratio.

[Insert [Table 4](#) about here]

We observe a significantly positive relation between IICR and Implied Asset Value / Total Assets. On average, a standard deviation increase in the Implied Asset Value / Total Assets leads to 0.008 increase in the IICR. This increase amounts to 6.35% of the average value of the IICR. Recall that our proxy for Tobin's Q, Implied Asset Value / Total Assets, which is based on the market valuation, measures the charter value more accurately so as to underscore the disciplining impact of the charter value on leverage. This result is consistent with our hypothesis about the positive relation between IICR and the charter value. This also indicates that the higher is the charter value, the less is the bank's leverage, but the disciplining effect is smaller for bigger banks. Unlike the case of FFC, we do not observe a significant size effect on IICR, as only the coefficient for Large Banks is significant among the four groups. We find a statistically significant negative relationship between IICR and Total Deposits / Total Liabilities for the full sample of 565 banks which appears to be driven by the Large Banks subsample.

In Online Appendix H, we report results obtained using additional control variables such as a funding-liquidity measure (Short-Term Borrowing & Other Short-Term Liabilities to Total Assets Ratio), a performance measure (ROA), and a traditional banking activities measure (Total Loans to Total Assets Ratio). The main messages above remain unchanged. In unreported exercises, we replace Idiosyncratic Volatility (GDP Growth) with Asset Volatility and Beta (GDP Output Gap and S&P 500 Index), then rerun the same regressions. We also exclude data from the crisis period and rerun the same regressions. The results, available from the authors upon request, deliver the same stories as discussed above.

6.3.3 Results with systemic risk

Finally, we provide additional results on how our IFFC and ICIR are related to the systemic risk of a bank. As discussed in more detail in [Moore and Zhou \(2013\)](#), [Laeven et al. \(2014\)](#), [Engle et al. \(2014\)](#), and many others in this growingly large literature, the systemic risk is most relevant for large banks and macro-prudential governance. Therefore, for compactness, here we only consider the Large Banks group consisting of banks with Total Assets of at least 7.4 billion USD and with average Total Assets of 149 billion USD. Using the U.S. banks' daily stock returns data, we calculate the commonly adopted systemic risk proxy, the *Marginal Expected Shortfall* (MES) proposed by [Brownlees and Engle \(2012\)](#), which utilizes the GARCH and Corrected Dynamic Conditional Correlation (CDCC) methods to better capture correlations between the stock returns of individual banks and the stock index returns. The MES, which captures the marginal exposure of a banking firm to a system-wide collapse, is defined as the negative mean net equity return of a bank conditional on the U.S. S&P 500 index experiencing extreme downward movements. We follow the mechanics with all the assumptions summarized in Annex 4 of [Laeven et al. \(2014\)](#) to estimate our two metrics of systemic risk MES and SRISK. Technical details are presented in Online Appendix I.

We replace Idiosyncratic Volatility in [Eq. \(7a\)](#) and [Eq. \(7b\)](#) with MES, and also exclude log Total Assets from both equations in light of the previous finding on the insignificant

size effect of Large Banks on FFC and ICR.³¹ Specifically, we estimate the following two equations

$$\text{FFC}_{j,t} = f_1 \begin{pmatrix} \text{FFC}_{j,t-\frac{1}{4}}, \text{ICR}_{j,t}, \\ \text{MES}_{j,t}, \text{GDP Growth}_t, \\ \text{Control Variables} \end{pmatrix} + \epsilon_{j,t}, \quad (8a)$$

$$\text{ICR}_{j,t} = f_2 \begin{pmatrix} \text{ICR}_{j,t-\frac{1}{4}}, \text{FFC}_{j,t}, \\ \text{MES}_{j,t}, \text{GDP Growth}_t, \\ \text{Control Variables} \end{pmatrix} + \epsilon_{j,t}, \quad (8b)$$

where the Control Variables include $\frac{\text{Implied Asset Value}_{j,t}}{\text{Total Assets}_{j,t}}$ and $\frac{\text{Total Deposits}_{j,t}}{\text{Total Liabilities}_{j,t}}$. To perform the regressions, we use the 2SGMM as described before. The results are reported in [Table 5](#).

[Insert [Table 5](#) about here]

We find a significantly positive relation between FFC and MES, while there is a significantly negative relation between ICR and MES. These results are in accordance with our hypotheses. Bank systemic risk is positively associated with capital forbearance. The lower is the bank capital, the higher is the bank's exposure to systemic risk. This finding is robust to an alternative systemic risk measure proposed by [Acharya et al. \(2012\)](#), SRISK, the capital shortfall, defined as a bank's contribution to the deterioration of the capitalization of the whole financial system (the dominant U.S. stock market) during a crisis.

³¹To better identify the coefficients we keep all the instrumental variables (both IV and GMM types) intact while adding MES as a GMM type instrument.

7 Conclusion

Ronn and Verma (1986) call the tolerance level below which the closure of a large, complex and interconnected insolvent bank is triggered the regulatory policy parameter. In this paper, we develop a two-factor model with Ronn and Verma (1986)'s bank regulatory policy parameter being stochastic and bank-specific. The model is calibrated using 565 U.S. banks' market capitalization and total liabilities data to infer the market impression about the regulatory closure rules for the period from 1990 to 2012. In accordance with economic intuition, the resulting forward-looking bank regulatory policy parameters show that the market expectation on the capital forbearance is significantly driven by bank-specific risk variables and business cycles. We find that the capital forbearance subsidy present in the largest banks could amount to 17% of the market value of equity and could go as high as 100% of a bank's stock value. The market believes in the "Too Big to Fail" paradigm and expects that banks with lower equity capital ratios will receive more capital forbearance or government assistance, congruent with the banking regulatory authority containing rescue costs. In effect, the market expects a strongly performing bank to receive less capital forbearance, and one with a high charter value and enjoying greater market power to cost less to bail out. Regarding idiosyncratic risk, the market believes that banks with higher volatility will benefit from more forbearance. The market expects banks to benefit from increases in capital forbearance during recessions. The market expectation of forbearance is also a positive function of banks' systemic risk, consistent with the expectation that banks with higher systemic risk will receive more capital forbearance from the government. Applying the model to the setting of fair market deposit insurance premiums would be a natural next step in future research. By means of the enhanced Ronn and Verma (1986) developed in this paper, one may study, for instance, the interaction between regulatory forbearance and market discipline in terms of equity valuation.

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Figure 1: Individual bank regulatory policy parameters time series: The case of Wells Fargo

This figure presents the results of the case study of Wells Fargo. The upper panel in this figure shows the time series of the actual market cap, fitted market cap, implied asset value, and smoothed total liabilities (y-axis is in billions of USD); the middle panel shows the time series of the Policy Parameter (ρ), the Effective Policy Parameter (ρ^*), and the Forbearance Fraction of Capital (FFC); the lower panel shows the time series of the Intrinsic Cap Ratio (ICR) and Book Equity / Total Assets (i.e., $1 - \text{Smoothed Total Liabilities} / \text{Smoothed Total Assets}$). The Intrinsic Market Cap is the market cap net of the capital forbearance value, and the Implied Asset Value is one of the state variables in our model, and represents the market-based asset value of a bank. The time series are from early 1998 to late 2012, and the data frequency is monthly.

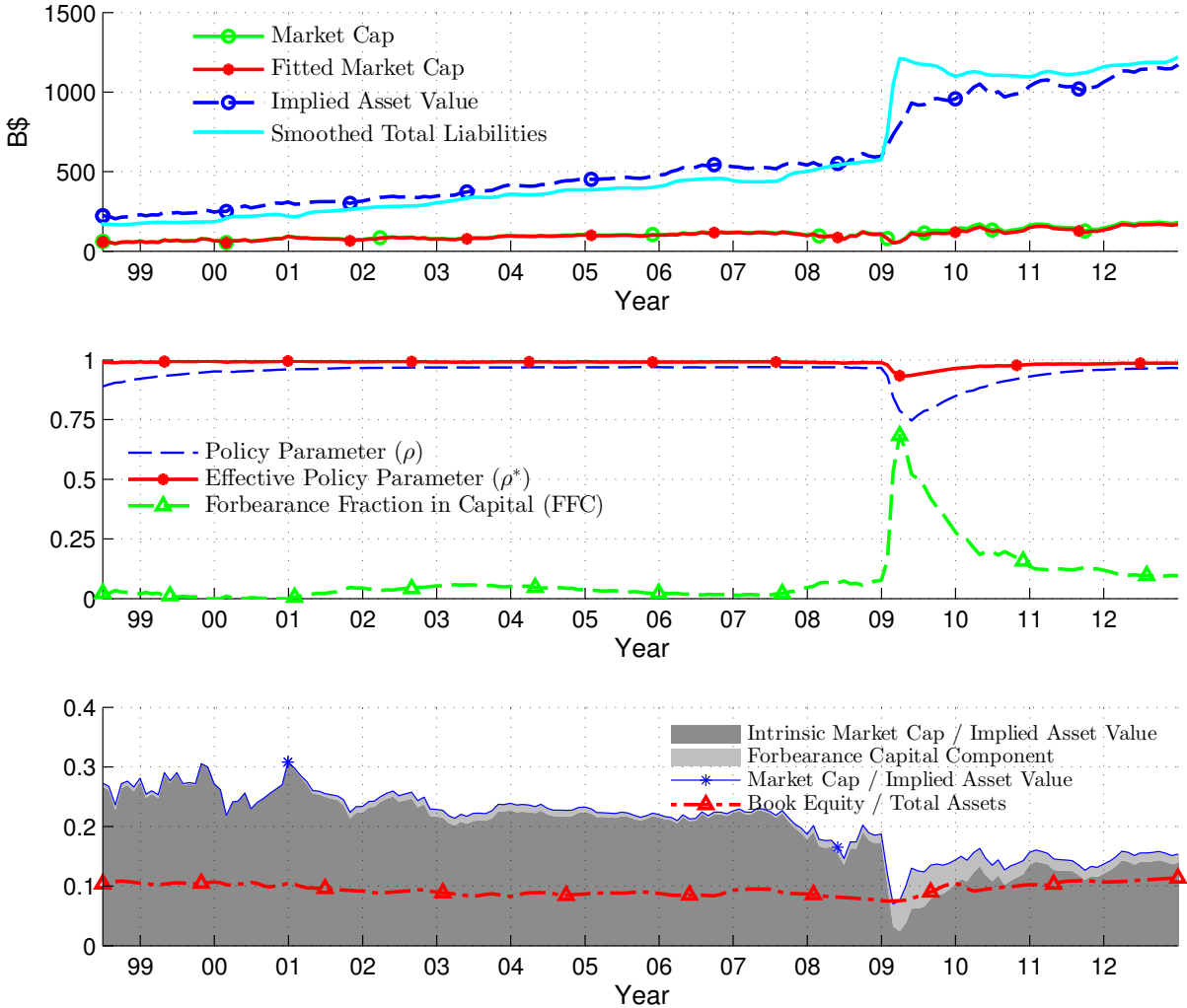


Figure 2: Forbearance Fraction of Capital (FFC) cross-sectional distribution dynamics

This figure shows the dynamics (from 1990 to 2012, monthly data) of the cross-sectional distributions of the Forbearance Fraction of Capital (FFC) for the four categories: Large Banks, Big Banks, Medium Banks, and All but Small Banks. The black line indicates the median over time, and the dark and light gray bands indicate the 25th to 75th and 10th to 90th percentile intervals, respectively.

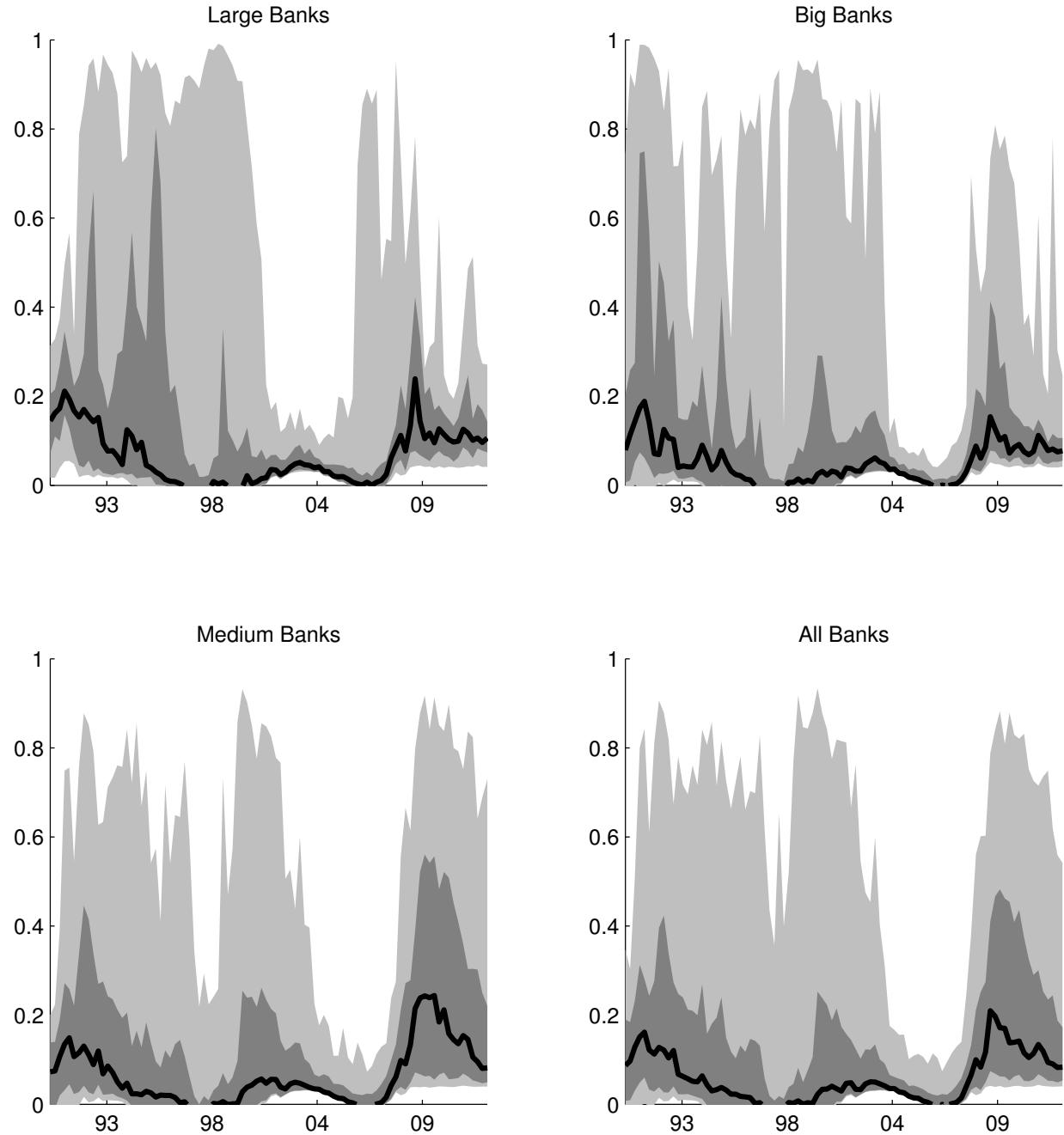


Figure 3: Forbearance Capital Value (IFC) cross-sectional distribution dynamics

This figure shows the dynamics (from 1990 to 2012, monthly data) of the cross-sectional distributions of the Forbearance Capital Value (IFC) (in billions of USD) for the four categories: Large Banks, Big Banks, Medium Banks, and All Banks. All the y-axes are in billions of USD. The black line indicates the median over time, and the dark and light gray bands indicate the 25th to 75th and 10th to 90th percentile intervals, respectively.

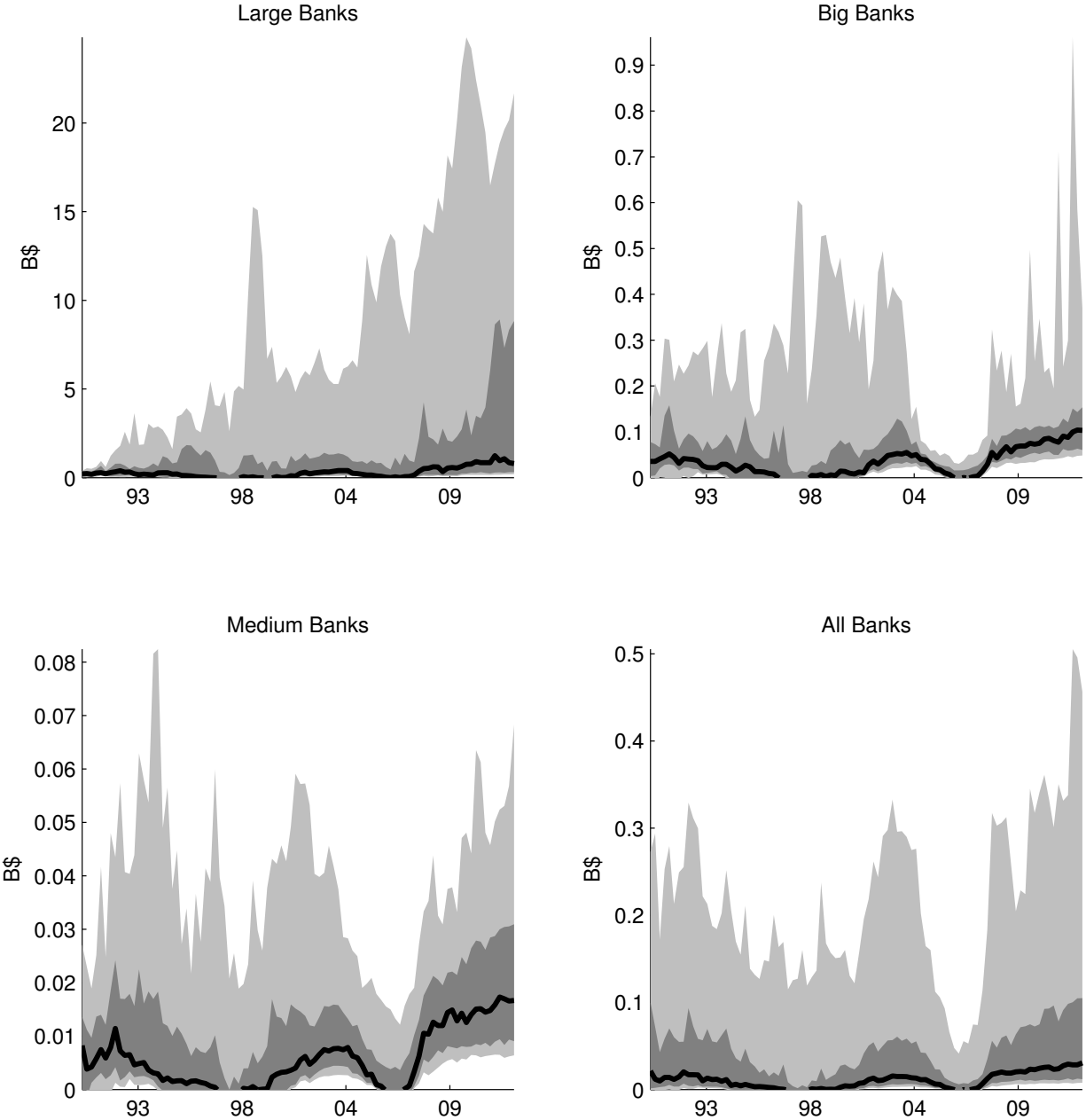


Figure 4: Effective Policy Parameter (ρ^*) cross-sectional distribution dynamics

This figure shows the dynamics (from 1990 to 2012, monthly data) of the cross-sectional distributions of the Effective Policy Parameter (ρ^*) for the four categories: Large Banks, Big Banks, Medium Banks, and All Banks. The black line indicates the median over time, and the dark and light gray bands indicate the 25th to 75th and 10th to 90th percentile intervals, respectively.

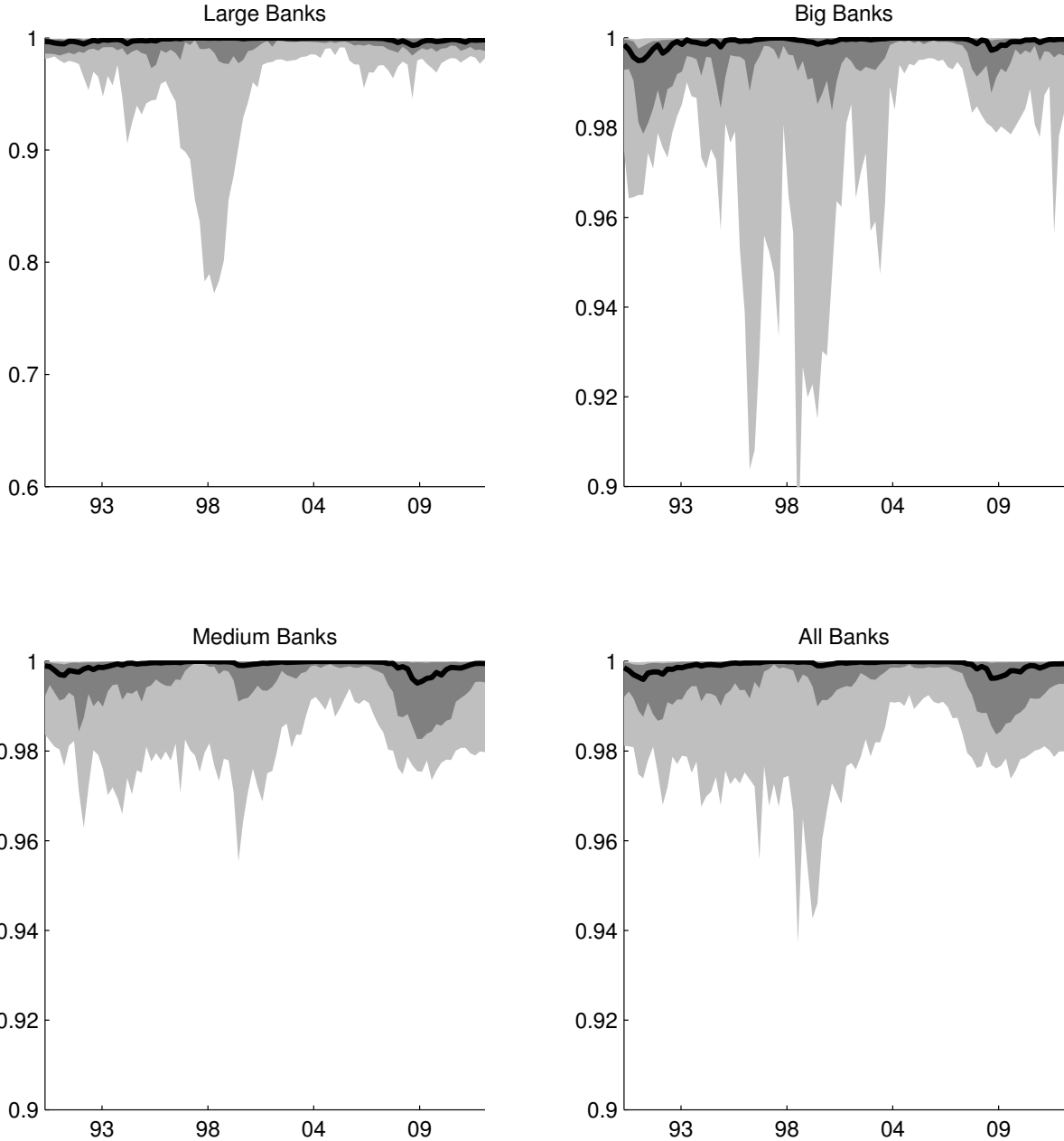
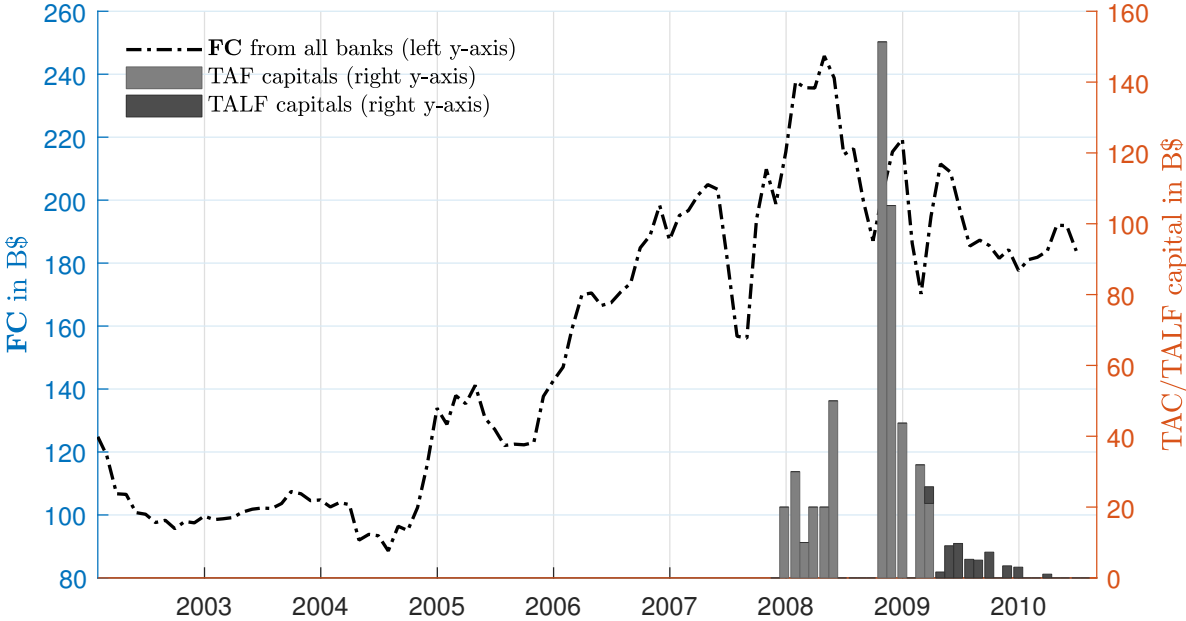


Figure 5: Forbearance Capital Value (FC) v.s. TAF and TALF Capitals

Panel (a) in this figure shows the monthly dynamics (from January 2002 to June 2010) of FC (left y-axis) in contrast with the timing and magnitude of the TAF/TALF capitals (right y-axis). Panel (b) in this figure shows the monthly changes (from July 2007 to July 2011) of FC in contrast with the monthly changes of the TAF+TALF outstandings. All the y-axes are in billions of USD. Data source of TAF/TALF capitals: [Hamilton \(2014\)](#).

(a) FC v.s. TAF+TALF



(b) Changes in FC v.s. changes in TAF+TALF outstandings

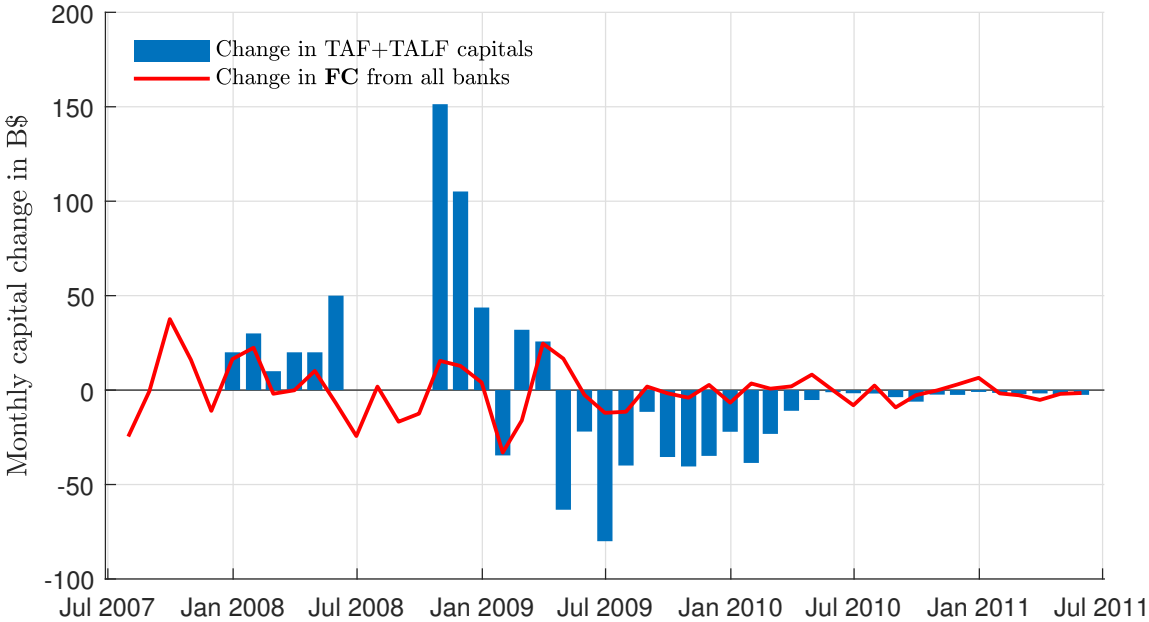


Figure 6: ICR v.s. Book Equity / Total Assets

This figure shows the dynamics of the cross-sectional mean values of ICR (Intrinsic Mkt Cap / Implied Asset Value) and Book Equity / Total Assets for the four different groups from the beginning of 1990 to the end of 2012. The data frequency is quarterly. The results for Large Banks, Big Banks, Medium Banks, and All Banks are shown in panels (a), (b), (c), (d), and (e) respectively.

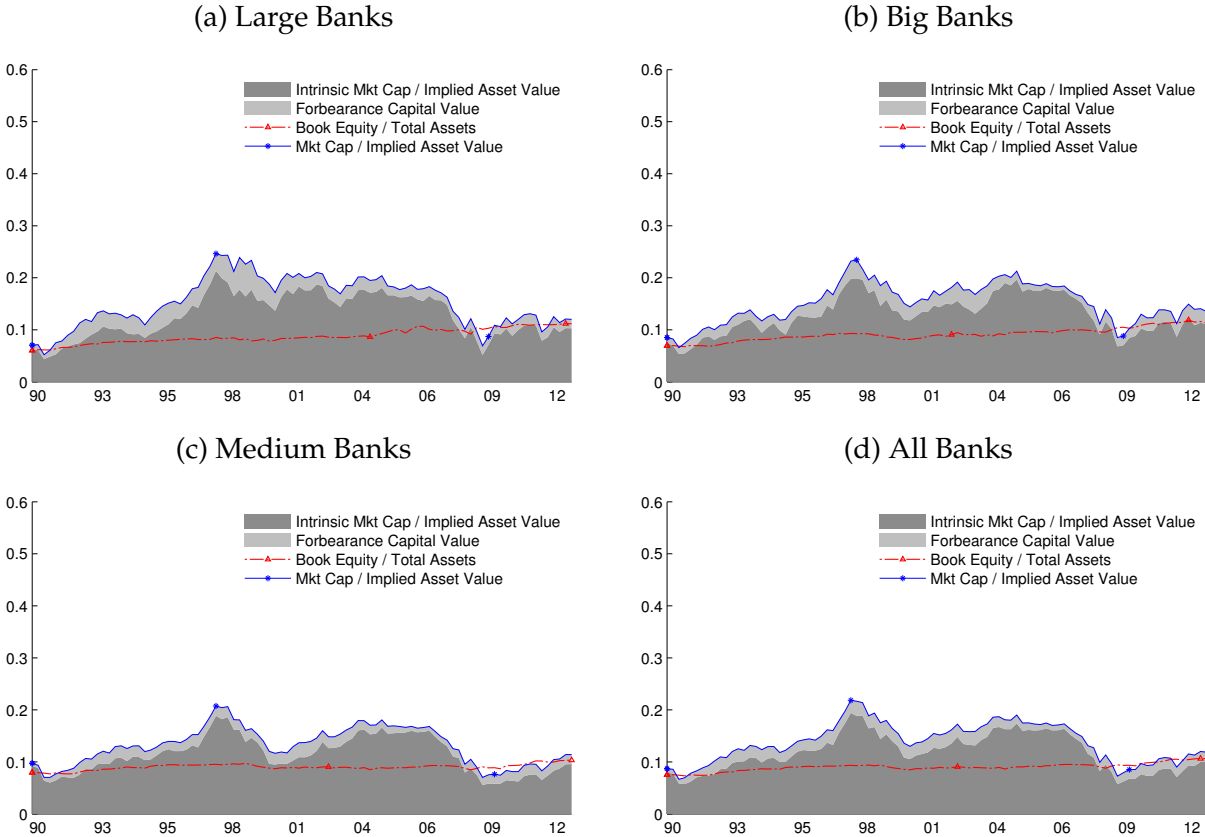


Figure 7: Time series of IFC differences

This figure shows the monthly dynamics (from February 1990 to April 2013) of the difference between IFC of [Ronn and Verma \(1986\)](#) and ours using our data. The 2008 financial crisis period is highlighted in gray (according to the NBER business cycle reference dates).

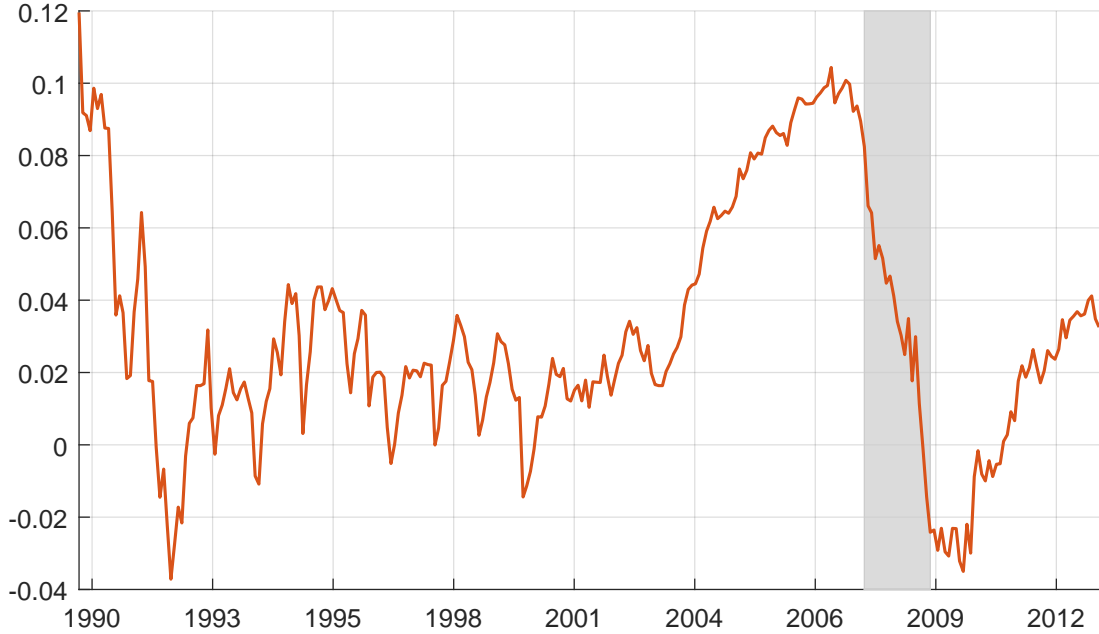


Figure 8: Receiver Operating Characteristic (ROC) Curve and Area Under the Curve (AUC)

This figure plots and compares the ROCs and AUCs of our model (Lai-Ye) and Ronn-Verma using the one-year in-sample probabilities of default. The data are based on 31 recorded defaults and 31 randomly selected active banks in our data.

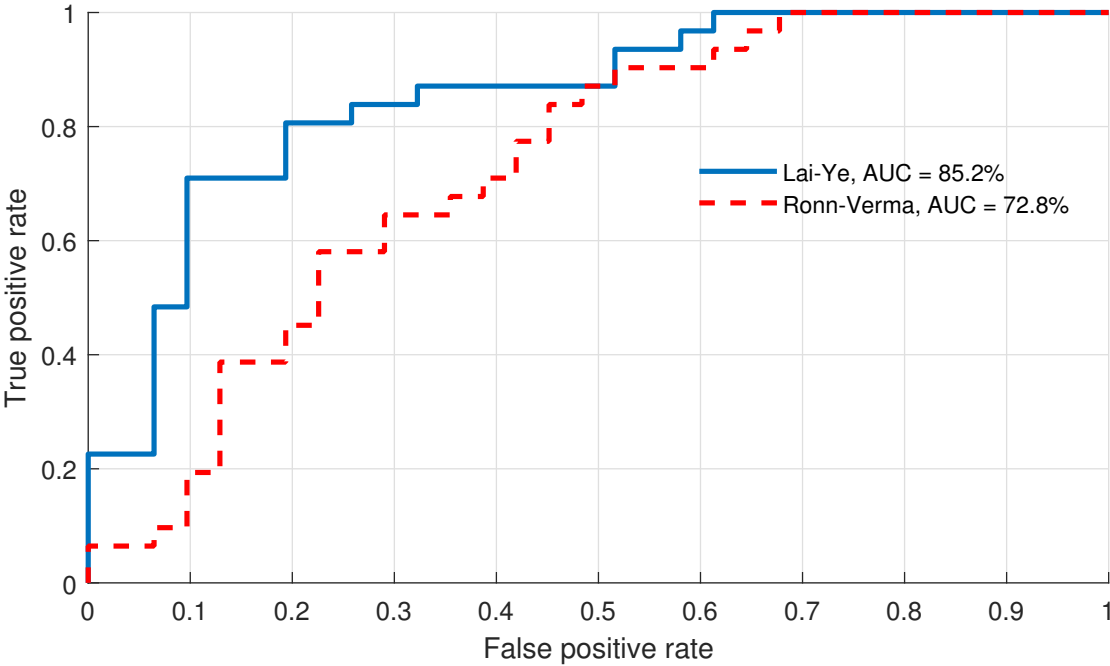


Table 1: Summary statistics of the variables

This table reports summary statistics for the Forbearance Fraction of Capital (FFC), Mkt Cap / Implied Asset Value (MC/IA), IICR, Book Equity / Total Assets (BE/TA), Idiosyncratic Volatility (Idio-Vol), Implied Asset Value / Total Assets (IA/TA), log Total Assets (log(TA)), Total Deposits / Total Liabilities (TD/TL), Short-Term Borrowing + Short-Term Other Liabilities / Total Assets (STB/TA), Net Income / Total Assets (ROA), and Total Loans / Total Assets (TLoan/TA) for all banks as well as three categories: Large Banks, Big Banks, and Medium Banks. Each variable is winsorized at 99% and 1%. The Mean, Standard Deviation (Std), Maximum (Max), 90%, 75%, 50%, 25%, 10% and Minimum (Min) of each category are reported. Note: Total Assets are in millions of USD and results for ROA are reported in percentages to avoid rounding imprecision. Summary statistics are calculated on a quarterly basis. For FFC, MC/IA, and IICR, the data of the last months in quarters are used.

	Mean	Std	Max	90%	75%	50%	25%	10%	Min
All Banks									
FFC	0.16	0.28	1.00	0.66	0.15	0.04	0.01	0.00	0.00
MC/IA	0.14	0.06	0.37	0.22	0.18	0.14	0.10	0.07	0.02
IICR	0.12	0.07	0.38	0.21	0.17	0.13	0.08	0.02	0.00
BE/TA	0.09	0.03	0.24	0.12	0.10	0.09	0.08	0.06	0.04
Idio-Risk	0.02	0.02	0.12	0.04	0.03	0.02	0.01	0.01	0.01
IA/TA	0.97	0.14	1.28	1.12	1.06	1.00	0.91	0.80	0.47
log(TA)	7.62	1.54	14.68	9.67	8.31	7.23	6.52	6.08	5.47
TD/TL	0.83	0.16	0.99	0.96	0.93	0.87	0.78	0.68	0.11
STB/TA	0.07	0.08	0.42	0.16	0.10	0.05	0.02	0.01	0.00
ROA(%)	0.24	0.24	0.63	0.41	0.34	0.27	0.19	0.08	-1.45
TLoan/TA	0.64	0.12	1.07	0.78	0.72	0.66	0.58	0.49	0.05
Sample Size	23908								

Continued on the next page

Table 1 –continued from the previous page

	Mean	Std	Max	90%	75%	50%	25%	10%	Min
Large Banks									
FFC	0.17	0.28	1.00	0.71	0.14	0.04	0.01	0.00	0.00
MC/IA	0.16	0.07	0.37	0.27	0.20	0.15	0.11	0.08	0.02
ICR	0.14	0.08	0.38	0.24	0.19	0.14	0.08	0.03	0.00
BE/TA	0.09	0.03	0.24	0.12	0.10	0.08	0.07	0.06	0.04
Idio-Risk	0.02	0.01	0.12	0.03	0.02	0.01	0.01	0.01	0.01
IA/TA	0.99	0.16	1.28	1.16	1.09	1.01	0.91	0.80	0.47
log(TA)	10.75	1.30	14.68	12.64	11.37	10.46	9.74	9.39	8.91
TD/TL	0.73	0.15	0.99	0.88	0.83	0.75	0.68	0.60	0.11
STB/TA	0.15	0.10	0.42	0.29	0.19	0.12	0.08	0.04	0.00
ROA(%)	0.29	0.18	0.63	0.45	0.37	0.32	0.25	0.14	-1.45
TLoan/TA	0.61	0.15	0.90	0.75	0.71	0.65	0.57	0.40	0.05
Sample Size	3121								
Big Banks									
FFC	0.16	0.28	1.00	0.68	0.12	0.04	0.01	0.00	0.00
MC/IA	0.16	0.06	0.37	0.24	0.20	0.15	0.12	0.09	0.02
ICR	0.14	0.07	0.38	0.23	0.18	0.14	0.09	0.03	0.00
BE/TA	0.09	0.03	0.24	0.12	0.10	0.09	0.08	0.07	0.04
Idio-Risk	0.02	0.01	0.12	0.04	0.02	0.02	0.01	0.01	0.01
IA/TA	0.99	0.15	1.28	1.14	1.08	1.01	0.92	0.81	0.47
log(TA)	8.52	0.51	10.01	9.20	8.89	8.53	8.13	7.82	7.44
TD/TL	0.81	0.15	0.99	0.94	0.91	0.85	0.77	0.69	0.11
STB/TA	0.09	0.08	0.42	0.18	0.12	0.07	0.04	0.02	0.00
ROA(%)	0.26	0.23	0.63	0.43	0.36	0.29	0.23	0.11	-1.45
TLoan/TA	0.62	0.12	1.07	0.75	0.70	0.64	0.56	0.46	0.06

Continued on the next page

Table 1 –continued from the previous page

	Mean	Std	Max	90%	75%	50%	25%	10%	Min
Sample Size	4861								
Medium Banks									
FFC	0.17	0.27	1.00	0.64	0.16	0.04	0.01	0.00	0.00
MC/IA	0.14	0.06	0.37	0.21	0.17	0.13	0.10	0.06	0.02
ICR	0.12	0.07	0.38	0.20	0.16	0.12	0.07	0.02	0.00
BE/TA	0.09	0.03	0.24	0.12	0.10	0.09	0.08	0.06	0.04
Idio-Risk	0.03	0.02	0.12	0.05	0.03	0.02	0.02	0.01	0.01
IA/TA	0.97	0.13	1.28	1.11	1.05	0.99	0.91	0.80	0.47
log(TA)	6.81	0.64	8.67	7.69	7.28	6.77	6.31	5.96	5.47
TD/TL	0.85	0.16	0.99	0.97	0.94	0.89	0.82	0.72	0.11
STB/TA	0.05	0.06	0.42	0.13	0.07	0.03	0.01	0.00	0.00
ROA(%)	0.22	0.25	0.63	0.40	0.33	0.26	0.18	0.07	-1.45
TLoan/TA	0.66	0.11	0.95	0.80	0.73	0.66	0.59	0.51	0.15
Sample Size	15330								

Table 2: Correlation coefficients of the variables

This table reports the correlation coefficient matrix of the following variables: the Forbearance Fraction of Capital (FFC), Intrinsic Cap Ratio (ICR), Idiosyncratic Volatility (Idio-Vol), Implied Asset Value / Total Assets (IA/TA), log Total Assets (logTA), Total Deposits / Total Liabilities (TD/TL), GDP Growth (GDPG), (Short-Term Borrowing + Other Short-Term Liabilities) / Total Assets (STB/TA), Net Income / Total Assets (ROA), and Total Loans / Total Assets (TLoan/TA). The correlation coefficients are computed in a pairwise manner using quarterly data.

	FFC	ICR	Idio Risk	GDPG	IA/TA	logTA	TD/TL	STB/TA	ROA	TLoan/TA
FFC	1									
ICR	-0.72	1								
Idio Risk	0.24	-0.30	1							
GDPG	-0.07	0.19	-0.23	1						
IA/TA	-0.70	0.78	-0.29	0.21	1					
logTA	0.02	0.05	-0.21	-0.09	-0.04	1				
TD/TL	-0.03	0.05	0.08	0.09	0.04	-0.32	1			
STB/TA	0.00	0.03	-0.10	0.01	0.05	0.44	-0.50	1		
ROA	-0.29	0.48	-0.43	0.23	0.46	0.03	0.04	0.06	1	
TLoan/TA	-0.02	-0.01	0.10	-0.13	-0.07	-0.14	0.16	-0.32	-0.07	1

Table 3: System GMM estimates of different groups of banks for Eq. (7a) with the LHS variable being the Forbearance Fraction of Capital (FFC)

This table reports the regression results for Eq. (7a).

$$\text{FFC}_{j,t} = f_1 \left(\begin{array}{c} \text{FFC}_{j,t-\frac{1}{4}}, \text{ICR}_{j,t}, \\ \text{Idios. Volatility}_{j,t}, \text{GDP Growth}_t, \\ \text{Control Variables} \end{array} \right) + \epsilon_{j,t},$$

where the Control Variables include $\frac{\text{Implied Asset Value}_{j,t}}{\text{Total Assets}_{j,t}}$, $\log \text{Total Assets}_{j,t}$, and $\frac{\text{Total Deposits}_{j,t}}{\text{Total Liabilities}_{j,t}}$. The LHS variable in the regression is the FFC, where the Implied Asset Value is one of the state variables in our model, and represents the market-based asset value of a bank. The first column contains all the RHS variable names, the second column reports the coefficients of the variables for all but small banks, and the third to sixth columns report those for the four categories: All but Small Banks, Large Banks, Big Banks, and Medium Banks. Sample sizes and instrument counts are also reported in the last two rows. Quarterly data are used in the regressions. Windmeijer (2005) corrected standard errors are in parentheses; *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively. Time fixed effects are controlled by including year dummies in the regressions.

VARIABLES	All	Large banks	Big banks	Medium banks
Lagged FFC	0.780*** (0.017)	0.824*** (0.038)	0.735*** (0.046)	0.778*** (0.023)
ICR	-0.229** (0.100)	-0.080 (0.243)	-0.081 (0.220)	-0.392*** (0.151)
Idio Risk	1.111*** (0.190)	0.948*** (0.349)	1.501*** (0.439)	0.834*** (0.241)
$\frac{\text{Implied Asset Value}}{\text{Total Assets}}$	-0.074 (0.045)	-0.050 (0.119)	-0.057 (0.117)	-0.066 (0.069)
$\log \text{Total Assets}$	0.012*** (0.004)	0.006 (0.009)	-0.000 (0.013)	0.015** (0.007)
$\frac{\text{Total Deposit}}{\text{Total Liability}}$	0.073*** (0.024)	0.117 (0.107)	0.089* (0.051)	0.073** (0.029)
GDP Growth	-0.568*** (0.203)	-0.845** (0.406)	-0.819* (0.420)	-0.368 (0.264)
Instrument Count	569	77	143	533
Sample Size	23908	3100	4857	15520

Table 4: System GMM estimates of different groups of banks for Eq. (7b) with the LHS variable being the Intrinsic Cap Ratio (ICR)

This table reports the regression results for Eq. (7b).

$$\text{ICR}_{j,t} = f_2 \left(\begin{array}{c} \text{ICR}_{j,t-\frac{1}{4}}, \text{FFC}_{j,t}, \\ \text{Idios. Volatility}_{j,t}, \text{GDP Growth}_t, \\ \text{Control Variables} \end{array} \right) + \varepsilon_{j,t},$$

where the Control Variables include $\frac{\text{Implied Asset Value}_{j,t}}{\text{Total Assets}_{j,t}}$, $\log \text{Total Assets}_{j,t}$, and $\frac{\text{Total Deposits}_{j,t}}{\text{Total Liabilities}_{j,t}}$. The LHS variable in the regression is ICR, where the Implied Asset Value is one of the state variables in our model, and represents the market-based asset value of a bank. The first column contains all the RHS variable names, the second column reports the coefficients of the variables for all but small banks, and the third to sixth columns report those for the four categories: All but Small Banks, Large Banks, Big Banks, and Medium Banks. Sample sizes and instrument counts are also reported in the last two rows. Quarterly data are used in the regressions. Windmeijer (2005) corrected standard errors are in parentheses; *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively. Time fixed effects are controlled by including year dummies in the regressions.

VARIABLES	All	Large banks	Big banks	Medium banks
Lagged ICR	0.584*** (0.022)	0.682*** (0.058)	0.607*** (0.048)	0.618*** (0.030)
FFC	-0.023*** (0.003)	-0.024** (0.011)	-0.021*** (0.008)	-0.019*** (0.004)
Idio Risk	-0.397*** (0.041)	-0.350*** (0.122)	-0.571*** (0.118)	-0.420*** (0.051)
$\frac{\text{Implied Asset Value}}{\text{Total Assets}}$	0.109*** (0.010)	0.061** (0.025)	0.073*** (0.027)	0.093*** (0.014)
$\log \text{Total Assets}$	-0.001 (0.001)	0.003 (0.003)	0.001 (0.003)	-0.003 (0.002)
$\frac{\text{Total Deposit}}{\text{Total Liability}}$	-0.008** (0.004)	-0.046** (0.022)	0.010 (0.012)	0.001 (0.004)
GDP Growth	0.059 (0.040)	0.320** (0.126)	0.120 (0.100)	0.081* (0.048)
Instrument Count	569	77	143	533
Sample Size	23908	3100	4857	15520

Table 5: System GMM estimates of the Marginal Expected Shortfall (MES) regressions Eq. (8a) and Eq. (8b), based on the Large Banks' data with the LHS variables being the FFC and ICR

This table reports the regression results of the Marginal Expected Shortfall (MES) regressions Eq. (8a) and Eq. (8b).

$$\text{FFC}_{j,t} = f_1 \begin{pmatrix} \text{FFC}_{j,t-\frac{1}{4}}, \text{ICR}_{j,t}, \\ \text{MES}_{j,t}, \text{GDP Growth}_t, \\ \text{Control Variables} \end{pmatrix} + \epsilon_{j,t},$$

$$\text{ICR}_{j,t} = f_2 \begin{pmatrix} \text{ICR}_{j,t-\frac{1}{4}}, \text{FFC}_{j,t}, \\ \text{MES}_{j,t}, \text{GDP Growth}_t, \\ \text{Control Variables} \end{pmatrix} + \epsilon_{j,t},$$

where the Control Variables include $\frac{\text{Implied Asset Value}_{j,t}}{\text{Total Assets}_{j,t}}$ and $\frac{\text{Total Deposits}_{j,t}}{\text{Total Liabilities}_{j,t}}$. The LHS variables in the regression are the Forbearance Fraction of Capital (FFC) and the Intrinsic Cap Ratio (ICR), where the Implied Asset Value is one of the state variables in our model, and represents the market-based asset value of a bank. The first column contains all the RHS variable names, the second and third columns report the results for FFC and ICR, respectively. Sample sizes and instrument counts are also reported in the last two rows. Quarterly data are used in the regressions. Windmeijer (2005) corrected standard errors are in parentheses; *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively. Time fixed effects are controlled by including year dummies in the regressions.

VARIABLES	FFC	ICR
Lagged FFC	0.811*** (0.043)	–
Lagged ICR	–	0.615*** (0.059)
FFC	–	–0.035*** (0.010)
ICR	–0.142 (0.238)	–
MES	0.534** (0.239)	–0.179*** (0.045)
GDP Growth	–0.792* (0.461)	0.208* (0.115)
$\frac{\text{Implied Asset Value}}{\text{Total Assets}}$	–0.021 (0.095)	0.091*** (0.020)
$\frac{\text{Total Deposits}}{\text{Total Liabilities}}$	0.172 (0.114)	–0.031 (0.020)
Instrument Count	79	79
Sample Size	3100	3100

Online Appendices for:
How Does the Stock Market View Bank Regulatory
Capital Forbearance Policies?

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A Alternative FFC estimate by simulation

In this appendix, we model the Forbearance Fraction of Capital FFC directly by defining the presumed FDIC payoffs/payments to the bank various constituencies at audit dates. In the numerical analysis, we estimate the forbearance value by simulation. The simulation results are then compared with those obtained from our analytical model.

The two state variables, ρ_t and V_t are specified in *Model setup* section in the main text. Similar to Pennacchi (1987)'s unlimited term deposit insurance case, the regulatory forbearance can be modeled as a put option with unlimited term payoffs at audit dates as long as $V_t > \rho_t D_t$. We model whether the FDIC has to meet the payoff to save the bank by way of $\Lambda(p)$, a binary random variable with probability of p being 1. The payoff F_t (where t is an audit date) and the total debt D_t are modeled respectively as follows:¹

$$F_t = \begin{cases} 0 & \text{if } V_t > D_t \\ (D_t - V_t)\Lambda(p) & \text{if } \rho_t D_t \leq V_t \leq D_t \\ 0 & \text{if } V_t < \rho_t D_t. \end{cases} \quad (\text{A1})$$

$$D_{t+} = \begin{cases} D_t & \text{if } V_t > D_t \\ D_t - (D_t - V_t)\Lambda(p) & \text{if } \rho_t D_t \leq V_t \leq D_t \\ 0 & \text{if } V_t < \rho_t D_t. \end{cases} \quad (\text{A2})$$

When $V_t > D_t$ the bank does not require forbearance; when $\rho_t D_t \leq V_t \leq D_t$, the FDIC will potentially have to settle the payoff to lower the bank debt level to keep it alive; when $V_t < \rho_t D_t$, the put option terminates and the insured bank is closed and all its debt liabilities will be indemnified by the FDIC. Regarding $\Lambda(p)$, it is reasonable to conjecture that p is small as the FDIC makes the payment only when debt holders request it. Normally, bank depositors do not withdraw their deposits unless there is a high likelihood of a bank run. Ideally, p should be time-varying and a function of D_t , V_t , and ρ_t , however, for simplicity, we assume p a constant. Given Eq. (A1) and Eq. (A2), the for-

¹ D_t is assumed to grow at the rate of r in each simulated path. It shifts downwards at an audit date to V_t whenever $\Lambda(p) = 1$ & $\rho_t D_t \leq V_t \leq D_t$, and to zero whenever $V_t < \rho_t D_t$.

bearance value is $FV_t = \mathbb{E}_t^Q \left\{ \sum_{s=t+1}^{\infty} F_s e^{-r(s-t)} \right\}$ where r is interest rate, the equity value $E_t = V_t + FV_t - D_t$, and $\mathbb{F}FC = FV_t/E_t$.

The value of FV_t is obtained by simulating under the risk neutral measure ρ_t and V_t . Specifically, 1000 paths of 200 audit dates are simulated for the two variables, and the present values of the payoffs at these dates are aggregated to obtain the value of FV_t .² Once FV_t is calculated, E_t and $\mathbb{F}FC$ can also be calculated. For the simulations, the parameters are specified as follows : $D = 15,000$, $V_0 = 20,000$, $\kappa = 0.17$, $\mu = r - \sigma_2^2/2$, $r = 0.04$, $\sigma_1 = 0.2$, and $\sigma_2 = 0.15$. To capture the impact of ρ on $\mathbb{F}FC$, θ is set to be $-\log \left(\frac{0.9+\rho}{2} \right)$. These figures are in line with the average parameter estimates from our empirical analysis to yield plausible results. Also, to capture the estimation uncertainty, we perform 300 sets of the above simulation.

To conduct the comparative analysis using our analytical model of $\mathbb{F}FC$, we have to specify p , the probability of the FDIC settling the payment at audit dates when the value of V_t lies within $[\rho_t D_t, D_t]$. As mentioned earlier, to keep the bank open, the FDIC does not have to fill the shortfall of $D_t - V_t$ all the times, hence indemnification frequency is likely very low. We show that our analytical model is consistent with the simulation when $p = 0.004$. This means that the FDIC settle the required payment roughly one out of 250 cases. Although estimating p empirically is desirable, it is out of the scope of this paper.

The comparative results are shown in [Figure A1](#). Note that our analytical model estimation of $\mathbb{F}FC$ sits well within the 95% confidence interval of the simulated values. Comparing the analytical $\mathbb{F}FC$ with the simulation mean, we also find that the analytical $\mathbb{F}FC$ is more linear with respect to ρ_t than is the simulated $\mathbb{F}FC$ mean. Due to the nonlinearity of the simulation mean, the analytical $\mathbb{F}FC$ slightly overstates the forbearance value when ρ_t is within $[0.4, 0.9]$ (on average around 0.4% higher than the simulated mean) and understates when $\rho_t < 0.4$ (in average around 0.3% lower) or > 0.9 (in average around 0.05% lower). As these biases are all within the confidence interval, they are deemed to be insignificant. Therefore, in our simulation exercise, we find that given the low frequency that the FDIC have to indemnify, the bias in using our analytical model of $\mathbb{F}FC$ is insignif-

² For each path, 200×20 points are simulated, then F_t is calculated at all audit dates, i.e., F_t every 20 points.

icant.

[Insert **Figure A1** about here]

B Definition of G function

$G_{\mathbf{b},\mathfrak{d}}(z; X_t, \tau)$ is defined as follows:

$$G_{\mathbf{b},\mathfrak{d}}(z; X_t, \tau) = \frac{\psi(\mathbf{b}, X_t, \tau)}{2} - \frac{1}{\pi} \int_0^\infty \frac{\text{Im}[\psi(\mathbf{b} + is\mathfrak{d}, X_t, \tau) z^{is}]}{s} ds, \quad (\text{A3})$$

where \mathbf{b} and \mathfrak{d} are 1×2 row vectors. G^P has the same functional form as G , but all the parameters are under the measure P , $\text{Im}(\cdot)$ denotes the imaginary part of \cdot , $i = \sqrt{-1}$, and for any 1×2 row vector $u = [u_1, u_2]$ (where u_1 and u_2 can be complex numbers), the transform function ψ is³

$$\psi(u, X_t, \tau) = \exp(\alpha(\tau) + \beta(\tau)x_1(t) + u_2x_2(t)),$$

where α and β satisfy the ordinary differential Riccati equations (ODEs)

$$\begin{aligned} \frac{\partial \beta(\tau)}{\partial \tau} &= -\kappa\beta(\tau) + \frac{1}{2}\sigma_1^2\beta(\tau)^2 \\ \frac{\partial \alpha(\tau)}{\partial \tau} &= \kappa\theta\beta(\tau) + \mu u_2 + \frac{1}{2}\sigma_2^2 u_2^2 \\ \beta(0) &= u_1 \\ \alpha(0) &= 0. \end{aligned}$$

³ The function ψ is defined via the Fourier-Stieltjes transform of $G_{\mathbf{b},\mathfrak{d}}(\cdot; X_t, \tau)$, i.e.,

$$\psi(\mathbf{b} + is\mathfrak{d}, X_t, \tau) = \int_0^\infty e^{isz} dG_{\mathbf{b},\mathfrak{d}}(z; X_t, \tau) = \mathbb{E}_t \left(e^{(\mathbf{b} + is\mathfrak{d})X_{t+\tau}} \right).$$

When $u_2 \neq 0$ and $\varphi \neq 0$ ⁴

$$\begin{aligned}\beta(\tau) &= \frac{\kappa - Y\Omega}{\sigma_1^2} \\ \alpha(\tau) &= \frac{-\kappa\theta \left(-\kappa\tau + 2 \log \sigma_1 + \log \left(\frac{2\kappa u_1 - u_1^2 \sigma_1^2 - 2\varphi u_2}{(\kappa^2 - 2\varphi u_2 \sigma_1^2)(1+Y^2)} \right) \right)}{\sigma_1^2} + \left(\frac{\sigma_2^2 u_2^2}{2} + u_2 \mu \right) \tau \\ Y &= \tan \left(-\frac{1}{2} \tau \Omega + \arctan \left(\frac{\kappa - u_1 \sigma_1^2}{\Omega} \right) \right) \\ \Omega &= \sqrt{2\varphi u_2 \sigma_1^2 - \kappa^2}.\end{aligned}$$

Otherwise

$$\begin{aligned}\beta(\tau) &= \frac{2\kappa u_1}{\sigma_1^2 (1 - e^{\kappa\tau}) u_1 + 2\kappa e^{\kappa\tau}} \\ \alpha(\tau) &= -\frac{2\kappa\theta \log \left(1 + u_1 \sigma_1^2 \frac{e^{-\kappa\tau} - 1}{2\kappa} \right)}{\sigma_1^2} + \left(\frac{\sigma_2^2 u_2^2}{2} + u_2 \mu \right) \tau.\end{aligned}$$

C Moment conditions

Assume X_t follows a system of stochastic differential equations (SDE) under the (real) physical probability measure P as follows:

$$dX_t = \left[\begin{pmatrix} \kappa\theta \\ \mu^P \end{pmatrix} + \begin{pmatrix} -\kappa^P & 0 \\ 0 & 0 \end{pmatrix} X_t \right] dt + \begin{pmatrix} \sigma_1 \sqrt{x_1(t)} & 0 \\ 0 & \sigma_2 \end{pmatrix} dW_t^P.$$

This assumption implies that the above SDE dynamics are derived from [Eq. \(A4\)](#)

$$dX_t = d \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \left[\begin{pmatrix} \kappa\theta \\ \mu \end{pmatrix} + \begin{pmatrix} -\kappa & 0 \\ \varphi & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \right] dt + \begin{pmatrix} \sigma_1 \sqrt{x_1(t)} & 0 \\ 0 & \sigma_2 \end{pmatrix} d \begin{pmatrix} w_1(t) \\ w_2(t) \end{pmatrix} \quad (\text{A4})$$

⁴ Note that the results hold even when $2\varphi u_2 \sigma_1^2 < \kappa^2$ if we allow Ω to take values that are imaginary numbers. This is due to the fact that

$$\tan(x) = \frac{e^{2xi} - 1}{i(e^{2xi} + 1)}, \quad \arctan = \frac{1}{2}i \log \left(\frac{1 - xi}{1 + xi} \right).$$

Regardless of whether Ω is an imaginary number or not, $\beta(\tau)$ and $\alpha(\tau)$ are always real for any $\tau > 0$.

by employing the essentially affine market prices of risks specification of [Duffee \(2002\)](#) for X_t . This specification allows compensations for risks of X_t to vary independently of X_t .

C.1 The conditional expectation

The conditional expectation under measure P satisfies a system of ordinary differential equations (ODE) ($s > t$)

$$\frac{d\mathbb{E}_t(X_s)}{ds} = \begin{pmatrix} \kappa\theta \\ \mu^P \end{pmatrix} + \begin{pmatrix} -\kappa^P & 0 \\ 0 & 0 \end{pmatrix} \mathbb{E}_t(X_s)$$

with the initial condition

$$\mathbb{E}_t(X_t) = X_t.$$

Therefore

$$\begin{aligned} \mathbb{E}_t(x_1(s)) &= x_1(t) e^{-\kappa^P(s-t)} + \left(1 - e^{-\kappa^P(s-t)}\right) \frac{\kappa\theta}{\kappa^P} \\ \mathbb{E}_t(x_2(s)) &= \mu^P(s-t) + x_2(t) \end{aligned}$$

i.e.,

$$\mathbb{E}_t(X_s) = \begin{pmatrix} e^{-\kappa^P(s-t)} & 0 \\ 0 & 1 \end{pmatrix} X_t + \begin{pmatrix} \left(1 - e^{-\kappa^P(s-t)}\right) \frac{\kappa\theta}{\kappa^P} \\ \mu^P(s-t) \end{pmatrix} \quad (\text{A5})$$

C.2 The conditional variance-covariance

Let us consider $\mathbb{E}_t(x_1^2(s))$, $\mathbb{E}_t(x_2^2(s))$, and $\mathbb{E}_t(x_1(s)x_2(s))$:

$$\begin{aligned} \frac{d\mathbb{E}_t(x_1^2(s))}{ds} &= -2\kappa^P \mathbb{E}_t(x_1^2(s)) + (2\kappa\theta + \sigma_1^2) \mathbb{E}_t(x_1(s)) \\ \frac{d\mathbb{E}_t(x_2^2(s))}{ds} &= 2\mathbb{E}_t(x_2(s)) \mu^P + \sigma_2^2 \\ \frac{d\mathbb{E}_t(x_1(s)x_2(s))}{ds} &= \mu^P \mathbb{E}_t(x_1(s)) + \kappa\theta \mathbb{E}_t(x_2(s)) - \kappa^P \mathbb{E}_t(x_1(s)x_2(s)) \end{aligned}$$

with initial conditions

$$\begin{aligned}\mathbb{E}_t \left(x_1^2(t) \right) &= x_1^2(t) \\ \mathbb{E}_t \left(x_2^2(t) \right) &= x_2^2(t) \\ \mathbb{E}_t \left(x_1(t) x_2(t) \right) &= x_1(t) x_2(t).\end{aligned}$$

Then

$$\begin{aligned}\mathbb{E}_t \left(x_1^2(s) \right) &= x_1^2(t) e^{-2\kappa^P(s-t)} + \left(2\kappa\theta + \sigma_1^2 \right) \int_t^s \mathbb{E}_t \left(x_1(\tau) \right) e^{-2\kappa^P(s-\tau)} d\tau \\ &= x_1^2(t) e^{-2\kappa^P(s-t)} \\ &\quad + \left(2\kappa\theta + \sigma_1^2 \right) e^{-2\kappa^P(s-t)} \int_0^{s-t} f \left(x_1(t), v \right) e^{2\kappa^P v} dv \\ \mathbb{E}_t \left(x_2^2(s) \right) &= x_2^2(t) + 2\mu^P \int_t^s \mathbb{E}_t \left(x_2(\tau) \right) d\tau + \sigma_2^2 (s-t) \\ &= x_2^2(t) + \int_0^{s-t} g \left(x_1(t), x_2(t), v \right) dv + \sigma_2^2 (s-t) \\ \mathbb{E}_t \left(x_1(t) x_2(t) \right) &= x_1(t) x_2(t) e^{-\kappa^P(s-t)} \\ &\quad + \int_t^s \left(\mu^P \mathbb{E}_t \left(x_1(\tau) \right) + \kappa\theta \mathbb{E}_t \left(x_2(\tau) \right) \right) e^{-\kappa^P(s-\tau)} d\tau \\ &= x_1(t) x_2(t) e^{-\kappa^P(s-t)} \\ &\quad + e^{-\kappa^P(s-t)} \int_0^{s-t} m \left(x_1^2(t), x_1(t), x_2(t), v \right) e^{\kappa^P v} dv\end{aligned}$$

where

$$\begin{aligned}f \left(x_1(t), v \right) &= \mathbb{E}_t \left(x_1(t+v) \right) \\ g \left(x_1(t), x_2(t), v \right) &= 2\mathbb{E}_t \left(x_2(t+v) \right) \mu^P \\ m \left(x_1^2(t), x_1(t), x_2(t), v \right) &= \mu^P \mathbb{E}_t \left(x_1(t+v) \right) + \kappa\theta \mathbb{E}_t \left(x_2(t+v) \right).\end{aligned}$$

Hence

$$\text{Var}_t \left(X_s \right) = \begin{pmatrix} \mathbb{E}_t \left(x_1^2(s) \right) & \mathbb{E}_t \left(x_1(t) x_2(t) \right) \\ \mathbb{E}_t \left(x_1(t) x_2(t) \right) & \mathbb{E}_t \left(x_2^2(s) \right) \end{pmatrix} - \begin{pmatrix} \mathbb{E}_t^2 \left(x_1(s) \right) & \mathbb{E}_t \left(x_1(t) \right) \mathbb{E}_t \left(x_2(t) \right) \\ \mathbb{E}_t \left(x_1(t) \right) \mathbb{E}_t \left(x_2(t) \right) & \mathbb{E}_t^2 \left(x_2(s) \right) \end{pmatrix} \quad (\text{A6})$$

D ρ and the stochasticity of the volatility of V_t

To test whether ρ_t estimated from the data is contaminated by the uncontrolled stochastic volatility of the bank value if any, we conduct a simulation exercise in which our model is calibrated with the data generated from a stochastic volatility option model with a constant ρ . Specifically, we use a [Heston \(1993\)](#) type of stochastic volatility model to artificially generate bank equity values. Therefore, these bank equity values by design carry both stochastic volatility information and premium but no ρ premium. Then, we calibrate our model with these values and estimate ρ_t and σ_2 . Obviously, this is a distorted experiment where we purposefully fit a wrong model to some controlled data. However, this also serves as a stress test for our model. If our model is robust, we would expect that most of the bank equity value stochastic volatility impact will be subsumed into σ_2 while the estimated ρ_t 's fluctuate around the true constant ρ .

In [Heston \(1993\)](#) model, the variance of the bank asset value is modelled as a CIR process:

$$dVar_t = \kappa_{Var} (\theta_{Var} - Var_t) dt + \sigma_{Var} \sqrt{Var_t} dW_{Var,t}.$$

For simplicity, Var_t is set to be independent of the bank asset value V_t . In our simulation, the following parameter values are used for [Heston \(1993\)](#) model: $D = 1,500$, $V_0 = 2,000$, $\kappa_{Var} = 1$, $\theta_{Var} = 0.01$, and $\sigma_{Var} = 0.1$.⁵ A constant $\rho = 0.95$ is also added in the model account for a constant policy parameter. 24 months of V_t are first generated. To focus on the impact of the bank asset value stochastic volatility on ρ_t estimation, for each simulation, we only re-simulate Var_t but fix V_t 's at their values initially generated and combine them to generate bank equity values using the Heston model. Also, we assume V_t , $\kappa = 0.17$, $\mu = r - \sigma_2^2/2$, $r = 0.04$, and $\sigma_1 = 0.2$ are known and only estimate σ_2 , θ , and ρ_t in the calibration.

In total, 100 rounds of simulations and estimations are conducted. Based on these results, the nonparametric density functions of the estimated ρ_t and estimated σ_2 are calculated and plotted in [Figure A2](#). The true level of the bank asset value stochastic volatility and the true value of the constant ρ are also plotted for comparison. The results coincide

⁵ These values are consistent with the numerical examples in [Heston \(1993\)](#).

very much with our expectation. From Panel (a) in [Figure A2](#), we see that even we fit our model, which has a constant volatility σ_2 , to the bank equity values generated by Heston's stochastic volatility model, the estimated ρ_t is always close to the true value. From Panel (b) in [Figure A2](#), we see that the impact of the bank asset stochastic volatility is absorbed in the estimated σ_2 which is much higher than the true level of the simulated stochastic volatility. This echoes the observation that the implied volatility is usually higher than the realized volatility as the constant volatility parameter has to match the stochastic volatility premium implicit in the option price.

[Insert [Figure A2](#) about here]

To further ensure that the estimated time series of ρ_t does not pick up the dynamic of the bank asset value stochastic volatility, we compute the time series correlations between the 24 stochastic volatilities and the 24 estimated ρ_t 's in all 100 simulations then average them. The average level (first difference) correlation is 0.05 (-0.01). This correlation combined with the results in [Figure A2](#) Panel (a) confirms that the estimated ρ_t is not a proxy of the uncontrolled bank value stochastic volatility. One might be curious that where the time variation of the bank asset value stochastic volatility goes if it is not captured by either σ_2 or ρ_t . To resolve this curiosity, we turn to the estimation errors. Similar to the above correlation computation, we compute the time series correlations between the 24 stochastic volatilities and the 24 absolute values of the percentage estimation errors (model equity value/simulated equity value - 1) in all 100 simulations then average them. The average correlation is -0.51. Therefore, it is clear that the time variation of the bank asset value stochastic volatility when present will be left in the estimation errors (even though the magnitude of the estimation errors is only as low as 1-2%) but not contaminate the ρ_t estimates.

E Unscented Kalman Filter

The Unscented Kalman Filter (UKF) is a well-developed technique, widely applied in state estimation, neural networks, and nonlinear dynamic systems (see, e.g., [Haykin et al.](#),

2001 and Simon, 2006). Since, in this paper, the measurement equations in the state space formulae are highly nonlinear, the UKF is the natural choice for our estimation procedure. The state space (ω -dimensional transitions and m -dimensional measurements) is given by the following system (for notational clarity, we normalize the time interval to one):

Transition equation

$$X_t = \mathcal{T}X_{t-\Delta t} + \Theta + \sqrt{\mathbf{V}_t}e_t, \quad e_t \sim N(\mathbf{0}_{\omega \times 1}, \mathbf{I}_{\omega \times \omega})$$

where \mathcal{T} and Θ are given by Eq. (A5), \mathbf{V}_t is given by Eq. (A6), and \mathbf{I} is an identity matrix.

Measurement equation

$$y_t = \mathcal{G}(X_t) + \zeta_t, \quad \zeta_t \sim N(\mathbf{0}_{m \times 1}, \mathcal{S}_{m \times m}),$$

where \mathcal{S} is a diagonal covariance matrix with positive and distinct elements on the diagonal.

The essence of the UKF (Chow et al., 2007) used in this paper can be summarized briefly as follows. For each measurement occasion t , a set of deterministically selected points, termed *sigma points*, is used to approximate the distribution of the current state⁶ estimated at time t using a normal distribution with a mean vector $X_{t|t-\Delta t}$, a covariance matrix that is a function in the state covariance matrix, $P_{X,t-\Delta t|t-\Delta t}$, and conditional covariance \mathbf{V}_t . Sigma points are specifically selected to capture the dispersion around $X_{t|t-\Delta t}$, and are then projected using the measurement function $\mathcal{G}(\cdot)$, weighted, and then used to update the estimates in conjunction with the newly observed measurements at time t to obtain $X_{t|t}$ and $P_{X,t|t}$.

Next we outline the detailed algorithm of UKF:

⁶ In the typical UKF setting, both transition and measurement equations are nonlinear. Hence, to compute the ex ante predictions of the state variables' mean and variance, sigma points are needed to approximate the distribution of previous state estimates. However, in our paper, the transition equations are linear, so we can directly compute the ex ante predictions as in the classic Kalman Filter, and do not need sigma points at this stage.

1. Initialization⁷

$$X_{0|0} = \text{Constants}$$

$$P_{0|0} = \mathbf{V}^*$$

2. Ex ante predictions of states

$$X_{t|t-\Delta t} = \mathcal{T}X_{t-\Delta t|t-\Delta t} + \Theta$$

$$P_{X,t|t-\Delta t} = \mathcal{T}P_{X,t-\Delta t|t-\Delta t}\mathcal{T}' + \mathbf{V}_t$$

3. Selecting sigma points

Given a $\omega \times 1$ vector of *ex ante* predictions of states $X_{t|t-\Delta t}$, a set of $\omega \times (2\omega + 1)$ sigma points are selected as follows:

$$\chi_{t|t-\Delta t} = \begin{bmatrix} \chi_{0,t-\Delta t} & \chi_{+,t-\Delta t} & \chi_{-,t-\Delta t} \end{bmatrix}$$

where

$$\begin{aligned} \underbrace{\chi_{0,t-\Delta t}}_{\omega \times 1} &= X_{t|t-\Delta t} \\ \underbrace{\chi_{+,t-\Delta t}}_{\omega \times \omega} &= \mathbf{1}_{1 \times \omega} \otimes X_{t|t-\Delta t} + \sqrt{(\omega + \vartheta)} \left(\mathcal{T} \sqrt{P_{X,t-\Delta t|t-\Delta t}} + \sqrt{\mathbf{V}_t} \right) \\ \underbrace{\chi_{-,t-\Delta t}}_{\omega \times \omega} &= \mathbf{1}_{1 \times \omega} \otimes X_{t|t-\Delta t} - \sqrt{(\omega + \vartheta)} \left(\mathcal{T} \sqrt{P_{X,t-\Delta t|t-\Delta t}} + \sqrt{\mathbf{V}_t} \right). \end{aligned}$$

The term ϑ is a scaling constant and given by

$$\vartheta = \eta^2 (\omega + \varrho) - \omega$$

where η and ϱ are user-specified constants in this paper, with $\eta = 0.001$, and $\varrho = 3 - \omega$. Since the values of these constants are not critical in our case, we omit a

⁷ Refer to [item 7](#) in [Appendix F](#).

detailed description for the sake of saving space. Readers are referred to [Chow et al. \(2007\)](#) or Chapter 7 in [Haykin et al. \(2001\)](#) for further details.

4. Transformation of sigma points by way of the measurement function (predictions of measurements)

$\chi_{t|t-\Delta t}$ is propagated through the nonlinear measurement function $\mathcal{G}(\cdot)$

$$Y_{t|t-\Delta t} = \mathcal{G}(\chi_{t|t-\Delta t}),$$

where the dimension of $Y_{t|t-\Delta t}$ is $m \times (2\omega + 1)$. Then define the set of weights for covariance estimates as

$$W^{(c)} = \text{diag} \left[\frac{\vartheta}{\omega + \vartheta} + 1 - \eta^2 + 2, \underbrace{\frac{1}{2(\omega + \vartheta)}, \dots, \frac{1}{2(\omega + \vartheta)}}_{2\omega} \right]_{(2\omega + 1) \times (2\omega + 1)} ;$$

obtain weights for the mean estimates as follows:

$$W^{(m)} = \begin{bmatrix} \frac{\vartheta}{\omega + \vartheta} \\ \frac{1}{2(\omega + \vartheta)} \\ \vdots \\ \frac{1}{2(\omega + \vartheta)} \end{bmatrix}_{(2\omega + 1) \times 1} .$$

The predicted measurements and associated variance and covariance matrices are computed as follows:

$$\begin{aligned} y_{t|t-\Delta t} &= Y_{t|t-\Delta t} W^{(m)} \\ P_{y_{t|t-\Delta t}} &= \left[Y_{t|t-\Delta t} - \mathbf{1}_{1 \times (2\omega + 1)} \otimes y_{t|t-\Delta t} \right] W^{(c)} \left[Y_{t|t-\Delta t} - \mathbf{1}_{1 \times (2\omega + 1)} \otimes y_{t|t-\Delta t} \right]' + \mathcal{S} \\ P_{X_t, y_t} &= \left[\chi_{t|t-\Delta t} - \mathbf{1}_{1 \times (2\omega + 1)} \otimes X_{t|t-\Delta t} \right] W^{(c)} \left[Y_{t|t-\Delta t} - \mathbf{1}_{1 \times (2\omega + 1)} \otimes y_{t|t-\Delta t} \right]' \end{aligned}$$

5. Kalman gain and ex-post filtering state update

With the output from Step 4, actual observations are finally brought in and the dis-

crepancy between the model's predictions and the actual observations is weighted by a Kalman gain Ξ_t function to yield ex-post state and covariance estimates as follows:

$$\begin{aligned}\Xi_t &= P_{X_t, y_t} P_{y_t|t-\Delta t}^{-1} \\ X_{t|t} &= X_{t|t-\Delta t} + \Xi_t (y_t - y_{t|t-\Delta t}) \\ P_{X_t, t|t} &= P_{X_t, t|t-\Delta t} - \Xi_t P_{y_t|t-\Delta t} \Xi_t' \\ y_{t|t} &= \mathcal{G}(X_{t|t}).\end{aligned}$$

F Technical details and assumptions of the model calibration

All the assumptions we make below are for purpose of either tractability or model identification. We emphasize that our empirical results are not at all driven by any of the assumptions here. In other words, relaxing or altering the assumptions makes model estimation tougher but has no material impact on our empirical results.

1. φ is assumed to be zero for the sake of stable calibration.
2. The numerical integration in [Eq. \(A3\)](#) is performed by way of the Gauss-Kronrod quadrature and truncated at 50. Given the data and parameters, it proves to be fast, accurate, and reliable for our calibration.
3. To apply the filtering techniques, we assume under the physical measure (measure P) that X_t follows

$$\begin{aligned}dx_1(t) &= (\kappa\theta - \kappa^P x_1(t)) dt + \sigma_1 \sqrt{x_1(t)} dw_1^P(t) \\ dx_2(t) &= \mu^P dt + \sigma_2 dw_2^P(t).\end{aligned}$$

This is equivalent to assuming essentially affine market prices of risks ([Duffee, 2002](#)) for X_t .

4. The drift term μ_V of V_t under measure Q is set to 0.04, which is roughly the sample average of the risk-free interest rate. Therefore, μ is set at $0.04 - \frac{\sigma_2^2}{2}$. Consistent with [Merton \(1974\)](#), we assume the bank asset to be tradable so that its drift term under measure Q is the risk-free interest rate.
5. Although the dynamics of x_1 are different for each individual bank, it is reasonable to assume that the parameters of x_1 , $\{\kappa, \kappa^P, \theta, \text{ and } \sigma_1\}$, reflect common market views of characteristics of the forbearance provider, the government. Therefore $\{\kappa, \kappa^P, \theta, \text{ and } \sigma_1\}$ are assumed to be common to all banks.
6. θ and κ^P are hard to pin down, and are set to $-\log(0.9)$ and $\frac{\log(0.9)}{\log(0.97)}\kappa$, respectively. This means that the long term mean of $x_1(t)$ is $-\log(0.9)$ under the measure Q and $-\log(0.97)$ under the measure P. Roughly speaking (ignoring Jensen's inequality), this also means that the long-term mean of ρ is 0.9 under the measure Q and 0.97 under the measure P. This assumption implies that there is a negative risk premium associated with $x_1(t)$, meaning bank equity holders regard downward movements in ρ as unfavorable shocks to the investment opportunity. This is consistent with the risk premium identified for interest rates and default risk factors in the literature (see, e.g., [Jarrow et al., 2010](#); [Filipović and Trolle, 2013](#)). Unlike the studies of interest rates and credit risk, we do not have term structure data here. This might be the reason behind the non-identifiability of θ and κ^P .
7. The initial values for x_1 and x_2 used to start up the UKF are, respectively, $-\log(0.97)$ and the log of the individual market cap at the first data point. The initial covariance \mathbf{V}^* is given by

$$\begin{bmatrix} -\log(0.97)\sigma_1^2\Delta t & 0 \\ 0 & \sigma_2^2\Delta t \end{bmatrix}$$
 where $\Delta t = 1/12$.
8. σ_1 and κ are calibrated using the average time series of market cap and total liabilities across all banks. When calibrating $\{\mu^P, \sigma_2\}$ of x_2 for each individual bank, $\{\kappa, \kappa^P, \theta, \text{ and } \sigma_1\}$ of x_1 are fixed.

G Time-series analysis of the countercyclical nature of bank regulatory capital forbearance

The results in *Results and discussion* section show that $\mathbb{F}FC$ is countercyclical with cross-reference to the GDP growth. In this appendix, we show that this countercyclical nature is also detected in the time series of the aggregate $\mathbb{F}FC$. Countercyclical forbearance implies that bank regulators try to ensure that banks build up capital buffers during good times so that they can draw it down in bad times. If this is true, a time series analysis of the aggregate forbearance should reveal that relatively high capital forbearance in bad times predicts relatively low capital forbearance in better times. To test this prediction, we construct two time series using the average $\mathbb{F}FC$ and GDP growth and run a time series regression.

Specifically, the two time series are the relative levels of average $\mathbb{F}FC$ at bad times, $\mathbb{F}FC_t^{\text{BT}}$, and good times, $\mathbb{F}FC_t^{\text{GT}}$. They are defined as:

$$\mathbb{F}FC_t^{\text{BT}} = \begin{cases} \mathbb{F}FC_t - \overline{\mathbb{F}FC}_{t-\Delta t} & \text{if } \mathbb{F}FC_t > \overline{\mathbb{F}FC}_{t-\Delta t} \text{ and } GDPG_t < \overline{GDPG}_{t-\Delta t} \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{F}FC_t^{\text{GT}} = \begin{cases} \overline{\mathbb{F}FC}_{t-\Delta t} - \mathbb{F}FC_t & \text{if } \mathbb{F}FC_t < \overline{\mathbb{F}FC}_{t-\Delta t} \text{ and } GDPG_t > \overline{GDPG}_{t-\Delta t} \\ 0 & \text{otherwise} \end{cases}$$

where $\mathbb{F}FC_t$ ($GDPG_t$) is the average $\mathbb{F}FC$ (the GDP growth) at period t , and $\overline{\mathbb{F}FC}_t$ (\overline{GDPG}_t) is the moving average of the average $\mathbb{F}FC$ (the GDP growth) up to period t , and Δt is one quarter. These moving averages are brought in to define the relative level. We then run the following regression:

$$\mathbb{F}FC_t^{\text{GT}} = C_0 + C_1 \mathbb{F}FC_{t-\Delta t}^{\text{BT}} + \epsilon_t.$$

The OLS estimate of C_1 is -0.2 with Newey-West standard error of 0.06 , meaning the estimate is highly significant (P-value = 0.5%). R^2 of the regression is 6% . A negative and significant C_1 indicates that a higher relative level of average $\mathbb{F}FC$ at bad times tends to

lead to a lower relative level of average FFC at good times in the next period. This verifies the prediction and confirms the countercyclical nature of regulatory capital forbearance.

H Results of regression specifications with additional control variables

In this appendix, in addition to the results presented in the main text, we present results from other regression specifications: Specifications (S2), (S3), and (S4). In (S2) we add Funding Liquidity (STB/TA) to the RHS of [Eq. \(A7a\)](#) and [Eq. \(A7b\)](#),

$$\text{FFC}_{j,t} = f_1 \left(\begin{array}{c} \text{FFC}_{j,t-\frac{1}{4}}, \text{ICR}_{j,t}, \\ \text{Idios. Volatility}_{j,t}, \text{GDP Growth}_t, \\ \text{Control Variables} \end{array} \right) + \epsilon_{j,t} \quad (\text{A7a})$$

$$\text{ICR}_{j,t} = f_2 \left(\begin{array}{c} \text{ICR}_{j,t-\frac{1}{4}}, \text{FFC}_{j,t}, \\ \text{Idios. Volatility}_{j,t}, \text{GDP Growth}_t, \\ \text{Control Variables} \end{array} \right) + \epsilon_{j,t} \quad (\text{A7b})$$

in (S3) we add STB/TA and ROA, and in (S4) we add STB/TA, ROA, and Total Loans / Total Assets. STB/TA is defined as the ratio (Short-Term Borrowing + Other Short-Term Liabilities) / Total Assets and ROA is the ratio Net Income / Total Assets. The results are reported in [Table A1](#) and [Table A2](#).

I Technical details on estimating the systemic risk

We confine the usual notations defined herein to this appendix only. That is, some notations might be used elsewhere in this paper, but these described here apply exclusively to this appendix and should not be confused with the same notations employed elsewhere in the paper.

Following [Brownlees and Engle \(2012\)](#), the Marginal Expected Shortfall, MES, is expressed as a function of volatility, correlation and tail expectations of the standardized

innovations distribution

$$\text{MES}_{i,t} = \sigma_{i,t} \rho_{i,t} \mathbb{E}(\epsilon_{m,t} | \epsilon_{m,t} < p) + \sigma_{i,t} \sqrt{1 - \rho_{i,t}^2} \mathbb{E}(\epsilon_{i,t} | \epsilon_{m,t} < p)$$

where $\epsilon_{m,t} = \frac{r_{m,t}}{\sigma_{m,t}}$, $\epsilon_{i,t} = \frac{r_{i,t}}{\sigma_{i,t}}$; and $\sigma_{i,t}$ is the conditional volatility of i th bank at time t ; $r_{m,t}$ and $r_{i,t}$ are daily returns of S&P 500 index and i th bank's market cap at time t , respectively; $\rho_{i,t}$ is the conditional correlation between the returns of i th bank and those of S&P 500 index; $\mathbb{E}(\epsilon_{i,t} | \epsilon_{m,t} < p)$ is the tail expectation of i th bank's standardized innovations of return given the standardized innovation of returns of S&P 500 index is less than p ; $\mathbb{E}(\epsilon_{m,t} | \epsilon_{m,t} < p)$ is the tail expectation of S&P 500 index's standardized innovation of returns conditional on it is less than p . As in [Brownlees and Engle \(2012\)](#), we set p to the 5th percentile of the empirical unconditional distribution of $\epsilon_{m,t}$ in the whole sample.

$\sigma_{m,t}$ s and $\sigma_{i,t}$ s are estimated using the Threshold ARCH (TARCH) specification ([Rabemananjara and Zakoian, 1993](#)). $\rho_{i,t}$ s are estimated using the Corrected Dynamic Conditional Correlation (CDCC) approach, which is a Dynamic Conditional Correlation (DCC) framework of [Engle \(2002\)](#) enhanced by [Aielli \(2013\)](#). The tail expectations are computed as the averages of the two standardized innovations in all cases that satisfy the condition ($\epsilon_{m,t}$ is less than the 5th percentile of the empirical unconditional distribution of $\epsilon_{m,t}$ in the whole sample).

Given the MES, we follow [Acharya et al. \(2012\)](#), and define the SRISK as

$$\text{SRISK}_{i,t} = k \frac{\text{TL}}{\text{TA}_{i,t}} + (1 - k) e^{-18 \text{MES}_{i,t}} \text{MC}_{i,t}$$

where $\frac{\text{TL}}{\text{TA}_{i,t}}$ is i th bank's total liabilities to total assets ratio at time t ; $\text{MC}_{i,t}$ is the market value of equity for i th bank at time t , which is represented by market cap; as in [Acharya et al. \(2012\)](#), we set k to 8%.

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Figure A1: Simulated FFC v.s. analytical FFC

This figure shows comparison between the simulated FFC and the analytical FFC at different level of ρ_t (from 0.3 to 0.95 with increment of 0.05). The parameters are specified as follows in the simulation: $\kappa = 0.17$, $\mu = r - \sigma_2^2/2$, $r = 0.04$, $\sigma_1 = 0.2$, $\sigma_2 = 0.15$, $p = 0.004$, and $\theta = -\log\left(\frac{0.9+\rho}{2}\right)$. The lower and upper dash lines capture the 95% confidence interval of the simulated FFC, the dash-dot line is the simulation mean of FFC, and the dash line with triangles is the analytical FFC.

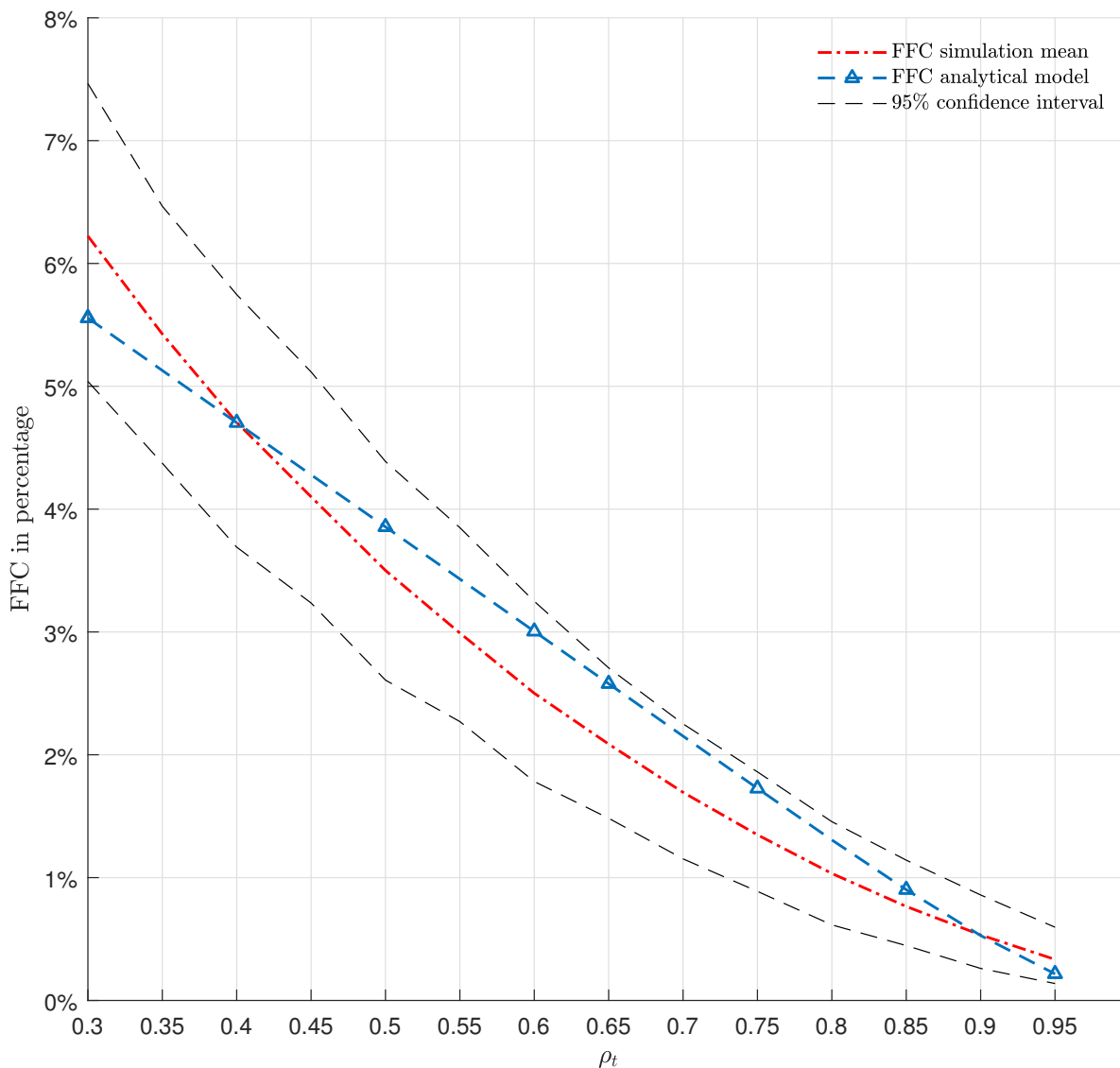


Figure A2: Estimated ρ_t and σ_2 v.s. their true levels

This figure shows the nonparametric density functions of the estimated ρ_t (Panel a) and estimated σ_2 (Panel b) based on the results of the 100 simulations conducted in [Appendix D](#). The true level of the stochastic volatility and the true value of the constant ρ are also plotted in the graphs for comparison.

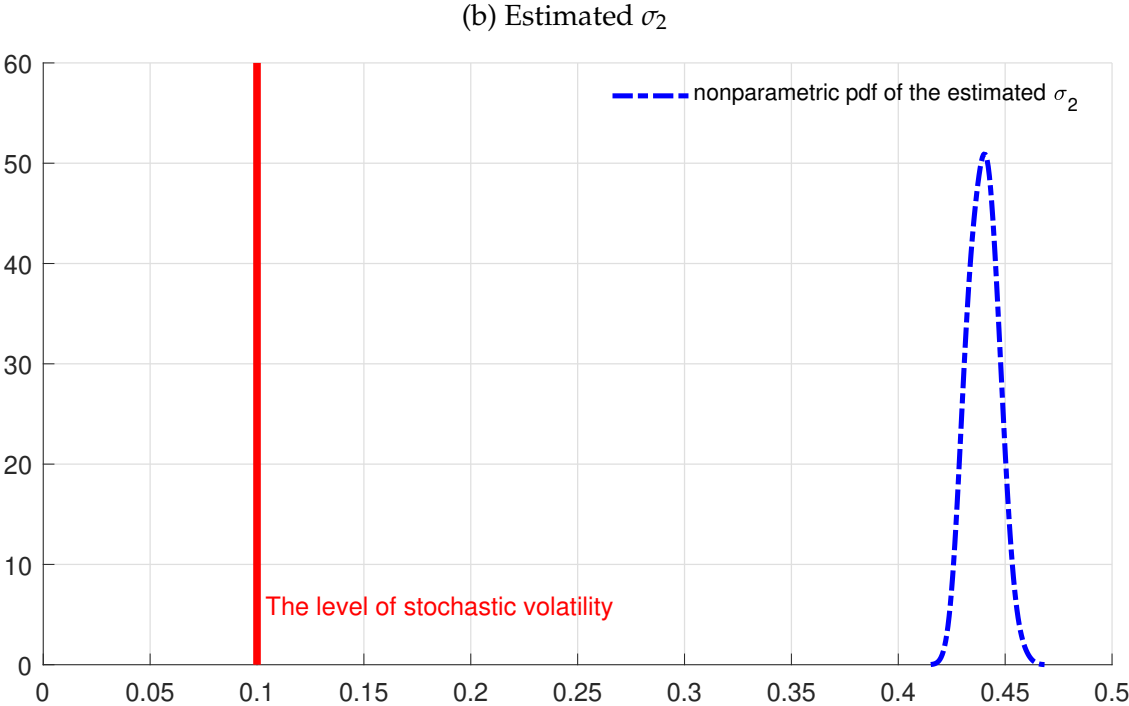
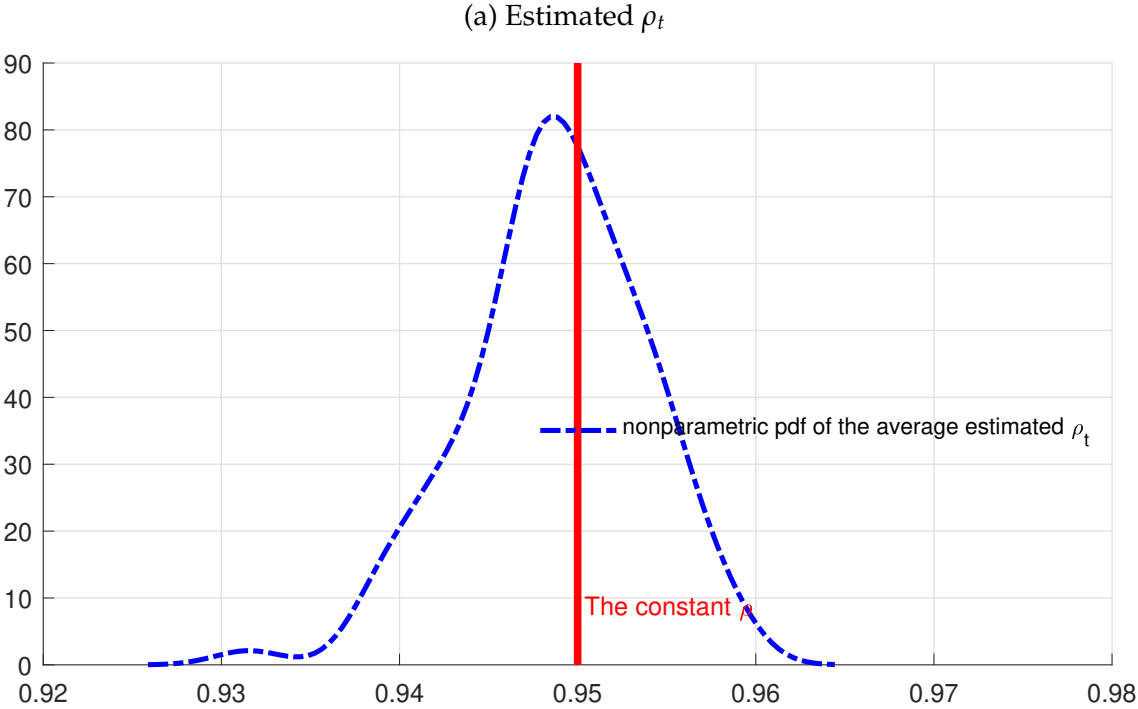


Table A1: System GMM estimates of additional regression specifications for the Forbearance Fraction of Capital (FFC)

This table reports the regression results of specifications (S2), (S3), and (S4) for FFC. The first column contains all the variable names; from the second to the last column, the table reports the coefficients of the variables for all four groups. Each group has three regressions, which are labelled (S2), (S3), and (S4). Sample sizes and instrument counts are also reported in the last two rows. Quarterly data are used in the regressions. [Windmeijer \(2005\)](#) corrected standard errors are in parentheses; *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively. Time fixed effects are controlled by including year dummies in the regressions.

VARIABLES	All			Large banks			Big banks			Medium banks		
	(S2)	(S3)	(S4)	(S2)	(S3)	(S4)	(S2)	(S3)	(S4)	(S2)	(S3)	(S4)
Lagged FFC	0.77*** (0.02)	0.77*** (0.02)	0.77*** (0.02)	0.82*** (0.04)	0.82*** (0.06)	0.83*** (0.06)	0.73*** (0.05)	0.73*** (0.05)	0.73*** (0.05)	0.76*** (0.03)	0.76*** (0.02)	0.76*** (0.02)
ICR	-0.17 (0.11)	-0.10 (0.11)	-0.29** (0.13)	0.08 (0.25)	-0.41 (0.26)	-0.36 (0.25)	-0.10 (0.30)	-0.03 (0.30)	-0.01 (0.30)	-0.34** (0.16)	-0.33** (0.16)	-0.53*** (0.17)
Idio Risk	0.98*** (0.21)	1.09*** (0.24)	1.36*** (0.24)	0.70 (0.46)	1.25* (0.71)	1.44* (0.87)	1.36*** (0.40)	0.79 (0.54)	0.67 (0.62)	0.82*** (0.27)	0.80*** (0.28)	1.01*** (0.29)
$\frac{\text{Implied Asset Value}}{\text{Total Assets}}$	-0.15*** (0.06)	-0.23*** (0.07)	-0.14* (0.08)	-0.16 (0.15)	-0.06 (0.15)	-0.11 (0.17)	-0.09 (0.17)	-0.27 (0.20)	-0.27 (0.21)	-0.14* (0.08)	-0.17* (0.09)	-0.07 (0.09)
log Total Assets	0.01*** (0.00)	0.02*** (0.01)	0.04*** (0.01)	0.01 (0.01)	0.01 (0.01)	0.01 (0.02)	-0.00 (0.01)	0.03 (0.02)	0.02 (0.02)	0.02*** (0.01)	0.02*** (0.01)	0.05*** (0.01)
$\frac{\text{Total Deposit}}{\text{Total Liability}}$	0.09*** (0.03)	0.12*** (0.04)	0.13*** (0.04)	0.13 (0.13)	0.08 (0.19)	0.02 (0.19)	0.13* (0.07)	0.21* (0.12)	0.21* (0.12)	0.09*** (0.03)	0.09*** (0.04)	0.11*** (0.04)
GDP Growth	-0.44** (0.21)	-0.42* (0.22)	-0.76*** (0.25)	-0.79* (0.43)	-0.52 (0.62)	-0.47 (0.64)	-0.80 (0.60)	-0.58 (0.49)	-0.56 (0.52)	-0.19 (0.28)	-0.24 (0.28)	-0.67** (0.31)
Liquidity	0.39*** (0.14)	0.31** (0.14)	0.29** (0.14)	0.44 (0.28)	-0.02 (0.41)	-0.04 (0.44)	0.05 (0.30)	-0.30 (0.40)	-0.31 (0.41)	0.43** (0.17)	0.28** (0.13)	0.26* (0.14)
ROA	-	0.74 (1.85)	1.14 (1.82)	-	8.46 (9.83)	9.26 (10.13)	-	-3.01 (5.30)	-3.39 (5.39)	-	0.90 (2.04)	1.64 (2.07)
$\frac{\text{Total Loan}}{\text{Total Assets}}$	-	-	-0.29*** (0.09)	-	-	0.14 (0.32)	-	-	0.07 (0.22)	-	-	-0.35*** (0.11)
Instrument Count	569	536	536	77	77	77	143	143	143	533	530	530
Sample Size	23646	20537	20527	3096	2483	2483	4815	4065	4065	15311	13618	13608

Table A2: System GMM estimates of additional regression specifications for the Intrinsic Cap Ratio (ICR)

This table reports the regression results of specifications (S2), (S3), and (S4) for ICR. The first column contains all the variable names; from the second to the last column, the table reports the coefficients of the variables for all four groups. Each group has three regressions, which are labelled (S2), (S3), and (S4). Sample sizes and instrument counts are also reported in the last two rows. Quarterly data are used in the regressions. [Windmeijer \(2005\)](#) corrected standard errors are in parentheses; *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively. Time fixed effects are controlled by including year dummies in the regressions.

VARIABLES	All			Large banks			Big banks			Medium banks		
	(S2)	(S3)	(S4)	(S2)	(S3)	(S4)	(S2)	(S3)	(S4)	(S2)	(S3)	(S4)
Lagged ICR	0.56*** (0.02)	0.54*** (0.02)	0.49*** (0.03)	0.68*** (0.06)	0.61*** (0.08)	0.61*** (0.07)	0.60*** (0.06)	0.56*** (0.06)	0.56*** (0.06)	0.59*** (0.03)	0.59*** (0.03)	0.55*** (0.03)
FFC	-0.02*** (0.00)	-0.02*** (0.00)	-0.02*** (0.00)	-0.02* (0.01)	-0.04** (0.02)	-0.05*** (0.02)	-0.02** (0.01)	-0.02* (0.01)	-0.02* (0.01)	-0.02*** (0.00)	-0.02*** (0.00)	-0.02*** (0.01)
Idio Risk	-0.36*** (0.04)	-0.31*** (0.05)	-0.18*** (0.06)	-0.24** (0.11)	0.09 (0.33)	0.27 (0.30)	-0.54*** (0.13)	-0.44*** (0.13)	-0.41*** (0.14)	-0.42*** (0.05)	-0.35*** (0.05)	-0.30*** (0.07)
$\frac{\text{Implied Asset Value}}{\text{Total Assets}}$	0.13*** (0.01)	0.15*** (0.01)	0.18*** (0.02)	0.08*** (0.03)	0.08** (0.03)	0.09** (0.04)	0.09** (0.04)	0.13*** (0.04)	0.13*** (0.04)	0.11*** (0.01)	0.12*** (0.02)	0.14*** (0.02)
log Total Assets	-0.00 (0.00)	-0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	-0.00 (0.00)	-0.00 (0.00)	-0.00 (0.00)	-0.01*** (0.00)	-0.00 (0.00)
$\frac{\text{Total Deposit}}{\text{Total Liability}}$	-0.01*** (0.01)	-0.03*** (0.01)	-0.02*** (0.01)	-0.05** (0.02)	-0.04 (0.04)	-0.03 (0.04)	-0.01 (0.02)	-0.04 (0.03)	-0.04 (0.03)	-0.00 (0.00)	-0.00 (0.01)	-0.00 (0.01)
GDP Growth	-0.01 (0.05)	-0.07 (0.05)	-0.19*** (0.06)	0.32** (0.14)	0.08 (0.13)	0.14 (0.15)	0.05 (0.13)	-0.15 (0.15)	-0.16 (0.15)	0.04 (0.05)	0.00 (0.05)	-0.09 (0.06)
Liquidity	-0.13*** (0.03)	-0.17*** (0.03)	-0.16*** (0.03)	-0.09** (0.04)	-0.06 (0.09)	-0.02 (0.10)	-0.14* (0.08)	-0.13 (0.09)	-0.13 (0.09)	-0.07** (0.03)	-0.00 (0.03)	-0.01 (0.03)
ROA	-	1.13*** (0.39)	1.20*** (0.40)	-	5.46 (4.01)	5.16* (2.90)	-	3.05** (1.23)	3.08** (1.30)	-	0.61* (0.32)	0.72** (0.35)
$\frac{\text{Total Loan}}{\text{Total Assets}}$	-	-	-0.11*** (0.02)	-	-	-0.10 (0.07)	-	-	-0.02 (0.04)	-	-	-0.07** (0.03)
Instrument Count	569	536	536	77	77	77	143	143	143	533	530	530
Sample Size	23646	20537	20527	3096	2483	2483	4815	4065	4065	15311	13618	13608