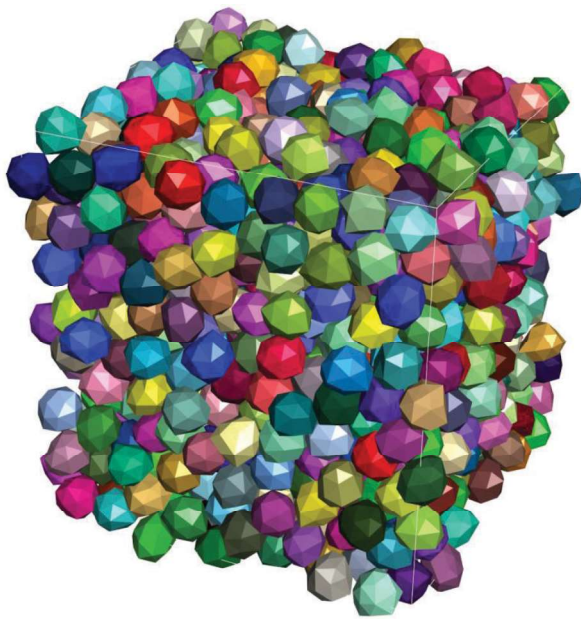


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## **Thermo-dependent properties of model parameters in yield-stress fluids: a survey**

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Non-isothermal flows of materials which display yield-stress-fluid type behaviour can occur frequently. Such flows are often simplistically represented using the Bingham or Hershel-Bulkley models and their analysis often assumes that the model properties, namely yield stress, consistency and shear-thinning indices but also thermal conductivities and diffusivities, remain temperature independent. Testing of the validity of this assumption is hampered by a lack of consensus regarding the precise role of temperature on these properties for different classes of yield stress materials. In this article we survey the available data in the literature on such temperature-dependence for different classes of yield stress materials.

### **1. Introduction**

There are numerous materials which can adequately be described in some circumstances using a “yield stress” fluid framework [1–3]. Namely, below some critical yield stress they can resist deformation and behave for practical purposes as a solid material and, once stressed beyond this critical value, flow in a liquid-like manner. Such yield-stress fluids are often well-characterised using rather simple inelastic and time independent expressions such as the Bingham or Hershel-Bulkley models (defined in Section 2). Although the different molecular architectures of the various materials exhibiting yield stress behaviours often mean they are significantly more complex than these simple models suggest, in particular with regards to effects of thixotropy or aging, the steady shear rheology of many materials, both natural and man-made, have been shown to be well approximated by such models; fruit juices [4] or purees [5,6], concentrated solutions of hydrocolloids such as xanthan gum [7], mud suspensions [8] and foodstuffs such as mayonnaises or mustards [9]. Many other examples are given in Refs [1–3].

Here our interest in such fluids is restricted to a rather narrow question: what role does temperature play and what effect may it have on the parameters in these simplistic models, i.e. yield stress, consistency and power-law index, and can these effects be generalised in any manner for particular classes of yield stress fluids? The motivation for this question arises from the fact that often one is interested in the fluid flow and heat transfer characteristics of such models in various flow situations. For example, a number of experimental [10,11], theoretical [12–16] and numerical [17–25] studies have been interested in studying both the onset of natural convection in such yield-stress fluids on the convective heat transfer (the “Nusselt” number) well beyond such onset conditions. The experimental studies to date have used a model polymeric yield-stress material, aqueous solutions of a commercial polyacrylic acid called “Carbopol” – whose thermo-rheological

properties are discussed below – whereas the other studies listed above assumed temperature-independent rheological properties. Weber et al [26] studied the thermo-dependent properties of Carbopol 980, in a detailed and systematic manner across a fairly modest temperature range of 12 - 38°C. For this particular yield stress fluid, a temperature invariance of most of the Herschel-Bulkley model parameters was found with the exception of the yield stress where a complex weakly non-monotonic dependence was observed. Other studies [27,28], admittedly over wider temperature ranges, have found slightly different behaviour for a different grade of Carbopol and we will discuss these differences in Section 4. Although such results are extremely useful in guiding the choice of temperature-dependent properties in numerical simulations *whose specific aim is to model this fluid in this temperature range* the purpose of this survey is to try to understand if such results are entirely general or if more broad-based trends can be observed for different classes of yield-stress fluids.

In addition to the studies concerned with natural convection effects in yield stress materials discussed above, there is also a fairly significant literature concerned with forced/mixed convection in such fluids which have investigated amongst other situations: flow through pipes experimentally [28,29] and analytically [30–34]; numerical studies of forced convection through parallel plates [35] including the effects of viscous dissipation [36]; through annuli [37,38] and past heated spheres [39], cylinders [40] and square cylinders [41]. In all of these papers thermo-independent properties were assumed. However a limited number of numerical studies have tried to incorporate temperature-dependent properties into their analyses e.g. [42–45]. Essentially most of these papers [43–45] assumed that the yield stress and power-law index remained temperature independent but that the consistency index  $K$  (defined in Section 2) varied with temperature according to an Arrhenius type ( $K = a \exp(-bT)$ ) relationship where  $a$  and  $b$  are model constants. In contrast, Forrest and Wilkinson [42] assumed a relationship of the form

$K = K_{REF} / [1 + \beta(T - T_{REF})^n]$  where  $\beta$  is a constant which characterises the temperature dependent properties of the fluid and  $n$  is the power-law index in the Herschel-Bulkley model (Section 2).

The remainder of the paper is organised as follows. Firstly the model parameters are defined for the Herschel-Bulkley model. The thermo-dependence of Newtonian fluids is then briefly discussed followed by the results of the survey on the thermo-dependence of yield stress fluids. The paper ends with some conclusions and a call for more experimental data for such systems.

## 2. Model yield stress behaviour

Although readily familiar to most workers in the field, as the majority of papers which contain information on the thermal dependence of yield stress fluids fit

either a Bingham or Herschel-Bulkley (HB) model to their data set, we here formally define the HB model (as the Bingham simply represents a special case). The Herschel-Bulkley model [46], which for a one-dimensional flow, is given by

$$\begin{aligned} \dot{\gamma} &= 0, & \tau &\leq \tau_Y^{HB} \\ \tau^{HB} &= \tau_Y^{HB} + K\dot{\gamma}^n, & \tau &> \tau_Y^{HB} \end{aligned} \quad (1)$$

where  $\tau_Y^{HB}$  is the Herschel-Bulkley yield stress,  $K$  is the so-called consistency index and  $n$  the power-law index. For  $n = 1$ , the model reduces to the Bingham model [47] (and  $K$  is replaced by the Bingham plastic viscosity  $\mu_B$ ). The use of different superscripts for the yield stress in both the Bingham and Herschel-Bulkley model in this paper is used to highlight to the reader that, unless  $n = 1$ , fitting the same experimental rheological data set to the different models may produce different values for such quantities. Thus, some care must be exercised in treating these fitting parameters as true material properties especially for the Bingham model where agreement with experimental data is usually not fully quantitative unless over quite narrow shear rate ranges.

### 3. Thermo-dependence of Newtonian fluids as a guide for yield stress fluid behaviour and property variations for water

Before surveying the limited literature concerned with the thermo-dependence of yield-stress materials, it is useful to firstly briefly review the well-known thermo-dependence of Newtonian liquids together with the empirical relationships which are often used to model such behaviour. Such information is beneficial from two key perspectives: it may be argued that to zeroth-order, in the absence of physical measurements, the temperature dependence of the viscous properties of some yield-stress materials may be approximated by Newtonian behaviour; secondly, as many yield stress materials are comprised of a Newtonian liquid as a solvent or “continuous” phase (e.g. aqueous solutions of Carbopol or aqueous suspensions of foodstuffs) it could be argued that some of the thermal properties of the resulting yield-stress materials remain essentially the same as the solvent or continuous phase. For other material types which exhibit yield stress, for example certain emulsions, where the viscous properties of the solvent are essentially irrelevant, such an assumption would be poor. Indeed it is known that for dilute and moderately concentrated aqueous solutions of commonly-used polymers including carboxymethyl cellulose, polyethylene oxide, carbopol, polyacrylamide, the resulting density, specific heat and thermal conductivity remain, to within 10%, essentially that of water [48]. As both are transport properties, and dependent on structure, it might be expected that thermal conductivity would exhibit some shear rate dependence, but although limited measurements by Loulou et al. [49] on Carbopol solutions and of Lin et al. [50] on fruit juices confirm this assertion, the effect is small. Based on this information, Chhabra and Richardson [48] conclude “For engineering design calculations, there will be little error in assuming that all the above physical properties (i.e. thermal conductivity and specific heat) of aqueous polymer solutions, except apparent viscosity, are equal to the values for

water". In contrast, for slurries and pastes exhibiting strong non-Newtonian behaviour, the thermo-physical properties can deviate significantly from those of its constituents: see the detailed discussed in Chhabra and Richardson [48] or consult the review article of Dutta and Mashelkar [51] for thermal conductivity information.

For Newtonian liquids, their viscosity is often a fairly strong function of temperature. For example the dynamic viscosity of water decreases by a factor of 6.3 between 0 and 100°C. For Newtonian liquids generally a good approximation [52] to such behaviour is

$$\mu = \mu_{REF} \exp \left[ C \left( \frac{T_{REF}}{T} - 1 \right) \right], \quad (2)$$

where  $\mu_{REF}$  is the dynamic viscosity at a reference temperature  $T_{REF}$  (in Kelvin) and  $C$  is a constant for a particular liquid. For water at 20°C ( $T_{REF} = 293\text{K}$ ),  $\mu_{REF} = 1.002 \times 10^{-3} \text{Pa}\cdot\text{s}$  and  $C = 5.9$ . As noted in the Introduction, such an Arrhenius-type equation has already been used in the literature to model the Bingham plastic viscosity and consistency index for yield-stress materials. Often this Arrhenius-type equation is of a simpler form than Eqn (2):

$$\mu = a \exp(-bT), \quad (3)$$

where  $a$  and  $b$  are fitting parameters.

For water at atmospheric pressure, the thermal conductivity  $k_c$  has been found to monotonically weakly *increase* with temperature. The standard reference [53] captures this variation empirically using a quadratic equation of the form  $k_c^* = b_0 + b_1 T^* + b_2 T^{*2}$  where  $k_c^*$  and  $T^*$  are the non-dimensional thermal conductivity and temperature respectively ( $T^* = T/298.15$  and  $k_c^* = k_c(T)/k_c(T = 298.15)$  where  $k_c(T = 298.15) = 0.6065 \text{W/mK}$ ):  $b_0 = -1.48445$ ,  $b_1 = 4.12292$ , and  $b_2 = 1.63866$ . Over the temperature range across which water remains as a liquid the variation in  $k_c$  is less than 16%. In the liquid phase the specific heat of water remains constant to within 1% [54].

#### 4. Thermo-dependence of yield-stress materials

Given the scarcity in the literature of studies which investigate the dependence of yield stress fluid properties on temperature, in this section we will collate all such data as reported by the papers in question. However it should be borne in mind that the majority of the experimental techniques used in these studies are perhaps open to question as the possible influence of slip is often not considered [55,56]. Additionally the yield stress values reported are rarely those measured in a direct matter and often just represent the fitted Bingham or Herschel-Bulkley values the quality of which often depends crucially on the shear rate range over which the data is fit (see e.g. the detailed discussions in Nguyen and Boger [55]). The values reported are also often unphysically small. An overview of all of the available data

sets used here, together with a brief description of the experimental technique and method of determining the yield stress, is provided in **Tables 1** and **2** (please note, in order to save space, Tables are available online or directly from the author). In an attempt to compare across data sets with significantly different yield stress values and temperature ranges we also plot non-dimensional yield stress, plastic viscosity/consistency index and temperature values (denoted with an asterisk), where the yield stress (plastic viscosity/consistency index) is made non-dimensional with the quoted yield stress (plastic viscosity/consistency index<sup>1</sup>) at the lowest temperature reported (i.e.  $\tau_Y^* = \tau_Y/\tau_Y(T_{REF})$  and  $\mu_B^* = \mu_B/\mu_B(T_{REF})$ ,  $K^* = K/K(T_{REF})$ ). The temperature is made non-dimensional using

$$T^* = \frac{T - T_{REF}}{T_{REF}}, \quad (4)$$

where all temperatures are in Kelvin and  $T_{REF}$  is again the lowest temperature for which data is reported. Thus, for a water-based fluid with a reference temperature of 10°C, assuming only fluid behaviour between 0 and 100°C,  $T^*$  can vary at most between -0.035 and 0.318.

#### 4.1 Fruit juices and purees

For relatively high fruit concentrations both fruit juice and fruit purees in aqueous solution have been shown to exhibit yield stress behaviour with yield stresses in the region of 0.5-30 Pa dependent on fruit content and temperature. Telis-Romero and co-workers have probed the temperature dependence of both orange juice [4] and passion fruit [57] concentrates. In both studies smooth concentric cylinder geometries were used and the data was fit to the Herschel-Bulkley model. The temperature varied from just above zero to about 60°C and the fruit content between 34-73% for orange juice and 50-90% for passion fruit. Both studies found that the yield stress and consistency index decreased with temperature whereas the power-law index was much less sensitive. For orange juice they were able to correlate the model parameters in terms of both temperature and water content using the following equations

$$\tau_Y^{HB} = 6.28 \times 10^{-8} \exp(A/T)(100 - X_W)^{2.68}, \quad (5)$$

for the yield stress in the range  $0.24 \text{ Pa} \leq \tau_Y^{HB} \leq 6.63 \text{ Pa}$  and  $X_W$  is the w/w percentage of water and  $A = 2008 \text{ K}$ . Unfortunately the equation provided for the power-law index contains a typographical error [58] and so is not reproduced here. The consistency index is correlated using

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<sup>1</sup> We recognise that this method of “non-dimensionalising”  $K$  is rather naïve as the units of  $K$  are  $\text{Pa}\cdot\text{s}^n$ . Thus our  $K^*$  will only be truly non-dimensional if  $n$  is temperature-independent. As the dependence of  $n$  on temperature is indeed rather weak we believe this definition is reasonable and simpler than a truly non-dimensional one e.g.  $K^* = \tau_Y(ref)/(K/\tau_Y(ref))^{1/n}$ .

$$\tau_Y^{HB} \left( \frac{K}{\tau_Y^{HB}} \right)^{1/n} = 2.76 \times 10^{-10} \exp(E_a/RT)(100 - X_W)^{4.22}, \quad (6)$$

for the range  $0.25 \text{ Pa}\cdot\text{s} \leq \tau_Y^{HB} \left( \frac{K}{\tau_Y^{HB}} \right)^{1/n} \leq 42.14 \text{ Pa}\cdot\text{s}$  where  $E_a$  is the activation energy of the flow ( $=17900 \text{ J/mol}$ ) and  $R$  is the gas constant ( $=8.314 \text{ J/molK}$ ). For passion fruit concentrates [57] a double exponential correlation was proposed

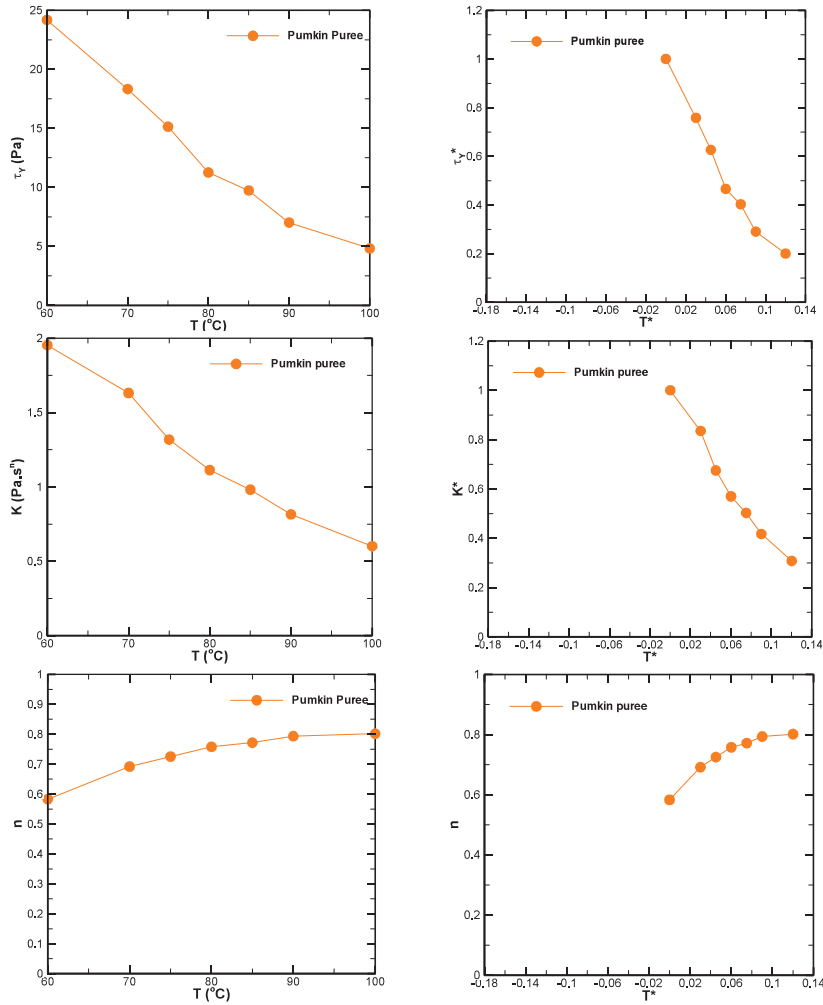
$$x = A \exp(E_a/RT) \exp(BX_W), \quad (7)$$

where  $x$  is either  $\tau_Y^{HB}$ ,  $K$  or  $n$  and then the fitting parameters  $A$ ,  $E_a$  and  $B$  are different for each Herschel-Bulkley parameter (see [57] for values and the range of validity of Eqn. 7). For fixed fruit weight concentration, the yield stress and consistency index decrease with temperature whereas  $n$  slightly increases. For example when  $X_W = 0.5$ ,  $n$  varies from 0.43 to 0.52 and with  $X_W = 0.9$ ,  $n$  varies from 0.57 to 0.69.

Dutta et al [6] report limited measurements of the effect of temperature in the range  $60\text{-}100^\circ\text{C}$  on pumpkin purees as part of a larger study investigating thermal degradation effects on beta-carotene and visual colour of pumpkin puree. A smooth concentric cylinder geometry was used and the resulting rheology data fit to the Herschel-Bulkley model: the associated data sets are shown graphically in **Figure 1**. For this material a decrease in both the yield stress and the consistency index is observed with temperature whereas the power-law index weakly increases with temperature. The yield stress variation is well captured assuming an Arrhenius-type (i.e. Eqn 3) with constants  $a_{pY} = 330 \text{ Pa}$  and  $b_{pY} = -0.0420^\circ\text{C}^{-1}$  and the consistency index with constants  $a_{pV} = 12.8 \text{ Pa}\cdot\text{s}^n$  and  $b_{pV} = -0.0305^\circ\text{C}^{-1}$ . Although both quantities decay approximately linearly in the plots shown in Figure 1, the quality of fit in both cases is slightly higher with an exponential fit rather than a simpler linear fit. In contrast the power-law index of the Herschel-Bulkley model is seen to increase with temperature going from 0.58 at  $60^\circ\text{C}$  to 0.80 at  $100^\circ\text{C}$ : the variation is fit reasonably well with a polynomial function  $n = -1.57 \times 10^{-2}T^2 + 3.05 \times 10^{-2}T - 0.678$  (with  $T$  in  $^\circ\text{C}$ ).

Guerrero and Alzamora [5] conducted a comprehensive study on the temperature dependence of banana purees with varying fruit content and pH for  $10 \leq T \leq 55^\circ\text{C}$ . They used a smooth co-axial cylinder device but checked, via a roughened geometry, that slip effects were not significant for these fluids. **Figure 2** illustrates the data in graphical form. For 100% fruit content the yield stress is seen to be nearly constant (within 20%) between  $10\text{-}40^\circ\text{C}$  before decreasing at  $60^\circ\text{C}$ . For lower concentrations a decrease in yield stress is observed with increasing temperature which is well-captured in non-dimensional form by an Arrhenius function. Similar trends are observed for the consistency index. The power-law index is approximately constant across the whole temperature between 0.4 and 0.6

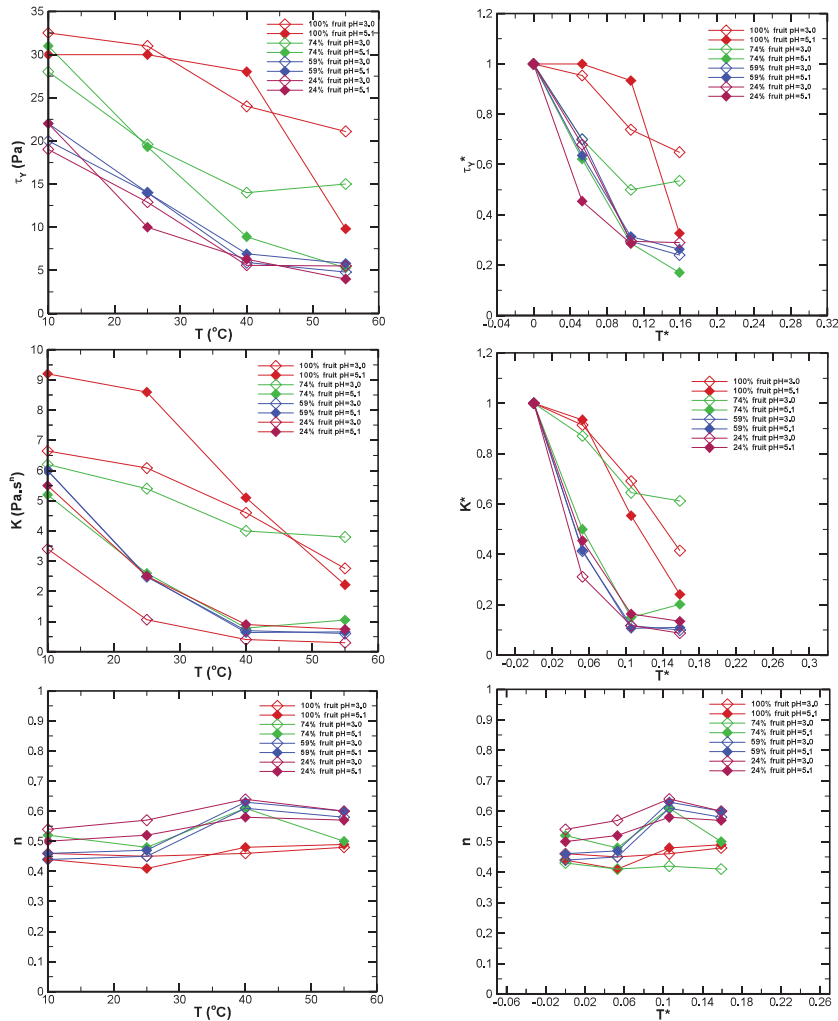
for all concentrations and fluids. Whilst there is some non-monotonic behaviour (e.g. 74% fruit pH=5.1), generally the weak increase in  $n$  is characteristic of the other fruit data sets.



**Figure 1.** Variation of Herschel-Bulkley model parameters of pumpkin puree products with temperature from Dutta et al [6]; (a) Dimensional yield stress (Pa) versus temperature (°C); (b) Non-dimensional yield stress versus non-dimensional temperature; (c) Dimensional consistency index (Pa.s<sup>n</sup>) versus temperature (°C); (d) Non-dimensional consistency index versus non-dimensional temperature; (e)

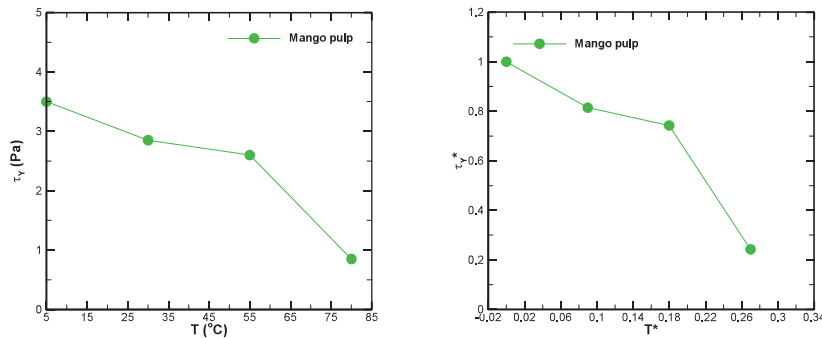


power-law index versus temperature ( $^{\circ}\text{C}$ ); (f) power-law index versus non-dimensional temperature.



**Figure 2.** Variation of Herschel-Bulkley model parameters of banana puree products with temperature from Guerrero and Alzamora [5]; (a) Dimensional yield stress (Pa) versus temperature ( $^{\circ}\text{C}$ ); (b) Non-dimensional yield stress versus non-dimensional temperature; (c) Dimensional consistency index ( $\text{Pa}\cdot\text{s}^n$ ) versus temperature ( $^{\circ}\text{C}$ ); (d) Non-dimensional consistency index versus non-dimensional

temperature; (e) power-law index versus temperature ( $^{\circ}\text{C}$ ); (f) power-law index versus non-dimensional temperature.



**Figure 3.** Variation of Herschel-Bulkley model parameters of mango pulp with temperature from Bhattacharya [59]; (a) Dimensional yield stress (Pa) versus temperature ( $^{\circ}\text{C}$ ); (b) Non-dimensional yield stress versus non-dimensional temperature

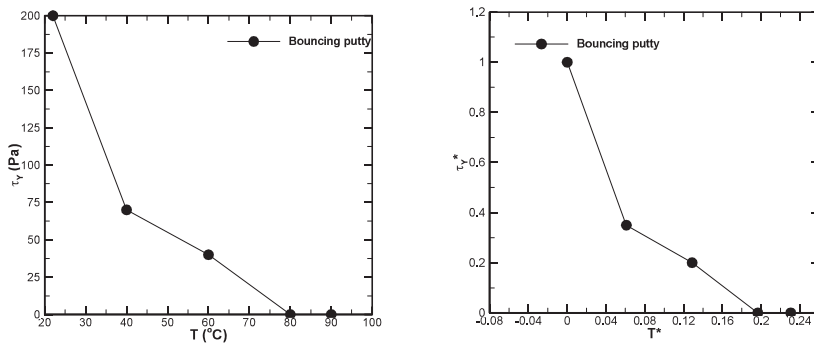
Bhattacharya [59] measured the temperature dependence of the yield stress for mango pulp (total solids content  $\sim 20\%$ ) using a direct method of stress relaxation in a smooth co-axial cylinder between 5 and 80  $^{\circ}\text{C}$ . In this method a shear-rate of  $3\text{s}^{-1}$  was applied for 120s, rotation stopped, and 300s was allowed for relaxation while continuously monitoring stress values: the yield stress was the stress measured at the end of this relaxation period (or averaged over this period, see e.g. Figure 4 in [59]). This limited data set of four points is plotted in **Figure 3** where it can be seen that the yield stress decreases only quite weakly by about 25% between 5 and 60 $^{\circ}\text{C}$  before a more rapid drop off at 80 $^{\circ}\text{C}$ . This data-set is not well fit by either a linear or Arrhenius function across the entire temperature range.

The effect of Ohmic or Joule heating on the rheology of quince nectar was studied by Bozkhurt and Icier [60]. They measured the rheology of this nectar at 20 $^{\circ}\text{C}$  and then after both conventional and Ohmic heating at 65, 70 and 75 $^{\circ}\text{C}$ . Although they state that “Significantly higher magnitude of yield was observed for nonheated nectar as compared to heated samples” the value of yield stress at 20 $^{\circ}\text{C}$  is unfortunately not provided in the paper: between 65-75 $^{\circ}\text{C}$  there was no effect on power-law index but the consistency index and yield stress were observed to decrease weakly with temperature.

#### 4.2 Bouncing putty

Hailemariam and Mulugeta [62] measured the temperature-dependent rheology of “bouncing putties” (silastic silicone rubber) used as rock analogues in the

temperature range 23-90°C using a capillary rheometer (shown in **Figure 4**). They found that yield stress decreases significantly with temperature, dropping from ~200Pa at 22°C to 40Pa at 60°C and disappearing altogether at  $T \sim 80^\circ\text{C}$ . An Arrhenius fit over the range  $22 \leq T \leq 60^\circ\text{C}$  works reasonably well with constants  $a_{BP} = 456\text{Pa}$  and  $b_{BP} = -0.042^\circ\text{C}^{-1}$ .



**Figure 4.** : Variation of Herschel-Bulkley model parameters of “bouncing putty” (silastic silicone rubber) with temperature from Hailemariam and Mulugeta [62]; (a) Dimensional yield stress (Pa) versus temperature ( $^\circ\text{C}$ ); (b) Non-dimensional yield stress versus non-dimensional temperature

#### 4.3 Mud suspensions

Coussot and Piau [8] measured the rheology of natural fine mud suspensions (water-Riffol mixtures ( $C_v = 40.1\%$ ) and water-Verdarel mixtures ( $C_v = 31.8\%$ ) where  $C_v$  is solid concentration by volume). A roughened parallel plate geometry was used to remove the possibility of slip and over a temperature range of 5-20°C temperature was observed to have a “negligible effect on the flow curve”.

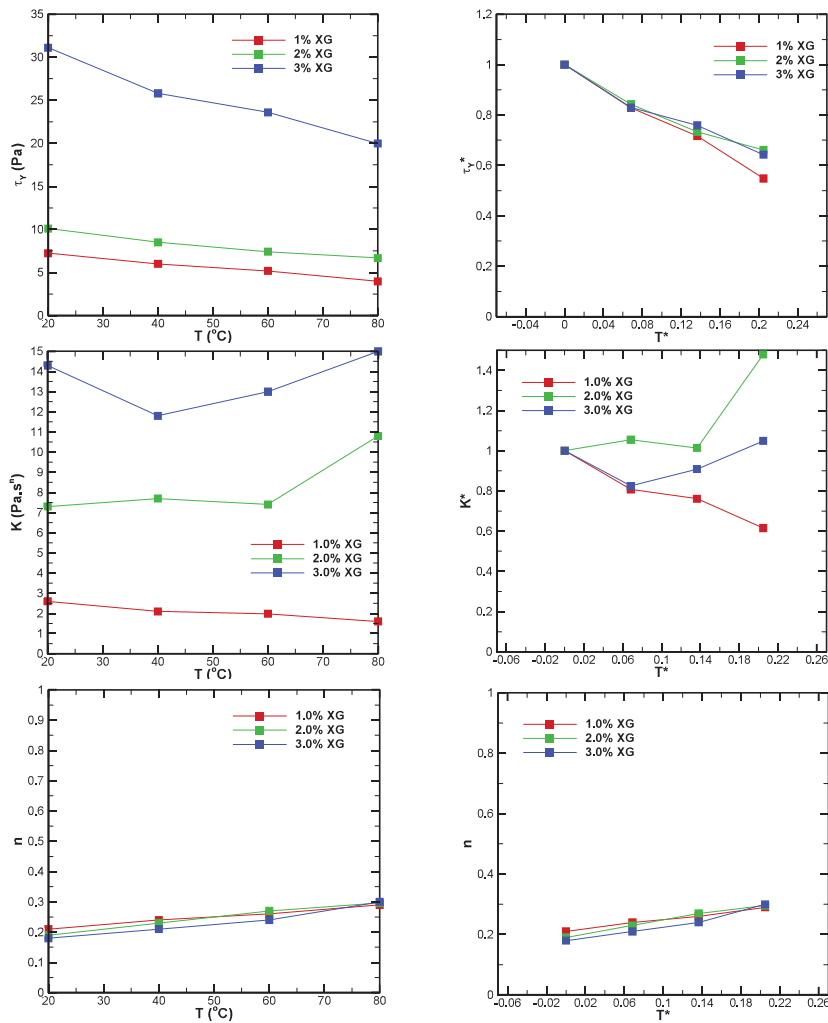
#### 4.4 Hydrocolloids

At high enough concentration certain hydrocolloids have been claimed to exhibit viscoplastic behaviour. Marcotte et al [7] studied the effect on temperature on a xanthan gum at three concentrations (1, 2 and 3%) across a temperature range of 20 -80°C using a smooth co-axial cylinder device (the paper also contains data for a number of other hydrocolloids but, for these data sets, no yield stress was reported). The xanthan gum data set is plotted in **Figure 5** and for arabic gum/guar gum and arabic gum/xanthan gum mixtures in **Figure 6** [64]. For the yield stress and power-law index clear trends are apparent and very nice data collapse is observed in non-dimensional form: the yield stress decreasing exponentially (with the exception of the lowest value occurring for lowest concentration and highest temperature) and the power-law index very weakly increases. The variation of the

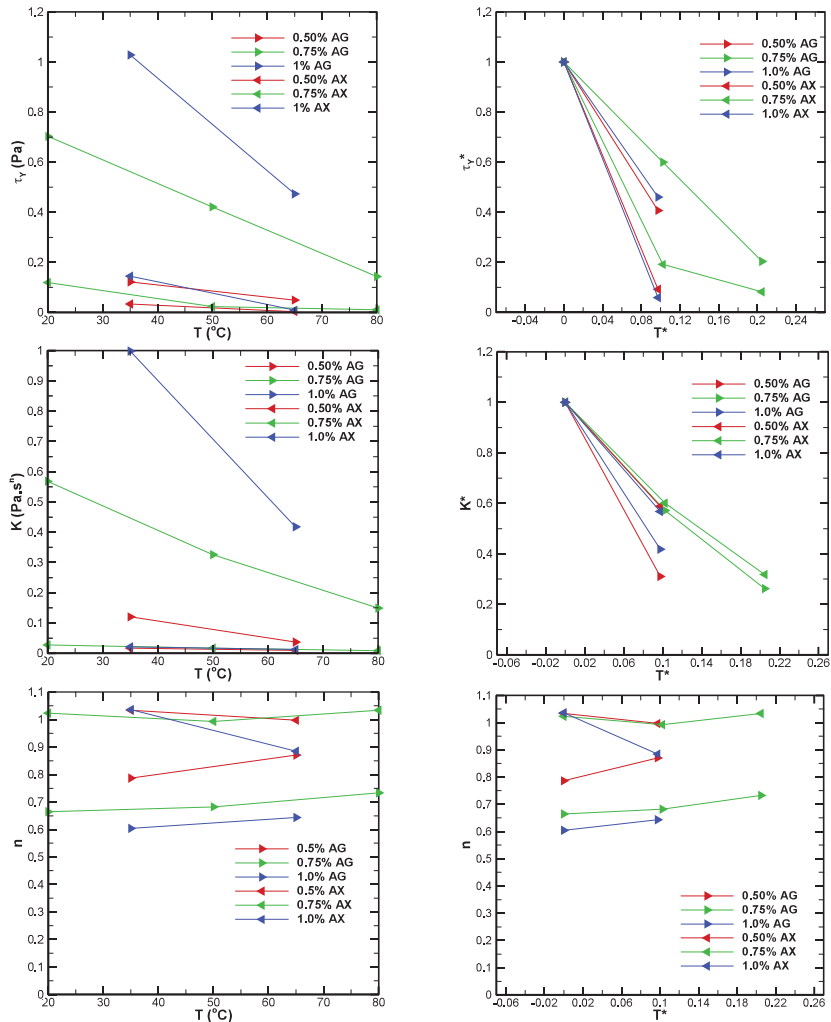
consistency

index

data



**Figure 5.** Variation of Herschel-Bulkley model parameters of xanthan gum with temperature from Marcotte et al. [7]; (a) Dimensional yield stress (Pa) versus temperature (°C); (b) Non-dimensional yield stress versus non-dimensional temperature; (c) Dimensional consistency index (Pa.s<sup>*n*</sup>) versus temperature (°C); (d) Non-dimensional consistency index versus non-dimensional temperature; (e) power-law index versus temperature (°C); (f) power-law index versus non-dimensional temperature.



**Figure 6.** Variation of Herschel-Bulkley model parameters of arabic gum/guar gum (AG) and arabic gum/ xanthan gum (AX) mixtures with temperature from Ahmed et al. [64]; (a) Dimensional yield stress (Pa) versus temperature (°C); (b) Non-dimensional yield stress versus non-dimensional temperature; (c) Dimensional consistency index (Pa.s<sup>*n*</sup>) versus temperature (°C); (d) Non-dimensional consistency index versus non-dimensional temperature; (e) power-law index versus temperature (°C); (f) power-law index versus non-dimensional temperature.

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is more confusing and is seen to both increase and decrease with temperature for different concentrations.

#### **4.5 Electro-rheological fluids**

Two papers have reported the effect of temperature on different electro-rheological fluids with very high yield stresses (on the order of kPa). Zhang et al [62] reported data for strontium titanate microparticles in silicon oil measured using a smooth parallel-plate geometry in the range 20-80°C. At 20°C the yield stress was ~10kPa and was seen to increase up to 20kPa at 40°C and then remain approximately independent of temperature from 40-80°C (constant within 20%). Lu and Zhao [63] reported data for two different electro-rheological fluids: polyaniline (PANI) and montmorillonite (MMT) nanocomposites both in silicone oil in the range 10-100°C using a smooth co-axial cylinder device. Yield stress was found to decrease with temperature for MMT from ~0.5 kPa at 10°C to ~0.25 kPa at 100°C. The yield stress was found to exhibit complex non-monotonic variation for PANI-MMT but the variation about the mean value (~3kPa) is less than 10% across the entire temperature range.

#### **4.6 Carbopol**

As already discussed in Section 1, Weber et al [26] studied the thermo-dependent properties of “the” model yield-stress material, aqueous solutions of the commercial polyacrylic acid – Carbopol - in a detailed and systematic manner across a fairly modest temperature range of 12 - 38°C. For three different Carbopol concentrations (0.10, 0.15 and 0.20%), they observed a temperature invariance of most of the Herschel-Bulkley model parameters with the exception of the yield stress where a complex weakly non-monotonic dependence was observed although the variation about a mean value was only approximately  $\sim\pm 15\%$ . Other studies from a different laboratory [27,28], admittedly over wider temperature ranges of 10-60°C [27] and 5-85°C [28] and for different grade of Carbopol, have found different behaviour for 0.2% Carbopol namely that the yield stress and power-law index remained approximately constant and the consistency index exhibited Arrhenius-type variation ( $a = 2.77\text{Pa}\cdot\text{s}^n$  and  $b = -0.011^\circ\text{C}^{-1}$  [28]). The discrepancies between these two results are probably attributable to the differing focus of the various papers: the purpose of the study of Weber et al [26] was to investigate precisely this issue and they went to great care to avoid slip by using “cleated” parallel plates and, to avoid issues associated with evaporation, limited the temperature range of the experiments. In contrast both of the other studies were primarily focused on fluid flow/heat transfer problems, used smooth geometries and were more interested in broad trends across wide temperature ranges. Indeed the scatter in the rheology data of Peixinho et al [28] in the same temperature range as Weber et al [26], i.e 12- 38°C, is greater than the measured differences observed by Ref. [26] for the yield stress. In addition the consistency index of Peixinho et al [28] across the same range could easily be fit by a constant



value. Thus, the small differences in conclusions between the studies are due to the different temperature ranges and experimental approaches used. Given that the effect of temperature between 12 -3 °C on the yield stress is minor ( $\sim\pm 15\%$ ) and smaller on the consistency index and power-law index, it would seem reasonable to assume temperature-independent properties for this model yield stress system in this temperature range.

## 5. Conclusions

Analytical and numerical studies of combined heat transfer and fluid flow problems involving materials which exhibit yield stress fluid behaviour are varied and numerous; encompassing both mixed/forced convection and, more recently, solely natural convection. With few exceptions, most of these studies have assumed that the yield stress model parameters, either the Bingham or Herschel-Bulkley model constants, remain temperature independent. The few studies which did try to incorporate such effects assumed an Arrhenius-type thermo-dependence of the consistency index and assumed the yield stress and power-law index remain temperature invariant. In the current paper we have attempted to survey the available experimental literature for temperature dependence of the rheological properties of various classes of materials which exhibit yield stress fluid type behaviour such as concentrated fruit juices/purées, mud suspensions, hydrocolloids, a model system (Carbopol) and electro-rheological fluids. Although care must be taken in over interpreting the data across different classes of materials where the physical origin of the yield stress differs and as many of the available data sets have been obtained using smooth geometries, and therefore may contain slip artefacts, some general patterns do emerge from the data. Generally yield stresses and plastic viscosity/consistency index values appear to decrease with increasing temperature whilst the power-law index appears to be less affected although generally slightly increasing with temperature. However there are sufficient examples of behaviour which differs significant from this simple picture: with either temperature-invariance of the yield stress or even weak non-monotonic effects for certain yield stress fluids, as to make entirely general claims impossible (as might be anticipated given the different physical origins of the yielding behaviour in different classes of materials). Given that most of the data reported in the literature, and summarised here, for the effect of temperature on “yield stress” fluids has not used the most robust methods either to avoid potential issues arising due to slip or to directly measure the yield stress, there is a clear need for careful experiment studies, along the lines of the recent study of Weber et al [26], for a range of different yield stress fluids. In addition, for a number of the systems summarised here the yield stresses reported are so low (less than 0.1Pa) that it may be that more accurate/modern rheometers will reveal such materials to simply possess a constant zero shear-rate Newtonian viscosity on the order of 1-10Pa.s rather than exhibiting behaviour more in-line with the “yield stress” concept.

**Tables:** can be found online at <http://pcwww.liv.ac.uk/~robpoole/Tables.pdf> or obtained directly from author (robpoole@liv.ac.uk).

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### **BioBone: 9th International Workshop on Interfaces New Frontiers in Biomaterials, 16<sup>th</sup>-18<sup>th</sup> April 2018, Santiago de Compostela, Spain**

I recently had the opportunity to visit Santiago de Compostela in the northern Spanish region of Galicia to visit my first conference in April, after beginning my PhD in September 2017. Santiago is located in the Galician mountains, inside a valley with hills surrounding every side. The city itself is very old with small winding streets and big plazas with spectacular cathedrals. The conference brought together chemists, biochemists, material scientists and engineers from vastly different backgrounds to present their work on developing new biomaterials.

Over the course of the three days there were 45 talks in total and with two short poster sessions per day and a chance for each student presenting a poster to do a brief rapid fire presentation to pitch their work. Many talks focused on creating new structures for improving the quality of life for people with various conditions. Others focused on looking at natural materials and using biomimetics to understand and harness some of the intricate and beautiful structures found in nature. The third day was largely about 3D printing with a particular focus on robocasting and the 3D printing of ceramics.