Reconstruction Algorithm for Primary Channel Statistics Estimation Under Imperfect Spectrum Sensing

Ogeen H. Toma*†, Miguel López-Benítez*‡, Dhaval K. Patel§ and Kenta Umebayashi¶

*Department of Electrical Engineering and Electronics, University of Liverpool, United Kingdom

†Department of Energy Engineering, Duhok Polytechnic University, Iraq

‡ARIES Research Centre, Antonio de Nebrija University, Spain

§School of Engineering and Applied Science, Ahmedabad University, India

¶Graduate School of Engineering, Tokyo University of Agriculture and Technology, Japan

Email: ogeen.toma@liverpool.ac.uk, M.Lopez-Benitez@liverpool.ac.uk

***Abstract*—Statistical information of primary channels has received considerable research interest in the recent years. This is due to the important role that these statistics play in improving the performance of Dynamic Spectrum Access (DSA)/Cognitive Radio (CR) systems. Although a DSA/CR system has no initial knowledge about the statistical information of the primary channels, these statistics can be estimated from the observations of spectrum sensing. However, spectrum sensing is not perfect in the real world and sensing errors are likely to occur during DSA/CR operation, which in turn leads to incorrect estimation of primary channel statistics as well. As a result, several attempts have arisen to reconstruct the estimated periods of the primary channel occupancy patterns which are affected by the sensing errors, in order to provide more accurate estimation for the statistical information. However, all the reconstruction methods available in the literature assume the perfect knowledge of the primary users' minimum occupancy time. In this context, this work proposes the first reconstruction method that does not require any prior knowledge about the primary channel activity and inactivity patterns while achieving almost the same performance achieved by the latest reconstruction methods available in the literature, making it significantly attractive and feasible in practical implementation scenarios.**

***Index Terms*—Cognitive radio, dynamic spectrum access, spectrum sensing, primary channel statistics, reconstruction algorithms.**

I. Introduction

Due to the underutilization of the frequency spectrum resulting from frequency allocation policy, and to the ever-increasing need for frequency bands to satisfy the fast growing technologies in the wireless communications sector, many researchers' focus has been turned to the available spectrum holes [1] within the licensed spectrum in order to exploit the available spectrum resources more efficiently. Therefore, the emerged Dynamic Spectrum Access (DSA) [2] based on the Cognitive Radio (CR) [3] technology is a promising solution to allow the inactivity periods (i.e., spectrum holes) of the primary channels to be exploited by the secondary users (SUs) without causing any harmful interference to the primary users (PUs) [4]. Spectrum sensing is the key enabler technology for DSA/CR, by which the instantaneous state of the primary channel can be detected and therefore the activity and inactivity patterns (i.e., idle/busy periods) of the channel can be observed based on the outcomes of sensing decisions. Besides their main purpose, sensing decisions can be further exploited to provide a broad range of statistical information about the primary channels, which can be very beneficial for enhancing the performance of DSA/CR systems and therefore the efficiency of spectrum usage. Perfect Spectrum Sensing (PSS) can be assumed when the DSA/CR system operates under sufficiently high Signal-to-Noise Ratio (SNR) conditions. However, in practice this assumption is not realistic because DSA/CR receivers are more likely to operate in low SNR environments and sensing errors are likely to occur, which results in Imperfect Spectrum Sensing (ISS). Therefore, in ISS the observed idle/busy periods of the primary channel can be highly inaccurate and the estimated statistics from these periods will be inaccurate as well. In this context, several attempts have arisen in order to palliate the degrading effects of sensing errors on the estimated statistics by reconstructing the estimated idle/busy periods observed under ISS.

A reconstruction technique was first introduced in [5], which presented three different methods to reconstruct the idle/busy periods estimated under ISS in order to improve the estimation of the statistical information of the primary channel. Then [6] developed three additional Reconstruction Algorithms (RAs) which outperform the methods presented in [5]. However, all the above mentioned methods require perfect knowledge about the minimum period that the PU is active or in active within the primary channel. In a practical scenario such information may be unknown to DSA/CR systems. In this context, this work proposes a novel RA that can reach the performance of the latest RA, which is presented in [6], but without requiring any additional knowledge about the primary channel and assuming that the DSA/CR system is blind to the PUs activity patterns.

The rest of the paper is organised as follows. First, Section II presents the system model. Then Section III introduces the reconstruction method, which covers the previous RAs and the proposed one. The simulation methodology is given in Section

IV, whereas the simulation results for all the RAs are compared in Section V. Finally, conclusions are drawn in Section VI.

II. System Model

In this work we consider a single primary channel allocated to a single PU. The occupancy patterns of the PU within this channel are represented in the time-domain by a sequence of idle/busy periods. The durations of the idle/busy periods are random variables and can be modelled to follow a particular distribution. Based on the experimental measurements in [7], it is found that the Generalised Pareto (GP) distribution is the best fit representation for those periods. In the DSA/CR side, the activity and inactivity patterns of the primary channel are unknown. Therefore, spectrum sensing can be performed periodically, with a sensing period of T_s, to observe the instantaneous state of the channel. A binary decision can then be made at every sensing event $\mathcal{H}_i$ (where i indicates the state of the channel), to be either $\mathcal{H}_0$ for idle or $\mathcal{H}_1$ for busy state. Based on these binary decisions, the time durations of the idle/busy periods of the primary channel can be estimated as $\widehat{T}_i$, which represent the PSS estimation of the original periods T_i as shown in Fig. 1a. As it can be seen, the estimated periods under PSS (i.e., when no sensing errors are assumed within the sensing events $\mathcal{H}_i$) provides an acceptable level of accuracy to represent the original idle/busy periods of the channel, and this accuracy is only affected by the time resolution of the periodic sensing events (i.e., T_s), which is investigated in detail in [8]. However, in the real world, DSA/CR systems may operate under low SNR conditions and sensing errors are likely to occur within the sensing events $\mathcal{H}_i$, which results in ISS. These sensing errors can be either false alarms (when an idle state of the channel is detected as a busy state) or missed detections (when a busy state of the channel is detected as an idle state), which can occur at every sensing event with a probability of false alarm P_{fa} and probability of missed detection P_{md}. In ISS, when errors (false alarms and missed detections) occur in the sensing decisions, the idle/busy periods will be estimated as shorter fragments of the original periods. In other words, a single original period would lead to multiple of shorter idle/busy periods due to sensing errors. Therefore the estimated periods under ISS, referred to as $\breve{T}_i$, will be highly inaccurate as compared to the original periods as shown in Fig. 1b.

III. Reconstruction Methods

In order to understand how the reconstruction technique can help obtain more accurate estimation for the primary channel statistics under ISS, we consider the estimation of the statistical distribution of the primary channel periods. The estimated distribution of these periods under ISS will be highly inaccurate compared with the original distribution. As shown in Fig. 1b, a single false alarm error could corrupt the estimation of T_0 by producing three new shorter period durations, which are $\breve{T}_0, \breve{T}_1$, and $\breve{T}_0$. In addition, it can be noticed that the estimated busy period $\breve{T}_1$ is resulting from the false alarm itself with a duration equal to the sensing period

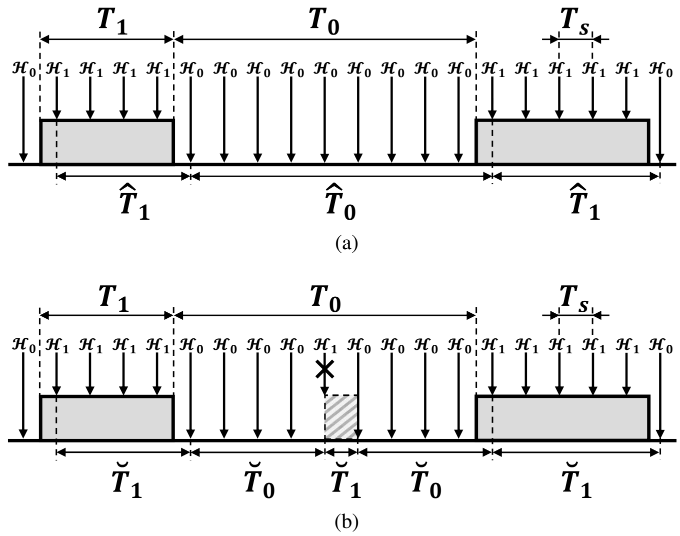

Fig. 1. Estimation of idle/busy periods based on spectrum sensing: (a) under perfect spectrum sensing (PSS), (b) under imperfect spectrum sensing (ISS) [5].

T_s, which should be shorter than the minimum period μ_i of the primary channel (i.e., $T_s < \mu_i$), where i refers to the state of the period ($i = 0$ for idle period and $i = 1$ for busy period). Since the majority of the sensing errors could appear as individual short periods with a duration of T_s, these errors can be easily identified when the minimum period μ_i of the channel is known. This inspired [5] to propose three methods to reconstruct the sensing decisions affected by the errors in order to provide more accurate estimation for the statistical distribution of the idle/busy periods under ISS. Given a set $\{\breve{T}_{i,n}\}_{n=1}^{N_{iss}}$ of N_{iss} periods observed under ISS, the following methods can be employed when the value of μ_i is perfectly known [5]:

A. Method 1 (from [5])

This method simply assumes that any estimated period under ISS shorter than the minimum period μ_i is an error. Therefore, it discards any period $\breve{T}_{i,n} < \mu_i$ and does not include it in the computation of the distribution of the channel periods under ISS.

B. Method 2 (from [5])

This method not only discards the periods which are $\breve{T}_{i,n} < \mu_i$, but also discards the preceding ($\breve{T}_{i,n-1}$) and the subsequent ($\breve{T}_{i,n+1}$) periods since these periods could be the remaining fragments of the original period as illustrated in Fig. 1b.

C. Method 3 (from [5])

Instead of discarding the periods, this method suggests to reconstruct the incorrect periods by joining all the possible fragments of an original period together. This can be performed by combining the periods which are $\breve{T}_{i,n} < \mu_i$ with the preceding and subsequent periods as $(\breve{T}_{i,n-1} + \breve{T}_{i,n} + \breve{T}_{i,n+1})$ and considering the resulting value as a single period of the opposite type to $\breve{T}_{i,n}$.

The explained methods of [5] can noticeably improve, to some extent, the accuracy of the estimated statistics of the primary channel under ISS. However, as shown in [5], Method 1 can perform better than the other methods. This is because, with the reconstruction technique in Method 3 the reconstructed periods could sometimes be incorrectly considered as the opposite type of the original type of the periods and this degrades the accuracy of statistics estimation. In this context, [6] developed three other RAs, which could outperform the methods in [5]. These RAs also assume perfect knowledge of the minimum period μ_i of the primary channel. In this work we consider the most significant RA proposed by [6] which can be referred to as Method 4.

D. Method 4 (from [6])

In this method, a threshold $\beta T_s < \mu_i$ is defined (where $\beta \in \mathbb{N}^+$), which can be tuned as explained in [6]. Starting from an initial observed period $\breve{T}_{i,n}$ that has a duration less than the threshold (i.e., $\breve{T}_{i,n} < \beta T_s$), all the subsequent periods (idle and busy) are checked until a period of the opposite type with a duration greater than the threshold (i.e, $\breve{T}_{1-i,n+N} > \beta T_s$) is found. These periods are then reconstructed by summing $(\breve{T}_{i,n} + ... + \breve{T}_{i,n+N-1})$ to form a single reconstructed period of the same type as $\breve{T}_{i,n}$. The next period $\breve{T}_{1-i,n+N}$, which is the opposite type of the previously reconstructed period, is then taken as the new initial period for a new reconstruction. This process is repeated over the whole sequence of the observed periods in an attempt to reconstruct the whole set of fragments of the original idle/busy periods.

E. Method 5 (proposed method)

All the previous discussed methods, including the latest RA in Section III.D, assume a perfect knowledge of the minimum period μ_i of the PU occupancy patterns within a particular channel. In a practical scenario such information may be unknown to the DSA/CR system. Therefore, this work proposes a novel reconstruction method which can reach the performance achieved by [6], but without requiring any additional knowledge and assuming that the DSA/CR system is blind to the PUs activity and inactivity patterns. In this method we develop a new algorithm that depends on another parameter, which is the mean of idle/busy periods m_i, rather than the minimum period μ_i. This algorithm takes the advantage of the novel estimator recently proposed in [9] for estimating the mean of idle/busy periods accurately even when the probability of error is high (i.e., no prior knowledge is required for this parameter since it can be estimated accurately based on the outcomes of ISS).

As explained in Section II, the estimated periods under ISS are divided into a number of shorter fragments due to sensing errors. As a result, the calculated mean of these periods will be much lower than its true value (i.e., the mean when there are no sensing errors). Based on this observation, the proposed algorithm reconstructs the periods in an iteration process and in each iteration the mean of the reconstructed periods will be calculated until it reaches the value of the mean obtained using the estimator proposed in [9]. To explain this in more detail, let us consider the first iteration as an example. In this iteration the shortest periods, which are $\breve{T}_{i,n} = T_s$, will be reconstructed first as ($\breve{T}_{i,n-1} = \breve{T}_{i,n-1} + \breve{T}_{i,n} + \breve{T}_{i,n+1}$), then the mean of the reconstructed periods will be calculated and compared with the estimated mean using the estimator in [9]. If the calculated mean is lower than the estimated one, the second iteration will take place where the second shortest periods, which are $\breve{T}_{i,n} = 2T_s$, will be reconstructed this time and in the same way as in the first iteration. Therefore, this process will be repeated until the calculated mean of the reconstructed periods reaches the estimated value of mean using the mean estimator proposed in [9]. This proposed method relies on the fact that the estimator proposed in [9] can produce a highly accurate estimation of the true mean period, even in the presence of sensing errors, and thus can be exploited as an indicator of when periods are reconstructed correctly, without any prior knowledge of the PU activity pattern. The details of the considered estimator for the mean period are here omitted but can be found in [9]. The steps of this method can be further illustrated in Algorithm 1. Notice that the mean period is here used as a reference to determine when the periods are correctly reconstructed, however once the process is finished, other statistics (not only the mean) can also be estimated.

Algorithm 1: Proposed Method 5

Input: $(\breve{T}_i)$ The estimated periods under ISS
Output: $(\bar{T}_i)$ The reconstructed periods

Calculate the mean $(\breve{m}_i)$ of the periods under ISS
Estimate the mean (m_i) of the periods using the estimator in [9]
$k = 0$
$\bar{T}_i = \breve{T}_i$
while $\breve{m}_i < m_i$ **do**
  $k = k + 1$
  for *each* $\breve{T}_{i,n} = kT_s$ **do**
    $\bar{T}_{i,n-1} = \breve{T}_{i,n-1} + \breve{T}_{i,n} + \breve{T}_{i,n+1}$
  end
  $\breve{m}_i = \mathbb{E}(\bar{T}_i)$ Calculate the mean of the reconstructed periods
end
return $(\bar{T}_i)$

IV. Simulation Methodology

To evaluate the performance of the proposed algorithm in Method 5 along with the previous methods we simulate the model using MATLAB, following the same simulation procedure as in [5]:

1) Generate a sequence of idle/busy periods with random durations T_0/T_1 that follow a generalised Pareto distribution.
2) Perform spectrum sensing on the generated periods in step 1 (assuming PSS) in order to obtain the sensing

decisions $\mathcal{H}_0/\mathcal{H}_1$ that would be observed by DSA/CR system when employing a sensing period of T_s.

3) Introduce random sensing errors (based on the selected values of P_{fa} and P_{md}) in the $\mathcal{H}_0/\mathcal{H}_1$ sequence obtained in step 2 to represent the ISS.
4) Calculate $\breve{T}_0/\breve{T}_1$ durations based on the new $\mathcal{H}_0/\mathcal{H}_1$ sequence obtained in step 3 to represent the estimated idle/busy periods under ISS.
5) Apply one of the reconstruction methods to reconstruct the ISS periods obtained in step 4 in order to retrieve the original periods. The reconstructed periods can be referred to as $\bar{T}_0/\bar{T}_1$.
6) Calculate the statistical Cumulative Distribution Function (CDF) of the reconstructed periods in step 5 as well as the unreconstructed periods in step 4, and compare them with the CDF of the original periods in step 1.

All discussed reconstruction methods in this work are implemented, and therefore the above simulation steps are repeated five times, each time with a different reconstruction method used in step 5. The accuracy of the reconstruction methods is evaluated in step 6 by comparing the estimated CDF of periods (after and before reconstruction) with the CDF of the original periods. Since the original period durations (T_i) in step 1 are continuous values, their CDF is continuous as well. In contrast, the periods in step 4 and 5 ($\breve{T}_i$ and $\bar{T}_i$ that are estimated under ISS before and after reconstruction, respectively) are discrete values, and their CDF therefore is discrete as well. Since it is impossible to compare between continuous and discrete distributions, the discrete distribution is interpolated in order to be comparable with the continuous one. This comparison can be performed using the Kolmogorov-Smirnov (KS) distance (e.g., comparing the CDF of the original periods in step 1 with the CDF of the ISS periods before reconstruction in step 4) as:

$$D_{KS} = \sup_{T} \mid F_T(T) - F_{\breve{T}}(T) \mid \tag{1}$$

where $F_T(T)$ represents the CDF of the original periods and $F_{\breve{T}}(T)$ is the interpolated version of the discrete distribution $F_{\breve{T}}(\breve{T})$, D_{KS} denotes the KS distance of the estimated distribution with respect to the original distribution.

Similarly, we can use the following to compare the CDF of the original periods in step 1 with the CDF of the periods after reconstruction in step 5:

$$D_{KS} = \sup_{T} \mid F_T(T) - F_{\bar{T}}(T) \mid \tag{2}$$

In this simulation we select the same parameters as in [5] and [6] in order to obtain a fair comparison. Therefore, we consider, without loss of generality, the idle periods drawn from a Generalised Pareto distribution using the following parameters: location (minimum period) $\mu_0 = 10$ t.u., scale $\lambda_0 = 30$ t.u., and shape $\alpha_0 = 0.25$. These parameters result in a mean period of $\mathbb{E}(T_0) = 50$ t.u. when using a duty cycle of $\Psi = 0.5$. However, for our proposed method these parameters are considered unknown to the DSA/CR system.

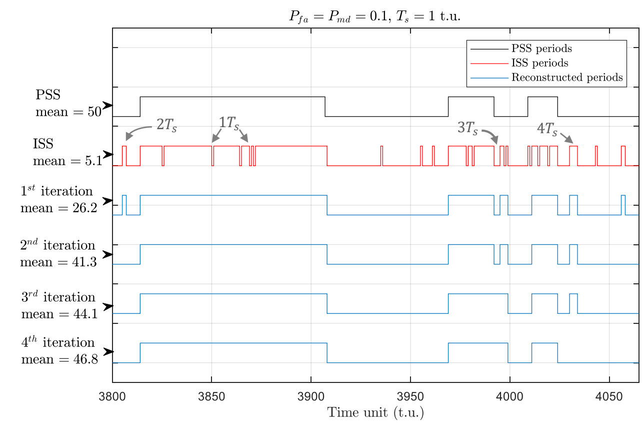

Fig. 2. Reconstruction process of the idle/busy periods in time-domain using the proposed algorithm and comparing it with the periods under PSS and ISS.

V. Simulation Results

A. Proposed Method Operation

In order to understand how the periods are reconstructed in the time-domain using the proposed RA of Method 5 and how the calculated mean converges to the true mean, a sequence of the estimated idle/busy periods under ISS selected from the simulation results is shown in Fig. 2 to illustrate the reconstruction process in each iteration and its corresponding calculated mean. We first generate an original sequence of the idle/busy periods with a mean equal to 50 time units (t.u.), then by employing a sensing period of $T_s = 1$ t.u. the estimated PSS sequence will be as shown in Fig. 2 (the first sequence). By applying sensing errors to this sequence with $P_{fa} = P_{md} = 0.1$ the ISS periods can be obtained as in the second sequence of the same figure. As it can be noticed, the calculated mean of the ISS sequence, about (5.1 t.u.), is much lower than its original value. The following sequences in Fig 2 represent the reconstruction process in each iteration. As it can be seen, the shortest periods, which are equal to $1T_s$, have been reconstructed in the first iteration and the calculated mean has become 26.2 t.u.. Similarly, the periods $2T_s, 3T_s$ and $4T_s$ have been reconstructed in the second, third and fourth iteration, respectively. In addition the calculated mean has converged gradually to the original value as $41.3, 44.1, 46.8$ t.u.. Since the original value of the mean should be considered unknown to the DSA/CR system, we use the mean estimator proposed in [9], which can provide a satisfactory accuracy for estimating the true mean. Therefore, in this case the estimated mean was $m = 44.4$ t.u., which was used as a threshold for the proposed algorithm to break the loop (i.e., stop the reconstruction process) whenever the calculated mean exceeds this value.

B. Performance Evaluation

Fig. 3 illustrates the accuracy of estimating the CDF under ISS using different methods, which is presented in terms of the KS distance versus the sensing period T_s. Regardless of

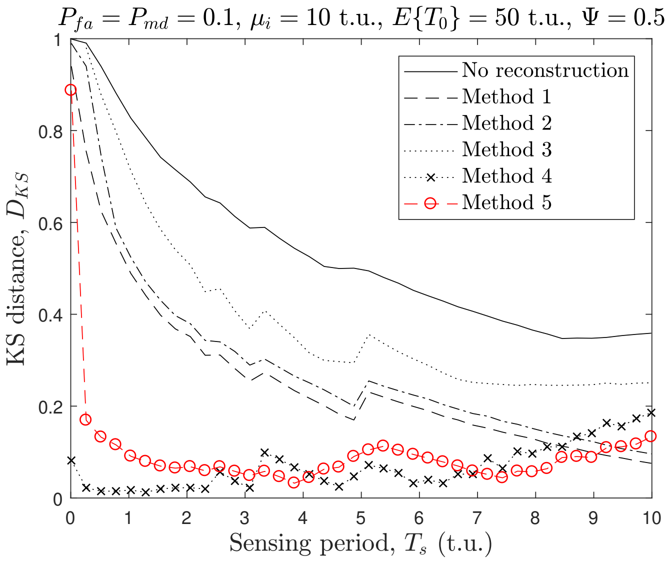

Fig. 3. Performance of the reconstruction methods.

which reconstruction method is used, the improvement in the CDF estimation can be easily noticed as compared with the case when no reconstruction method is used. Knowing that as the KS distance approaches zero as the estimated CDF under ISS approaches the original CDF value.

As it can be noticed, the methods proposed by [5] (Method 1,2 and 3) can improve, to some extent, the estimation of the CDF under ISS. However, Method 1 shows better performance than Method 2 and 3 as explained in Section III. On the other hand, the RA developed by [6] in Method 4 has further improved the accuracy of estimating the distribution under ISS, and it could outperform Method 1, 2 and 3 since its KS distance approaches closer to zero. The proposed algorithm in this work, which is referred to as Method 5, has been also examined and compared with the previous methods. As shown in Fig. 3, Method 5 approaches the performance achieved by Method 4, while it outperforms Method 1, 2 and 3 as well. It can also be noticed that Method 5 outperforms Method 4 when the value of the sensing period T_s is high, while it performs slightly worse when T_s is low. This is due to the fact that the accuracy of the mean estimator in [9] decreases as the employed sensing period T_s decreases, which in turn will degrade the performance of this method at the very low T_s. Meanwhile, the accuracy of the mean estimator in [9] increases as T_s increases (this is because there will be lower number of sensing errors for higher T_s, which is also explained in [5]), thus Method 5 outperforms Method 4 for higher sensing period T_s. Overall, the proposed Method 5 can approximately achieve the same accuracy of the latest reconstruction method in the literature (i.e., Method 4) without requiring any knowledge about the activity patterns of the primary channel, opposite to all the previous methods (1-4) which always assume a perfect knowledge of the minimum period μ_i of the primary channel.

VI. Conclusion

The performance of DSA/CR systems can be improved significantly by knowing the statistical information of the primary channel. Therefore, it is important to obtain this information accurately even when the DSA/CR receivers operate under low SNR conditions (i.e., under ISS). Reconstruction technique is a considerable solution to overcome the degrading effects of sensing errors in the estimation of the statistical parameters. However, all the available reconstruction methods in the literature assume a perfect knowledge of the minimum time that the primary channel is occupied. This assumption is impractical since the DSA/CR will require an external system to provide such information perfectly. On the other hand, this work has proposed the first reconstruction method that does not require any prior knowledge about the primary channel (i.e., DSA/CR is considered blind to the primary channel activity and inactivity patterns), and at the same time it could approach the performance achieved by the latest reconstruction method available in the literature as it has been proven by means of simulation.

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