A Variation with Risk-Averse Buyers and Demand Uncertainty

Comment

by

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1 Introduction

Inspired by Bar-Gill's (2020 [2]) novel twist on the classic Hotelling model, where buyers misperceive their demands, as well as the preceding analysis of the monopoly case by the same author (Bar-Gill, 2019 [1]), this note applies his idea to a slightly different setting. In particular, it suggests a model where the buyers' have imperfect knowledge of the good's value and their location on the Hotelling line. That is, the variables representing the value and the location will be treated as random; the buyers will only observe their distribution (or more specifically, the first two moments of the distribution). The buyers in the original paper (Bar-Gill, 2020) can be thought of as observing the realizations of these random variables, while their observation is different from the true realization, upon which the welfare analysis is based. Here, the buyers will make their purchasing decisions before observing a realization. Another way to compare both settings would be to say that the buyers "don't know that they don't know" in the original paper, while they "know that they don't know" in this note. The purpose of this note would be to shed light on whether such a change of frame can be made while maintaining the basic insights of the paper. The present analysis does not attempt to match the original model in either its depth nor completeness. Rather, it offers a variation that hopefully sheds light on the nature of misperceptions.

The buyers have a mean-variance type utility: their payoffs increase in the expectation of their net payoff and decrease linearly in the variance of such payoffs; the degree of risk aversion may vary (which, as we will see, corresponds to the magnitude of misperception in the original paper). The expectations are taken with respect to two variables, that capture the uncertainty - mirroring the original analysis of common and relative misperceptions. In Section 3 of his note, the value of the good is treated as a random variable - which corresponds to the "common misperception" and can also be referred to as vertical uncertainty. In Section 4, the buyer's location becomes random instead; this is the analogy

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to the "relative misperception", or horizontal uncertainty. A limitation of this analysis is on the assumption that the solutions are interior. An important contribution of the original paper is is its specific focus on corner solutions that are arguably important in practice.

The focus in this comment is to contribute a supplementary model for the analysis of misperceptions in a Hotelling model, rather than offer a complete analysis of consumer surplus and welfare, which is already done in the original paper. We show that in order to make a transition from the model outlined here to Bar-Gill's (2020) model of common misperceptions, one would assume that buyers are risk loving to obtain positive, and risk-averse, to obtain negative common misperceptions. One would expect the results of the original paper to apply in these cases.

2 The Model

A buyer's utility is given by $u(v - t_i(x) - p_i)$, if he buys from firm $i \in \{1, 2\}$ at price p_i, v is the good's value and $t_i(x)$ is the transportation cost and x is between 0 and 1. The value v and the location x on the Hotelling line may be uncertain. When they are treated as random variables, the notation becomes $\tilde{v}$ and $\tilde{x}$, respectively. In either case of uncertainty, the argument of $u(\cdot)$ is a random variable.

Suppose u is a mean-variance utility function, such that for the random payoff $\tilde{\xi}$,

$$u\left(\tilde{\xi}\right) = \mathbb{E}\left[\tilde{\xi}\right] - \gamma \mathrm{Var}\left[\tilde{\xi}\right]$$

The common misperception would correspond, in this model, to uncertainty about v, i.e.,

$$u(\tilde{v} - t_i(x) - p_i) = v^e - \gamma\sigma_v^2 - t_i(x) - p_i,$$

where $v^e = \mathbb{E}v$ and $\sigma_v^2 = \mathrm{Var}v$, $i = 1, 2$ refers to a firm.

In contrast, relative misperception would correspond to

$$u(v - t_i(\tilde{x}) - p_i) = v - t_i^e - \gamma s_i^2 - p_i,$$

where $t_i^e = \mathbb{E}[t_i(x)]$ and $s_i^2 = \mathrm{Var}[t_i(x)]$. Similar to the original article, I will treat the cases of common and relative misperception in isolation. I will start with the uncertainty about v, the case of common misperception, that I will refer to as vertical uncertainty.

3 Vertical Uncertainty

"Vertical uncertainty" mirrors common misperception in the original paper - it refers to the uncertainty about the good's value to the customer. The buyer's subjective expectation of the value is v^e and the variance is σ_v^2.

3.1 No Price Discrimination

Consider firm i that sets its uniform price p_i to maximize its revenue R_i, defined as follows.

$$\begin{aligned} R_i &= (p_i - c_i) \int_0^1 I\left(\left(\mathbb{E}U_i^B(x) > 0\right) \wedge \left(\mathbb{E}U_i^B(x) > \mathbb{E}U_j^B(x)\right)\right) d\Phi(x) \\ &= (p_i - c_i) \int_0^1 I\left(\left(v^e - \gamma\sigma_v^2 - p_i > t_i(x)\right) \wedge \right. \\ &\qquad \left. \wedge \left(p_j - p_i > t_i(x) - t_j(x)\right)\right) d\Phi(x) \qquad (1) \end{aligned}$$

where $I(\cdot)$ is the indicator function that takes value 1 if its argument is true, 0 if it's false; $\wedge$ is the logical *and* operator; Φ is the cumulative distribution function describing the locations of the mass of consumers, x measures the distance from firm i, and $d_i \equiv t_i - t_j$ is the difference in transportation costs for a given consumer, when he buys from firm i as opposed to firm j.

Suppose first that $v^e - \gamma\sigma_v^2$ is sufficiently low, then we have a monopoly situation (the market is not covered), and therefore, the first order condition for maximization is given by

$$\frac{\partial R_i}{\partial p_i} = \Phi\left(t_i^{-1}\left(v^e - \gamma\sigma_v^2 - p_i\right)\right) - (p_i - c_i)\frac{\phi\left(t_i^{-1}\left(v^e - \gamma\sigma_v^2 - p_i\right)\right)}{t_i'\left(v^e - \gamma\sigma_v^2 - p_i\right)} = 0 \qquad (2)$$

In Bar-Gill (2000), the distribution of locations is uniform, $\Phi(x) = x$, the Hotelling line has length 1, the transportation cost is linear, $t(x) = x$, and the marginal cost of production is zero, $c_i = 0$, for $i = 1, 2$. In this setting, the first order condition (2) implies

$$p_i = \frac{v^e - \gamma\sigma_v^2}{2}.$$

Skipping the intermediate case covered in the original paper, suppose that $v^e - \gamma\sigma_v^2$ is sufficiently large, then we have, for the first order condition,

$$\frac{\partial R_i}{\partial p_i} = \Phi\left(d_i^{-1}(p_j - p_i)\right) - (p_i - c_i)\frac{\phi\left(d_i^{-1}(p_j - p_i)\right)}{d_i'(p_j - p_i)} = 0. \qquad (3)$$

In the setting of the original paper, $d_i = t_i - t_j = x - (1 - x) = 2x - 1$ and thus $d_i^{-1}(y) = \frac{y+1}{2}$. Therefore, the first order condition (3) becomes

$$\frac{p_j - p_i + 1}{2} = p_i \quad \Leftrightarrow \quad \frac{p_i}{2} = \frac{p_j + 1}{2}.$$

which implies $p_1 = p_2 = 1$, the prices do not depend on $v^e - \gamma\sigma_v^2$.

We conclude that an increase in γ, σ_v^2, and a decrease in v^e make partial market coverage more likely. The effect of risk aversion is opposite to the effect of *negative* common misperception in the original paper. The case of positive common misperception is achieved in this model if buyers are risk-loving - in this case, they overstate their demand for a good with an uncertain value.

Risk loving consumers will buy more often than optimal. In contrast, risk aversion leads to reduced market coverage.

3.2 Price Discrimination

$$\begin{aligned}
R_i &= \int_0^1 (p_i(x) - c_i) I \left(\left(EU_i^B (x) > 0 \right) \wedge \left(U_i^B (x) > U_j^B (x) \right) \right) d\Phi (x) \\
&= \int_0^1 (p_i(x) - c_i) I \left(\left(v^e - \gamma\sigma_v^2 - p_i(x) - t_i (x) > 0 \right) \wedge \right. \\
&\qquad \left. \wedge \left(p_j(x) + t_j (x) - t_i (x) > p_i(x) \right) \right) d\Phi (x)
\end{aligned}$$

The analysis is similar to the original paper with the exception that $\hat{v}$, the perceived value, is replaced by $v^e - \gamma\sigma_v^2$. Positive misperception thus corresponds to the case of a risk-loving buyer.

4 Horizontal Uncertainty

In this subsection, I will consider the case of uncertainty about the buyer's own location x on the Hotelling line. Thus "horizontal uncertainty" mirrors relative misperception in the original paper.

4.1 No Price Discrimination

For simplicity, assume a linear transportation cost, $t_i(x) = x$ (where x denotes the distance from firm i).

$$\begin{aligned}
R_i &= (p_i - c_i) \int_0^1 I \left(\left(\mathbb{E}U_i^B (x) > 0 \right) \wedge \left(\mathbb{E}U_i^B (x) > \mathbb{E}U_j^B (x) \right) \right) dF (x^e) \\
&= (p_i - c_i) \int_0^1 I \left(\left(v - x^e - \gamma s_i^2 > p_i \right) \wedge \left(p_j - p_i > 2x^e - 1 \right) \right) dF (x^e) \qquad (4)
\end{aligned}$$

where the F is the distribution of the *expected* Hotelling locations x^e. Comparing (4) with equation (1) we see that σ_v^2 in (1) is replaced by s_i^2 in (4), Φ by F, but otherwise the equations are the same. Thus, the analysis would be similar. If beliefs are consistent and $F \equiv \Phi$ we will get the same effects as in the case of vertical uncertainty. If, however, instead of belief consistency, F first-order stochastically dominates Φ, implying that the buyers overestimate the distance from i, then we have compound effects of vertical and horizontal uncertainty.

In addition to the demand-lowering effect of risk aversion, as outlined in 3.1, we will obtain the effects similar to the relative misperception case studied in Bar-Gill (2020). In case of overestimation ($F \succ_{FOSD} \Phi$), the effect has the same direction as risk aversion, causing the consumers to buy less from the firm they perceive to be further than it is. If in contrast the buyers underestimate the distance from firm i, it works in the same way as risk loving: the demand of Firm i's product is exaggerated.

4.2 Price Discrimination

In the case of price discrimination, customers will be offered different prices depending on their expected location on the Hotelling line.

$$
\begin{aligned}
R_i &= \int_0^1 (p_i(x^e) - c_i)\, I\left(\left(EU_i^B(x) > 0\right) \wedge \left(U_i^B(x) > U_j^B(x)\right)\right) dF(x^e) \\
&= \int_0^1 (p_i(x^e) - c_i)\, I\left(\left(v - \gamma s_i^2 - p_i(x) > x^e\right) \wedge \left(p_j(x) - p_i(x) > 2x^e - 1\right)\right) dF(x^e)
\end{aligned}
$$

As in the case of price discrimination, we are back to the vertical uncertainty if beliefs are consistent, i.e., if $F \equiv \Phi$. There are compound effects, as in 4.1 in the case where one of the distributions dominates the other.

5 Conclusion

This note attempted to translate Bar-Gill's (2020) analysis can to a framework where the buyers are fully rational. In this framework, common misperception can be modelled as uncertainty about product value, buyer's risk aversion is then the analog of negative misperception, whereas negative risk aversion (i.e., if buyers are risk-loving) is the analog of positive misperception in the model. The mapping between relative misperception and the horizontal uncertainty proposed here is somewhat less straightforward. This is due to the symmetric effect of uncertainty in the present model when beliefs are consistent. To obtain the effects similar to Bar-Gill (2020) one would postulate belief inconsistency, effectively approaching the original setting. This note does not cover welfare comparisons, where one could expect important differences between the original misperception model and the uncertainty. It would be interesting to see further research exploring the role of misperceptions, and more generally, behavioral biases within the Hotelling model.

References

[1] Oren Bar-Gill. Algorithmic price discrimination: When demand is a function of both preferences and (mis) perceptions. *The University of Chicago Law Review*, 2020.

[2] Oren Bar-Gill. Consumer misperception in a hotelling model: With and without price discrimination. *JITE*, 2020.