Robust Semi-Blind Estimation of Channel and CFO for GFDM Systems

Yujie Liu*†, Xu Zhu*, Eng Gee Lim†, Yufei Jiang‡ and Yi Huang*
*Department of Electrical Engineering and Electronics, University of Liverpool, Liverpool, UK
Email: {yujieliu, xuzhu and yi.huang}@liverpool.ac.uk
†Department of Electrical and Electronic Engineering, Xi'an Jiaotong-Liverpool University, Suzhou, P. R. China
Email: enggee.lim@xjtlu.edu.cn
‡School of Electronic and Information Engineering, Harbin Institute of Technology, Shenzhen, P. R. China
Email: jiangyufei@hit.edu.cn

***Abstract*—We propose a robust semi-blind estimation scheme of channel and carrier frequency offset (CFO) for generalized frequency division multiplexing (GFDM) systems. This, to the best of our knowledge, is the first work to propose an integral solution to channel and full-range CFO for a wide range of GFDM systems. Based on the derived equivalent system model with CFO included implicitly, a subspace based method is proposed to perform initial channel estimation blindly, which requires only a small number of received symbols to achieve the second order statistics of the received signal. Then, CFO estimation and channel ambiguity elimination are undertaken in series by utilizing a small number of nulls and pilots in a single sub-symbol. Both channel and CFO estimations are more robust against inter-carrier interference (ICI) and inter-symbol interference (ISI) caused by the nonorthogonal filter of GFDM, compared to the existing methods. The proposed scheme achieves a bit error rate (BER) performance close to the ideal case with perfect CFO and channel estimations especially at medium and high signal-to-noise-ratios (SNRs).**

I. Introduction

Generalized frequency division multiplexing (GFDM), a generalized form of orthogonal frequency division multiplexing (OFDM), has been considered as a potential waveform candidate for the fifth generation (5G) wireless communications [1]–[13]. However, GFDM suffers from inter-carrier interference (ICI) and inter-symbol interference (ISI) caused by its nonorthogonal prototype filter, which makes its channel estimation and synchronization more challenging than in OFDM, and the solutions for OFDM [14]–[19] cannot be applied directly.

Channel estimation for GFDM systems has been studied in [1]–[5] assuming perfect frequency synchronization. Scattered-pilot aided channel estimation was presented in [1], where the interference was pre-canceled at the transmitter. However, its performance was vulnerable to frequency selective fading. Least squares (LS) channel estimation with a rectangular-pattern pilot was shown to outperform that with a block-pattern pilot [2]. However, they required high training overhead and were biased due to ICI and ISI. The prototype filters in [3]–[5] were modified to enable the interference-free pilot aided channel estimation, which however worked only for specific GFDM systems.

GFDM is sensitive to carrier frequency offset (CFO), including an integer CFO (iCFO) and a fractional CFO (fCFO), which are usually caused by the mismatch between local oscillators at the transmitter and receiver or a Doppler frequency shift [6], [14]. Most previous work on CFO estimation for GFDM systems have focused on fCFO only [7]–[9]. A pseudo noise (PN) based preamble with two identical sub-symbols was utilized to estimate fCFO in [7] and [8]. However, it required high training overhead and was sensitive to ICI and ISI. fCFO was blindly estimated utilizing cyclic prefix (CP) and maximum likelihood approach in [8] and [9]. However, their fCFO estimation range was limited. The only work that has considered iCFO estimation was [10], however, it was applicable to a specific GFDM system only and its performance degraded a lot in the multipath fading channel. Our previous work in OFDM systems [16]–[18] enabled full-range CFO estimation, however they can not be utilized to GFDM systems due to their nonorthogonal pulse shaping.

The aforementioned work [1]–[10] dealt with either channel estimation or CFO estimation, without considering their effect on each other. In [11], both estimations were considered, which however focused on small-scale CFO and was applicable only for unique-word (UW) GFDM systems. Semi-blind channel and CFO estimation for zero-padding (ZP) OFDM systems were presented in [15] and our previous work in [18], which however cannot be utilized for CP-GFDM systems.

In this paper, a robust semi-blind channel and CFO estimation (RSCCE) scheme is proposed for GFDM systems, where based on the derived equivalent system model with CFO included implicitly, a subspace based method is proposed to perform initial channel estimation blindly requiring only a small number of received symbols, and then CFO estimation and channel ambiguity elimination are undertaken in series by utilizing a small number of nulls and pilots in a single sub-

This work was supported in part by the Department for Digital, Culture, Media and Sport, U.K., through the Liverpool 5G Project, in part by the Science and Technology Innovation Commission of Shenzhen under Grant JCYJ20170307151258279, and in part by the Natural Science Foundation of Guangdong Province under grant 2018A030313344.

symbol only. Our work is different in the following aspects.

- This, to the best of our knowledge, is the first work to propose an integral solution to channel estimation and CFO estimation, considering their effect on each other, while in [1]–[10] only one of the issues was addressed assuming perfect estimation of the other.
- Our work is applicable to a wide range of GFDM systems and allows full-range CFO estimation, while the previous work either was applicable to specific GFDM systems such as interference-free GFDM in [3]–[5] and UW-GFDM in [11] or considered CFO estimation in a limited range only [7]–[9], [11].
- The proposed RSCCE scheme achieves a bit-error-rate (BER) performance close to the ideal case with perfect channel and CFO estimations especially from medium to high signal-to-noise-ratios (SNRs). Both channel and CFO estimations are more robust against ICI and ISI caused by the nonorthogonal filter of GFDM than the existing methods [2], [7], [8], thanks to the subspace based initial channel estimation and the ICI and ISI mitigation in subsequent CFO estimation and channel ambiguity elimination.
- The proposed semi-blind scheme utilizes a small number of nulls and pilots in a single sub-symbol only to enable CFO estimation and channel scalar ambiguity elimination respectively, with much lower training overhead than that required in [2], [7] and [8]. Also, the proposed blind channel estimation algorithm achieves the second order statistics of the received signal with only a small number of received symbols, about tens of times less than that required in [15], [18] and [19].

The rest of this paper is organized as follows. Section II presents the system model. The proposed RSCCE scheme is described in Section III. Complexity analysis and simulation results are given in Sections IV and V, respectively. Section VI draws conclusion.

Notations: Bold symbols represent vectors/matrices, and superscripts T, H and $\dagger$ denote the transpose, complex conjugate transpose and pseudo inverse of a vector/matrix. diag$\{\mathbf{a}\}$ is a diagonal matrix with vector $\mathbf{a}$ on its diagonal. $\mathbf{0}_{M\times N}$ is an $M\times N$ zero matrix. $\|\cdot\|_{\mathrm{F}}^2$ is the Frobenius norm. $\mathbb{E}\{\}$ is the expectation operator.

II. System Model

A. System Model

We consider a single-input-multiple-output (SIMO) GFDM system where the receiver is equipped with N_{r} receive antennas. Each GFDM symbol is divided into M sub-symbols each with K subcarriers and we define $N = KM$. Let $\mathbf{d}_i = [\mathbf{d}_{i,0}^T, \cdots, \mathbf{d}_{i,M-1}^T]^T$ denote the i-th GFDM symbol with $\mathbf{d}_{i,m} = [d_{i,0,m}, \cdots, d_{i,K-1,m}]^T$ where $d_{i,k,m}$ is the corresponding complex data transmitted on the k-th subcarrier in the m-th sub-symbol of the symbol i. Then, each $d_{i,k,m}$ is transmitted with the corresponding pulse shape [12]

$$g_{k,m}[n] = g[(n - mK) \bmod N] \cdot \exp(-j2\pi kn/K) \tag{1}$$

where n $(n = 0, \cdots, N-1)$ is the sampling index. Each $g_{k,m}[n]$ is a time and frequency shifted version of a prototype filter $g[n]$, where the modulo operation performs a circularly shifted version of $g_{k,0}(n)$ and the complex exponential makes $g_{k,m}(n)$ a frequency shifted version of $g_{0,m}(n)$. After pulse shaping, the transmit symbol $x_i[n]$ is given by

$$x_i[n] = \sum\nolimits_{k=0}^{K-1} \sum\nolimits_{m=0}^{M-1} g_{k,m}[n] d_{i,k,m} \tag{2}$$

Denote $\mathbf{x}_i = [x_i[0], \cdots, x_i[N-1]]^T$ as the transmit data vector, which can be expressed as

$$\mathbf{x}_i = \mathbf{A}\mathbf{d}_i \tag{3}$$

where $\mathbf{A} = [\mathbf{g}_{0,0}, \cdots, \mathbf{g}_{K-1,0}, \cdots, \mathbf{g}_{0,M-1}, \cdots, \mathbf{g}_{K-1,M-1}]$ is an $N \times N$ pulse shaping filter matrix with $\mathbf{g}_{k,m} = [g_{k,m}[0], \cdots, g_{k,m}[N-1]]^T$. A single CP with length L_{cp} is pre-pended to the GFDM symbol $\mathbf{x}_i$, obtaining $\mathbf{s}_i = [s_i[0], \cdots, s_i[L_{\mathrm{cp}}-1], s_i[L_{\mathrm{cp}}], \cdots, s_i[G-1]]^T = [x_i[N - L_{\mathrm{cp}}], \cdots, x_i[N-1], x_i[0], \cdots, x_i[N-1]]^T$, with $G = N + L_{\mathrm{cp}}$.

The channel is assumed to exhibit quasi-static block fading and the channel impulse response (CIR) remains constant within N_{s} GFDM symbols. Denote $\mathbf{h}^{n_{\mathrm{r}}} = [h^{n_{\mathrm{r}}}[0], \cdots, h^{n_{\mathrm{r}}}[L-1]]^T$ as the CIR for the n_{r}-th receive antenna with L being the length of CIR. $\phi = \phi_{\mathrm{f}} + \phi_{\mathrm{i}}$ $(\phi \in (-K/2, K/2])$ is defined as the CFO between the transmitter and receiver, where ϕ_{f} and ϕ_{i} are respectively the fCFO and iCFO. The time-domain received signal in the i-th symbol at the n_{r}-th receive antenna is written as

$$y_i^{n_{\mathrm{r}}}[g] = e^{j2\pi\phi g/K} \sum\nolimits_{l=0}^{L-1} h^{n_{\mathrm{r}}}[l] s_i[g-l] + w_i^{n_r}[g] \tag{4}$$

where $w_i^{n_r}[g]$ $(g = 0, \cdots, G-1)$ is the noise term.

B. Equivalent System Model with Implicit CFO

To enable robust semi-blind estimation of channel and CFO, an equivalent system model is derived, which includes CFO in the transmitted signal and channel implicitly. By incorporating the CFO into the transmitted signal and channel, (4) is equivalent to

$$y_i^{n_{\mathrm{r}}}[g] = \sum\nolimits_{l=0}^{L-1} \bar{h}^{n_{\mathrm{r}}}[l] \bar{s}_i[g-l] + w_i^{n_r}[g] \tag{5}$$

with $\bar{h}^{n_{\mathrm{r}}}[l] = e^{j2\pi\phi l/K} h^{n_{\mathrm{r}}}[l]$ and $\bar{s}_i[g] = e^{j2\pi\phi g/K} s_i[g]$ denoted as the CFO-included channel and transmitted signal.

Among the received signal samples in CP, $y_i^{n_{\mathrm{r}}}[L-1]$ to $y_i^{n_{\mathrm{r}}}[L_{\mathrm{cp}}-1]$ are free from ISI due to multipath, and thus are utilized alongside signal samples $y_i^{n_{\mathrm{r}}}[L_{\mathrm{cp}}]$ to $y_i^{n_{\mathrm{r}}}[M-1]$ for semi-blind estimation of CFO and channel. Collecting all these samples from N_{r} received antennas into a vector, we obtain $\mathbf{y}_i = [y_i^0[L-1], \cdots, y_i^{N_{\mathrm{r}}-1}[L-1], \cdots, y_i^0[G-1], \cdots, y_i^{N_{\mathrm{r}}-1}[G-1]]^T$, which is then given by

$$\mathbf{y}_i = \mathbf{H}\mathbf{s}_i + \mathbf{w}_i \tag{6}$$

where $\mathbf{H}$ of size $(G-L+1)N_{\mathrm{r}} \times G$ is defined as

$$\mathbf{H} = \begin{bmatrix} \bar{\mathbf{h}}(L-1) & \cdots & \bar{\mathbf{h}}(0) & \cdots & \cdots & \mathbf{0}_{N_{\mathrm{r}}\times 1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0}_{N_{\mathrm{r}}\times 1} & \cdots & \cdots & \bar{\mathbf{h}}(L-1) & \cdots & \bar{\mathbf{h}}(0) \end{bmatrix} \tag{7}$$

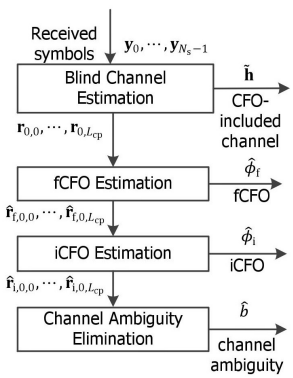

Fig. 1. Flow chart of the proposed RSCCE scheme for GFDM systems.

with $\bar{\mathbf{h}}(l) = [\bar{h}^0[l], \cdots, \bar{h}^{N_r-1}[l]]^T$ and $\mathbf{w}_i = [w_i^0[L-1], \cdots, w_i^{N_r-1}[L-1], \cdots, w_i^0[G-1], \cdots, w_i^{N_r-1}[G-1]]^T$.

Since $\mathbb{E}\{\mathbf{s}_i\mathbf{s}_i^H\}$ is rank deficient whose rank is N instead of G due to the redundancy from CP, (6) cannot be applied to the subspace based blind channel estimation approach. To address this problem, the consecutive $(N-L+1)N_r$ samples from $\mathbf{y}_i$ are collected as a subvector, and it is easily found that $(L_{cp}+1)$ such subvectors can be formed. The t-th $(t = 0, 1, \cdots, L_{cp})$ subvector is denoted as $\mathbf{y}_{i,t} = [y_i^0[L-1+t], \cdots, y_i^{N_r-1}[L-1+t], \cdots, y_i^0[N-1+t], \cdots, y_i^{N_r-1}[N-1+t]]^T$, which is written as

$$\mathbf{y}_{i,t} = \bar{\mathbf{H}}\mathbf{s}_{i,t} + \mathbf{w}_{i,t}, \quad t = 0, \cdots, L_{cp} \tag{8}$$

where $\bar{\mathbf{H}}$ follows the similar form to $\mathbf{H}$ but with a reduced size of $(N-L+1)N_r \times N$; $\mathbf{s}_{i,t} = [\bar{s}_i[t], \cdots, \bar{s}_i[N-1+t]]^T$ and $\mathbf{w}_{i,t} = [w_i^0[L-1+t], \cdots, w_i^{N_r-1}[L-1+t], \cdots, w_i^0[N-1+t], \cdots, w_i^{N_r-1}[N-1+t]]^T$.

III. Robust Semi-Blind Channel and CFO Estimation

Based on the derived equivalent system model in (8), an RSCCE scheme is proposed for GFDM systems in the presence of CFO, which mainly consists of three stages, as shown in Fig. 1: a) subspace based blind channel estimation is performed in the presence of CFO; b) both fCFO and iCFO in the full range are estimated; c) channel ambiguity due to blind estimation in stage a) is eliminated. The processes at stages b) and c) are semi-blind, which respectively utilize a small number of nulls and a small number of pilots in a single sub-symbol only, as illustrated in Fig. 2.

A. Blind Channel Estimation

We propose a subspace based blind channel estimator to enable the robust estimation of the CFO-included channel in (8). We assume that 1) noise samples are uncorrelated and 2) noise and signal samples are uncorrelated. By utilizing N_r receive antennas with $(N-L+1)N_r > N$, the proposed estimator is summarized in four steps below.

Step 1. $(N_s - 1)$ received symbols are used to compute the autocorrelation matrix of the received signal, obtaining

$$\mathbf{R}_y = \frac{1}{(N_s-1)(L_{cp}+1)} \sum_{i=1}^{N_s-1} \sum_{t=0}^{L_{cp}} \mathbf{y}_{i,t}\mathbf{y}_{i,t}^H \tag{9}$$

Note that the number of signal samples per received symbol utilized to compute the autocorrelation matrix of the received signal has been increased, thanks to the partition of the received signal vector $\mathbf{y}_i$ into a number of subvectors $\mathbf{y}_{i,t}$ when deriving the equivalent system model. Thus, the required number of received symbols to achieve the second order statistics of the received signal can be much smaller than the methods in OFDM [15], [18], [19].

Step 2. Eigenvalue decomposition (EVD) is performed on the autocorrelation matrix $\mathbf{R}_y$. The signal subspace has a dimension of N, regardless of the GFDM nonorthogonal filter. Consequently, the noise subspace has Q $(Q = (N-L+1)N_r - N)$ eigenvectors corresponding to the smallest Q eigenvalues of the matrix $\mathbf{R}_y$. Denote the q-th eigenvector as $\boldsymbol{\gamma}_q = [\boldsymbol{\gamma}_q^T(0), \cdots, \boldsymbol{\gamma}_q^T(N-L)]^T$ $(q = 0, \cdots, Q-1)$, where $\boldsymbol{\gamma}_q(m)$ is a column vector of size N_r. Due to the inherent orthogonality between the signal and noise subspaces, the columns of $\bar{\mathbf{H}}$ are orthogonal to each vector $\boldsymbol{\gamma}_q$, *i.e.*,

$$\boldsymbol{\gamma}_q^H \bar{\mathbf{H}} = \mathbf{0}_{1\times N} \tag{10}$$

Therefore, $\boldsymbol{\gamma}_q$ spans the left null space of $\bar{\mathbf{H}}$. As $\bar{\mathbf{H}}$ is formulated by the matrices $\bar{\mathbf{h}}(l)$, we can restrict the channel estimation to $\bar{\mathbf{h}}(l)$, instead of the whole matrix $\bar{\mathbf{H}}$.

Step 3. Three equations equivalent to (10) are derived:

$$\begin{aligned}
&\sum_{l=L-1-n}^{L-1} \boldsymbol{\gamma}_q^H(l-L+1+n)\bar{\mathbf{h}}(l) = 0,\\
&\text{for } n = 0, \cdots, L-2\\
&\sum_{l=0}^{L-1} \boldsymbol{\gamma}_q^H(n-L+1+l)\bar{\mathbf{h}}(l) = 0,\\
&\text{for } n = L-1, \cdots, N-L\\
&\sum_{l=0}^{N-1-n} \boldsymbol{\gamma}_q^H(n-L+1+l)\bar{\mathbf{h}}(l) = 0,\\
&\text{for } n = N-L+1, \cdots, N-1
\end{aligned} \tag{11}$$

They are formulated in the matrix form: $\boldsymbol{\Theta}_q\bar{\mathbf{h}} = \mathbf{0}_{N\times 1}$, and $\boldsymbol{\Theta}_q$ of size $N \times N_rL$ is given by

$$\boldsymbol{\Theta}_q = \begin{bmatrix}
\boldsymbol{\gamma}_q^H(a) & \mathbf{0}_{1\times N_r} & \cdots & \mathbf{0}_{1\times N_r}\\
\boldsymbol{\gamma}_q^H(a-1) & \boldsymbol{\gamma}_q^H(a) & \cdots & \mathbf{0}_{1\times N_r}\\
\vdots & \vdots & \ddots & \vdots\\
\boldsymbol{\gamma}_q^H(b+1) & \boldsymbol{\gamma}_q^H(b+2) & \cdots & \boldsymbol{\gamma}_q^H(a)\\
\boldsymbol{\gamma}_q^H(b) & \boldsymbol{\gamma}_q^H(b+1) & \cdots & \boldsymbol{\gamma}_q^H(a-1)\\
\vdots & \vdots & \ddots & \vdots\\
\boldsymbol{\gamma}_q^H(0) & \boldsymbol{\gamma}_q^H(1) & \cdots & \boldsymbol{\gamma}_q^H(L-1)\\
\mathbf{0}_{1\times N_r} & \boldsymbol{\gamma}_q^H(0) & \cdots & \boldsymbol{\gamma}_q^H(L-2)\\
\vdots & \vdots & \ddots & \vdots\\
\mathbf{0}_{1\times N_r} & \mathbf{0}_{1\times N_r} & \cdots & \boldsymbol{\gamma}_q(0)
\end{bmatrix} \tag{12}$$

where $a = N-L$, $b = N-2L$, and $\bar{\mathbf{h}} = [\bar{\mathbf{h}}^T(0), \cdots, \bar{\mathbf{h}}^T(L-1)]^T$ is with size $N_rL \times 1$.

Step 4. Considering all $\mathbf{\Theta}_q$ matrices as follows $\mathbf{\Theta} = [\mathbf{\Theta}_0^T, \cdots, \mathbf{\Theta}_{Q-1}^T]^T$, we can obtain

$$\mathbf{\Theta}\bar{\mathbf{h}} = \mathbf{0}_{NQ\times 1} \tag{13}$$

Hence, the CFO-included channel $\bar{\mathbf{h}}$ can be estimated by choosing the right singular vectors of $\mathbf{\Theta}$, denoted as $\tilde{\mathbf{h}}$. However, there exists a complex scalar ambiguity b between the real CFO-included channel $\bar{\mathbf{h}}$ and the blindly estimated CFO-included channel $\tilde{\mathbf{h}}$, *i.e.*, $\bar{\mathbf{h}} = \tilde{\mathbf{h}}b$.

It is noteworthy that the blind estimate of the CFO-included channel is robust against the ICI and ISI introduced by the nonorthogonal filter, as the proposed scheme is based on the inherent orthogonality between the signal and noise subspaces and their dimensions are not changed by the nonorthogonal filter. With the blind channel estimate, (8) can be rewritten as

$$\mathbf{y}_{i,t} = \tilde{\mathbf{H}}\mathbf{B}\mathbf{s}_{i,t} + \mathbf{w}_{i,t} \tag{14}$$

where $\tilde{\mathbf{H}}$ is defined as the same form to $\bar{\mathbf{H}}$ but with $\bar{\mathbf{h}}(l)$ replaced by $\tilde{\mathbf{h}}(l)$ and $\mathbf{B} = \text{diag}\{[b, \cdots, b]\}$ is with size $N \times N$. By performing equalization, the received signal $\mathbf{y}_{i,t}$ is multiplied with the pseudoinverse of $\tilde{\mathbf{H}}$, obtaining

$$\mathbf{r}_{i,t} = \mathbf{B}\mathbf{s}_{i,t} + \bar{\mathbf{w}}_{i,t} \tag{15}$$

where $\bar{\mathbf{w}}_{i,t} = \tilde{\mathbf{H}}^{\dagger}\mathbf{w}_{i,t}$. (15) is easily rewritten as

$$\mathbf{r}_{i,t} = \mathbf{E}_t(\phi)\mathbf{A}_t\mathbf{B}\mathbf{d}_i + \bar{\mathbf{w}}_{i,t} \tag{16}$$

where $\mathbf{A}_t = \mathbf{A}_{\text{all}}(t\colon N-1+t, :)$ with $\mathbf{A}_{\text{all}} = [\mathbf{A}(N-L_{\text{cp}}\colon N-1, :); \mathbf{A}]$ and $\mathbf{E}_t(\phi) = \text{diag}\{[e^{j2\pi\phi t/K}, \cdots, e^{j2\pi\phi(t+N-1)/K}]\}$. It is noteworthy that (16) looks like a GFDM system in the presence of flat-fading channel so that CFO estimation problem is much easier to solve, allowing the robust estimation of both fCFO and iCFO in the full range in the following.

B. First Symbol Design

Based on (16), the CFO estimation and channel ambiguity elimination can be achieved by designing a single sub-symbol of the first received symbol with a small number of nulls and pilots only. Assuming its first sub-symbol is utilized for training, $K_{\text{p,cfo}}$ and $K_{\text{p,cha}}$ ($K_{\text{p,cfo}} + K_{\text{p,cha}} \leq K$) subcarriers within it are transmitted as nulls and pilots to enable CFO estimation and channel ambiguity elimination, respectively. Note that in theory the values of $K_{\text{p,cfo}}$ and $K_{\text{p,cha}}$ can be as small as 1. Denote $\mathbb{K}_{\text{p,cfo}}$ and $\mathbb{K}_{\text{p,cha}}$ as the respective subcarrier index set for CFO estimation and channel ambiguity elimination. Fig. 2 provides a general example for the first symbol design with $K_{\text{p,cfo}} = 2$ and $K_{\text{p,cha}} = 2$.

C. CFO Estimation

After performing equalization with the blind channel estimate, both fCFO and iCFO in the full range can be easily estimated by utilizing the designed first symbol with a small number of nulls in a single sub-symbol only. By eliminating interference through zero-forcing (ZF) algorithm, both fCFO and iCFO estimations are robust against the ICI and ISI introduced by the GFDM nonorthogonal filter.

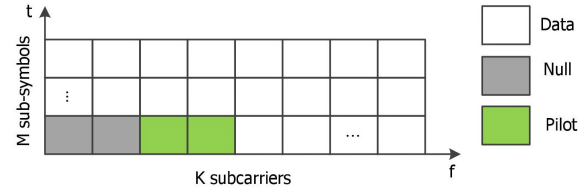

Fig. 2. Example of the first GFDM symbol structure: $K_{\text{p,cfo}} = 2$ nulls and $K_{\text{p,cha}} = 2$ pilots within the first sub-symbol are utilized for CFO estimation and channel ambiguity elimination, respectively.

1) fCFO Estimation: fCFO estimation is based on the rank-reduction criterion. Define $\mathbf{c}_t = \text{diag}\{\mathbf{A}_t^{\dagger}\mathbf{r}_{0,t}\}$. In the absence of fCFO ($\phi_{\text{f}} = 0$), the rank of $\mathbf{c}_t$ should be $(N - K_{\text{p,cfo}})$. This property is true even in the presence of iCFO, since iCFO is likely to induce the cyclic shift and would not change its rank. However, if fCFO exists ($\phi_{\text{f}} \neq 0$), this property is destroyed and the rank becomes N due to the fCFO-induced ICI.

Given a largest fCFO search range $[-0.5, 0.5)$ and a fCFO trial value $\tilde{\phi}_{\text{f}}$, we can obtain $\tilde{\mathbf{r}}_{\text{f},0,t}^{\tilde{\phi}_{\text{f}}} = \mathbf{E}_t(-\tilde{\phi}_{\text{f}})\mathbf{r}_{0,t}$. Therefore, with a good fCFO trial, the rank of $\mathbf{c}_{\text{f},t}^{\tilde{\phi}_{\text{f}}} = \text{diag}\{\mathbf{A}_t^{\dagger}\tilde{\mathbf{r}}_{\text{f},0,t}^{\tilde{\phi}_{\text{f}}}\}$ should be $(N - K_{\text{p,cfo}})$. Then, fCFO is determined following the steps below.

Step 1. Compute the autocorrelation matrix of $\mathbf{c}_{\text{f},t}^{\tilde{\phi}_{\text{f}}}$, obtaining $\mathbf{R}_{\text{c}}^{\tilde{\phi}_{\text{f}}} = \frac{1}{L_{\text{cp}}+1}\sum_{t=0}^{L_{\text{cp}}} \mathbf{c}_{\text{f},t}^{\tilde{\phi}_{\text{f}}}(\mathbf{c}_{\text{f},t}^{\tilde{\phi}_{\text{f}}})^H$;

Step 2. Perform EVD on $\mathbf{R}_{\text{c}}^{\tilde{\phi}_{\text{f}}}$, and the eigenvalue vector in an ascending order is determined as $\boldsymbol{\beta}^{\tilde{\phi}_{\text{f}}}$;

Step 3. fCFO is estimated by

$$\hat{\phi}_{\text{f}} = \arg \min_{\tilde{\phi}_{\text{f}} \in [-0.5, 0.5)} \parallel \boldsymbol{\beta}^{\tilde{\phi}_{\text{f}}}(1 : K_{\text{p,cfo}}) \parallel_{\text{F}}^2 \tag{17}$$

With fCFO compensation, we obtain $\hat{\mathbf{r}}_{\text{f},0,t} = \mathbf{E}_t(-\hat{\phi}_{\text{f}})\mathbf{r}_{0,t}$. Note that ZF algorithm has been performed on the received signal (*i.e.*, $\mathbf{A}_t^{\dagger}\mathbf{r}_{0,t}$), allowing the elimination of ICI and ISI from the nonorthogonal filter, assuming the filter matrix $\mathbf{A}$ is well-conditioned and its inverse exists [12]. Similarly, ZF is executed to the iCFO estimation and channel ambiguity elimination below to remove the ICI and ISI.

2) iCFO Estimation: The core idea of iCFO estimation is to minimize the signal power on the null subcarriers in the designed sub-symbol. Denote $\mathbf{e}_t = \mathbf{A}_t^{\dagger}\hat{\mathbf{r}}_{\text{f},0,t}$. Given that iCFO is absent ($\phi_{\text{i}} = 0$), the signal power of $\mathbf{e}_t$ on the specific subcarriers with nulls should be zero. However, this is not true if iCFO exists ($\phi_{\text{i}} \neq 0$).

Given a largest iCFO search range $[-K/2, K/2)$ and an iCFO trial value $\tilde{\phi}_{\text{i}}$, we can obtain an iCFO compensated $\hat{\mathbf{r}}_{\text{f},0,t}$ as $\tilde{\mathbf{r}}_{\text{i},0,t}^{\tilde{\phi}_{\text{i}}} = \mathbf{E}_t(-\tilde{\phi}_{\text{i}})\hat{\mathbf{r}}_{\text{f},0,t}$. After ZF algorithm execution, we obtain $\mathbf{e}_{\text{i},t}^{\tilde{\phi}_{\text{i}}} = \mathbf{A}_t^{\dagger}\tilde{\mathbf{r}}_{\text{i},0,t}^{\tilde{\phi}_{\text{i}}}$. Therefore, iCFO is determined by

$$\hat{\phi}_{\text{i}} = \arg \min_{\tilde{\phi}_{\text{i}} \in [-\frac{K}{2}, \frac{K}{2})} \sum\nolimits_{t=0}^{L_{\text{cp}}} \sum\nolimits_{k \in \mathbb{K}_{\text{p,cfo}}} \parallel \mathbf{e}_{\text{i},t}^{\tilde{\phi}_{\text{i}}}(k) \parallel_{\text{F}}^2 \tag{18}$$

With iCFO compensation, we obtain $\hat{\mathbf{r}}_{\text{i},0,t} = \mathbf{E}_t(-\hat{\phi}_{\text{i}})\hat{\mathbf{r}}_{\text{f},0,t}$. The integral CFO is estimated by $\hat{\phi} = \hat{\phi}_{\text{i}} + \hat{\phi}_{\text{f}}$.

D. Channel Ambiguity Elimination

By performing ZF algorithm, we obtain $\hat{\mathbf{d}}_{0,t} = \mathbf{A}_t^{\dagger}\hat{\mathbf{r}}_{\mathrm{i},0,t}$. With a small number of pilots in a single sub-symbol only, the complex channel ambiguity is computed by

$$\hat{b} = \frac{1}{K_{\mathrm{p,cha}}(L_{\mathrm{cp}}+1)} \sum_{t=0}^{L_{\mathrm{cp}}} \sum_{k\in\mathbb{K}_{\mathrm{p,cha}}} \frac{\hat{\mathbf{d}}_{0,t}(k)}{\mathbf{d}_0(k)} \tag{19}$$

The CFO-included channel is then estimated as $\hat{\mathbf{h}} = \hat{b}\tilde{\mathbf{h}}$. Hence, the channel ambiguity due to blind estimation has been eliminated.

According to (16), the transmitted signal is easily detected by ZF algorithm

$$\hat{\mathbf{d}}_{i,t} = (\mathbf{A}_t\hat{\mathbf{B}})^{\dagger}\mathbf{E}_t(-\hat{\phi})\mathbf{r}_{i,t} \tag{20}$$

where $\hat{\mathbf{B}} = \mathrm{diag}\{[\hat{b},\cdots,\hat{b}]\}$. Then, the averaged signal estimate is $\hat{\mathbf{d}}_i = \frac{1}{L_{\mathrm{cp}}+1}\sum_{t=0}^{L_{\mathrm{cp}}}\hat{\mathbf{d}}_{i,t}$.

It is worth noticing that ZF algorithm has been performed on the received signal in the above estimations, which is able to eliminate the ICI and ISI introduced by the nonorthogonal filter [12] for both CFO estimation and channel ambiguity elimination. Meanwhile, the blind channel estimate is regardless of the nonorthogonal filter and is not affected by its introduced ICI and ISI as discussed previously. Hence, the proposed scheme with semi-blind estimation of both channel and CFO are more robust against the ICI and ISI caused by the nonorthogonal filter than the existing methods [2], [7], [8].

IV. Complexity Analysis

In Table I, the computational complexity of the proposed RSCCE scheme is presented, in terms of the number of complex additions and multiplications. They are summarized into four aspects, namely blind channel estimation, fCFO estimation, iCFO estimation and channel ambiguity elimination. Regarding channel estimation, the proposed subspace based blind channel estimation dominates the whole complexity, resulting from the computations of the autocorrelation matrix of the received signal, EVD and etc. The complexity for channel ambiguity elimination can be negligible. Both fCFO and iCFO estimation might suffer from high complexity due to the exhaustive fCFO and iCFO searches. For instance, the complexity of fCFO estimation is inversely proportional to the fCFO search step size Δ. Moreover, iCFO estimation is likely to have a lower complexity than fCFO estimation, since iCFO has a search step size as large as 1.

TABLE I
Analytical Computational Complexity (est.: estimation, eli.: elimination)

item	proposed RSCCE scheme
blind channel est.	$2(N-L+1)^2N_{\mathrm{r}}^2N_{\mathrm{s}}(L_{\mathrm{cp}}+1) - N_{\mathrm{r}}^2L^2N^2$ $+N_{\mathrm{r}}^3L^2N(N-L+1) + (N-L+1)^3N_{\mathrm{r}}^3$
fCFO est.	$(4N^2(L_{\mathrm{cp}}+1) + N^3 + 2K_{\mathrm{p,cfo}})(\frac{1}{\Delta}+1)$
iCFO est.	$2(N^2+K_{\mathrm{p,cfo}})(L_{\mathrm{cp}}+1)(K+1)$
channel ambiguity eli.	$K_{\mathrm{p,cha}}(L_{\mathrm{cp}}+1)$

V. Simulation Results

Simulation results are used to demonstrate the performance of the proposed RSCCE scheme for GFDM systems. System parameters are set as follows: each GFDM symbol contains $M = 3$ sub-symbols each with $K = 16$ subcarriers; the number of receive antennas is $N_{\mathrm{r}} = 4$; the CP length is $L_{\mathrm{cp}} = 4$; the channel model follows an exponential profile with channel length $L = 3$; $N_{\mathrm{s}} = 30$ received symbols are used for semi-blind estimation, except for Fig. 6; the CFO value is randomly generated in $[-K/2, K/2)$; the first sub-symbol of the first GFDM symbol is used for training where $K_{\mathrm{p,cfo}} = 12$ and $K_{\mathrm{p,cha}} = 4$ are exploited for CFO estimation and channel ambiguity elimination respectively except for Fig. 7; the fCFO search step size is $\Delta = 0.001$; root raised cosine prototype filter with roll-off coefficient $\alpha = 0.5$ is utilized for GFDM except for Fig. 5; quadrature phase shift keying (QPSK) scheme is used. The mean square errors (MSEs) of channel and fCFO estimation are defined as $\mathrm{MSE}_{\mathrm{channel}} = \mathbb{E}\{\frac{1}{N_{\mathrm{r}}L} \parallel \hat{\mathbf{h}} - \bar{\mathbf{h}} \parallel_{\mathrm{F}}^2\}$ and $\mathrm{MSE}_{\mathrm{fCFO}} = \mathbb{E}\{(\hat{\phi}_{\mathrm{f}} - \phi_{\mathrm{f}})^2\}$, respectively.

Fig. 3 illustrates the BER performance of the proposed RSCCE scheme in comparison to the methods in [2], [7], [8] and [13]. Due to the lack of iCFO estimation approach for general GFDM systems in the literature, for fair comparison, the iCFO estimation approach in the proposed RSCCE scheme is performed with the existing methods for general GFDM systems including fCFO estimation in [7] and [8], channel estimation in [2] and ZF based signal detection in [13]. They are arranged into three groups, namely g1: CP [8]+LS [2]+ZF [13], g2: PN [7], [8]+LS [2]+ZF [13] and g3: ZF [13] with perfect CFO and channel estimations. Note that g1 and g2 have 1 and 3 sub-symbols in total for training, while the proposed RSCCE scheme needs a single sub-symbol only. The proposed RSCCE scheme has a much superior BER performance to the existing methods g1 and g2, and its BER also approaches to the ideal case with perfect CFO and channel estimations at the receiver as SNR increases. Due to the biased fCFO and channel estimation resulting from ICI and ISI as will be seen in Figs. 4 and 5, the existing methods g1 and g2 suffer from error floors as well in BER.

Fig. 4 shows the MSE performance of fCFO estimation by the proposed scheme, in comparison to the methods in [7] and [8]. The existing CP [8] and PN [7], [8] based algorithms are selected. The fCFO estimation range of CP based algorithm is $[-0.5/M, 0.5/M)$. When $M = 3$, its possible fCFO estimation range is $[-0.17, 0.17)$. Thus, its performance is the worst and an error floor is formed for GFDM systems with $M = 3$ and fCFO within $[-0.5, 0.5)$ in Fig. 4. Instead, the CP based algorithm has a good performance for OFDM systems and however also biased at high SNRs due to the multipath channel. Regarding the PN based method, the first two identical sub-symbols are utilized as pilots to determine the fCFO by exploring their phase shift relationship. Nevertheless, owing to the ICI and ISI from the nonorthogonal filter, its performance with about two-fold training overhead over

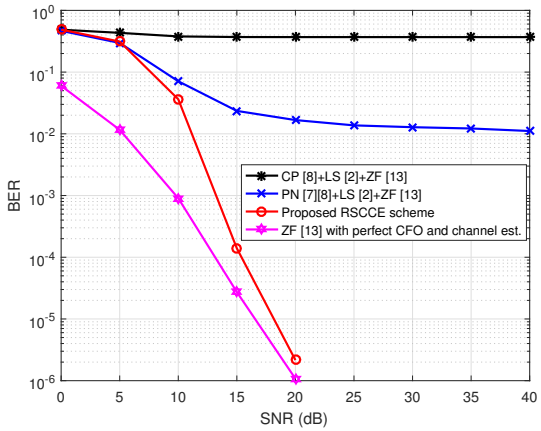

Fig. 3. BER performance of the proposed RSCCE scheme and the existing methods [2], [7], [8], [13] (est.: estimation).

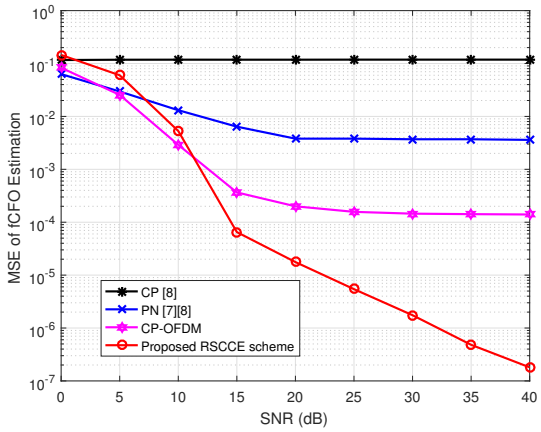

Fig. 4. MSE performance of fCFO estimation of the proposed RSCCE scheme and the existing CP and PN based methods [7], [8].

the proposed scheme is poor as well. The proposed RSCCE scheme outperforms the existing methods [7], [8] significantly especially from the medium to high SNRs and do not suffer from any error floors induced by the ICI and ISI.

Fig. 5 investigates the impact of the roll-off coefficient of the nonorthogonal filter α on the MSE performances of fCFO and channel estimation of the proposed scheme and the existing methods [2], [7], [8] at SNR=30 dB. To avoid the impact from CFO estimation when studying channel estimation, it is assumed that CFO has been compensated perfectly. A single sub-symbol of the first GFDM block is considered as pilots for channel estimation in LS based method [2], while the proposed scheme only utilizes a quarter of it for channel estimation. By utilizing the subspace method in the initial blind channel estimation and considering interference mitigation in the following CFO estimation and channel ambiguity elimination, the proposed scheme is shown to be more robust against the roll-off coefficient than the existing methods [2], [7], [8] in terms of both fCFO and channel estimation. The

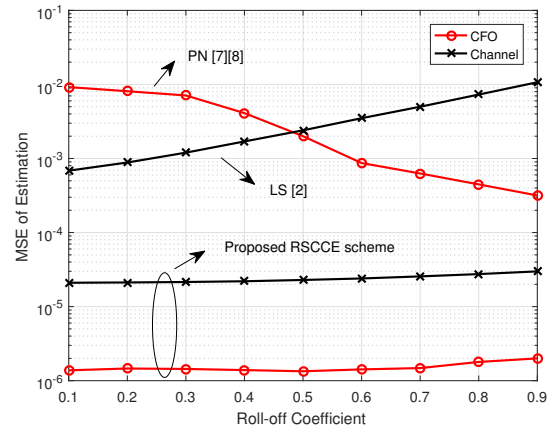

Fig. 5. MSE performance of fCFO and channel estimation of the proposed RSCCE scheme and the existing PN and LS based methods [2], [7], [8] versus the roll-off coefficient α at SNR=30 dB.

existing methods [2], [7], [8] suffer from ICI and ISI and their performances are susceptible to the nonorthogonal filter matrix $\mathbf{A}$ which changes with α. The existing fCFO estimator [7], [8] has a decreasing MSE as α increases, because a larger α could reduce the power difference between the received two identical sub-symbols. In contrast, the LS channel estimator [2] tends to perform worse at a larger α, owing to the noise enhancement. For a trade-off between channel and fCFO estimation for the existing methods [2], [7], [8], α is specified as 0.5 in Figs. 3, 4, 6 and 7.

Fig. 6 shows the MSE of channel performance of the proposed RSCCE scheme and the existing method in OFDM [15] versus the number of received symbols N_{s} at SNR=15 dB and SNR=20 dB, respectively. To the best of our knowledge, there is no semi-blind channel estimation method for GFDM systems in the literature. The proposed scheme is compared with the existing semi-blind channel estimation method for OFDM systems in [15], referred to as SB-OFDM. It is observed that $N_{\mathrm{s}} = 30$ symbols are sufficient to output a good performance for the proposed scheme, thanks to the increased number of signal samples per received symbol to compute the autocorrelation matrix of the received signal as discussed in Section III-A. In contrast, more than 200 symbols are required for the SB-OFDM method in OFDM systems [15].

Fig. 7 demonstrates the probability of correct iCFO estimation of the proposed RSCCE scheme and the existing method in [14]. Since full-range iCFO estimation technique for general GFDM systems is lacking in the literature, the existing pilot aided iCFO estimation method for OFDM systems [14] is chosen. The proposed scheme with $K_{\mathrm{p,cfo}} = 14$ is shown to have a higher iCFO detection probability than a symbol aided iCFO estimator [14] in OFDM systems which do not suffer from ICI and ISI introduced in GFDM systems. For example, at SNR=8 dB, the iCFO detection probability of the existing method [14] is enhanced approximately 17% by the proposed RSCCE scheme.

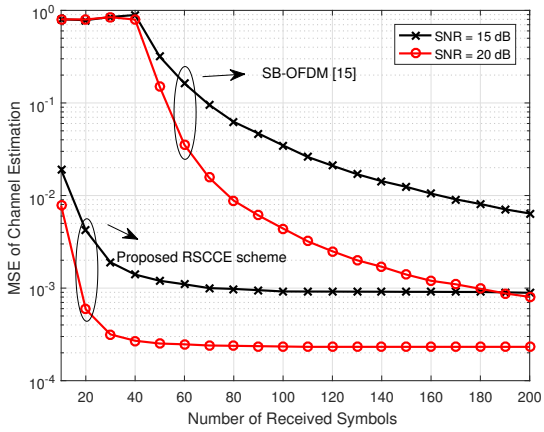

Fig. 6. MSE performance of channel estimation of the proposed RSCCE scheme and the existing SB-OFDM method [15] versus the number of received symbols N_s at SNR=15 dB and SNR=20 dB.

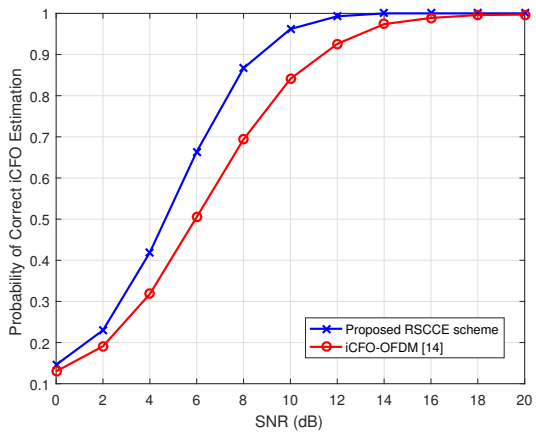

Fig. 7. Probability of correct iCFO estimation of the proposed RSCCE scheme and the pilot aided approach for OFDM [14].

VI. Conclusion

A semi-blind integral solution to channel and CFO estimation has been proposed for a wide range of GFDM systems, which enables high-accuracy channel estimation based on the second order statistics of a small number of received symbols and high-accuracy estimation of iCFO and fCFO in the full range, with a small number of subcarriers within a single sub-symbol only for training. With much higher spectral efficiency, the proposed semi-blind scheme not only outperforms the existing methods [2], [7], [8], [13] in terms of both CFO and channel estimation performance, but also achieves a BER performance close to the ideal case with perfect CFO and channel estimations at the receiver especially at medium and high SNRs. It is shown to be more robust against ICI and ISI resulted from the nonorthogonal filter than the previous methods [2], [7], [8]. The proposed RSCCE scheme is almost independent of the roll-off coefficients of the nonorthogonal filter. Also, a small number of received symbols are sufficient to achieve the second order statistics of the received signal, about tens of times less that that required in [15].

References

[1] U. Vilaipornsawai and M. Jia, "Scattered-pilot channel estimation for GFDM," in *Proc. IEEE WCNC 2014*, Istanbul, Turkey, Apr. 2014, pp. 1053–1058.

[2] S. Ehsanfar, M. Matthe, D. Zhang, and G. Fettweis, "Theoretical analysis and CRLB evaluation for pilot-aided channel estimation in GFDM," in *Proc. IEEE GLOBECOM 2016*, Washington, DC, USA, Dec. 2016, pp. 1–7.

[3] ——, "Interference-free pilots insertion for MIMO-GFDM channel estimation," in *Proc. IEEE WCNC 2017*, San Francisco, CA, USA, Mar. 2017, pp. 1–6.

[4] J. Jeong, Y. Park, S. Weon, J. Kim, S. Choi, and D. Hong, "Eigendecomposition-based GFDM for interference-free data transmission and pilot insertion for channel estimation," *IEEE Trans. Wireless Commun.*, vol. 17, no. 10, pp. 6931–6943, Oct. 2018.

[5] Z. Na, Z. Pan, M. Xiong, J. Xia, and W. Lu, "Soft decision control iterative channel estimation for the Internet of things in 5G networks," *IEEE IoT Journal*, pp. 1–1, 2018, (early access).

[6] B. Lim and Y. Ko, "SIR analysis of OFDM and GFDM waveforms with timing offset, CFO, and phase noise," *IEEE Trans. Wireless Commun.*, vol. 16, no. 10, pp. 6979–6990, Oct. 2017.

[7] I. S. Gaspar, L. L. Mendes, N. Michailow, and G. Fettweis, "A synchronization technique for generalized frequency division multiplexing," *EURASIP J. Adv. Signal Process.*, vol. 2014, no. 1, p. 67, May 2014.

[8] Z. Na, M. Zhang, M. Xiong, J. Xia, X. Liu, and W. Lu, "Pseudo-Noise sequence based synchronization for generalized frequency division multiplexing in 5G communication system," *IEEE Access*, vol. 6, pp. 14 812–14 819, Mar. 2018.

[9] P. Wang and D. W. Lin, "Maximum-likelihood blind synchronization for GFDM systems," *IEEE Signal Process. Lett.*, vol. 23, no. 6, pp. 790–794, Jun. 2016.

[10] T. Kadur, I. Gaspar, N. Michailow, and G. Fettweis, "Non-data aided frequency synchronization exploiting ICI in non-orthogonal systems," in *Proc. IEEE VTC 2014-Fall*, Vancouver, BC, Canada, Sep. 2014, pp. 1–5.

[11] D. Zhang, A. Festag, and G. P. Fettweis, "Performance of generalized frequency division multiplexing based physical layer in vehicular communications," *IEEE Trans. Veh. Technol.*, vol. 66, no. 11, pp. 9809–9824, Nov. 2017.

[12] N. Michailow, M. Matth, I. S. Gaspar, A. N. Caldevilla, L. L. Mendes, A. Festag, and G. Fettweis, "Generalized frequency division multiplexing for 5th generation cellular networks," *IEEE Trans. Commun.*, vol. 62, no. 9, pp. 3045–3061, Sep. 2014.

[13] A. Farhang, N. Marchetti, and L. E. Doyle, "Low-complexity modem design for GFDM," *IEEE Trans. Signal Process.*, vol. 64, no. 6, pp. 1507–1518, Mar. 2016.

[14] Y. Jiang, H. Minn, X. Gao, X. You, and Y. Li, "Frequency offset estimation and training sequence design for MIMO OFDM," *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, pp. 1244–1254, Apr. 2008.

[15] Y. Zeng, A. R. Leyman, and T. Ng, "Joint semiblind frequency offset and channel estimation for multiuser MIMO-OFDM uplink," *IEEE Trans. Commun.*, vol. 55, no. 12, pp. 2270–2278, Dec. 2007.

[16] Y. Liu, X. Zhu, E. G. Lim, Y. Jiang, and Y. Huang, "High-accuracy joint multi-CFO and multi-TOA estimation for multiuser SIMO OFDM systems," in *Proc. IEEE ICC 2018*, Kansas City, MO, USA, May 2018, pp. 1–6.

[17] Y. Liu, X. Zhu, E. G. Lim, Y. Jiang, Y. Huang, W. Xu, and H. Lin, "High-robustness and low-complexity joint estimation of TOAs and CFOs for multiuser SIMO OFDM systems," *IEEE Trans. Veh. Technol.*, vol. 67, no. 8, pp. 7739–7743, Aug. 2018.

[18] Y. Liu, X. Zhu, E. G. Lim, Y. Jiang, and Y. Huang, "Iterative semi-blind CFO estimation, SI cancelation and signal detection for full-duplex systems," in *Proc. IEEE Globecom 2018*, Abu Dhabi, UAE, Dec. 2018, pp. 1–7.

[19] F. Gao, Y. Zeng, A. Nallanathan, and T. S. Ng, "Robust subspace blind channel estimation for cyclic prefixed MIMO OFDM systems: algorithm, identifiability and performance analysis," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 2, pp. 378–388, Feb. 2008.