Model-Based Filtering of EEG Alpha Waves for Enhanced Accuracy in Dynamic Conditions and Artifact Detection

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***Abstract*—Electroencephalography (EEG) is the recording of brain electrophysiological activity, usually by electrodes placed on the scalp. The EEG signals contain useful information about the brain state, with specific states being associated with oscillations at specific frequencies (the so-called brain waves); hence, EEG signals are usually analyzed in terms of their frequency content. A notable example is the amplitude estimation of alpha waves (8-14 Hz). This paper proposes a model-based estimation approach, based on known physical properties of alpha waves, which allows enhanced robustness in presence of fast amplitude dynamics, as well as an automatic identification of possible artifacts or discontinuities in the alpha wave. The proposed method is illustrated in this paper with application to a clinical EEG signal, but it is particularly promising for wearable EEG applications, such as brain-computer interface (BCI), to name one, where no expert human supervision is available.**

***Index Terms*—Electroencephalography, Biomedical measurement, Signal processing, Time-domain analysis, Frequency-domain analysis, Digital filters, Brain-computer interfaces**

I. Introduction

Electroencephalography (EEG) is the recording of brain electrophysiological activity, which is usually performed in a non-invasive way by placing electrodes on the scalp [1]. The EEG signals contain useful information about the brain state, which is routinely used in clinical applications for the diagnosis of neurological conditions, but it is nowadays increasingly used also in non-clinical applications, e.g. for cognitive tests and brain-computer interfaces (BCI) [2]. The non-clinical applications are rapidly growing, owing to the increased availability of affordable wearable EEG systems. While they undoubtedly open the way for a number of promising applications, they also raise important concerns about the accuracy and reliability of brain activity measurements obtained from such devices, particularly when they are used without expert supervision.

EEG signals are very challenging to measure and to analyze, primarily because of their very low amplitude (typically less than 100 μV), which makes them prone to being significantly affected by noise and artifacts created by other electrophysiological activity in the body, such as muscular, ocular and cardiac activity. The high contact impedance between skin and electrodes also contributes to adding noise caused by electromagnetic interference. Furthermore, the EEG signals themselves can change rapidly, in both amplitude and frequency content, as the brain continuously changes its state. For these reasons, despite the vast research carried out on the subject, the EEG signal processing remains a challenging task and most clinical applications still rely on the visual analysis performed by a trained clinician as the most accurate way to interpret an EEG recording. While this may be acceptable (though time consuming) in a clinical environment, it is not feasible in non-clinical applications; in this case, automated analysis tools are required to extract relevant features from the signals.

EEG signals are traditionally analyzed in the frequency domain, because different frequencies are associated with different types of brain activity, often originating from different parts of the brain [1]. A notable example is the alpha activity, typically defined in the range from 8 to 13-14 Hz in adults, and associated with a state of wakeful relaxation or meditation [3]. The measurement of alpha activity has been proved to be useful for the investigation of cognitive states in a number of clinical studies [4] as well as non-clinical applications [5]. However, traditional signal processing methods in the time or frequency domains, suitable for periodic signals, can lead to significant errors in the amplitude estimation when the signal is characterized by fast dynamics and/or by the presence of artifacts. Signals are often pre-processed by means of digital band-pass filters to remove frequency components outside the alpha range, but the bandwidth of those filters is typically large enough to preserve fast amplitude oscillations that can pose significant challenges to the amplitude estimation; moreover, filters can make artifacts more difficult to detect, as they may look similar to alpha oscillations once they have been filtered.

Several general (not model-based) approaches exist and have been widely used to overcome the limitations of traditional Fourier analysis of oscillating signals with time-varying amplitude and/or frequency, such as the Hilbert transform, the Taylor-Fourier transform and the wavelet transform [6]–[9]. While some of them may offer some advantages in the analysis

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of EEG signals, they may also appear as excessively complex in some circumstances, and the results do not always allow a straightforward physical interpretation.

An alternative method for the estimation of the time-varying amplitude of alpha waves in EEG signals is therefore proposed in this paper, based on the *a priori* knowledge of some physical properties of those signals, such as the sinusoidal waveform and the almost constant frequency [10], [11]. Such a knowledge allows a more robust amplitude estimation in presence of fast amplitude dynamics and, more importantly, it also allows using the measured deviations from the model to identify artifacts and discontinuity in alpha activity. A preliminary implementation of the proposed method is illustrated in this paper, with application to a clinical EEG signal, but this approach is particularly promising for wearable EEG, where large artifacts are more likely to occur [12]. This method has the potential to improve the reliability of decision-making processes based on the extraction and interpretation of EEG signal features, e.g. in BCI, but it can be applied also to research studies, e.g. in cognitive and clinical neuroscience.

II. EEG Dynamics and Filter Design

A. Amplitude Modulation and Frequency Bandwidth

A typical EEG signal may show significant frequency components in a range from less than 1 Hz to more than 30 Hz. Those components are traditionally divided into bands (delta, theta, alpha, beta and gamma), corresponding to different types of brain activity, often originating from different parts of the brain [1]. A notable example is represented by the alpha waves, typically defined in the range from 8 to 13-14 Hz in adults, which are usually the most visible oscillations in an EEG signal due to their relatively large amplitude.

The alpha activity can be modeled, with good approximation, as a sinusoidal oscillation in the EEG signal, at a frequency f_α in the alpha range defined above:

$$s_\alpha\left(t\right) = A_\alpha\left(t\right)\sin\left(2\pi f_\alpha\left(t\right)t + \varphi_\alpha\left(t\right)\right) \tag{1}$$

where the amplitude, frequency and phase are generically functions of time. The frequency f_α may be different in different people and it may slightly change with their age; however, in an individual person, it is unlikely to show significant variations on short timescales [10]. When analyzing alpha waves in an EEG recording, the model in (1) can therefore be simplified by considering a constant frequency f_α and taking into account any residual frequency variation within the phase $\varphi_\alpha(t)$:

$$s_\alpha\left(t\right) = A_\alpha\left(t\right)\sin\left(2\pi f_\alpha t + \varphi_\alpha\left(t\right)\right) \tag{2}$$

The amplitude $A_\alpha(t)$ is likely to show large and fast variations, on timescales shorter than 1 s. Even when tracking those fast amplitude oscillations is not of practical interest, they should be taken into account to inform the design and choice of signal processing methods, including filters and amplitude estimation algorithms. For illustration purposes, and

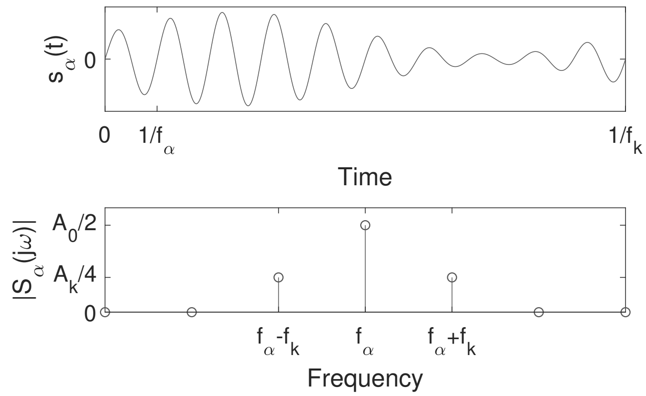

Fig. 1. Qualitative representation of a sinusoidal signal at frequency f_α, whose amplitude is modulated at a frequency f_k (top), and its frequency spectrum (bottom).

without loss of generality, a periodic amplitude modulation is considered, so that $A_\alpha(t)$ can be expressed by a Fourier series:

$$A_\alpha\left(t\right) = A_0 + \sum_k A_k \sin\left(2\pi f_k t + \varphi_k\right) \tag{3}$$

The amplitude oscillations at frequencies f_k in (3) produce two side-band frequency components in the spectrum of $s_\alpha(t)$ around the main frequency f_α, at frequencies $f_\alpha \pm f_k$. This is illustrated in Fig. 1 for a single modulation frequency f_k. The bandwidth of the alpha signal $s_\alpha(t)$ is therefore defined not only by the nominal frequency f_α, but also by the amplitude modulation dynamics, with higher frequencies f_k creating a larger bandwidth, from $f_\alpha - f_k$ to $f_\alpha + f_k$.

B. Digital Filter Design

The band-pass digital filter applied to EEG signals to extract alpha waves is usually designed based on the nominal alpha range, i.e. with a pass band from 8 Hz to 14 Hz (or similar values) [13]. The advantage of this choice is that the filter is suitable for all adults and it does not require to be adjusted based on the individual's alpha frequency. However, such a large pass band also means that fast amplitude oscillations, up to a few hertz, can pass through the filter. E.g., according to (2)-(3), if f_α is 11 Hz, the amplitude of the filtered signal will contain modulation frequencies f_k up to 3 Hz, not much lower than the alpha frequency itself. Such fast amplitude dynamics should be taken into account in the design of signal processing algorithms because they can lead to errors in the amplitude estimation, as it will be shown in Sec. III.

The amplitude dynamics should be taken into account also for the design of the filter, with regard to its response time. Digital filters are categorized into Infinite Impulse Response (IIR) and Finite Impulse Response (FIR) types. IIR filters are recursive, i.e. they use previous output values to calculate the new output at each time; this makes them computationally more efficient than FIR filters, but the practical duration of any transient created by input variations cannot be predicted (mathematically, it is infinite). Therefore, IIR filters are less

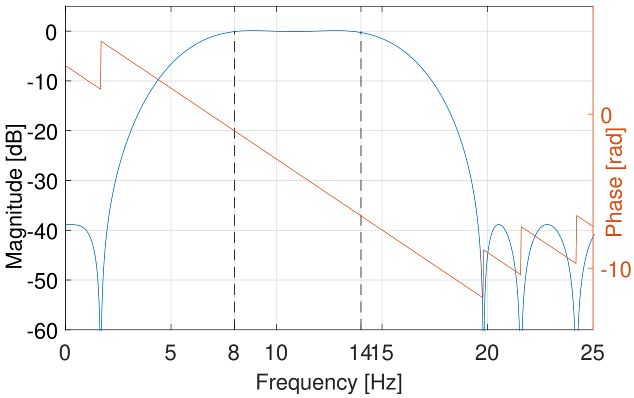

Fig. 2. Frequency response (magnitude, blue, and phase, red) of the designed band-pass FIR filter.

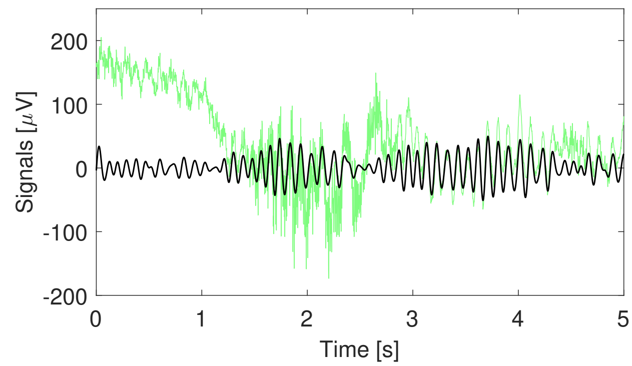

Fig. 3. Raw (green, thin line) and filtered (black, thick line) EEG signals, showing alpha activity with a noticeable amplitude modulation on short timescales.

convenient to use in EEG applications, where large artifacts can occur and can instigate long and unpredictable transients in the output. FIR filters, on the other hand, have a fixed response time, they are intrinsically stable and allow a linear phase filtering (i.e., the time delay is constant for all frequencies and can be compensated for), which are important properties for EEG signal processing [9], [14]. The main drawback of FIR filters is the high order, which requires a large computational effort, but this is usually acceptable in EEG applications, particularly when the processing is done offline.

The order of a FIR filter is equal to the number of input samples that are involved in the output calculation, and it is therefore proportional to the length of the time window that affects each output sample. The best order of the filter should be chosen as a trade-off between a good approximation of the desired frequency response (the higher the order the better) and a short time window (the lower the order the better). The suitable length of the time window, in turn, depends on the expected dynamics of the signal, to insure that an appropriate time resolution is maintained in the output signal.

Although filters are widely available from commercial EEG processing software packages, a dedicated FIR filter has been designed for this work, according to the principles explained above, and its frequency response is shown in Fig. 2. The filter is designed to extract alpha waves from an EEG signal acquired with a sampling rate of 512 samples/s; this is a common choice of sampling frequency, often used also in wearable EEG devices, but slightly different values would not affect the validity of the analysis presented in this paper. The chosen pass band is 8-14 Hz, whereas frequency components below 3 Hz and above 18.5 Hz are attenuated more than -20 dB. The order of the filter is 150, corresponding to a time window of approximately 0.3 s, compatible with the expected amplitude dynamics in the filtered signal. The phase of the filter is linear in the pass band, so the equivalent constant time delay is easily removed by post-processing.

As an example, the designed filter has been applied to a clinical EEG signal recorded from a 12-years-old boy, with a sampling rate of 512 samples/s; a channel in the occipital area (O2) has been selected, because the alpha activity is the highest in this area. The raw and filtered signals are reported in Fig. 3. The fast amplitude oscillations are well visible, and they correspond to the expected dynamics of the signal.

III. Challenges in Amplitude Estimation

Even after the application of the band-pass filter in Fig. 2, the signal reported in Fig. 3 is still far from being stationary, because of the large and fast amplitude oscillations, in addition to the possible phase variations and the residual presence of spurious signal components. Therefore, estimating the amplitude of the alpha wave poses significant challenges in terms of signal processing. The challenges are similar in the time and frequency domains, and they mainly arise from the lack of synchronism between the observation window and some or all of the frequency components in the signal (alpha and modulation frequencies).

The simplest method to estimate the signal amplitude, often employed, consists in calculating the power (or the Root Mean Square value) of the signal and estimating the equivalent amplitude from it, assuming a sinusoidal waveform. The total signal power can be conveniently calculated in the time domain, by selecting a time window $T_w = LT_s$ (being T_s the sampling time) and averaging the signal squared over that window:

$$P_{tot}\left(L\right) = \frac{A_{eq}\left(L\right)^2}{2} = \frac{1}{L}\sum_{k=1}^{L} s\left(kT_s\right)^2 \tag{4}$$

If the signal $s(t)$ is periodic and the window T_w corresponds to its period (or an integer multiple of it), the calculation of P_{tot} does not depend on the position of the window and it corresponds to the true power of the signal. If, on the contrary, this condition is not satisfied, the calculation of P_{tot} according to (4) will lead to some errors, which will vary with the window position.

In the case of the considered EEG signal, with fast amplitude oscillations, two types of errors may arise, depending on the length of the window. A short window, approximately equal to the alpha period ($T_w \approx 1/f_\alpha$), would provide a

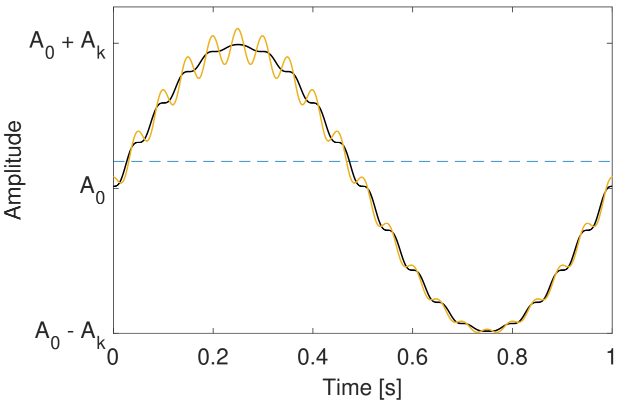

Fig. 4. Estimation of the amplitude of the signal shown in Fig. 1 from the total power calculation (4), with three choices of the time window T_w: 0.09 s (yellow line), 0.1 s (black line) and 1 s (blue dashed line).

suitable time resolution to follow the fast amplitude dynamics, but it can lead to errors if the window is not synchronous with the alpha period, and also if it is, because the amplitude oscillations would not be synchronous with that window. On the other hand, the choice of a much larger window ($T_w \gg 1/f_\alpha$) would decrease the errors caused by a lack of synchronism, but it would include the power associated with the amplitude oscillations in the calculation of the total power of the signal (4); therefore, in this case, the errors in the amplitude estimation would arise from the assumption of sinusoidal waveform, which is no longer a good approximation on long timescales.

The errors described above are illustrated in Fig. 4, based on an artificial signal defined according to (2)-(3), with a sampling rate of 512 samples/s, f_α = 10 Hz, a single modulation frequency f_k = 1 Hz, and A_k = $0.8A_0$; φ_α and φ_k are chosen equal to zero to simplify the analysis (this signal is used here for illustrative purposes only, and it should be noted that real signals are affected by more complex, non-periodic amplitude variations with a large frequency bandwidth). Three different values for the time window T_w are chosen, approximately equal to 0.09 s (L = 46), 0.1 s (L = 51) and 1 s (L = 512). The two shortest time windows can follow the amplitude modulation, but with some errors due to the lack of synchronism with all frequency components in the signal (T_w = 0.09 s) or only with the amplitude modulation (T_w = 0.1 s). On the other hand, the longest time window (T_w = 1 s) is perfectly synchronous with all signal components, but it provides an average amplitude that is higher than A_0 because it includes the contribution from A_k in the total power calculation; moreover, this choice of the time window is not suitable to track the amplitude modulation, so it may not be appropriate for some applications.

Similar issues arise if the signal power is calculated in the frequency domain, after applying the Fourier Transform to the signal, again on a time window $T_w = LT_s$. The total power of the signal can be calculated by summing the powers associated with each frequency component (positive and negative) in the spectrum $S(j\omega)$:

$$P_{tot}\left(L\right) = \frac{A_{eq}\left(L\right)^2}{2} = \sum_{i=-L/2}^{L/2} \left|S\left(j\omega_i\right)\right|^2 \tag{5}$$

The expressions in (4) and (5) are equivalent; therefore, the estimation of the signal amplitude from the calculation of the total power in the frequency domain is affected by the same errors discussed above for the time-domain estimation.

In principle, the frequency-domain analysis may offer some advantages because it allows separating the power associated with different frequency components. When the time window is large enough to acquire all signal components synchronously (e.g., T_w = 1 s in the example considered above), a correct estimation of the average amplitude A_0 can be obtained by considering only the power at frequency f_α in the signal spectrum. In practice, however, the required time window to acquire all signal components synchronously (or at least with negligible errors) may not be acceptable in many applications because of the corresponding poor time resolution. If shorter time windows are considered, the resulting limited frequency resolution will not be enough to separate the spectral component at f_α from the side-bands at $f_\alpha \pm f_k$, and the results will be similar to those obtained from the total power calculation. Moreover, the spectral leakage resulting from the poor frequency resolution may add further errors, particularly when the time window is not synchronous with f_α. The numerical results in the considered example are not reported because they are almost identical to those in Fig. 4, with a mean difference of $0.006A_0$ for the 0.1 s window and $0.02A_0$ for the 0.09 s window.

More advanced signal processing methods may be able to overcome some of the limitations above, but they are mainly designed for different types of signals. E.g., the Hilbert transform is usually applied to signals with time-varying frequencies [9], the wavelet transform is more suitable for signals with a large bandwidth [6], whereas the Taylor-Fourier transform is used to estimate time-varying harmonic content of periodic signals [7], [8]. For these reasons, an alternative approach is proposed in the next section, specifically designed for the considered application.

IV. Model-Based Amplitude Estimation

In order to overcome some of the issues discussed in the previous section, a model-based signal processing method is proposed to estimate the amplitude of alpha waves, with a short time resolution suitable for the analysis of signals with fast dynamics. The method is based on the *a priori* knowledge that the alpha activity produces a sinusoidal waveform with an almost constant frequency and limited phase modulation. Therefore, such a sinusoidal waveform can be identified by means of a model-based fitting of the EEG signal, after an initial filtering with the band-pass filter designed in Sec. II-B.

This approach offers a number of advantages: 1) It allows a short time resolution, without being affected by the significant errors arising from an asynchronous window, because it does not require processing an integer number of periods of the signal; 2) It is less sensitive to other residual frequency components not completely filtered out by the band-bass filter, because in each window it extracts only the signal component at the alpha frequency; 3) Any significant deviation of the signal from the model can reveal that the signal is not a continuous undisturbed alpha wave and can therefore be used for an automatic identification of artifacts or discontinuity in alpha activity.

The method is based on the following four steps:

1) The filtered signal is divided into windows of length $T_w = LT_s$, and the signal in each window k is fitted with the following model, using a least-square-errors approach:

$$m_k(t) = A_k \sin(2\pi f_k t + \varphi_k) \tag{6}$$

The fitting relies on the assumption that the noise not included in the model is unbiased and uncorrelated with the alpha signal.

2) As the frequency of the alpha wave is expected to be almost constant, its value is identified as the median of the frequencies f_k estimated in the windows where the amplitude A_k is higher than a defined threshold:

$$f_\alpha = \text{median}(f_k | A_k > A_{min}) \tag{7}$$

The threshold on the amplitude (here empirically set at 20 μV) is used to discard the windows with negligible alpha activity, where the frequency estimation is less accurate. The median is chosen instead of the mean to decrease the effect of possible outliers.

3) The signal in each window is then fitted again with the same model in (6), but with the known frequency f_α, in order to estimate only the amplitude A_k and phase φ_k.

4) Finally, the amplitudes and phases estimated in the step above for each window are interpolated, in order to estimate the functions $A(t)$ and $\varphi(t)$ with the same time resolution as the original signal.

Steps 1 and 2 above may require further work to optimize the procedure, but this is beyond the scope of this paper, which illustrates only a preliminary implementation of the method.

The resulting $A(t)$ for the EEG signal shown in Fig. 3 is reported in Fig. 5, for two different choices of the time window T_w (0.09 s and 0.1 s, corresponding to L = 46 and 51, respectively), and it is compared to the filtered signal. The estimated constant f_α for this signal is 9.9 Hz, so the 0.1 s window corresponds to almost synchronous conditions, whereas the 0.09 s window corresponds to asynchronous conditions. The results confirm not only that the amplitude estimation is accurate, but also that this method is robust with respect to the choice of the time window T_w, because the model-based fitting does not require a selection of an integer number of periods to obtain an accurate result. The optimal choice of T_w is a trade-off between model accuracy and noise rejection: A too long time window may jeopardize the validity of the constant-parameter model, due to the time-varying nature of the signal, whereas a too short time window

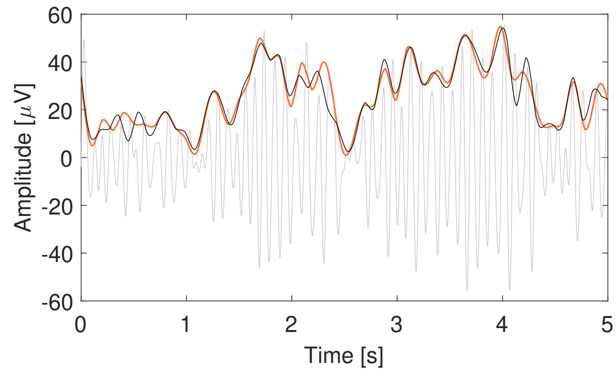

Fig. 5. Band-pass filtered EEG signal (gray thin line, same as in Fig. 3), compared to its estimated amplitude $A(t)$ according to the model-based method described in Sec. IV, with two choices of the time window T_w: 0.09 s (black line) and 0.1 s (orange line).

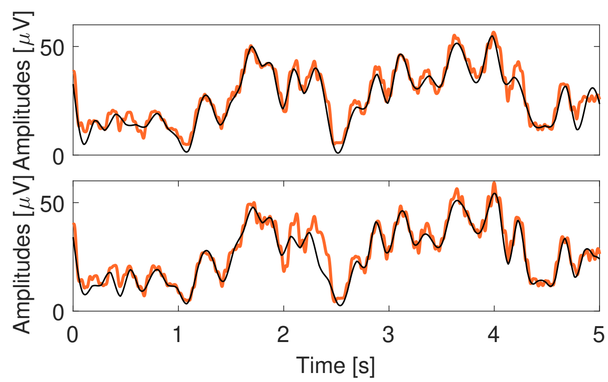

Fig. 6. Comparison between the alpha wave amplitude estimation according to the proposed model-based fitting method (black lines) and from the total signal power (4) (orange lines), with two choices of the time window T_w: 0.1 s (top plot) and 0.09 s (bottom plot).

may be too sensitive to noise. A time window close to the alpha period (around 0.1 s) appears to be a reasonable choice, but it does not have to be strictly synchronous with the alpha period, as Fig. 5 confirms.

In Fig. 6, the same $A(t)$ curves shown in Fig. 5 are reported again, compared to the amplitudes estimated from the total signal power according to (4), for the same choices of the time window T_w (0.09 s and 0.1 s). This shows that the amplitude estimation obtained with the proposed method is not affected by the oscillations that would appear from the power calculation, particularly large in case of an asynchronous window.

V. Artifact Detection

According to the method presented in Sec. IV, the amplitude A_k and phase φ_k estimated in each time window are independent of the values in adjacent windows. While large variations in the amplitude between two windows can be expected, as the alpha activity is characterized by fast and large amplitude oscillations, the variation in the phase is expected to be much more limited. Therefore, the analysis of such phase

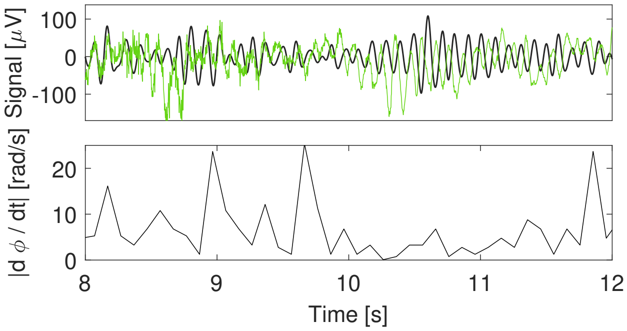

Fig. 7. Raw (green, thin line) and filtered (black, thick line) EEG signals (top plot), compared to the derivative of the estimated phase $\varphi(t)$ of the alpha wave according to the proposed method (bottom plot).

variations can be used to detect the presence of spurious signal components, identified by large and erratic phase variations between consecutive windows, or a discontinuity between intervals of alpha activity.

This is illustrated in Fig. 7, where the absolute value of the numerical derivative of $\varphi(t)$, calculated from the estimated values φ_k, is plotted together with the EEG signal (raw and filtered). This signal is the same as the one used in previous figures, but it is shown in a different time interval (8-12 s), which represents a better example for the purposes of this analysis. In the interval 10-11.5 s, the phase derivative is almost zero, with small oscillations. This means that the signal in that time interval is an authentic alpha wave. On the other hand, in the interval 8-10 s, the phase derivative shows large peaks, which are not compatible with the expected features of an alpha wave. This indicates that the EEG signal is likely to be affected by artifacts or discontinuities in the alpha activity during that time. It is worth noting that the filtered EEG signal looks similar in the two intervals; the raw signal, on the contrary, reveals some differences between the two intervals, particularly in terms of high-frequency components, which support the conclusion that the interval 8-10 s was affected by spurious transients.

This preliminary result confirms that the phase derivative can be effectively used to identify the regions of the signal that contain an authentic, continuous and undisturbed alpha wave, where the estimated amplitude is more accurate, because it is based on a model that correctly represents the signal.

VI. Conclusions

This paper addressed the challenges that arise from the amplitude estimation of alpha waves in EEG signals, in presence of fast and large amplitude oscillations that are likely to affect those signals. Although alpha waves are characterized by a sinusoidal waveform with an almost constant frequency, the digital band-pass filters that are commonly used to extract alpha waves from raw EEG signals have a relatively large bandwidth (8-14 Hz), which was shown to allow fast amplitude oscillations to pass through the filter. To prevent errors arising from such amplitude oscillations in traditional signal processing, a model-based method was proposed to estimate the time-varying amplitude of the alpha wave, based on the *a priori* knowledge of some of its physical properties (sinusoidal waveform, constant frequency and limited phase oscillations). This method was shown to be more robust than traditional methods with respect to the choice of the time window used for the amplitude estimation and the presence of fast amplitude modulation dynamics, thus decreasing errors arising from asynchronous windows. More importantly, the proposed method allows also the automatic identification of artifacts or discontinuity in the alpha wave, based on the mismatch between measured and expected signal features. The method was illustrated with application to a clinical EEG signal, but its properties make it very promising also for wearable EEG applications without expert human supervision.

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