Dynamic Effects of Minimum Wage on Growth and Innovation in a Schumpeterian Economy

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Abstract

We explore the dynamic effects of minimum wage in a Schumpeterian model with endogenous market structure and obtain the following results. First, raising the minimum wage decreases the employment of low-skill workers and increases the unemployment rate. Second, it decreases the level of output. Third, it decreases the transitional growth rate of output but does not affect the steady-state growth rate. Our quantitative analysis shows that the magnitude of the negative effects of minimum wage is sharply increasing in low-skill labor intensity in production and that employed low-skill workers gain initially but might suffer from slower growth in future wages.

1 Introduction

In the US, some states have recently passed legislation to raise the minimum wage significantly. For example, California and Massachusetts will raise the minimum wage to $15 per hour by 2023, whereas Illinois will do so by 2025.[1] This study explores the effects of minimum wage on unemployment and economic growth. We incorporate minimum wage into a Schumpeterian model with endogenous market structure and obtain the following results. First, raising the minimum wage decreases the employment of low-skill workers and increases the unemployment rate. Second, it decreases the level of output. Third, it decreases the transitional growth rate of output but does not affect the steady-state growth rate.[2] The negative effect of minimum wage on the level of output is in line with standard theory, whereas endogenizing both the equilibrium growth rate and the market structure gives rise to a transitional growth deceleration and amplifies the traditional level effect of minimum wage.

The intuition of the above results can be explained as follows. A binding minimum wage decreases the demand for low-skill workers and causes their employment level to fall below full employment, which leads to unemployment. Lower employment reduces the level of output. The smaller market size in turn reduces the incentives for innovation and leads to a decrease in the transitional growth rate of output. Over time, the smaller market size causes some firms to exit the market, and the market size of each firm gradually returns to the initial level. Given that it is the average firm size (rather than the aggregate market size) that determines the incentives for innovation,[3] the steady-state growth rate of output also returns to the initial level. Therefore, raising the minimum wage has a temporary stifling effect on growth, but not a permanent one.

Calibrating our model to US data for a quantitative analysis, we find that raising the minimum wage by 1% increases the unemployment rate by at least 0.3% and decreases the growth rate of output by at least 0.1% on impact. Furthermore, the magnitude of these effects is sharply increasing in the intensity of low-skill labor in production. Therefore, a more skill-intensive economy is more immune to the negative effects of minimum wage. Finally, we simulate the path of low-skill wages and find that employed low-skill workers gain initially but might suffer from slower growth in future wages.

This study relates to the literature on innovation and economic growth. The seminal R&D-based growth model developed by Romer (1990) focuses on the invention of new products (i.e., horizontal innovation). The Schumpeterian model developed by Aghion and Howitt (1992) focuses on the quality improvement of products (i.e., vertical innovation). Subsequent studies, such as Smulders and van de Klundert (1995), Peretto (1998, 1999) and Howitt (1999), combine the two dimensions of innovation and develop the second-generation Schumpeterian model.[4] This study contributes to the literature by incorporating minimum wage into this vintage of the Schumpeterian model with endogenous market structure to

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[1] The current federal minimum wage in the US is $7.25 per hour.

[2] Although empirical studies on minimum wage tend to find contrasting effects, these results are consistent with recent studies by Sabia (2014, 2015), who finds that minimum wage reduces low-skill employment and causes a short-run decline in state GDP generated by low-skill industries.

[3] See Laincz and Peretto (2006) for empirical evidence.

[4] See Ha and Howitt (2007) and Madsen (2010) for evidence that supports the second-generation model.

explore its effects on unemployment and innovation.[5]

This study also relates to the vast literature on labor markets and economic growth; see Topel (1999) for a comprehensive survey. Our study relates most closely to studies that explore minimum wage in growth models. Early studies by Cahuc and Michel (1996), Ravn and Sorensen (1999) and Askenazy (2003) identify interesting mechanisms through which minimum wage has ambiguous effects on growth; see also Tamai (2009) and Fanti and Gori (2011) for more recent studies and a discussion of the subsequent literature. A recent study by Agenor and Lim (2018) develops an ambitious quantitative framework based on the Romer model with horizontal innovation to explore the dynamic effects of minimum wage and also finds that it increases unemployment and reduces growth. This study complements these previous studies by considering a Schumpeterian model with two dimensions of innovation to analytically derive and quantify the complete transitional effects of minimum wage on growth and showing that endogenous market structure changes its effects at different time horizons.

2 A Schumpeterian model with minimum wage

The Schumpeterian model with in-house R&D and endogenous market structure is based on Peretto (2007, 2011a). We incorporate minimum wage into the Peretto model to explore its effects on unemployment and innovation.

2.1 Household

The representative household has the following utility function:

$$U = \int_0^\infty e^{-\rho t} \ln c_t dt, \tag{1}$$

where c_t is the household's consumption of final good (numeraire) and $\rho > 0$ determines the rate of subjective discounting. The household maximizes (1) subject to

$$\dot{a}_t = r_t a_t + \omega_t H + \overline{w}_t l_t + b_t (L - l_t) - c_t - \tau_t. \tag{2}$$

a_t is the value of assets owned by the household. r_t is the real interest rate. The household has $H + L$ members.[6] Each of the H members supplies one unit of high-skill labor and earns ω_t, which is above the minimum wage and determined as an equilibrium outcome in the high-skill labor market. Each of the L members supplies one unit of low-skill labor. Employed low-skill workers l_t earn $\overline{w}_t$, which is determined by the minimum wage set by the government. Unemployed low-skill workers $L - l_t$ receive an unemployment benefit $b_t < \overline{w}_t$.

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[5]For other approaches of incorporating unemployment into the innovation-driven growth model, see Mortensen and Pissarides (1998) for search frictions, Parello (2010) for efficiency wage, Peretto (2011b) for wage bargaining, and Ji *et al.* (2016) and Chu *et al.* (2016, 2018) for trade unions.

[6]Our results are robust to allowing for population growth. Derivations are available upon request.

The household pays a lump-sum tax τ_t to the government. From dynamic optimization, the Euler equation is

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{3}$$

2.2 Final good

Competitive firms produce final good Y_t using the following production function:

$$Y_t = \int_0^{N_t} X_t^\theta(i)[Z_t^\alpha(i)Z_t^{1-\alpha}l_t^\gamma h_t^{1-\gamma}/N_t]^{1-\theta}di, \tag{4}$$

where $\{\theta, \alpha, \gamma\} \in (0,1)$. $X_t(i)$ is the quantity of intermediate good $i \in [0, N_t]$, where N_t is the mass of available intermediate goods at time t. The productivity of $X_t(i)$ depends on its own quality $Z_t(i)$ and also on the average quality $Z_t \equiv \frac{1}{N_t}\int_0^{N_t} Z_t(i)di$ of all intermediate goods capturing technology spillovers. The degree of technology spillovers is $1 - \alpha$. We follow Romer (1990) to assume that the production of final good uses both high-skill labor h_t and low-skill labor l_t with a unitary elasticity of substitution. The term $l_t^\gamma h_t^{1-\gamma}/N_t$ captures a congestion effect of variety and removes the scale effect in the model.[7]

Profit maximization yields the following conditional demand functions for $\{l_t, h_t, X_t(i)\}$:

$$l_t = \gamma(1 - \theta)Y_t/\overline{w}_t, \tag{5}$$

$$h_t = (1 - \gamma)(1 - \theta)Y_t/\omega_t, \tag{6}$$

$$X_t(i) = \left[\frac{\theta}{p_t(i)}\right]^{1/(1-\theta)} Z_t^\alpha(i)Z_t^{1-\alpha}l_t^\gamma h_t^{1-\gamma}/N_t, \tag{7}$$

where $p_t(i)$ is the price of $X_t(i)$. Competitive firms pay $\theta Y_t = \int_0^{N_t} p_t(i)X_t(i)di$ for intermediate goods. The market-clearing condition for high-skill labor implies $h_t = H$. The minimum wage in the low-skill labor market implies unemployment (i.e., $l_t < L$).

2.3 Intermediate goods and in-house R&D

The monopolistic firm in industry $i \in [0, N_t]$ uses $X_t(i)$ units of final good to produce $X_t(i)$ units of intermediate good i and incurs $\phi Z_t^\alpha(i)Z_t^{1-\alpha}$ units of final good as a fixed operating cost. The firm also invests $R_t(i)$ units of final good to improve the quality $Z_t(i)$ of its product. The in-house R&D process is

$$\dot{Z}_t(i) = R_t(i). \tag{8}$$

In industry i, the profit flow (before R&D investment) at time t is

$$\Pi_t(i) = [p_t(i) - 1]X_t(i) - \phi Z_t^\alpha(i)Z_t^{1-\alpha}. \tag{9}$$

The value of the monopolistic firm in industry i is

$$V_t(i) = \int_t^\infty \exp\left(-\int_t^s r_u du\right)[\Pi_s(i) - R_s(i)]\,ds. \tag{10}$$

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[7]Our results are robust to parameterizing this congestion effect as $l_t^\gamma h_t^{1-\gamma}/N_t^{1-\xi}$, where $\xi \in (0,1)$. See the discussion in footnote 10.

The firm maximizes (10) subject to (7)-(9). The current-value Hamiltonian is

$$H_t(i) = \Pi_t(i) - R_t(i) + \eta_t(i)\dot{Z}_t(i), \tag{11}$$

where $\eta_t(i)$ is the co-state variable on (8). We solve this optimization problem in Appendix A and derive the profit-maximizing price $p_t(i) = 1/\theta > 1$.

We follow previous studies to consider a symmetric equilibrium in which $Z_t(i) = Z_t$ and $X_t(i) = X_t$ for $i \in [0, N_t]$.[8] From (7) and $p_t(i) = 1/\theta$, the quality-adjusted firm size is

$$\frac{X_t}{Z_t} = \theta^{2/(1-\theta)}\frac{H^{1-\gamma}}{N_t}l_t^\gamma. \tag{12}$$

We define the following transformed variable:

$$x_t \equiv \theta^{2/(1-\theta)}\frac{H^{1-\gamma}}{N_t}, \tag{13}$$

which is a state variable. Lemma 1 derives the rate of return on quality-improving R&D, which is increasing in the firm size $x_t l_t^\gamma$.

Lemma 1 *The rate of return to in-house R&D is*

$$r_t^q = \alpha\frac{\Pi_t}{Z_t} = \alpha\left(\frac{1-\theta}{\theta}x_t l_t^\gamma - \phi\right). \tag{14}$$

Proof. See Appendix A. ∎

2.4   Entrants

We follow previous studies to assume that entrants have access to aggregate technology Z_t to ensure symmetric equilibrium at any time t. A new firm incurs βX_t units of final good to set up its operation and enter the market with a new intermediate good. $\beta > 0$ is a cost parameter, and the entry cost βX_t is increasing in the size of initial production. The asset-pricing equation determines the rate of return on assets as

$$r_t = \frac{\Pi_t - R_t}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{15}$$

The entry condition is

$$V_t = \beta X_t. \tag{16}$$

Substituting (8), (9), (13), (16) and $p_t(i) = 1/\theta$ into (15) yields the return on entry as[9]

$$r_t^e = \frac{1}{\beta}\left(\frac{1-\theta}{\theta} - \frac{\phi + z_t}{x_t l_t^\gamma}\right) + \gamma\frac{\dot{l}_t}{l_t} + \frac{\dot{x}_t}{x_t} + z_t, \tag{17}$$

where $z_t \equiv \dot{Z}_t/Z_t$ is the growth rate of aggregate quality.

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[8]Symmetry also implies $\Pi_t(i) = \Pi_t$, $R_t(i) = R_t$ and $V_t(i) = V_t$.

[9]We treat entry and exit symmetrically (i.e., the scrap value of exiting an industry is also βX_t); therefore, $V_t = \beta X_t$ always holds, which implies $r_t^e = r_t$ for all t. Alternatively, one can consider a random exit shock to generate exit dynamics.

2.5 Government

The government sets a minimum wage $\overline{w}_t$ that is binding in the low-skill labor market. We follow previous studies to assume that $\overline{w}_t$ is proportionally higher than the equilibrium wage w_t^* that would have prevailed without the minimum wage; i.e.,

$$\overline{w}_t = (1 + \mu)w_t^*, \tag{18}$$

where $\mu > 0$ is the minimum-wage policy instrument. The government levies a lump-sum tax τ_t on the household to balance the fiscal budget $\tau_t = b_t(L - l_t)$.

2.6 Equilibrium

We define the equilibrium in Appendix B.

2.7 Aggregation

The resource constraint on final good is

$$Y_t = c_t + N_t(X_t + \phi Z_t + R_t) + \dot{N}_t \beta X_t. \tag{19}$$

Substituting (7) and $p_t(i) = 1/\theta$ into (4) and imposing symmetry yield the level of output:

$$Y_t = \theta^{2\theta/(1-\theta)} Z_t l_t^\gamma H^{1-\gamma}. \tag{20}$$

Substituting (5) and (20) into (18) yields

$$\gamma(1 - \theta)\theta^{2\theta/(1-\theta)} Z_t \left(\frac{H}{l_t}\right)^{1-\gamma} = (1 + \mu)\gamma(1 - \theta)\theta^{2\theta/(1-\theta)} Z_t \left(\frac{H}{L}\right)^{1-\gamma}, \tag{21}$$

which implies

$$l_t = \frac{L}{(1 + \mu)^{1/(1-\gamma)}} < L. \tag{22}$$

Equation (22) shows that low-skill employment is stationary (i.e., $l_t = l$ for all t). Therefore, the growth rate of output is simply determined by the quality growth rate; i.e., $\dot{Y}_t/Y_t = z_t$.[10]

2.8 Dynamics

To analyze the dynamics of the economy, we derive the consumption-output ratio first.

Lemma 2 *The consumption-output ratio jumps to a unique and stable steady-state value:*

$$\frac{c_t}{Y_t} = \rho\beta\theta^2 + 1 - \theta. \tag{23}$$

Proof. See Appendix A. ∎

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[10]Parameterizing the congestion effect in (4) as $l_t^\gamma h_t^{1-\gamma}/N_t^{1-\xi}$ would yield $Y_t = \theta^{2\theta/(1-\theta)} Z_t N_t^\xi l^\gamma H^{1-\gamma}$ in which case the growth rate of output is given by $\dot{Y}_t/Y_t = z_t + \xi\dot{N}_t/N_t$, where $\xi\dot{N}_t/N_t$ would capture an additional negative effect of minimum wage on productivity. The overall output growth rate $\dot{Y}_t/Y_t$ is nonetheless determined by the rate of return r_t^q on in-house R&D in (14) as (24) shows.

Lemma 2 implies that consumption and output grow at the same rate given by

$$g_t \equiv \frac{\dot{Y}_t}{Y_t} = \frac{\dot{c}_t}{c_t} = r_t - \rho, \tag{24}$$

where the last equality uses (3). Substituting (14) and (22) into (24) yields

$$g_t = z_t = \alpha \left[\frac{1-\theta}{\theta} \frac{L^\gamma x_t}{(1+\mu)^{\gamma/(1-\gamma)}} - \phi \right] - \rho, \tag{25}$$

which is positive if and only if

$$x_t > \overline{x} \equiv \frac{\theta}{1-\theta} \left(\frac{\rho}{\alpha} + \phi \right) \frac{(1+\mu)^{\gamma/(1-\gamma)}}{L^\gamma}. \tag{26}$$

Intuitively, each firm's market size needs to be large enough for in-house R&D to be profitable. For the rest of the analysis, we assume that $x_t > \overline{x}$, which implies $z_t > 0$ and $r_t^q = r_t$ for all t. Lemma 3 derives the dynamics of x_t, which in turn determines the number of firms (i.e., $N_t = \theta^{2/(1-\theta)} H^{1-\gamma}/x_t$) in the market.

Lemma 3 *The dynamics of x_t is determined by an one-dimensional differential equation:*

$$\dot{x}_t = \frac{(1-\alpha)\phi - \rho}{\beta L^\gamma} (1+\mu)^{\gamma/(1-\gamma)} - \left[\frac{(1-\alpha)(1-\theta)}{\beta\theta} - \rho \right] x_t. \tag{27}$$

Proof. See Appendix A. ∎

Proposition 1 *Under the parameter restriction $\rho < \min\{(1-\alpha)\phi, (1-\alpha)(1-\theta)/(\theta\beta)\}$, the dynamics of x_t is globally stable and x_t gradually converges to a unique steady-state value. The steady-state values $\{x^*, g^*\}$ are given by*

$$x^*(\underset{+}{\mu}) = \frac{(1-\alpha)\phi - \rho}{(1-\alpha)(1-\theta)/\theta - \beta\rho} \frac{(1+\mu)^{\gamma/(1-\gamma)}}{L^\gamma} > \overline{x}, \tag{28}$$

$$g^* = \alpha \left[\frac{1-\theta}{\theta} \frac{(1-\alpha)\phi - \rho}{(1-\alpha)(1-\theta)/\theta - \beta\rho} - \phi \right] - \rho > 0. \tag{29}$$

Proof. See Appendix A. ∎

Proposition 1 shows that given an initial x_0, the state variable x_t gradually converges to its steady-state x^*. Equation (25) shows that when x_t converges to x^*, the equilibrium growth rate g_t also converges to its steady-state g^*.

3 Dynamic effects of minimum wage

Equation (22) shows that raising the minimum wage μ decreases low-skill employment l_t and increases the unemployment rate u_t given by

$$u_t \equiv \frac{L - l_t}{H + L} = \frac{L}{H + L} \left[1 - \frac{1}{(1 + \mu)^{1/(1-\gamma)}} \right] \qquad (30)$$

for all t. Substituting (22) into (20) yields the equilibrium level of output Y_t, which is decreasing in μ for a given technology level Z_t. In other words, raising the minimum wage leads to a contemporaneous drop in output by decreasing employment. Equation (25) shows that the equilibrium growth rate g_t is also decreasing in μ for a given x_t. Intuitively, raising the minimum wage decreases employment l_t and the firm size $x_t l_t^\gamma$, which in turn decreases the rate of return to in-house R&D in (14) and the equilibrium growth rate in (25). However, the lower level of employment reduces the aggregate market size and leads to an exit of firms, which in turn gives rise to a gradual increase in x_t as implied by (27). When x_t reaches the new steady-state x^*, which is increasing in μ as shown in (28), the steady-state growth rate returns to the initial level because g^* in (29) is independent of μ. Therefore, raising the minimum wage has a temporary stifling effect on growth, but not a permanent one. Figure 1 illustrates the transitional path of g_t when the government raises μ at time t. Proposition 2 summarizes these results.

Proposition 2 *Raising the minimum wage has the following effects: (a) it permanently decreases the employment of low-skill workers, (b) it permanently increases the unemployment rate; (c) it decreases the level of output for a given Z_t; (d) it decreases the transitional growth rate of output but does not affect the steady-state growth rate.*
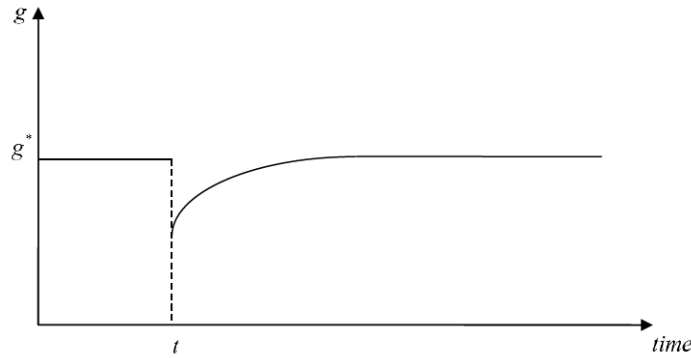
Proof. Proven in text. ∎

Figure 1: Transition dynamics of the growth rate

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3.1 Quantitative analysis

We calibrate the model to simulate the effects of raising μ by 1%. The model features the following parameters: $\{\rho, \alpha, \gamma, \theta, \beta, \phi, \mu, L/H\}$. We set the discount rate ρ to 0.04. We follow Iacopetta *et al.* (2019) to set the degree of technology spillovers $1-\alpha$ to 0.833. From the estimates in Heathcote *et al.* (2010), we consider $\gamma \in [0.25, 0.75]$ as a plausible range for the intensity of low-skill labor. Then, we calibrate $\{\theta, \beta, \phi, \mu, L/H\}$ by matching the following moments in the US. First, labor income share is 60%. Second, the consumption share of output is 64%. Third, the growth rate g_t is 2%. Fourth, the unemployment rate u_t is 6%. Fifth, the skill premium $\omega_t/\overline{w}_t$ is 1.40.

Table 1 presents the calibrated parameter values and the simulated effects on u_t. Figure 2 presents the simulated paths of g_t. We find that raising the minimum wage μ by 0.01 increases the unemployment rate u_t permanently by at least 0.34% (i.e., from 6% to 6.34%) and decreases the growth rate g_t on impact by at least 0.10% (i.e., from 2% to 1.90%). The magnitude of these effects is sharply increasing in γ because minimum wage affects low-skill labor. Therefore, a more skill-intensive economy (i.e., a smaller γ) is more immune to the negative effects of minimum wage.

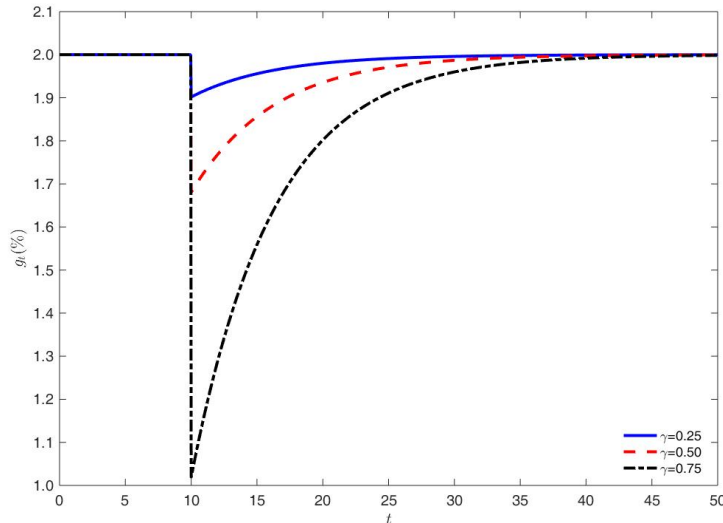| Table 1: Calibration and simulation								
γ	ρ	α	θ	β	ϕ	L/H	μ	Δu_t
0.250	0.040	0.167	0.400	6.250	1.676	0.560	0.147	0.34%
0.500	0.040	0.167	0.400	6.250	1.676	1.553	0.053	1.03%
0.750	0.040	0.167	0.400	6.250	1.676	4.532	0.019	2.91%

Figure 2: Simulated paths of the growth rate

3.2 Dynamic effects on low-skill wages

Finally, we explore the dynamic effects of μ on the path of minimum wages:

$$\ln \overline{w}_t = \ln[\gamma(1-\theta)] + \ln(Y_t/l_t), \tag{31}$$

where $\ln(Y_t/l_t)$ is

$$\ln(Y_t/l_t) = \ln(Y_0/l_0) + \int_0^t g_s ds = \ln\left[\theta^{2\theta/(1-\theta)} Z_0 \left(H/L\right)^{1-\gamma}\right] + \underbrace{\ln(1+\mu)}_{\uparrow \text{ in } \mu} + \underbrace{\int_0^t g_s ds}_{\downarrow \text{ in } \mu}, \tag{32}$$

which uses (20) and (22). Figure 3 presents the simulated paths of $\Delta \ln \overline{w}_t$ given by

$$\Delta \ln \overline{w}_t \approx \underbrace{\Delta\mu}_{\text{"+" level effect}} + \underbrace{\int_0^t \Delta g_s ds}_{\text{"−" growth effect}} \tag{33}$$

when μ increases by 1%. It shows that although employed low-skill workers receive higher wages initially, even these intended beneficiaries might experience a loss in future wages due to the temporary slow down in growth.
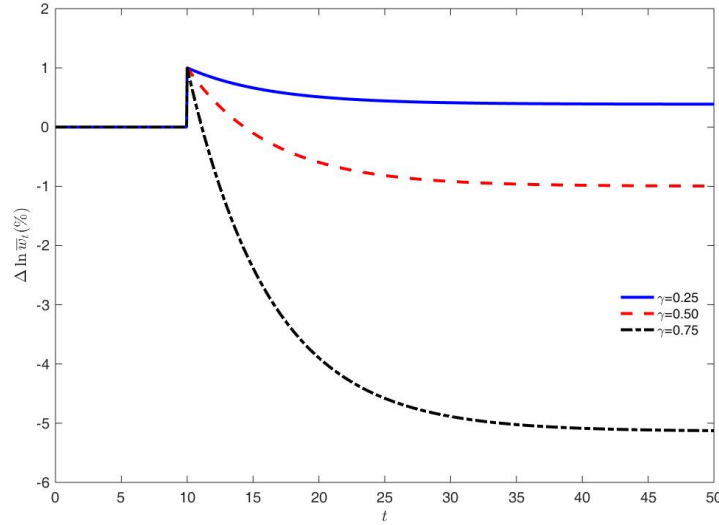

Figure 3: Simulated paths of changes in low-skill wages

4 Conclusion

This study analyzes minimum wage in the Peretto model of Schumpeterian growth. Although our finding that minimum wage reduces employment and growth is not surprising, it is consistent with recent empirical evidence; see e.g., Sabia (2014, 2015). Finally, this study

focuses on quality-improving innovation, which determines long-run growth,[11] and leaves the important implications on automation (i.e., labor-saving technologies) to future research.[12]

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[11]See Peretto and Connolly (2007) on why long-run growth must be driven by vertical innovation.

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Proof of Lemma 1. The current-value Hamiltonian for monopolistic firm i is given by (11). Substituting (7)-(9) into (11), we can derive

$$\frac{\partial H_t(i)}{\partial p_t(i)} = 0 \Rightarrow \frac{\partial \Pi_t(i)}{\partial p_t(i)} = 0, \tag{A1}$$

$$\frac{\partial H_t(i)}{\partial R_t(i)} = 0 \Rightarrow \eta_t(i) = 1, \tag{A2}$$

$$\frac{\partial H_t(i)}{\partial Z_t(i)} = \alpha \left\{ [p_t(i) - 1] \left[\frac{\theta}{p_t(i)} \right]^{1/(1-\theta)} \frac{l_t^\gamma h_t^{1-\gamma}}{N_t} - \phi \right\} Z_t^{\alpha-1}(i) Z_t^{1-\alpha} = r_t \eta_t(i) - \dot\eta_t(i). \tag{A3}$$

(A1) yields $p_t(i) = 1/\theta$. Substituting (A2), (13) and $p_t(i) = 1/\theta$ into (A3) and imposing symmetry yield (14). ∎

Proof of Lemma 2. Substituting (16) into the total asset value $a_t = N_t V_t$ yields

$$a_t = N_t \beta X_t = \theta^2 \beta Y_t, \tag{A4}$$

where the second equality uses $\theta Y_t = N_t X_t / \theta$.[13] Differentiating (A4) with respect to t yields

$$\frac{\dot Y_t}{Y_t} = \frac{\dot a_t}{a_t} = r_t + \frac{\omega_t H + \overline{w}_t l_t}{a_t} - \frac{c_t}{a_t}, \tag{A5}$$

where the second equality uses (2) and $\tau_t = b_t(L - l_t)$. Using (3) for r_t, (5) for $\overline{w}_t$, (6) for ω_t and (A4) for a_t, we can rearrange (A5) to obtain

$$\frac{\dot c_t}{c_t} - \frac{\dot a_t}{a_t} = \frac{c_t}{a_t} - \left(\rho + \frac{1-\theta}{\beta\theta^2} \right), \tag{A6}$$

which is increasing in c_t/a_t with a strictly negative y-intercept. Therefore, c_t/a_t must jump to the steady state. Then, we have (23), noting (A4). ∎

Proof of Lemma 3. Substituting $z_t = g_t = r_t - \rho = r_t^e - \rho$ into (17) yields

$$\frac{\dot x_t}{x_t} = \rho - \frac{1}{\beta} \left(\frac{1-\theta}{\theta} - \frac{\phi + z_t}{x_t l_t^\gamma} \right), \tag{A7}$$

which also uses $\dot l_t = 0$ from (22). Then, we use the expression of z_t in (25) to derive (27). ∎

Proof of Proposition 1. One can rewrite (27) simply as $\dot x_t = d_1 - d_2 x_t$. This linear system for x_t has a unique (non-zero) steady state that is globally (and locally) stable if

$$d_1 \equiv \frac{(1-\alpha)\phi - \rho}{\beta L^\gamma} (1 + \mu)^{\gamma/(1-\gamma)} > 0, \tag{A8a}$$

$$d_2 \equiv \frac{(1-\alpha)(1-\theta)}{\beta\theta} - \rho > 0, \tag{A8b}$$

from which we obtain $\rho < \min\{(1-\alpha)\phi, (1-\alpha)(1-\theta)/(\theta\beta)\}$. Then, $\dot x_t = 0$ yields the steady-state value $x^* = d_1/d_2$, which gives (28). Substituting (28) into (25) yields (29). ∎

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[13] We derive this by using $p_t(i) = 1/\theta$ and $X_t(i) = X_t$ for $\theta Y_t = \int_0^{N_t} p_t(i) X_t(i) di$.

Appendix B (online appendix)

The equilibrium is a time path of allocations $\{a_t, c_t, Y_t, l_t, h_t, X_t(i), R_t(i)\}$ and a time path of prices $\{r_t, \omega_t, \overline{w}_t, p_t(i), V_t(i)\}$ such that the following conditions are satisfied:

- the household maximizes utility taking $\{r_t, \omega_t, \overline{w}_t\}$ as given;

- competitive firms produce Y_t and maximize profits taking $\{p_t(i), \omega_t, \overline{w}_t\}$ as given;

- a monopolistic firm produces $X_t(i)$ and chooses $\{p_t(i), R_t(i)\}$ to maximize $V_t(i)$ taking r_t as given;

- entrants make entry decisions taking V_t as given;

- the government balances the fiscal budget;

- the value of monopolistic firms is equal to the value of the household's assets such that $N_t V_t = a_t$;

- the final-good market clears such that $Y_t = c_t + N_t(X_t + \phi Z_t + R_t) + \dot{N}_t \beta X_t$;

- the high-skill labor market clears such that $h_t = H$; and

- the demand for low-skill labor is less than its supply such that $l_t < L$.