Wind Speed Field Simulation via Stochastic Harmonic Function Representation based on Wavenumber-Frequency Spectrum

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ABSTRACT

Simulation of fluctuating wind speed field is of paramount significance in the design of large flexible structures. To circumvent the difficulty due to the decomposition of cross power spectral density (PSD) matrix and the interpolation between discretized spatial points, a wavenumber-frequency joint spectrum based spectral representation method (SRM) has been developed recently. To further improve the efficiency and accuracy, the stochastic harmonic function (SHF) representation is extended in the present paper for the simulation of stationary and nonstationary fluctuating wind fields in two spatial dimensions. In contrast to the SRM, besides the phase angles, the frequencies and wavenumbers are also random variables over partitioned wavenumber-frequency subdomains. Further, a strategy of dependent random frequencies and wavenumbers based on the SHF is proposed so that the number of random variables can be considerably reduced by 3/7. A new acceptance-rejection criterion, which avoids the artificial intervene, is suggested based on the *p*-power joint spectrum, and the subdomains are correspondingly determined by the Voronoi cell partitioning. For illustrative purposes, two numerical examples for the simulation of stationary and nonstationary fluctuating wind speed fields in two spatial dimensions are addressed, demonstrating the effectiveness of the proposed method in considerably reducing the random variables as well as the computational efforts.

Key words: random wind field; wavenumber-frequency joint spectrum; stochastic harmonic function; dependent random frequency-wavenumber points; stationary and nonstationary

INTRODUCTION

Simulation of random fluctuating wind speed field has received a long-term attention due to its significant impact on the safety design of long-span and high-flexible structures, such as large bridges, tall buildings and wind turbines, etc. (Kareem 2008; Li et al. 2017). The spectral representation method (SRM) has been investigated and widely employed for more than four decades in the simulation of wind fields due to its high accuracy and simple algorithm (Shinozuka & Jan 1972; Di Paola 1998; Chen & Kareem 2005; Zeng et al. 2017). In the conventional methods, the space is firstly discretized into a series of spatial points, then the wind speeds at these points are regarded as correlated random vector processes. Correspondingly, the cross power spectral density (PSD) matrix is introduced to describe the statistical characteristics of the random vector process. In the simulation of wind fields, decompositions of the cross PSD matrix are needed at each discretized frequency, which is computationally inefficient if the number of discretized spatial points is large (Tao et al. 2018), or even numerically ill-posed (Benowitz & Deodatis 2015). Besides, to obtain wind speeds at other arbitrary spatial points, interpolations between the discretized spatial points are needed. This will induce additional errors (Tao et al. 2017).

Since the fluctuating wind speed varies with time and space simultaneously, it is essentially a continuous temporal-spatial multi-dimensional random field. In fact, in early 1970's, Shinozuka (1971) regarded the wind speed field in one-spatial dimension as a two-dimensional (2D) random process, and derived its wavenumber-frequency joint spectrum. Unfortunately, to this method almost no attention has been paid for decades until Benowitz & Deodatis (2015) simulated the homogeneous wind speed field in one-spatial dimension along this line. In this method, the decomposition of the cross-PSD matrix and the interpolations involved in the conventional SRM are not needed. Besides, the fast Fourier transform (FFT) technique can be adopted to considerably improve the efficiency. This method was then quickly extended to nonhomogeneous and nonstationary cases in one spatial dimension (Peng et al. 2017) and homogeneous and nonhomogeneous cases in two spatial dimensions (Chen et al. 2018b; Song et al. 2018).

Despite the above advances in the SRM based on joint wavenumber-frequency spectrum, the number of the involved harmonic components is extremely large, leading to a large amount of random phase angles simultaneously (Deodatis 1996). It is usually cumbersome to handle a large number of random variables in a stochastic system. Consequently, to reduce the number of random variables while maintaining the accuracy is a critical task in the analysis of stochastic systems (Spanos et al. 2007; Li et al. 2012; Liu et al.

2018). A stochastic harmonic function (SHF) representation for one-dimensional (1D) stationary random process was proposed by Chen et al. (2013), and has been extended to 1D non-stationary random processes (Chen et al. 2017) and 2D homogenous random fields (Chen et al. 2018a). In this method, both the phase angles and discretized frequencies are regarded as random variables. It was proved that the SHF representation can reproduce the target PSD exactly no matter how many harmonic components are retained.

In this paper, the SHF representation will be extended to three-dimensional (3D) random fields, and then integrated with the wavenumber-frequency joint spectra to simulate fluctuating wind fields in two spatial dimensions. The remaining sections in this paper are organized as follows. The wavenumber-frequency joint spectra for fluctuating wind fields and its expression with the SRM is firstly revisited briefly. Then, the unified form of the SHF representation for 3D homogeneous and nonhomogeneous random fields is derived, and the strategy of dependent random frequencies and wavenumbers is proposed. Further, the implementation procedures of the SHF representation are elaborated. To demonstrate the effectiveness of the proposed method, two numerical examples for simulation of stationary and nonstationary fluctuating wind speed fields are addressed. Concluding remarks pertaining to the entire study are provided.

WAVENUMBER-FREQUENCY JOINT SPECTRA FOR WIND FIELDS AND ITS SRM EXPRESSION

For clarity, the spatial-temporal coordinate system is denoted as (x, y, z, t), in which the x, y, z axes indicate the longitudinal, lateral and vertical spatial direction, respectively, and t is the time. The longitudinal component of the fluctuating wind speed, denoted by $u(x, y, z, t)$, is essentially a four-dimensional spatial-temporal random field. In fact, because of Taylor's frozen hypothesis, only a 3D random field $u(y, z, t)$ needs to be considered (Simiu & Scanlan 1996), e.g., in the analysis of rotating blades of a wind turbine (Chen et al. 2018b). For convenience, $u(y, z, t)$ is called the wind speed field in two spatial dimensions and is the focus of this paper. To describe the characteristics of the 3D random field, the wavenumber-frequency joint spectra were developed recently and are briefly outlined below.

The joint spectrum for the homogeneous fluctuating wind speed field in two spatial dimensions was given by Chen et al. (2018b)

$$S^{(\text{W-F})}(k_z,k_y,\omega)=S^{\text{Dav}}(\omega)\cdot\rho^{(\text{W-F})}(k_z,k_y,\omega) \tag{1}$$

where $S^{(\text{W-F})}(k_z,k_y,\omega)$ denotes the joint spectrum, $S^{\text{Dav}}(\omega)$ denotes the Davenport spectrum, ω is the circular frequency, k_z,k_y are the wavenumbers in z,y direction, respectively; $\rho^{(\text{W-F})}(k_z,k_y,\omega)$ is the two-fold Fourier transform of the Davenport's coherence function $\rho(\xi_z,\xi_y,\omega)$ with respect to ξ_z,ξ_y, i.e.,

$$\begin{aligned}\rho^{(\text{W-F})}(k_z,k_y,\omega)&=\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\rho(\xi_z,\xi_y,\omega)e^{-\mathrm{i}(\xi_z k_z+\xi_y k_y)}\mathrm{d}\xi_z\mathrm{d}\xi_y\\&=\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\exp\left(-\tfrac{1}{2\pi U_{10}}|\omega|\sqrt{C_z^2{\xi_z}^2+C_y^2{\xi_y}^2}\right)e^{-\mathrm{i}(\xi_z k_z+\xi_y k_y)}\mathrm{d}\xi_z\mathrm{d}\xi_y\\&=\frac{1}{2\pi C_z C_y}\frac{1}{\left(\tfrac{1}{2\pi U_{10}}|\omega|\right)^2}\frac{1}{\left(1+\left[\left(\tfrac{1}{C_y}k_y\right)^2+\left(\tfrac{1}{C_z}k_z\right)^2\right]\Big/\left(\tfrac{1}{2\pi U_{10}}|\omega|\right)^2\right)^{\frac{3}{2}}}\end{aligned} \tag{2}$$

in which ξ_z,ξ_y are the spatial coordinate differences, i.e., $\xi_z=z_1-z_2,\xi_y=y_1-y_2$, C_z,C_y are the exponential decay coefficients in z,y direction, respectively, U_{10} is the mean wind speed at 10m high, and i denotes the imaginary unit.

It was soon extended to nonhomogeneous case by introducing the concept of evolutionary spectrum (Song et al. 2018). In this case, the joint spectrum depends on the height

$$S^{(\text{W-F})}(z,k_z,k_y,\omega)=S^{\text{Kai}}(z,\omega)\cdot\rho^{(\text{W-F})}(k_z,k_y,\omega) \tag{3}$$

where $S^{\text{Kai}}(z,\omega)$ denotes the two-sided Kaimal spectrum (Kaimal et al. 1972)

$$S^{\text{Kai}}(z,\omega)=\frac{50zu_*^2}{\pi U(z)\left(1+\frac{50z}{2\pi U(z)}|\omega|\right)^{5/3}} \tag{4}$$

in which $U(z)$ is the mean wind speed at the height z, and u_* is the shear velocity.

For clarity, the joint spectra of wind speed fields in two spatial dimensions, i.e. Eqs.(1) and (3), can be written in a unified form as

$$S(z,k_z,k_y,\omega)=S_0(z,\omega)\cdot\rho^{(\text{W-F})}(k_z,k_y,\omega)$$
$$=S_0(z,\omega)\cdot\frac{1}{2\pi C_z C_y}\frac{1}{\left(\frac{1}{2\pi U_{10}}|\omega|\right)^2}\frac{1}{\left(1+\left[\left(\frac{1}{C_y}k_y\right)^2+\left(\frac{1}{C_z}k_z\right)^2\right]\Big/\left(\frac{1}{2\pi U_{10}}|\omega|\right)^2\right)^{\frac{3}{2}}} \quad (5)$$

It is noted that, besides the Davenport spectrum and the Kaimal spectrum, the auto-PSD function $S_0(z,\omega)$ can take any other wind spectra, e.g., the von Karman spectrum (Benowitz & Deodatis 2015). In addition, the coherence function $\rho_{u_1u_2}(\omega)$ can take other models such as the Krenk model (Benowitz & Deodatis 2015) and the IEC 61400-1 model (Peng et al. 2017).

Since the spectrum for the 3D random field is obtained, the spectral representation method (SRM) can be directly utilized to generate wind speed field samples (Shinozuka & Deodatis 1996). To reduce the computational efforts, the acceptance-rejection non-uniform discretization method is suggested (Song et al. 2018) and the random wind field is correspondingly expressed as

$$\begin{aligned} u(z,y,t)=\sum_{j=1}^{N}&\sqrt{4S(z,k_j^{(z)},k_j^{(y)},\omega_j)V_j} \\ &\times[\cos(k_j^{(z)}z+k_j^{(y)}y+\omega_j t+\varphi_j^{(1)})+\cos(k_j^{(z)}z+k_j^{(y)}y-\omega_j t+\varphi_j^{(2)}) \\ &+\cos(k_j^{(z)}z-k_j^{(y)}y+\omega_j t+\varphi_j^{(3)})+\cos(k_j^{(z)}z-k_j^{(y)}y-\omega_j t+\varphi_j^{(4)})] \end{aligned} \quad (6)$$

where N is the number of discretized wavenumber-frequency points in the 3D wavenumber-frequency domain, and $(k_j^{(z)},k_j^{(y)},\omega_j)$ is the *j*-th discretized point; V_j is the representative volume of the point $(k_j^{(z)},k_j^{(y)},\omega_j)$, which can be determined by the Voronoi cells through the schemes similar to the calculation of assigned probabilities of Li and Chen (2009). $\varphi_j^{(1)}$, $\varphi_j^{(2)}$, $\varphi_j^{(3)}$ and $\varphi_j^{(4)}$ are four different sets of independent random phases uniformly distributed in [0, 2π].

In this way, approximate 1.5×10^5 discretized wavenumber-frequency points are needed to obtain a satisfactory simulation result. Correspondingly, the number of random phases is as large as 6×10^5. Though much smaller compared to the direct SRM, the number of random variables is still too large. In the present paper, the stochastic harmonic function representation is adopted and extended, and the computational efforts as well as the number of random variables can be further considerably reduced.

STOCHASTIC HARMONIC FUNCTION REPRESENTATION FOR WIND SPEED FIELDS

A stochastic harmonic function (SHF) representation was proposed by Chen et al. (2013) for 1D stationary random process. It has been extended to 1D nonstationary random processes (Chen et al. 2017) and 2D homogeneous random fields (Chen et al. 2018a). In the SHF representation of the previous studies, the frequencies and wavenumbers are mutually independent random variables. In this section, the SHF representation is extended to the 3D random field case since $u(z,y,t)$ is a 3D temporal-spatial random field, then, a new strategy of the dependent frequencies and wavenumbers is proposed to further reduce the number of random variables. To make it clear, the basic idea of the SHF representation for 1D random process is briefly revisited firstly.

The SHF representation for 1D random process

In the SHF representation, both the phase angles and discretized frequencies are taken as random variables, distinguishing it from the SRM, in which only the phase angles are random variables. According to Chen et al. (2013; 2017), the SHF representation for 1D (non-)stationary random process can be expressed in a unified form as

$$Y_N^{\mathrm{SHF}}(t)=\sum_{j=1}^{N}A(\Omega_j,t)\cos(\Omega_j t+\varphi_j) \tag{7}$$

where $Y_N^{\mathrm{SHF}}(t)$ denotes the 1D stationary or nonstationary random process, N is the number of the harmonic components, Ω_j 's are independent random frequencies with the probability density functions (PDFs) $p_{\Omega_j}(\omega)$ valued on the partitioned subintervals (distribution domain) $[\omega_j^{\mathrm{L}},\omega_j^{\mathrm{U}})_{j=1}^{N}$. The subintervals $[\omega_j^{\mathrm{L}},\omega_j^{\mathrm{U}})$ are non-overlapping such that $[\omega_j^{\mathrm{L}},\omega_j^{\mathrm{U}})\bigcap[\omega_k^{\mathrm{L}},\omega_k^{\mathrm{U}})=\varnothing, \forall j\neq k$ and $\bigcup_{j=1}^{N}[\omega_j^{\mathrm{L}},\omega_j^{\mathrm{U}})=[\omega^{\mathrm{L}},\omega^{\mathrm{U}})$, where $\omega^{\mathrm{L}},\omega^{\mathrm{U}}$ are the lower and upper cut-off frequencies, respectively. φ_j 's are identically independent random phase angles uniformly distributed over [0, 2π]. When Ω_j 's are uniformly distributed over $[\omega_j^{\mathrm{L}},\omega_j^{\mathrm{U}})_{j=1}^{N}$, the amplitude $A(\Omega_j,t)$ is derived as $A(\Omega_j,t)=\sqrt{4S(\Omega_j,t)(\omega_j^{\mathrm{U}}-\omega_j^{\mathrm{L}})}$, in which $S(\cdot)$ is the PSD function of the random process. It was proved that the SHF representation could reproduce the target power spectral density functions even the number of harmonic components are finite and small.

The SHF Representation for 3D Random Field

For simplicity of writing, define the following operational rules for two 3D vectors $\boldsymbol{a}=(a_1,a_2,a_3)$ and $\boldsymbol{b}=(b_1,b_2,b_3)$,

$$\begin{cases} \boldsymbol{a}\cdot^{++}\boldsymbol{b}=a_1b_1+a_2b_2+a_3b_3 \\ \boldsymbol{a}\cdot^{+-}\boldsymbol{b}=a_1b_1+a_2b_2-a_3b_3 \\ \boldsymbol{a}\cdot^{-+}\boldsymbol{b}=a_1b_1-a_2b_2+a_3b_3 \\ \boldsymbol{a}\cdot^{--}\boldsymbol{b}=a_1b_1-a_2b_2-a_3b_3 \end{cases} \tag{8}$$

Similar to the previous studies (Chen et al. 2013; 2017; 2018a), the SHF representation for the 3D random field $u(z,y,t)$ can be expressed as

$$\begin{aligned} u_N^{\mathrm{SHF}}(\boldsymbol{x})=\sum_{j=1}^{N}A(z,\boldsymbol{K}_j)\times[&\cos(\boldsymbol{K}_j\cdot^{++}\boldsymbol{x}+\varphi_j^{(1)})+\cos(\boldsymbol{K}_j\cdot^{+-}\boldsymbol{x}+\varphi_j^{(2)}) \\ &+\cos(\boldsymbol{K}_j\cdot^{-+}\boldsymbol{x}+\varphi_j^{(3)})+\cos(\boldsymbol{K}_j\cdot^{--}\boldsymbol{x}+\varphi_j^{(4)})] \end{aligned} \tag{9}$$

where $\boldsymbol{x}=(z,y,t)$, $u_N^{\mathrm{SHF}}(\boldsymbol{x})$ denotes the spatially homogeneous or nonhomogeneous random wind field; $\boldsymbol{K}_j=(K_j^z,K_j^y,\Omega_j)$ ($j=1,2,\cdots N$) are independent 3D random vectors with the probability density functions (PDFs) $p_{\boldsymbol{K}_j}(\boldsymbol{k}_j)$ valued on the partitioned subdomains (distribution domain) D_j $(j=1,2,...,N)$; The subdomains D_j $(j=1,2,...,N)$ are non-overlapping such that $D_0=\bigcup_{j=1}^{N}D_j$ and $D_j\bigcap D_m=\varnothing, \forall j\neq m$, where $D_0=[k_z^{\mathrm{L}},k_z^{\mathrm{U}}]\times[k_y^{\mathrm{L}},k_y^{\mathrm{U}}]\times[\omega^{\mathrm{L}},\omega^{\mathrm{U}}]$ is the 3D wavenumber-frequency domain of interest; $k_z^{\mathrm{L}},k_z^{\mathrm{U}}$ are the lower and upper cut-off wavenumbers of k_z, respectively, similar symbols used for wavenumber k_y and frequency ω. The amplitudes $A(z,\boldsymbol{K}_j)$'s are the functions of random wavenumber-frequency points and height for spatially nonhomogeneous cases, while they are not dependent on the height for homogeneous cases.

Based on Eq.(9) and noting that $\boldsymbol{K}_j$'s and φ_j's are independent, one can easily derive the correlation function of $u_N^{\mathrm{SHF}}(\boldsymbol{x})$

$$
\begin{aligned}
R_{u_N^{\mathrm{SHF}}}(z,\boldsymbol{\xi}) &= E[u_N^{\mathrm{SHF}}(\boldsymbol{x})u_N^{\mathrm{SHF}}(\boldsymbol{x}+\boldsymbol{\xi})] \\
&= \sum_{j=1}^{N} E\Big\{A(z,\boldsymbol{K}_j)\,A(z+\xi_z,\boldsymbol{K}_j)\cdot\frac{1}{2}[\cos(\boldsymbol{K}_j\cdot^{++}\boldsymbol{\xi})+\cos(\boldsymbol{K}_j\cdot^{+-}\boldsymbol{\xi}) \\
&\qquad +\cos(\boldsymbol{K}_j\cdot^{-+}\boldsymbol{\xi})+\cos(\boldsymbol{K}_j\cdot^{--}\boldsymbol{\xi})]\Big\} \\
&= \frac{1}{2}\sum_{j=1}^{N}\int_{D_j} A(z,\boldsymbol{k}_j)\,A(z+\xi_z,\boldsymbol{k}_j)[\cos(\boldsymbol{k}_j\cdot^{++}\boldsymbol{\xi})+\cos(\boldsymbol{k}_j\cdot^{+-}\boldsymbol{\xi}) \\
&\qquad +\cos(\boldsymbol{k}_j\cdot^{-+}\boldsymbol{\xi})+\cos(\boldsymbol{k}_j\cdot^{--}\boldsymbol{\xi})]\}\,p_{\boldsymbol{K}_j}(\boldsymbol{k}_j)\mathrm{d}\boldsymbol{k}_j
\end{aligned}
\tag{10}
$$

where $\boldsymbol{\xi}=(\xi_z,\xi_y,\tau)=(z_1-z_2,y_1-y_2,t_1-t_2)$ and $E(\cdot)$ is the expectation operator.

Meanwhile, the correlation function of the target stochastic process $u(\boldsymbol{x})$ can be obtained from (Chen et al. 2017)

$$
\begin{aligned}
R_u(z,\boldsymbol{\xi}) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{S(z,k_z,k_y,\omega)}\sqrt{S(z+\xi_z,k_z,k_y,\omega)}e^{\mathrm{i}k_z\xi_z}e^{\mathrm{i}k_y\xi_y}e^{\mathrm{i}\omega\tau}\mathrm{d}k_z\mathrm{d}k_y\mathrm{d}\omega \\
&= 8\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\sqrt{S(z,\boldsymbol{k})}\sqrt{S(z+\xi_z,\boldsymbol{k})}\cos(k_z\xi_z)\cos(k_y\xi_y)\cos(\omega\tau)\mathrm{d}k_z\mathrm{d}k_y\mathrm{d}\omega \\
&= 8\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\sqrt{S(z,\boldsymbol{k})}\sqrt{S(z+\xi_z,\boldsymbol{k})}\cdot\frac{1}{4}[\cos(\boldsymbol{k}\cdot^{++}\boldsymbol{\xi})+\cos(\boldsymbol{k}\cdot^{+-}\boldsymbol{\xi}) \\
&\qquad +\cos(\boldsymbol{k}\cdot^{-+}\boldsymbol{\xi})+\cos(\boldsymbol{k}\cdot^{--}\boldsymbol{\xi})]\mathrm{d}\boldsymbol{k} \\
&= 2\sum_{j=1}^{N}\int_{D_j}\sqrt{S(z,\boldsymbol{k}_j)}\sqrt{S(z+\xi_z,\boldsymbol{k}_j)}[\cos(\boldsymbol{k}_j\cdot^{++}\boldsymbol{\xi})+\cos(\boldsymbol{k}_j\cdot^{+-}\boldsymbol{\xi}) \\
&\qquad +\cos(\boldsymbol{k}_j\cdot^{-+}\boldsymbol{\xi})+\cos(\boldsymbol{k}_j\cdot^{--}\boldsymbol{\xi})]\mathrm{d}\boldsymbol{k}_j
\end{aligned}
\tag{11}
$$

By comparing Eqs.(10) and (11) for each component, one can immediately find that if $\sqrt{1/2\,p_{\boldsymbol{K}_j}(\boldsymbol{k}_j)}A(z,\boldsymbol{k}_j)=\sqrt{2S(z,\boldsymbol{k}_j)}$ for $\boldsymbol{k}_j\in D_j$, then the correlation function of $u_N^{\mathrm{SHF}}(\boldsymbol{x})$ is identical to that of $u(\boldsymbol{x})$. Therefore, $A(z,\boldsymbol{K}_j)$ in Eq.(9) should satisfy

$$
A(z,\boldsymbol{K}_j)=\sqrt{\frac{4S(z,\boldsymbol{K}_j)}{p_{\boldsymbol{K}_j}(\boldsymbol{K}_j)}},\ \text{for } \boldsymbol{K}_j\in D_j \tag{12}
$$

It should be noted that in the derivation of $A(z,\boldsymbol{K}_j)$, $R_{u_N^{\mathrm{SHF}}}(z,\boldsymbol{\xi})$ exactly equals to $R_u(z,\boldsymbol{\xi})$ without any restrictions on the value of N and the distribution type of Ω_j's.

Since $p_{\boldsymbol{K}_j}(\boldsymbol{k}_j)$ can be chosen arbitrarily, the uniform distribution is usually taken for convenience. Such scheme is called the SHF of the second kind (SHF-II) (Chen et al. 2013, 2017, 2018a) and is adopted in this paper

$$p_{\boldsymbol{K}_j}(\boldsymbol{k}_j)=\frac{1}{V_j}\cdot I\{\boldsymbol{k}_j\in D_j\} \tag{13}$$

where V_j is the volume of the subdomain D_j. $I\{\cdot\}$ is the indicator function, $I\{a\}=1$ if *a* is true; otherwise, $I\{a\}=0$.

Therefore, the amplitude $A(z,\boldsymbol{K}_j)$ is

$$A(z,\boldsymbol{K}_j)=\sqrt{4S(z,\boldsymbol{K}_j)V_j} \tag{14}$$

In this case, the total number of random variables (frequencies and phases) is 7*N*, in which *N* for K_j^z's, K_j^y's, Ω_j's, $\varphi_j^{(1)}$'s, $\varphi_j^{(2)}$'s, $\varphi_j^{(3)}$'s and $\varphi_j^{(4)}$'s, respectively.

It is noted, interestingly, that in Eq.(10) the independence of random phases are necessary, but there is no requirement on whether the random wavenumber-frequency points should be independent or not. Therefore, it is promising to further reduce the number of random variables by using dependent random wavenumber-frequency points.

The SHF Representation with Dependent Random Wavenumber-Frequency Points

To this end, the random wavenumber-frequency vector $\boldsymbol{K}_j$ can be written as the functions of basic random vectors $\boldsymbol{\lambda}_j$'s, i.e.,

$$\boldsymbol{K}_j=\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\ \ (j=1,2\cdots N) \tag{15}$$

where $\boldsymbol{\lambda}_j=(\alpha_j,\beta_j,\gamma_j)$, α_j's, β_j's and γ_j's are three sets of dependent random variables identically uniformly distributed over [0, 1] with the PDFs $p_{\alpha_j}(\alpha)=1$, $p_{\beta_j}(\beta)=1$ and $p_{\gamma_j}(\gamma)=1$, respectively. However, the components of $\boldsymbol{\lambda}_j$, i.e. α_j, β_j and γ_j, are independent. Therefore, the PDFs for $\boldsymbol{\lambda}_j(j=1,2,...,N)$ is $p_{\boldsymbol{\lambda}_j}(\boldsymbol{\lambda}_j)=1$ with the support domain $D_{\boldsymbol{\lambda}_j}=[0,1]\times[0,1]\times[0,1]$.

In this case Eq.(9) becomes

$$\begin{aligned}u_N^{\mathrm{SHF}}(\boldsymbol{x})=\sum_{j=1}^{N}A[z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)]\{&\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{++}\boldsymbol{x}+\varphi_j^{(1)}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{+-}\boldsymbol{x}+\varphi_j^{(2)}]\\&+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{-+}\boldsymbol{x}+\varphi_j^{(3)}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{--}\boldsymbol{x}+\varphi_j^{(4)}]\}\end{aligned} \tag{16}$$

Accordingly, Eqs.(10) and (11) are rewritten as, respectively,

$$
\begin{aligned}
R_{u_N^{\mathrm{SHF}}}(z,\boldsymbol{\xi}) &= E[u_N^{\mathrm{SHF}}(\boldsymbol{x})u_N^{\mathrm{SHF}}(\boldsymbol{x}+\boldsymbol{\xi})] \\
&= \sum_{j=1}^{N} E\Big\{A[z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)]A[z+\xi_z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)]\cdot\frac{1}{2}\{\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{++}\boldsymbol{\xi}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{+-}\boldsymbol{\xi}] \\
&\qquad +\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{-+}\boldsymbol{\xi}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{--}\boldsymbol{\xi}]\}\Big\} \\
&= \frac{1}{2}\sum_{j=1}^{N}\int_{D_{\boldsymbol{\lambda}_j}} A[z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)]A[z+\xi_z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)]\{\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{++}\boldsymbol{\xi}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{+-}\boldsymbol{\xi}] \\
&\qquad +\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{-+}\boldsymbol{\xi}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{--}\boldsymbol{\xi}]\}\,p_{\boldsymbol{\lambda}_j}(\boldsymbol{\lambda}_j)\mathrm{d}\boldsymbol{\lambda}_j \\
&= \frac{1}{2}\sum_{j=1}^{N}\int_{D_{\boldsymbol{\lambda}_j}} A[z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)]A[z+\xi_z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)]\{\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{++}\boldsymbol{\xi}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{+-}\boldsymbol{\xi}] \\
&\qquad +\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{-+}\boldsymbol{\xi}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{--}\boldsymbol{\xi}]\}\,\mathrm{d}\boldsymbol{\lambda}_j
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
R_u(z,\boldsymbol{\xi}) &= 2\sum_{j=1}^{N}\int_{D_j}\sqrt{S[z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)]}\sqrt{S[z+\xi_z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)]}\{\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{++}\boldsymbol{\xi}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{+-}\boldsymbol{\xi}] \\
&\qquad +\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{-+}\boldsymbol{\xi}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{--}\boldsymbol{\xi}]\}\,\mathrm{d}\boldsymbol{K}_j(\boldsymbol{\lambda}_j) \\
&= 2\sum_{j=1}^{N}\int_{D_{\boldsymbol{\lambda}_j}}\sqrt{S[z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)]}\sqrt{S[z+\xi_z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)]}\{\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{++}\boldsymbol{\xi}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{+-}\boldsymbol{\xi}] \\
&\qquad +\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{-+}\boldsymbol{\xi}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{--}\boldsymbol{\xi}]\}\,|J_j(\boldsymbol{\lambda}_j)|\,\mathrm{d}\boldsymbol{\lambda}_j
\end{aligned}
\tag{18}
$$

where $|J_j(\boldsymbol{\lambda}_j)|$ is the Jacobian determinate

$$
|J_j(\boldsymbol{\lambda}_j)| = \left|\frac{\partial \boldsymbol{K}_j(\boldsymbol{\lambda}_j)}{\partial \boldsymbol{\lambda}_j}\right| = \begin{vmatrix} \dfrac{\partial K_j^z}{\partial \alpha_j} & \dfrac{\partial K_j^y}{\partial \alpha_j} & \dfrac{\partial \Omega_j}{\partial \alpha_j} \\ \dfrac{\partial K_j^z}{\partial \beta_j} & \dfrac{\partial K_j^y}{\partial \beta_j} & \dfrac{\partial \Omega_j}{\partial \beta_j} \\ \dfrac{\partial K_j^z}{\partial \gamma_j} & \dfrac{\partial K_j^y}{\partial \gamma_j} & \dfrac{\partial \Omega_j}{\partial \gamma_j} \end{vmatrix}
\tag{19}
$$

Comparing Eqs.(17) and (18) term to term yields

$$
A[z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)] = \sqrt{4S[z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)]|J_j(\boldsymbol{\lambda}_j)|}
\tag{20}
$$

In this case, $u(z,y,t)$ is represented by

$$u_N^{\mathrm{SHF}}(\boldsymbol{x}) = \sum_{j=1}^{N} \sqrt{4S[z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)]|J_j(\boldsymbol{\lambda}_j)|}\{\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{++}\boldsymbol{x}+\varphi_j^{(1)}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{+-}\boldsymbol{x}+\varphi_j^{(2)}] + \cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{-+}\boldsymbol{x}+\varphi_j^{(3)}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{--}\boldsymbol{x}+\varphi_j^{(4)}]\} \tag{21}$$

Since $\boldsymbol{\lambda}_j = (\alpha_j, \beta_j, \gamma_j), (j=1,2,...,N)$ are random vectors identically uniformly distributed over $[0,1]\times[0,1]\times[0,1]$, one can easily find that: (1) when $\boldsymbol{\lambda}_j$'s are mutually independent, the number of the random variables for Eq. (21) is 7*N* (*N* for α_j's, β_j's, γ_j's, $\varphi_j^{(1)}$'s, $\varphi_j^{(2)}$'s, $\varphi_j^{(3)}$'s and $\varphi_j^{(4)}$'s, respectively), which means the random wavenumber-frequency points are mutually independent and is the same as those in the preceding section ; and (2) when $\boldsymbol{\lambda}_j$'s take the same value, i.e. $\boldsymbol{\lambda}_j = \boldsymbol{\lambda}_0 = (\alpha_0, \beta_0, \gamma_0)$ $(j=1,2,...,N)$, the number of the random variables is only 4*N*+3 (4*N* for $\varphi_j^{(1)}$'s, $\varphi_j^{(2)}$'s, $\varphi_j^{(3)}$'s and $\varphi_j^{(4)}$'s, and 3 for α_0, β_0 and γ_0), reduced by a factor of almost 3/7.

To determine the Jacobian determinate $|J_j(\boldsymbol{\lambda}_j)|$, the function relationships between $\boldsymbol{K}_j$'s and $\boldsymbol{\lambda}_j$'s, i.e., $\boldsymbol{K}_j(\boldsymbol{\lambda}_j)$ must be given, which will be specified in next section.

It is noted that such generated stochastic processes are non-ergodic, as in the previous SHF scheme (Chen et al 2013). However, because the number of harmonic components is not as few as only several, this will not be a problem for practical applications. On the other hand, it should also be noted that the ergodicity is an extra property or assumption compared to the stationarity. For non-stationary processes the ergodicity does not exist.

IMPLEMENTATION PROCEDURES OF THE SHF REPRESENTATION FOR FLUCTUATING WIND FIELD SIMULATION

According to the discussions in the preceding section, to adopt the SHF representation for wind field simulation, three key steps need to be implemented and some parameters should be specified, i.e.,

(1) Determine the cut-off wavenumbers and frequency to construct D_0;

(2) Determine the subdomains D_j's, i.e. how to partition D_0 into a set of non-overlapping

subdomains; and

(3) Generate the frequency-wavenumber point $\boldsymbol{K}_j$ in the subdomain D_j for a given basic random vector $\boldsymbol{\lambda}_j$, i.e., determine the function $\boldsymbol{K}_j(\boldsymbol{\lambda}_j)$ such that the Jacobian determinate $|J_j(\boldsymbol{\lambda}_j)|$ can be specified simultaneously.

The implementation procedures are interpreted in the following three subsections, respectively.

Determination of D_0

The lower cut-off frequency and wavenumbers usually take zero. While for the upper cut-off values, on the one hand, they depend on the frequency of the structures subjected to the wind field (Ke et al. 2015), on the other hand, the following criterion can be adopted (Shinozuka & Deodatis 1996).

$$\int_0^{k_z^{\mathrm{U}}}\int_0^{k_y^{\mathrm{U}}}\int_0^{\omega^{\mathrm{U}}} S(z,k_z,k_y,\omega)\mathrm{d}\omega\mathrm{d}k_y\mathrm{d}k_z = (1-\varepsilon)\int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty} S(z,k_z,k_y,\omega)\mathrm{d}\omega\mathrm{d}k_y\mathrm{d}k_z \quad (22)$$

where ε denotes a truncated error which is far less than 1, e.g., ε = 0.05 or 0.01.

Determination of D_j's

Theoretically, the partition of D_0 is arbitrary as long as D_j 's satisfy $D_0 = \bigcup_{j=1}^{N} D_j$ and $D_j \bigcap D_m = \varnothing, \forall j \neq m$. A simplest case is the cuboid grid partitioning, in which each subdomain D_j is a small cuboid

$$D_j = [\underline{k}_j^z, \overline{k}_j^z]\times[\underline{k}_j^y, \overline{k}_j^y]\times[\underline{\omega}_j, \overline{\omega}_j] \quad (23)$$

where $\underline{k}_j^z, \overline{k}_j^z$ are the lower and upper bounds of wavenumber k^z of the subdomain D_j, similar symbols used for wavenumber k^y and frequency ω. However, a large number of subdomains are needed in such partition because for multi-dimensional random fields a tensor product scheme is essentially adopted here.

An alternative partition scheme is the Voronoi cell partitioning (Li & Chen 2009), in which the subdomain D_j is usually a convex polyhedron. To this end, a set of representative points $\boldsymbol{K}_j^* = (K_j^{*z}, K_j^{*y}, \Omega_j^*) \in D_0\ (j = 1, 2, ..., N)$ should be specified such that D_0 can be partitioned into a number of N Voronoi subdomains. $\boldsymbol{K}_j^*$'s can be obtained by the acceptance-rejection method (Li & Chen 2009; Song et al. 2018), which results in taking denser representative points where the joint PSD value is greater. In other words, the region where the joint PSD is greater will be partitioned into more subdomains.

However, it is found from the radial formulation of wind speed joint PSD (Song et al. 2018) that in the range close to the origin the spectral value is far greater by several orders of magnitude than that in the range away from the origin (Fig.1). When the acceptance-rejection method is directly adopted over D_0, there is almost no representative points distributed in the range away from the origin as shown in Fig.2. This is unreasonable and will induce errors in simulation. To alleviate this problem, Song et al. (2018) and Chen et al. (2018b) partition D_0 into some (no more than five) regular subdomains firstly, then implement the acceptance-rejection method over each subdomains. However, such artificial intervening may lead to multiple empirical trials which reduces the efficiency. In the present paper, a new acceptance-rejection scheme is proposed as follows: the p-power of $S(z,k_z,k_y,\omega)$ ($0 < p < 1$) is suggested to be used for the acceptance-rejection criterion instead of $S(z,k_z,k_y,\omega)$ itself being used in the original acceptance-rejection method. The value of p is suggested to take 0.5~0.6 according to experiences. In this case, the difference of the values of $[S(z,k_z,k_y,\omega)]^p$ over D_0 is not that huge so that the acceptance-rejection (A-R) can be performed over D_0 directly.

To this end, a set of uniformly scattered points $M_n = \{\boldsymbol{\eta}_i = (\zeta_i, k_i^z, k_i^y, \omega_i)\}_{i=1}^n$ in the four-dimensional hyper-rectangle $[0,a]\times[0,k_z^{\mathrm{U}}]\times[0,k_y^{\mathrm{U}}]\times[0,\omega^{\mathrm{U}}]$ should be firstly specified, where n is the number of points in M_n. Here $a > \max\{[S(z_{\max},k_z,k_y,\omega)]^p\}$, $z_{\max}$ is the maximum vertical coordinate of the positions to be simulated. This point set can be specified by an affine transform of the point set $\tilde{M}_n = \{\tilde{\boldsymbol{\eta}}_i = (\eta_i^{(1)}, \eta_i^{(2)}, \eta_i^{(3)}, \eta_i^{(4)}) \in [0,1]^4\}_{i=1}^n$ uniformly scattered over the unit cube $[0,1]^4$, i.e. $\zeta_i = a\eta_i^{(1)}$, $k_i^z = k_z^{\mathrm{U}}\eta_i^{(2)}$, $k_i^y = k_y^{\mathrm{U}}\eta_i^{(3)}$, $\omega_i = \omega^{\mathrm{U}}\eta_i^{(4)}$. The Sobol' point set, which features a small discrepancy (Dick & Pillichshammer 2010), can be chosen as $\tilde{M}_n$. Then the following criterion is adopted: if $\zeta_i > [S(z_{\max},k_i^z,k_i^y,\omega_i)]^p$, the point $\boldsymbol{\eta}_i$ will be deleted from the point set M_n. For clarity, the remaining points in M_n are denoted by $M' = \{\boldsymbol{\eta}_j = (\zeta_j, k_j^z, k_j^y, \omega_j)\}_{j=1}^{N}$, where N denotes the number of points in M'. Then the projection of this point set in the wavenumber-frequency space, i.e. $(k_j^z, k_j^y, \omega_j)_{j=1}^{N}$ is the finally determined representative point set, i.e., $\boldsymbol{K}_j^* = (K_j^{*z}, K_j^{*y}, \Omega_j^*)_{j=1}^{N}$. The value of $[S(z,k_r,\omega)]^p$ and the selected representative points based on $[S(z,k_z,k_y,\omega)]^p$ when p = 0.6 are shown in Fig.3 and Fig.4, respectively.

In Fig.1 and Fig.3, the radial formulation of the joint spectrum has the following expression (Song et al. 2018)

$$\begin{aligned} S(z,k_r,\omega) &= S(z,\omega)\,\rho^{(\text{W-F})}(k_r,\omega) \\ &= S(z,\omega)\frac{1}{C_z C_y \left(\frac{|\omega|}{2\pi U_{10}}\right)^2}\frac{k_r}{\left(1 + k_r^2 \big/ \left(\frac{1}{2\pi U_{10}}|\omega|\right)^2\right)^{3/2}} \end{aligned} \tag{24}$$

where $k_r = \sqrt{(k_y/C_y)^2 + (k_z/C_z)^2}$ is the radial coordinate. It is noted that the parameters needed for implementing the acceptance-rejection in the above two cases take the values of the first numerical example in the section of numerical examples.

Since $\boldsymbol{K}_j^*$'s (j =1,2,…,N) have been specified, D_0 can be partitioned into a number of N Voronoi subdomains as shown in Fig.5.

Generation of $\boldsymbol{K}_j$'s for Given $\boldsymbol{\lambda}_j$'s

When the cuboid grid partitioning is adopted, $\boldsymbol{K}_j(\boldsymbol{\lambda}_j)$ can be specified by the following simple transform in each small cuboid subdomain $D_j = [\underline{k}_j^z, \overline{k}_j^z] \times [\underline{k}_j^y, \overline{k}_j^y] \times [\underline{\omega}_j, \overline{\omega}_j]$

$$\begin{cases} K_j^z(\alpha_j,\beta_j,\gamma_j) = \underline{k}_j^z + \alpha_j(\overline{k}_j^z - \underline{k}_j^z) \\ K_j^y(\alpha_j,\beta_j,\gamma_j) = \underline{k}_j^y + \beta_j(\overline{k}_j^y - \underline{k}_j^y) \\ \Omega_j(\alpha_j,\beta_j,\gamma_j) = \underline{\omega}_j + \gamma_j(\overline{\omega}_j - \underline{\omega}_j) \end{cases} \tag{25}$$

Therefore, the Jocobian determinate is

$$|J_j(\boldsymbol{\lambda}_j)| = \left|\frac{\partial \boldsymbol{K}_j(\boldsymbol{\lambda}_j)}{\partial \boldsymbol{\lambda}_j}\right| = \begin{vmatrix} \frac{\partial K_j^z}{\partial \alpha_j} & \frac{\partial K_j^y}{\partial \alpha_j} & \frac{\partial \Omega_j}{\partial \alpha_j} \\ \frac{\partial K_j^z}{\partial \beta_j} & \frac{\partial K_j^y}{\partial \beta_j} & \frac{\partial \Omega_j}{\partial \beta_j} \\ \frac{\partial K_j^z}{\partial \gamma_j} & \frac{\partial K_j^y}{\partial \gamma_j} & \frac{\partial \Omega_j}{\partial \gamma_j} \end{vmatrix} = (\bar{k}_j^z - \underline{k}_j^z)(\bar{k}_j^y - \underline{k}_j^y)(\bar{\omega}_j - \underline{\omega}_j) \quad (26)$$

When the Voronoi cell partitioning is adopted, the scheme of determining $\boldsymbol{K}_j(\boldsymbol{\lambda}_j)$ is shown in Fig.6 and interpreted as follows:

(a) Determine the lower and upper bounds of frequency ω in the subdomain D_j, which is denoted by $\underline{\omega}_j$ and $\bar{\omega}_j$, respectively. Then, specify Ω_j by the following transform

$$\Omega_j(\alpha_j, \beta_j, \gamma_j) = \underline{\omega}_j + \alpha_j(\bar{\omega}_j - \underline{\omega}_j) \quad (27)$$

a simple case is shown in Fig.6(a), in which the subdomain D_j is a pentagon prismoid.

(b) Determine the bounds of the intersections between the plane $\omega = \Omega_j$ and the subdomain D_j, which form a convex polygon denoted as B_j^α and is shown in Fig.8(a) and Fig.8 (b). Then K_j^z can be specified by

$$\begin{aligned} K_j^z(\alpha_j, \beta_j, \gamma_j) &= K_j^{z\mathrm{L}} + \beta_j(K_j^{z\mathrm{U}} - K_j^{z\mathrm{L}}) \\ &= K_j^{z\mathrm{L}}(\alpha_j) + \beta_j[K_j^{z\mathrm{U}}(\alpha_j) - K_j^{z\mathrm{L}}(\alpha_j)] \end{aligned} \quad (28)$$

where $K_j^{z\mathrm{L}}, K_j^{z\mathrm{U}}$ are the lower and upper bounds of k_z in B_j^α, respectively.

(c) Determine the two points $(K_j^z, K_j^{y\mathrm{L}})$ and $(K_j^z, K_j^{y\mathrm{U}})$, $K_j^{y\mathrm{L}} < K_j^{y\mathrm{U}}$, which are the intersections between the line $k_z = K_j^z$ and the bounds of B_j^α. Therefore, K_j^y can be specified by

$$\begin{aligned} K_j^y(\alpha_j, \beta_j, \gamma_j) &= K_j^{y\mathrm{L}} + \gamma_j(K_j^{y\mathrm{U}} - K_j^{y\mathrm{L}}) \\ &= K_j^{y\mathrm{L}}(\alpha_j, \beta_j) + \gamma_j[K_j^{y\mathrm{U}}(\alpha_j, \beta_j) - K_j^{y\mathrm{L}}(\alpha_j, \beta_j)] \end{aligned} \quad (29)$$

Thus, the Jacobian determinate is

$$|J_j(\boldsymbol{\lambda}_j)| = \left|\frac{\partial \boldsymbol{K}_j(\boldsymbol{\lambda}_j)}{\partial \boldsymbol{\lambda}_j}\right| = \begin{vmatrix} \dfrac{\partial K_j^z}{\partial \alpha_j} & \dfrac{\partial K_j^y}{\partial \alpha_j} & \dfrac{\partial \Omega_j}{\partial \alpha_j} \\ \dfrac{\partial K_j^z}{\partial \beta_j} & \dfrac{\partial K_j^y}{\partial \beta_j} & \dfrac{\partial \Omega_j}{\partial \beta_j} \\ \dfrac{\partial K_j^z}{\partial \gamma_j} & \dfrac{\partial K_j^y}{\partial \gamma_j} & \dfrac{\partial \Omega_j}{\partial \gamma_j} \end{vmatrix} = (\overline{\omega}_j - \underline{\omega}_j)(K_j^{z\mathrm{U}} - K_j^{z\mathrm{L}})(K_j^{y\mathrm{U}} - K_j^{y\mathrm{L}}) \quad (30)$$

In this way, $\boldsymbol{K}_j$ can be specified using Eqs.(27) (28) and (29) for a given random vector $\boldsymbol{\lambda}_j$ (j=1,2,…,N).

NUMERICAL EXAMPLES

For illustrative purposes, two numerical examples of wind field simulation are addressed. The first one is a stationary and nonhomogeneous case for the rotating blades of wind turbines in two spatial dimensions, and the second one is a nonstationary and homogeneous case in two spatial dimensions. For each cases, the SRM, the SHF with different $\boldsymbol{\lambda}_j$'s and the SHF with the same $\boldsymbol{\lambda}_0$ are adopted for comparisons. Since the Voronoi cell partitioning scheme integrated with the acceptance-rejection method is more efficient (Song et al. 2018), it will be employed in the SRM and SHF for the two cases.

Stationary and Nonhomogeneous Case

Consider a 5-MW wind turbine (Jonkman et al. 2009), the hub is at the height of 90m, and the diameter of blades is about 120m. In this case, the joint spectrum in Eq.(5) is adopted, in which $S_0(z,\omega)$ takes the Kaimal spectrum in Eq.(4). The other parameters are: $U_{10} = 20$ m/s, $u_* = 1.691$ m/s, $z_0 = 0.005$ m; $C_z = C_y = 7$ (Chen et al. 2018b); $k_z^{\mathrm{U}} = k_y^{\mathrm{U}} = \pi$ rad/m, $\omega^{\mathrm{U}} = 2\pi$ rad/s (Ke et al. 2015); $T = 600$ s, and $\Delta t = 0.5$ s. For illustrative purposes, the fluctuating wind speeds at the spatial points P1(0,30), P2(60,90), and P3(0,150) in the rotating blade plane are to be simulated for a wind field sample, which are shown in Fig.7.

The SRM in Eq.(6) and the SHF in Eq.(31) are adopted for the simulation, respectively.

$$u_N^{\mathrm{SHF}}(\boldsymbol{x})=\sum_{j=1}^{N}\sqrt{4S[z,\boldsymbol{K}_j(\boldsymbol{\lambda}_j)](\overline{\omega}_j-\underline{\omega}_j)(K_j^{z\mathrm{U}}-K_j^{z\mathrm{L}})(K_j^{y\mathrm{U}}-K_j^{y\mathrm{L}})}\{\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{++}\boldsymbol{x}+\varphi_j^{(1)}]$$
$$+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{+-}\boldsymbol{x}+\varphi_j^{(2)}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{-+}\boldsymbol{x}+\varphi_j^{(3)}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{--}\boldsymbol{x}+\varphi_j^{(4)}]\} \quad (31)$$

It seems that all the needed parameters have been specified and the random field is readily to be simulated by now. However, the number of the harmonic components, N, is still undetermined, which may have effects on the accuracy of simulation results. In this numerical example, two cases with different N are considered.

Case 1: N=4900

In this case, take p=0.6 and a total of 9×10^8 basic four-dimensional Sobol's points is involved, around 4900 points (i.e., N = 4900) are retained after the acceptance-rejection operation, which is shown in Fig.4. Totally, 500 samples are generated by the three methods, respectively. The typical fluctuating wind speed time histories at P3 are shown in Fig.8. Then, the auto-PSD function, the cross correlation function and the coherence function of the fluctuating wind speed process can be estimated (Bendat & Piersol 2010; Chen et al 2018b). The comparison between the reproduced auto-PSD at P3 by 500 samples and the target Kaimal spectrum is shown in Fig.9. The comparisons between the reproduced and target cross-correlation function and coherence function between the fluctuating wind speed processes at P1 and P2 are shown in Fig.10 and Fig.11, respectively.

It is seen that the time histories of fluctuating wind speed generated by the three methods are almost identical. However, the performance of the SRM in the assemble characteristics including the auto-PSD function, the cross correlation and the coherence function is not as good as the SHF representation. In addition, the accuracy of the simulation results by the SHF with independent and dependent random wavenumber-frequency points is almost the same.

Codes are written in MATLAB platform and run on a PC with Intel (R) Core (TM) i7-4790K CPU @ 4.00GHz and 16GB main memory under the WIN7 operating environment. The multi-core parallel computing technique is activated during the simulation. The consumed time for the simulation of such a wind field sample of the three methods is listed in Table 1. It is seen that with the same N, the SRM is more efficient than the SHF. Besides, the consumed time of the SHF with independent and dependent random wavenumber-frequency points is nearly identical. Therefore, only the SHF with dependent wavenumber-frequency points is compared with the SRM in case 2.

Case 2: N=9,800 and 98,000

In this case, take p=0.6 and a total of 1.8×10^{9} and 1.8×10^{10} basic four-dimensional Sobol's points are involved, respectively. Then around 9,800 and 98,000 points (i.e., N=9,800 and 98,000) are respectively retained after the acceptance-rejection operation. The simulation results by the SRM and the SHF with dependent wavenumber-frequency points are shown from Fig.12 through Fig.15.

It is seen that the accuracy of the SRM with N = 98,000 is almost identical with that of the SHF with N = 9,800, both of which are quite consistent with the target values. In contrast, there exists obvious difference between the results of the SRM with N = 9,800 and the target values. Therefore, the number of discretized wavenumber-frequency points for the SRM and the SHF is suggested to be around 100,000 and 10,000, respectively.

The consumed time for the simulation of such a wind field is listed in Table 1. It can be seen that the simulation efficiency of the SHF with N = 9,800 is quite close to that of the SRM with N = 98,000.

Nonstationary and Homogeneous Case

To verify the effectiveness of the proposed method for the simulation of nonstationary wind speed field, the fluctuating wind speed time histories at the three points in Fig.7 during a typhoon process is simulated in this section.

According to Huang et al. (2015), it is reasonable to simulate the fluctuating wind speed process during a typhoon as a uniformly modulated process. A simplified nonstationary wind spectrum model is suggested as (Huang et al. 2015)

$$S(n,t)=\sigma^{2}(t)S_{\mathrm{nor}}(n);\quad S_{\mathrm{nor}}(n)=\frac{An^{\gamma}}{(1+Bn^{\alpha})^{\beta}} \tag{32}$$

where $S(n,t)$ is the evolutionary spectrum of the nonstationary wind speed; n denotes the natural frequency (Hz); $\sigma(t)$ is the time-varying standard deviation of the fluctuating wind speed; $S_{\mathrm{nor}}(n)$ is the normalized wind spectrum and its integration with n is unity; A, B, α, β and γ are constants and can be identified based on observation data.

The Davenport coherence model is still valid for the fluctuating wind field during a typhoon process (Huang et al. 2015). Therefore, the wavenumber-frequency joint spectrum for the nonstationary wind speed field in two spatial dimensions can be expressed as

$$\begin{aligned} S(k_z,k_y,\omega,t) &= S(\omega,t)\cdot\rho^{(\text{W-F})}(k_z,k_y,\omega) \\ &= \frac{\sigma^2(t)}{2\pi}\frac{A(\omega/2\pi)^{\gamma}}{\left[1+B(\omega/2\pi)^{\alpha}\right]^{\beta}}\cdot\frac{1}{2\pi C_z C_y}\frac{1}{\left(\frac{1}{2\pi U_{10}}|\omega|\right)^2}\frac{1}{\left(1+\left[\left(\frac{1}{C_y}k_y\right)^2+\left(\frac{1}{C_z}k_z\right)^2\right]\Big/\left(\frac{1}{2\pi U_{10}}|\omega|\right)^2\right)^{\frac{3}{2}}} \end{aligned} \tag{33}$$

Correspondingly, the nonstationary wind field represented by the SRM and SHF is given by, respectively,

$$\begin{aligned} u(z,y,t) = \sum_{j=1}^{N}\sqrt{4S(k_j^{(z)},k_j^{(y)},\omega_j,t)V_j} \\ \times[\cos(k_j^{(z)}z+k_j^{(y)}y+\omega_j t+\varphi_j^{(1)})+\cos(k_j^{(z)}z+k_j^{(y)}y-\omega_j t+\varphi_j^{(2)}) \\ +\cos(k_j^{(z)}z-k_j^{(y)}y+\omega_j t+\varphi_j^{(3)})+\cos(k_j^{(z)}z-k_j^{(y)}y-\omega_j t+\varphi_j^{(4)})] \end{aligned} \tag{34}$$

$$\begin{aligned} u_N^{\text{SHF}}(\boldsymbol{x}) = \sum_{j=1}^{N}\sqrt{4S[\boldsymbol{K}_j(\boldsymbol{\lambda}_j),t](\overline{\omega}_j-\underline{\omega}_j)(K_j^{z\text{U}}-K_j^{z\text{L}})(K_j^{y\text{U}}-K_j^{y\text{L}})}\{\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{++}\boldsymbol{x}+\varphi_j^{(1)}] \\ +\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{+-}\boldsymbol{x}+\varphi_j^{(2)}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{-+}\boldsymbol{x}+\varphi_j^{(3)}]+\cos[\boldsymbol{K}_j(\boldsymbol{\lambda}_j)\cdot^{--}\boldsymbol{x}+\varphi_j^{(4)}]\} \end{aligned} \tag{35}$$

The time-varying standard deviation $\sigma(t)$ is identified from the wind speed records during a typhoon process (Huang et al. 2015). In the present simulation, only a duration of 1200s of $\sigma(t)$ is employed, which can be fitted by the superposition of finite sinusoidal series,

$$\sigma(t) = a_1\sin(b_1 t+c_1)+a_2\sin(b_2 t+c_2)+a_3\sin(b_3 t+c_3) \tag{36}$$

in which $a_1=18.76$, $b_1=0.002057$, $c_1=0.3878$, $a_2=15.59$, $b_2=0.002271$, $c_2=3.455$, $a_3=0.1864$, $b_3=0.006924$, $c_3=3.216$. The process of the 1200s duration of $\sigma(t)$ is shown in Fig.16.

The parameters in Eq.(32) are identified as $A=45.135$, $B=51.474$, $\alpha=0.9242$, $\beta=1.788$ and $\gamma=6.376\times10^{-4}$ (Huang et al. 2015). The other parameters for the simulation are the same as the above stationary cases.

Case 3: $N = 4600$

Based on the acceptance-rejection method, about 4600 points (i.e., $N = 4600$) are retained. A number of 5000 wind speed field samples are generated by the three methods, respectively. The time histories of fluctuating wind speed at P3 of a wind field sample are shown in Fig.17.

Then, the time dependent auto-correlation function of P3 can be estimated by (Bendat & Piersol 2010)

$$R_{\mathrm{es}}(t,\tau) = E\left[u\left(t-\frac{\tau}{2}\right)u\left(t+\frac{\tau}{2}\right)\right] \tag{37}$$

The comparisons between the estimated and target auto-correlation function of P3 at different time are shown in Fig.18.

Further, the estimated value of time-varying auto-PSD function of P3 is obtained by (Bendat & Piersol 2010)

$$S_{\mathrm{es}}(t,\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{\mathrm{es}}(t,\tau)\,e^{-\mathrm{i}\omega\tau}\,\mathrm{d}\tau \tag{38}$$

The comparisons between the estimated and target auto-PSD function of P3 at different time are shown in Fig.19.

Besides, the cross-correlation function between P1 and P2 can be estimated by (Bendat & Piersol 2010)

$$R_{12}^{\mathrm{es}}(t,\tau) = E\left[u_1\left(t-\frac{\tau}{2}\right)u_2\left(t+\frac{\tau}{2}\right)\right] \tag{39}$$

The comparisons between the estimated and target cross-correlation function between P1 and P2 at different time are shown in Fig.20.

As can be seen from Fig.17 to Fig.20, the results are quite similar to the temporal stationary cases. The performance of the three methods in a single wind speed field sample is almost identical, while the SHF representation exhibits obvious advantages over the SRM in the reproduction of the second order statistics. Besides, there is no influence on the accuracy of the simulation results of the SHF whether the wavenumber-frequency points are independent or dependent. The consumed time for the three methods in this case is listed in Table 1.

Case 4: N=9,200 and 92,000

In this case, more discretized wavenumber-frequency points are used for the simulation. The dependent random wavenumber-frequency points are used in the SHF in this case. The time histories of wind speeds and the comparisons between estimated and target values of auto-correlation function, auto-PSD function as well as the cross-correlation function are shown from Fig.21 through Fig.24.

It is seen that the simulation results of the SHF with N = 9200 and the SRM with N = 92000 are well consistent with the target values, while the results of the SRM with N = 9200 are not that satisfactory. As a result, approximate 10,000 and 100,000 discretized points are suggested for the SHF and SRM respectively, in the simulation of nonstationary wind speed field. The consumed time in this case is also listed in Table 1.

In addition, the number of the random variables in all above cases is included in Table 1. One can observe that in the case that around 10,000 dependent random wavenumber-frequency points are used for the SHF and around 100,000 discretized points are used for the SRM, the consumed time and the accuracy of simulation results are quite similar. However, the number of random variables in the SRM is ten times that of the SHF, and theoretically the latter can reproduce the target spectrum exactly when the number of harmonic components are finite or even small.

CONCLUDING REMARKS

The stochastic harmonic function (SHF) representation method has been extended to 3D random field cases so as to simulate the fluctuating wind speed fields in two spatial dimensions. In this method, based on the wavenumber-frequency joint spectra, both the phase angles and frequencies (wavenumbers) are regarded as random variables. In particular, the random frequencies (wavenumbers) can be dependent so that the number of random variables can be further reduced considerably. The p-power of the joint spectrum is adopted in the acceptance-rejection method for the determination of uneven discretized points in the wavenumber-frequency domain. The Voronoi cells, and thus the supports of random frequencies (wavenumbers), are then determined accordingly. Simulation of stationary and nonstationary wind speed fields are carried out. The conclusions include:

(1) The SHF representation method for random fields can reproduce the target PSD and EPSD exactly by a very finite number of harmonic components. Numerical examples show that the integration of wavenumber-frequency joint spectrum and SHF representation for fluctuating wind field simulation is of high accuracy and efficiency.

(2) The introduction of *p*-power joint spectra in the acceptance-rejection method provides a rational approach that almost no artificial intervening is needed in the determination of the Voronoi cells. Very importantly, the value $p = 0.6$ is suitable for different wind speed spectra.

(3) There is almost no effect on the efficiency and accuracy of the SHF representation whether the random wavenumber-frequency points are independent or not, while the number of random variables can be reduced by 3/7 when the dependent wavenumber-frequency points are adopted.

(4) To obtain the accuracy that is not much different from the target values, the consumed time of the SRM and the SHF is quite similar, while the number of random variables of the SRM is around ten times that of the SHF with dependent random wavenumber-frequency points.

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Table 1. Consumed time and number of random variables in different simulation cases

Case	Number of discretized points (N)	Method	Consumed time for single sample (s)	Number of random variables
Stationary (600s)	4900	SHF-independent	4.2	$7N=3.43\times10^4$
		SHF-dependent	4.5	$4N+3\approx1.96\times10^4$
		SRM	0.4	$4N=1.96\times10^4$
	9800	**SHF-dependent**	**8.4**	$\mathbf{4\boldsymbol{N}+3\approx3.92\times10^4}$
		SRM	0.7	$4N=3.92\times10^4$
	98000	**SRM**	**7.3**	$\mathbf{4\boldsymbol{N}=3.92\times10^5}$
Nonstationary (1200s)	4600	SHF-independent	4.5	$7N=3.22\times10^4$
		SHF-dependent	4.6	$4N+3\approx1.84\times10^4$
		SRM	0.9	$4N=1.84\times10^4$
	9200	**SHF-dependent**	**9.0**	$\mathbf{4\boldsymbol{N}+3\approx3.68\times10^4}$
		SRM	1.5	$4N=3.68\times10^4$
	92000	**SRM**	**10.7**	$\mathbf{4\boldsymbol{N}=3.68\times10^5}$