Reliability and Importance Analysis of Uncertain System with **Common Cause Failures Based on Survival Signature**

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Abstract

Redundant design has become the commonly used technique for ensuring the reliability of complex systems, which calls for great concern to common cause failure problems in such systems. Incomplete data in combination with vague judgments from experts introduce imprecision and epistemic uncertainties in the performance characterization of components. These issues need to be taken into account for assessing the system reliability. In this paper, a comprehensive reliability assessment method is presented by adopting the concept of survival signature to estimate the reliability of complex systems with multiple types of components. Particular attention is devoted to common cause failures (CCFs), which are modeled and quantified by decomposed partial α -decomposition method. Uncertainties caused by incomplete data for CCF events are reduced by hierarchical Bayesian inference. The component importance measure is enhanced to assess the importance of various possible CCF scenarios and to identify their potential impact on system reliability. The presented method is used to analyze the reliability of a dual-axis pointing mechanism for communication satellite, which is a commonly used satellite antenna control mechanism. The engineering application demonstrates the effectiveness of the method.

Keywords: reliability analysis, survival signature, common cause failure, hierarchical Bayesian inference, epistemic uncertainty, dual-axis pointing mechanism.

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1. Introduction

Redundant design has become one of the critical measures to ensure the high reliability and long lifetime requirement of large complex systems, such as nuclear systems [1]-[3], aerospace systems[4][5], etc., especially for nonrepairable systems[6]. Common cause failure (CCF), which is failure or degradation of multiple components that triggered by shared causes [7], has become the dominating type of dependent failure in modern complex system. CCFs can lead to a decrease of system reliability, which is critical in view of the original intention of improving system reliability by redundant design. Dependent failure and CCF have attracted a large number of concerns in past decades. In the early stage, the CCFs modeling was introduced for probability safety assessment (PSA) of systems in nuclear industry, explicit and implicit methods have proposed for modeling of CCF [8]. Many parametric models such as the basic parameters. Hokstad and Rausand [9] presented a review and trends of CCF modeling in 2008, especially focused on the development of the β -factor model and its extension.

After the aforementioned CCF quantification parameter models have been proposed, comprehensive work on CCF modelling and evaluate the effect of CCFs is devoted to system reliability. O'Connor summarized the CCF quantification models and proposed an extended α -factor model and a general dependency model based on a Bayesian network (BN) for system risk and reliability assessment [10][11]. Some extension works have been implemented to integrate the effect of CCF with other impact factors, such as uncertainties, on system reliability. Mi et al. [12][13] proposed an evidential network (EN)-based method and a Belief universal generation function (UGF)-based method for reliability analysis of complex multi-state systems (MSSs) with CCF and epistemic uncertainty. Le Duy and Vasseur [1] put forward a new practical method of modelling multi-unit CCFs in a nuclear PSA context. To further investigate the influence of coupling causes on CCFs and system reliability, Zheng et al. [14] proposed a α -factor decomposition method to combine the coupling common cause information in traditional models. Troffaes et al. [15] presented a robust Bayesian approach to modelling epistemic uncertainty in α -factors. Zubair and Amjad [16] used an α -factor model and Bayes theory to calculate and update the system unavailability with consideration of CCF. Recently, George-Williams, et al. [17] has investigated the sensitivities of multiple component failure modes to system survivability, and the critical common cause component group can be identified. However, this

sensitivity work performed till common-cause component groups level, and did not drill down to the vaiours CCF event and common cause layers. Therefore, all those works investigated the relationship between CCFs with system reliability without distinguishing various scenario of CCFs which are caused by several coupling common causes. It is necessary to find a proper method to model the system reliability with different CCF modes and quantify the importance of various CCF scenario to system reliability.

Except for the classical reliability modeling methods, i.e. binary decision diagram (BDD) [18][19], fault tree (FT) model [20], Bayesian networks (BNs) [21][22] etc., survival signature has been proposed by Coolen and Coolen-Maturi [23] based on the concept of system signature [24], as an effective method for system reliability modeling especially for redundant systems with multiple types of component groups. Furthermore, they quantified the uncertainty and dependability of a system with several kinds of components, where the lifetime follows different distributions, by survival signature [25]. When CCFs are considered in system, this research included an investigation of CCFs with non-parametric predictive inference method for system reliability [26]. Moreover, an efficient simulation-based reliability analysis method was proposed by Feng et al. [27] for complex non-repairable systems following CCFs. Some other extension works are presented by Liu et al. [28][29] to analyze stress-strength reliability (SSR) and dynamic SSR of systems with multiple types of components based on survival signature. Survival signature has the prominent advantage that can separate the structure of system from the failure time distribution of its components, and provides a better way to integrate the CCFs and imprecise into system survival function with less time consuming.

In association with system reliability analysis, component importance analysis is useful for system design, reliability improvement and system control. Depending on the purpose of the analysis, a number of different importance measures have been defined. The most commonly used importance measures address structural importance, probability importance and critical importance. Birnbaum [30] categorized importance measures into three classes, including structural importance measures, reliability measures and lifetime importance measures. Kuo and Zhu [31] gave a review of reliability importance measures. Wei et al. [32]-[34] published a comprehensive review on variable importance analysis, and performed considerable work on importance analysis of structural system. Feng et al. [35] integrated the modelling advantage of survival signature, presented a new component importance measures and quantified the effect of impression on the system survival function. Further, Eryilmaz et

al. [36][37] reported considerable developments on joint reliability importance for all kind of systems under several complex system characters, and proposed an extension on marginal and joint reliability importance based on survival signature. Although the research achievements on importance measures are considerable, most of the works are concentrated on the components importance measure. For the importance analysis of CCFs, Guey [38] gave a review of the state-of-the-art of CCF analysis methods, from the sensitivity study of CCF parameters, a conclusion that prevention of CCFs is more important than other analysis technical was given. To solve the problem that how to prevent CCFs, Pan and Nonaka [39] was firstly extended the importance analysis method to the field of CCF analysis, and attempted to find a better time and resources allocation strategy for system reliability analysis. Kamyab [40] et al. performed sensitivity analysis and estimated the importance measures of software CCFs, through this method, the specific contribution of software CCF in the trip failure probability can be revealed. All those works did not distinguish the various CCF types caused by different coupling common causes, there is not yet a proper method to identify the importance of CCF events associated with common causes to system reliability.

Synthesis above works, for complex system with various kinds of redundant mechanisms, such as aerospace system which always consist of components which belong to different types, CCFs are of great importance in reliability evaluation of such systems. There are mainly three problems within the above research works: (1) several coupling common causes will lead to various CCF scenarios which increase the modeling difficulty. For instance, when system reliability is modeled by fault tree or Bayesian network, the relationship between failure causes and events should be analyzed, then the basic common cause events or nodes should be added, which increases the size and complexity of models. (2) Evaluate the system reliability with consideration of CCFs, especially caused by several coupling factors, is time consuming. After the modeling of CCFs in system reliability model, the computing of minimal cut sets for fault trees and the reasoning of Bayesian networks will take much more time corresponding to complex model structures. (3) The research on importance of CCF events caused by different causes needs to further investigate, especially when influenced by other factors, such as missing data, uncertain information, etc. Therefore, it is necessary to perform reliability and importance analysis on system susceptible to CCFs with various scenarios. The importance measure of CCFs would help engineers to find the most significant factor and most efficient defence strategies against CCFs.

In this paper, we aim for advancement in this direction and propose a comprehensive system reliability and importance analysis method that quantifies information and uncertainty of CCF effects driven by the coupling mechanisms in the system. The remainder of this paper is organized as follows. Section 2 presents the problems to be addressed in this paper. Then, a comprehensive system reliability analysis approach will be proposed in Section 3. In this section, a decomposed partial α factor model is deduced, system reliability model, the importance of components and CCF events are developed based on survival signature. Reliability evaluation and importance analysis of an aerospace subsystem with consideration of dependent failure and coupling causes are discussed in Section 4. Section 5 gives a brief conclusion as well as directions for future work.

2. Problem statement

This paper considers the problem of evaluating the reliability and CCF importance of complex redundant system with incomplete data. For most of the situations, it is possible to observe the total frequencies of common cause and CCF events occurrence by experiment or engineering statistics, but it is not easy to investigate the exact common cause of one particular CCF events. This causes an uncertainty challenge for CCF analysis in engineering system, especially when estimating the parameters of CCF models. The importance measure of interest in this paper has two levels of meaning, including the importance of components and importance of different common cause scenarios. Both of them will be analyzed with respect to the comprehensive system reliability. The assumptions for the problem are listed as follows [39][41].

1) The system and components are binary, and only have two states: complete work or failed.

2) The occurrences of different coupling common causes are mutually *s*-independent, and each cause has two states: appear or disappear.

3) Each component can be affected by multiple common causes. When no common cause exists, the failures of components are mutually *s*-independent.

4) A hypothetical database includes common cause occurrence frequencies and CCF events occurrence frequencies is given.

3. Comprehensive system reliability analysis approach

3.1 Methodologies overview

In redundant system, because there are several types of components and the number of each type of components are more than one, one impact factor always can cause two or more than two components with the same type fail simultaneously. So, the common cause failures (CCFs) must be considered in such systems.

There are existing several kinds of CCF models which can be preliminarily divided into four categories including direct estimates, ratio models, shock models and interference models. The most commonly used category of models in engineering practice is ratio models, which can be estimated by specific collected failure data base and success data is not required. On most simple and popular model named β factor model, which was proposed by Fleming in 1975 [42], has been widely used in all kind of engineering practice outside the nuclear industry. This model ignores the size of common cause component groups and only use one parameter β to express the common cause part. When it is used in practice, β factor always defined as a real number which usually between 0.01 to 0.3, it makes this model has a strong dependence on expert's experience which brings subjective uncertainty in systems. The α factor model, which was first proposed by Mosleh and Siu in 1987 [11], is another famous ratio model for CCF modeling. Various multiplicities of failure can be modeled by a series of α factors which can be directly calculated from observed failure data. Some extension models are also proposed based on β factor model and α factor model. In this paper, in order to measure the impact of different CCF modes and various coupling factors on reliability of whole system, α factor model and decomposed partial α factor model will be introduced in Section 3.2.1.

Survival signature, which was proposed by Coolen and Coolen-Maturi in 2012 [23], has proved to be an efficient method for system reliability analysis, especially for redundant systems with multiple types of components and multiple lifetime distributions. Survival signature has the prominent advantage that can separate the structure of system from the failure time distribution of its components [35]. This provide an effective way to deal with the system with imprecise failure time of components, the details can refer from Ref. [35]. In this paper, the survival signature method is introduced to integrate CCFs into system reliability model, the detail will be illustrated in Section 3.2.2

3.2 Proposed comprehensive reliability analysis approach

3.2.1 Common cause failure modeling with incomplete data

- $3.2.1.1 \alpha$ factor model extension for common cause failure modelling
- (1) Global α factor

In this section, the α factor parameter model is introduced to quantify the CCFs in system. Global α factors are defined as fractions of failure probability for particular groups of components without consider the common causes.

For systems with several redundant components, a failure event ratio-based model called α factor model has become a popular quantification model for CCFs. All components which may fail simultaneously by one common cause can be classified within the same common cause component group (CCCG). For the system with *K* types of components and the *k*-th type of components are grouped in a CCCG with m_k components, the common cause factor $\alpha_j^{m_k}$ represents the frequency of there are j ($1 \le j \le m_k$) components failing within this CCCG. Then the α factors can be defined and estimated using the following maximum likelihood estimator,

$$\hat{\alpha}_{j}^{m_{k}} = \left(\sum_{j=1}^{m_{k}} n_{j}^{m_{k}}\right)^{-1} n_{j}^{m_{k}} = \frac{n_{j}^{m_{k}}}{n_{total}^{m_{k}}}$$
(1)

where $n_j^{m_k}$ is the number of failure events with j ($1 \le j \le m_k$) components failing within a CCCG with m_k components. Further, $\sum_{j=1}^{m_k} n_j^{m_k}$ is the total number of failure events caused by a common cause, and $\sum_{j=1}^{m_k} \alpha_j^{m_k} = 1$. $n_j^{m_k}$ can be calculated by the weighted impact vector method [12][43], which can further reflect the multiple failure modelling ability of the α factor model. Besides, the ability of integrating experts' judgments of system and past data makes α factor model be a more proper parameter model in practice engineering than other parameter models, such as β factor model can only get an approximate scope by engineering experiences.

For staggered test data, when $Q_{total}^{m_k}$ is the total probability of failure accounting for both the CCF and the independent failure, the probability of a common cause basic event involving failure of *k* components in the CCCG of m_k components can be calculated by,

$$Q_j^{m_k} = \frac{1}{\binom{m_k - 1}{j - 1}} \alpha_j^{m_k} Q_{total}^{m_k}$$
(2)

For non-staggered test data,

$$Q_{j}^{m_{k}} = \frac{j}{\binom{m_{k}-1}{j-1}} \frac{\alpha_{j}^{m_{k}}}{\alpha_{total}} Q_{total}^{m_{k}} = \frac{j}{\binom{m_{k}-1}{j-1}} \frac{\alpha_{j}^{m_{k}}}{\sum_{j=1}^{m_{k}} j\alpha_{j}^{m_{k}}} Q_{total}^{m_{k}}$$
(3)

(2) Gamma factor: Risk-significance measurement of common causes

Assume that there are *w* coupling factors representing common causes can lead to failure of a component, and all causes have the ability to propagate within CCCGs. In order to represent the portion of system failures which have the potential to propagate through common causes, the Gamma factors are introduced and the maximum likelihood estimate for a Gamma factor of common cause C_i $(1 \le i \le w)$ is [11]

$$\gamma_{i} = \frac{P\left(X_{k}^{m_{k}} | C_{i}\right) P\left(C_{i}\right)}{P\left(X_{k}^{m_{k}}\right)} = E\left(\frac{n_{i,total}}{\sum_{i=1}^{w} n_{i,total}}\right) = E\left(\frac{n_{i,total}}{n_{total}}\right)$$
(4)

where $X_k^{m_i}$ is the failure event of component type k, $n_{i,total}$ is the total number of failure events caused by coupling factor *i*, and n_{total} is the total number of failure events in the whole system. From the definition of γ_i in Eq.(4), it is obvious that the γ_i have practical engineering meaning; they represent the occurrence rate of common cause *i* over the occurrence of all causes. That is there are $\gamma_i \times 100\%$ failures generated by cause *i* among all failures. This index can be used to measure the risksignificance of various causes [14].

(3) Decomposed partial α factors

The inchoate α factor model only considers the concept of an impact vector and cannot make full use of information about failure causes. The record of failure causes for a single failure data has provided the opportunity to model the influence of common causes for CCCGs. Then the α decomposition method and partial α factor model was developed to assess the events and comprehensively considering the knowledge of failure causes [10][14][16].

When there are *w* coupling factors or shared causes which all have a potential to induce the failure of a CCCG with m_k components, the partial α factor $\alpha_{i,j}^{m_k}$ is the portion of system failure events, which includes j ($1 \le j \le m_k$, $1 \le k \le K$) failure components resulting from the *i* -th ($1 \le i \le w$) failure cause (coupling factor). $n_{j,i}$ is the number of failure events which resulted in j ($1 \le j \le m$) component failures by coupling factor *i*. Then the maximum likelihood estimation of the partial α factor model is

$$\alpha_{i,j}^{m_{k}} = \frac{P(FM_{k,j} | C_{i})}{P(X_{k}^{m_{k}} | C_{i})} = E\left(\frac{n_{i,j}^{m_{k}}}{\sum_{j=1}^{m_{k}} n_{i,j}^{m_{k}}}\right) = E\left(\frac{n_{i,j}^{m_{k}}}{n_{i,total}}\right)$$
(5)

provides $FM_{k,j}$ is the failure event of component type k caused by coupling factor j; $n_{i,total}$ is the total number of failure events caused by coupling factor i. The partial α factor represents the probability of a failure to propagate to other components which in the same CCCG through a particularly common cause. $\alpha_{i,j}^{m_k}$ is a risk characteristic of possible causes which also represent different CCF triggering abilities.

Figure 1 shows the decomposition of global α factors and representation of partial α factors. The global α factors are affected by decomposed partial α factors and coupling causes.



Figure 1 Global a factors decompose into partial a factors

For a component $X_k^{m_k}$ in CCCG k, the probability of component can be expressed by

$$P(X_{k}^{m_{k}}) = P(X_{k}^{m_{k}}|C_{1})P(C_{1}) + P(X_{k}^{m_{k}}|C_{2})P(C_{2}) + \dots + P(X_{k}^{m_{k}}|C_{w})P(C_{w})$$
(6)

And the probability of component caused by cause 1 (C_1) can be expressed as the sum of different failure mode caused by cause 1, which is

$$P(X_{k}^{m_{k}}|C_{1}) = P(FM_{k,1}|C_{1}) + P(FM_{k,2}|C_{1}) + \dots + P(FM_{k,m_{k}}|C_{1})$$
(7)

Then, from Eqs. (5)-(7), the relation among decomposed α factors for common cause *i* can be further deduced and

$$\frac{P(FM_{k,1}|C_i)}{P(X_k^{m_k}|C_i)} + \dots + \frac{P(FM_{k,m_k}|C_i)}{P(X_k^{m_k}|C_i)} = \sum_{j=1}^{m_k} \alpha_{i,j}^{m_k} = 1$$
(8)

The probability of the *j*-th failure mode (*j* components fail in *k*-th CCCG) is

$$P(FM_{k,j}) = P(FM_{k,j}|C_1)P(C_1) + P(FM_{k,j}|C_2)P(C_2) + \dots + P(FM_{k,j}|C_w)P(C_3)$$

= $\alpha_{1,j}^{m_k}P(X_k^{m_k}|C_1)P(C_1) + \dots + \alpha_{1,j}^{m_k}P(X_k^{m_k}|C_w)P(C_w)$ (9)

Because $P(FM_{k,j})/P(X_k^{m_k}) = \alpha_j^{m_k}$, the Eq. (9) can be written as the following equation when both sides are divided by $P(X_k^{m_k})$,

$$\alpha_{j}^{m_{k}} = \alpha_{1,j}^{m_{k}} \frac{P(X_{k}^{m_{k}} | C_{1}) P(C_{1})}{P(X_{k}^{m_{k}})} + \dots + \alpha_{w,j}^{m_{k}} \frac{P(X_{k}^{m_{k}} | C_{w}) P(C_{w})}{P(X_{k}^{m_{k}})}$$
(10)

The following simple form of global α factors $\alpha_j^{m_k}$ can be considered as weighted averages of all decomposed partial α factors $\alpha_{i,j}^{m_k}$, and can be obtained by taken Eq.(5) into Eq. (10),

$$\alpha_j^{m_k} = \sum_{i=1}^{w} \alpha_{i,j}^{m_k} \gamma_i \tag{11}$$

Then, a common cause matrix $\mathbf{M}_{CCCG}^{m_k}$ is defined to represent the effect degree of different common cause on each failure mode in a common cause component group with m_k components of type k.

$$\mathbf{M}_{CCCG}^{m_{k}} = \begin{bmatrix} \alpha_{1,1}^{m_{k}} & \dots & \alpha_{1,j}^{m_{k}} & \dots & \alpha_{1,1}^{m_{k}} \\ \dots & \dots & \dots & \dots & \dots \\ \alpha_{i,1}^{m_{k}} & \dots & \alpha_{i,j}^{m_{k}} & \dots & \alpha_{1,m_{k}}^{m_{k}} \\ \dots & \dots & \dots & \dots & \dots \\ \alpha_{w,1}^{m_{k}} & \dots & \alpha_{w,j}^{m_{k}} & \dots & \alpha_{1,m_{k}}^{m_{k}} \end{bmatrix}$$
(12)

When the Gamma factors are packed into matrix $\gamma = [\gamma_1, ..., \gamma_i, ..., \gamma_w]$, a global common cause matrix $\mathbf{M}_{CCCG}^{\prime m_k}$ will be gotten and

$$\mathbf{M}_{CCCG}^{\prime m_{k}} = \boldsymbol{\gamma} \times \mathbf{M}_{CCCG}^{m_{k}}$$
$$= \left[\sum_{i=1}^{w} \gamma_{i} \alpha_{i,1}^{m_{k}}, ..., \sum_{i=1}^{w} \gamma_{i} \alpha_{i,j}^{m_{k}}, ..., \sum_{i=1}^{w} \gamma_{i} \alpha_{i,m}^{m_{k}}\right]$$
$$= \left[\alpha_{1}^{m_{k}}, ..., \alpha_{j}^{m_{k}}, ..., \alpha_{m_{k}}^{m_{k}}\right]$$
(13)

For example, for a common cause group with 4 components, there are two coupling causes, for each cause, the collected failure data vectors are $\mathbf{I}_1 = [45, 6, 2, 3]$ and $\mathbf{I}_2 = [35, 4, 3, 2]$, then the Gamma factors can be calculated as $\gamma = [\gamma_1 = 56/(56+44), \gamma_2 = 44/(56+44)] = [0.56, 0.44]$, and the decomposed partial α factors are calculated as,

$$\mathbf{M}_{CCCG} = \begin{bmatrix} \alpha_{1,1} = 45/56 & \alpha_{1,2} = 6/56 & \alpha_{1,3} = 2/56 & \alpha_{1,4} = 3/56 \\ \alpha_{2,1} = 35/44 & \alpha_{2,2} = 4/44 & \alpha_{2,3} = 3/44 & \alpha_{2,4} = 2/44 \end{bmatrix}$$
(14)

Then the global α factor matrix \mathbf{M}'_{CCCG} of the CCF of common cause group will be calculated by Eq. (13) and

$$\mathbf{M}_{ccccg}' = \gamma \times \mathbf{M}_{ccccg} = \begin{bmatrix} 0.45 + 0.35 & 0.06 + 0.04 & 0.02 + 0.03 & 0.03 + 0.02 \end{bmatrix}$$

=
$$\begin{bmatrix} 0.8 & 0.1 & 0.05 & 0.05 \end{bmatrix}$$
 (15)

And the former matrix \mathbf{M}'_{cccg} can be split into two matrix based on coupling causes, and for cause 1 and 2 the partial α factor matrixes are [0.45 0.06 0.02 0.03] and [0.35 0.04 0.03 0.02] respectively.

3.2.1.2 Hierarchical Bayesian inference with incomplete data

In this section, a standard Bayesian inference approach is used to reduce the epistemic uncertainty caused by incomplete data in decomposed α factor estimation. Different data sources of CCF failure events and coupling causes occurrence can be combined by a Bayesian regression model and obtained posterior distributions so that a result with less uncertainty. The detail two-stage hierarchical Bayesian inference process is shown in Figure 2.



Figure 2 Hierarchical Bayesian inference process

The prior distributions for decomposed partial α factors $\alpha_{i,j}^{m_k}$ are modelled by noninformative distributions. A precise Dirichlet distribution with all parameters δ_i equal to 1 is most commonly chosen as the prior distribution which is conjugate to multinomial likelihood $(\alpha_{i,j}^{m_k}[1:m_k] \sim ddirich(\delta[1:m_k]))$. The global α factors $\alpha_j^{m_k}$ is also assumed as a Dirichlet distribution $(\alpha_j^{m_k}[1:m_k] \sim ddirich(\theta[1:m_k]))$. Then through updating the Dirichlet prior with observed data, the posterior distribution can be obtained, which is still a Dirichlet distribution [14][44]-[46]. The data of CCF events **t** is assumed as a multinomial distribution $(t[k,1:m_k] \sim dmulti(\alpha_i^{m_k}[k,1:m_k],T[k]), T[k]$

is the total failure events in *k*-th CCCG). The Bayes' theorem for decomposed partial α factors can be given by

$$\pi_{2}\left(\boldsymbol{\alpha}^{C_{i}}\left|\mathbf{r},\boldsymbol{\theta},\boldsymbol{\alpha},\mathbf{t}\right.\right)=\frac{P\left(\boldsymbol{\alpha}^{C_{i}},\mathbf{r},\boldsymbol{\theta},\boldsymbol{\alpha},\mathbf{t}\right)}{P\left(\mathbf{r},\boldsymbol{\theta},\boldsymbol{\alpha},\mathbf{t}\right)}=\frac{L\left(\mathbf{r},\boldsymbol{\theta},\boldsymbol{\alpha},\mathbf{t}\left|\boldsymbol{\alpha}^{C_{i}}\right.\right)\pi_{1}\left(\boldsymbol{\alpha}^{C_{i}}\right)}{P\left(\mathbf{r},\boldsymbol{\theta},\boldsymbol{\alpha},\mathbf{t}\right)}$$
(16)

where \mathbf{r} , $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}^{C_i}$ are the vectors of the common causes' occurrence rates, global α factors and decomposed partial α factors, respectively. $\pi_1(\boldsymbol{\alpha}^{C_i})$ is the prior distribution for decomposed partial α factors $\boldsymbol{\alpha}_{i,j}^{m_k}$, π_2 is the posterior distribution, and $L(\mathbf{r}, \boldsymbol{\theta}, \boldsymbol{\alpha}, \mathbf{t} | \boldsymbol{\alpha}^{C_i})$ is the likelihood of parameters. The posterior joint probability density can be written as

$$\pi_{2}\left(\boldsymbol{\alpha}^{C_{i}},\boldsymbol{\alpha}|\mathbf{r},\boldsymbol{\theta},\mathbf{t}\right) \propto p_{t}\left(\mathbf{t}|\boldsymbol{\alpha}\right)p_{\alpha}\left(\boldsymbol{\alpha}|\boldsymbol{\alpha}^{C_{i}},\mathbf{r},\boldsymbol{\theta}\right)p_{\theta}\left(\boldsymbol{\theta}|\boldsymbol{\alpha}^{C_{i}},\mathbf{r}\right)\pi_{1}\left(\boldsymbol{\alpha}^{C_{i}}\right)$$
(17)

where $p_t(\mathbf{t}|\boldsymbol{\alpha})$ is likelihood function, $p_{\alpha}(\boldsymbol{\alpha}|\boldsymbol{\alpha}^{C_i},\mathbf{r},\boldsymbol{\theta})$ is called prior distribution and $p_{\theta}(\boldsymbol{\theta}|\boldsymbol{\alpha}^{C_i},\mathbf{r})$ is hyper prior distribution. Based on Eqs. (6)-(11), the parameters in the hyper prior distribution $p_{\theta}(\boldsymbol{\theta}|\boldsymbol{\alpha}^{C_i},\mathbf{r})$ can be expressed by decomposed partial α factors and occurrence rates, which can be represented as $\theta[k, j] = (\sum_{i=1}^{w} \alpha_{i,j}^{m_k} \times r[k,i])T[k]$. Therefore, the marginal density of global α factors and decomposed partial α factors can be computed

$$\pi_{2}\left(\boldsymbol{\alpha}|\mathbf{r},\boldsymbol{\theta},\mathbf{t}\right) = \int \pi_{2}\left(\boldsymbol{\alpha}^{C_{i}},\boldsymbol{\alpha}|\mathbf{r},\boldsymbol{\theta},\mathbf{t}\right) d\boldsymbol{\alpha}^{C_{i}}$$
(18)

$$\pi_{2}\left(\boldsymbol{\alpha}^{C_{i}} | \mathbf{r}, \boldsymbol{\theta}, \mathbf{t}\right) = \int \pi_{2}\left(\boldsymbol{\alpha}^{C_{i}}, \boldsymbol{\alpha}^{C_{i}} | \mathbf{r}, \boldsymbol{\theta}, \mathbf{t}\right) d\boldsymbol{\alpha}$$
(19)

Finally, the posterior distribution of decomposed partial α factors and global α factors can be calculated. All the calculation of hierarchical Bayesian inference is conducted by OpenBUGS version 3.2.3, the detailed script for this inference process can refer to Ref. [14].

3.2.2 Comprehensive system reliability modeling by survival signature with common cause failures

In [27], Feng et al. had proposed a simulation method to analyze system reliability with CCFs, this section proposes a simple theoretical method for system reliability analysis with consideration of CCF for redundant systems. The CDF of type k component is assumed to be $F_k(t)$, which can be obtained from independent experimental data in the design stage. Thus, it is reasonable to consider the original CDF of the component as the independent part of the component failure probability. The dependent part of failures is caused by external environmental; the interior component ageing; the design, manufacturing, and installation quality; and human errors, etc. [12]. The corresponding weighted

impact vector of different impact factor can be calculated by the stress strength interference (SSI) model and the classical life distribution of components, which have described in Ref. [12] in detail. Furthermore, based on the definition of the α factor model, the total failure probability of each component with consideration of CCF will be

$$F_{k,CCF}(t) = \binom{m_k - 1}{1 - 1} (\alpha_1^{m_k})^{-1} F_k(t) = (\alpha_1^{m_k})^{-1} F_k(t)$$
(20)

And the failure probability of a common cause event with *j* components failure will be

$$Q_{j}\left(t\right) = {\binom{m_{k}-1}{j-1}}^{-1} \frac{\alpha_{j}^{m_{k}}}{\alpha_{1}^{m_{k}}} F_{k}\left(t\right)$$
(21)

Then, the Eq. (20) can be further derived by the following process,

$$F_{k,CCF}(t) = \sum_{j=1}^{m_{k}} {\binom{m_{k}-1}{j-1}} Q_{j}(t)$$

$$= \sum_{j=1}^{m_{k}} {\binom{m_{k}-1}{j-1}} {\binom{m_{k}-1}{j-1}}^{-1} \frac{\alpha_{j}^{m_{k}}}{\alpha_{1}^{m_{k}}} F_{k}(t)$$

$$= \sum_{j=1}^{m_{k}} \frac{\alpha_{j}^{m_{k}}}{\alpha_{1}^{m_{k}}} F_{k}(t) = (\alpha_{1}^{m_{k}})^{-1} \sum_{j=1}^{m_{k}} \alpha_{j}^{m_{k}} F_{k}(t)$$

$$= (\alpha_{1}^{m_{k}})^{-1} F_{k}(t)$$

(22)

Based on the definition of survival probability in Ref. [23] and [35], the survival probability of the *k*-th CCCG with $C_k(t)$ ($C_k(t) = l_k$) components working at time *t* can be expressed as the following equation, and this is based on the assumption that the failure components of different type are independent.

$$P\left(\left\{C_{k}\left(t\right)=l_{k}\right\}, CCF\right)=\binom{m_{k}}{l_{k}}\left[F_{k,CCF}\left(t\right)\right]^{m_{k}-l_{k}}\left[1-F_{k,CCF}\left(t\right)\right]^{m_{k}}$$
(23)

Based on the definition of survival signature in Ref. [23], the system survival function with CCF can be extended and will be expressed as,

$$P_{CCF}(T_{s} > t) = \sum_{l_{1}=0}^{m_{1}} \dots \sum_{l_{K}=0}^{m_{K}} \Phi(l_{1}, \dots, l_{K}) P\left(\bigcap_{k=1}^{K} \{C_{k}(t) = l_{k}\}, CCF\right)$$

$$= \sum_{l_{1}=0}^{m_{1}} \dots \sum_{l_{K}=0}^{m_{K}} \Phi(l_{1}, \dots, l_{K}) \prod_{k=1}^{K} \binom{m_{k}}{l_{k}} [F_{k,CCF}(t)]^{m_{k}-l_{k}} [1 - F_{k,CCF}(t)]^{m_{k}}$$
(24)

where $\Phi(l_1,...,l_K)$ is the survival signature of system. The survival signature defines the probability that the system functions when l_k of its m_k components of type k function, where $l_k = 0, 1, ..., m_k$ for k = 1, ..., K. It can be defined by the following equation [23][25][26]

$$\Phi(l_1,\dots,l_K) = \left[\prod_{k=1}^{K} \binom{m_k}{l_k}^{-1}\right] \times r_{m_1,\dots,m_K}(l_1,\dots,l_K)$$
(25)

where $r_{m_1,...,m_k}(l_1,...,l_k)$ denotes the total number of path sets for a system with l_k components of type k working, and $1 \le k \le K$. Then, combining Eqs. (23) and (25) with Eq.(24), the system survival probability can be computed. For this purpose, to calculate the system survival signature and reliability with high efficiency, a number of methods are available. The most advanced developments in this regard the R package "ReliabilityTheory" [47] and the sampling approach proposed in [48]. An alternative approach to compute the exact signature for systems with reduced computation time was proposed by Reed [49], based on reduced order binary decision diagrams (ROBDDs).

Using the numerical example shown as the Fig. 2 in Ref. [35]. The 6 components are grouped into to two CCCGs by the type of components. The lifetime distribution of each type of components is assumed to follow exponential distribution, and $F_1(t) = 1 - e^{-0.8t}$, $F_2(t) = 1 - e^{-1.6t}$.

For component type 1 (X_{11} , X_{12} , X_{13}), the corresponding α factor are assumed and $\alpha_1^1 = 0.8$, $\alpha_2^1 = 0.1$, $\alpha_3^1 = 0.1$; and for type 2 components, $\alpha_1^2 = 0.9$, $\alpha_2^2 = 0.05$, $\alpha_3^2 = 0.05$. Then the survival function of system with CCF and without CCF can be gotten by Eqs. (24) and (25) which is shown in Figure 3. From Figure 3 we can see that the survival function is reduced by consideration of CCF, which means CCFs have remarkable influence on system reliability, it is necessary to take into account of CCFs when analyzing system reliability.



Figure 3 Survival function of sample system

3.2.3 Importance analysis

^{3.2.3.1} Importance of component to system

For estimating the importance of component i at time t in the system, Feng et al. proposed the Birnbaum's measure-based relative importance index [27][30], which can be used to identify the most critical components for system reliability, and this relative importance index can be expressed as,

$$RI_{i}(t|P) = \frac{\partial R_{s}(t)}{\partial R_{i}(t)}$$

$$= P(T_{s} > t|T_{i} > t) - P(T_{s} > t|T_{i} < t)$$

$$= \sum_{l_{i}=0}^{m_{i}} \dots \sum_{l_{K}=0}^{m_{K}-1} \Phi_{X_{i}=1}(l_{1}, \dots, l_{K}) P\left(\bigcap_{k=1}^{K} \{C_{k}(t) = l_{k}\}\right)$$

$$- \sum_{l_{i}=0}^{m_{i}} \dots \sum_{l_{K}=0}^{m_{K}-1} \Phi_{X_{i}=0}(l_{1}, \dots, l_{K}) P\left(\bigcap_{k=1}^{K} \{C_{k}(t) = l_{k}\}\right)$$

$$= \sum_{l_{i}=0}^{m_{i}} \dots \sum_{l_{K}=0}^{m_{K}-1} (\Phi_{X_{i}=1}(l_{1}, \dots, l_{K}) - \Phi_{X_{i}=0}(l_{1}, \dots, l_{K})) P\left(\bigcap_{k=1}^{K} \{C_{k}(t) = l_{k}\}\right)$$
(26)

Take the components in Section 3.2.2 as an example, the procedure of computing the importance of X_{23} can be illustrated as following steps: 1) X_{23} is set to be functioning, replace X_{23} by a forward path in the system reliability diagram, calculate the system survival signature and expressed as $\Phi_{X_{23}=1}(l_1, l_2)$; 2) X_{23} is set to be failed, X_{23} should be taken away, and the path should be disconnected in system reliability diagram, the system survival signature will be expressed as $\Phi_{X_{23}=0}(l_1, l_2)$. Then based on the definition of relative importance index in Eq. (26) the importance of X_{23} can be calculated by

$$RI_{i}(t|P) = P(T_{s} > t|T_{i} > t) - P(T_{s} > t|T_{i} < t)$$

$$= \sum_{l_{1}=0}^{m_{1}} \sum_{l_{2}=0}^{m_{2}-1} (\Phi_{X_{23}=1}(l_{1}, l_{2}) - \Phi_{X_{23}=0}(l_{1}, l_{2})) P\left(\bigcap_{k=1}^{2} \{C_{k}(t) = l_{k}\}\right)$$
(27)

Similarly, the relative importance indexes of all components in this example can be calculated and shown in Figure 4(a), when CCF are considered in this system, the importance of components can be shown as Figure 4(b). In order to shown the difference between importance of components with consideration of CCF and without CCF, the importance of X_{13} is shown in Figure 5.



Figure 4 Importance of components (a) without CCF; (b) with CCF



Figure 5 Importance of component X_{13}

3.2.3.2 Importance of common cause failure to system

In order to measure the effect of two or more components on system performance, the time dependent joint reliability importance (JRI) is induced to measure the joint contribution of common cause event to system reliability. And based on Ref. [37] the JRI of two components is defined as

$$JRI(i, j) = P(T_s > t | T_i > t, T_j > t) - P(T_s > t | T_i > t, T_j \le t)$$

$$-P(T_s > t | T_i \le t, T_j > t) + P(T_s > t | T_i \le t, T_j \le t)$$
(28)

where, JRI > 0 means one component will become more important when the other is functioning; JRI = 0 means the importance is unchanged; and JRI < 0 means one component will become less important when the other is functioning. This definition can straightforwardly be extended to several common cause events. In this paper, only the effect of common causes to the same types of components are considered, which means the failure of different type of components are assumed to be time independent. On the basis of definition of *JRI* and *RI*, in order to further identify the potential impact of different various possible CCF scenarios on the system reliability, the relative importance index of CCF events can be defined as,

$$RI_{CCF}\left(x_{i}^{m_{k}}, x_{j}^{m_{k}}\right) = P_{CCF}\left(T_{s} > t \left| T\left(x_{i}^{m_{k}}\right) > t, T\left(x_{j}^{m_{k}}\right) > t\right) - P_{CCF}\left(T_{s} > t \left| T\left(x_{i}^{m_{k}}\right) \le t, T\left(x_{j}^{m_{k}}\right) \le t\right)\right)$$

$$(29)$$

where,

$$P_{CCF}\left(T_{s} > t \left| T\left(x_{i}^{m_{k}}\right) \leq t, T\left(x_{j}^{m_{k}}\right) \leq t \right) = \sum_{l_{1}=0}^{m_{1}} \dots \sum_{l_{k}=0}^{m_{k}-2} \dots \sum_{l_{k}=0}^{m_{K}} \Phi_{x_{i}^{m_{k}} = 0, x_{j}^{m_{k}} = 0}\left(l_{1}, \dots, l_{K}\right) P\left(\bigcap_{k=1}^{K} \left\{C_{k}\left(t\right) = l_{k}\right\}\right)$$
(30)

Here, only the CCFs that occur in the same type of components are considered. That is, the failures of different type of components are still assumed to be independent of one another.

4. Case study

4.1 System description : Dual-axis pointing mechanism for communication satellite

As a key part of realizing the large scope of satellite antenna rotation and high precise positioning, dual-axis positioning mechanism is prone to fail, therefore, its reliability analysis is of a great significance. According to different functions, the entire dual-axis positioning mechanism can be divided into two subsystems: the transmission system and the control system. The transmission system achieves accurate positioning of satellite antenna system through adjusting the direction of the pitch axis and azimuth axis. Each axis is mainly composed of a motor, a reducer, and the shafts. The proper control voltage transmits to stepper motor and drives shaft to rotate, then the force and torque go through a harmonic reducer, and drive the antenna to rotate around the corresponding axis to the predetermined angle. After precise direction adjustment of pitch axis and azimuth axis, the accurate positioning of whole satellite can be performed. According to the basic operating principle and the structure of the transmission system, the reliability block diagram can be expressed as a parallel-series structure and shown in Figure 6. Each axis consists of stepper motor, drive shaft, and harmonic reducer which are in series. In this paper, the pitch axis and azimuth axis are simplified as parallel units, i.e. one axis failure does not cause the failure of the entire system [42].



Figure 6 The reliability block diagram of transmission subsystem

Since the pitch axis and the azimuth axis of the positioning mechanism are in a parallel structure, the relationship between the states of the two sets of components (A_1 is the state of the pitch axis, A_2 is the state of the azimuth axis) and the system state (X) is as follows: the failure of both sets of components will cause the failure of the whole transmission system. If one set of components is failed and the other is partially failed, the system is failed too. The system is working properly only when at least one of the two sets of components are working perfectly.

The control system is responsible for the movement control of dual-axis pointing mechanism, including power supply subsystem, operation circuit and control computer. Those three parts are modeled as series in system reliability block diagram. In order to improve the system reliability, standby strategies are generated in these subsystems. The power supply components are composed by one main component and two hot-standby components. The operation circuit and control computer use the cold-standby strategy, which are both including one main component and one standby component. Based on above structure and function analysis, the reliability block diagram of control subsystem of dual-axis pointing mechanism can be obtained and shown in Figure 7.



Figure 7 The reliability block diagram of control subsystem

4.2 Reliability and importance analysis

In order to simply the problem, in this section, the dual-axis pointing mechanism is supposed to be static system, which means the standby strategies are translated into a series and parallel structure. The resulting representation of the dual-axis pointing mechanism system with 6 types of components are shown in Figure 8. Due to the complexity of the dual-axis positioning mechanism of satellite antenna and the shortage of data, the prior probability of each type of components at mission time t = 3000h are given based on experience [42]. When there are two common cause factors to make the components with the same type failure, the data of 6 CCCGs including the occurrence of causes and CCF events are listed in Table 1. Based on the partial α factor model introduced in Section 3.2.1, when lifetime distributions of components are assumed to be exponential distribution, then the failure rates of components and CCF model factors can be calculated and listed in Table 2.



Figure 8 System static logic block diagram

Table 1 Database of CCF events

Component type	Common cause frequency			CCF event frequency		
(CCFGs)	Cause 1	Cause 2	Total	single	2_Comp.	3_comp.
Type 1 (X11, X12)	14 (70%)	6 (30%)	20	19	1	/
Type 2 (X21, X22)	4 (40%)	6 (60%)	10	9	1	/
Type 3 (X31, X32)	11(55%)	9 (45%)	20	19	1	/
Type 4 (X41, X42, X43)	8 (80%)	2 (20%)	10	18	1	1
Type 5 (X51, X52)	3 (30%)	7 (70%)	10	9	1	/
Type 6 (X61, X62)	12 (60%)	8 (40%)	20	19	1	/

Table 2 Components lifetime distribution parameters

Comp. type	Dist. type	Dist. Para. (10 ⁻⁷)	CCF Para. $M_{\rm CCCG}$
1	Exp	1.00015	{0.95,0.05}
2	Exp	2.33415	{0.9,0.1}
3	Exp	2.00060	{0.95,0.05}
4	Exp	1.33360	{0.9,0.05,0.05}
5	Exp	1.16687	{0.9,0.1}
6	Exp	3.00135	{0.95,0.05}

Based on the Bayesian inference approach which introduced in Section 3.2.1, the estimated global

 α factors and decomposed partial α factors for each type of components can be obtained by using the database of CCF events in Table 1. The posterior distributions for decomposed partial α factors are summarized and listed in Table 3, and the risk-significance of two common causes are also listed in this table. Figure 9 represents the standard deviations of global α factors for each type of components (CCCGs), the standard deviations of global α factors without cause information is larger than that with consideration of causes information. It is clear that the uncertainty in the model parameter estimation has been reduced by integration of all kind of information, including failure occurrence rate, failure coupling causes etc.

Posterior $\alpha_{i,j}$	Mean	SD	2.5%	Median	97.5%
alpha.c1[1]	0.8841	0.06343	0.7387	0.8931	0.979
alpha.c2[1]	0.8149	0.09607	0.6017	0.8254	0.9665
alpha.c1[2]	0.07816	0.05661	0.003541	0.06764	0.213
alpha.c2[2]	0.1439	0.08889	0.0111	0.1325	0.3432
alpha.c1[3]	0.03771	0.02984	0.001891	0.0307	0.1136
alpha.c2[3]	0.04122	0.03879	0.00113	0.0299	0.1436
γ.c1	0.57778		γ.c2	0.42222	

Table 3 Posterior distributions for decomposed partial a factors



Figure 9 Comparison of uncertainty in global a factors

To analyze the system reliability, firstly, when CCFs are not considered in this system, the system survival function can be calculated with the data presented in Table 1. Then we analyze the system survival function with consideration of CCFs in all CCCGs. Furthermore, CCF and coupling causes are both taken into account, the system survival function is updated by the renewed global α factors for each CCCGs, and the results of those three conditions are shown in Figure 10. The reliability of this aerospace subsystem is decreased with CCF, which means CCF has significant influence on the system reliability. After implementing coupling cause information in the α factors model, the reliability of the system experiences a little decrease. Considering cause information can lead to a relatively conservative result of the system reliability analysis. Without considering CCF and coupling causes, the reliability of system would be estimated with a much more optimistic result, which leads to a potential hazard for the whole system, which is particularly critical for an aerospace system with high requirement of reliability and long lifetime.



Figure 10 Survival function of aerospace subsystem

The importance of each type of components without CCFs can be computed by Eq. (26), as shown in Figure 11(a). Figure 11(b) represents the importance of all types of components under CCFs. The ranking of component importance is

$$RI(Type2) > RI(Type3) > RI(Type1) > RI(Type5) \approx RI(Type6) > RI(Type4)$$
(31)

As shown in Figure 11, the importance of component type 4 stays at a low level. This is because there are two hot-standby power supply components, they both have lower importance in the entire system, and the change of their reliability will don't have obvious influence on the reliability of whole system. The importance ranking of all type of components can give correct and effective guidance for system renewal design. In order to clearly present the effect of CCFs to component importance, importance of component type 3 and 4 before and after considering CCFs are extracted and shown in Figure 12(a) and (b), respectively. We can see that when the CCFs and coupling causes information are both considered, the importance of components will lead to an adjustment but remain in the original trend.







Then based on the definition of importance of CCFs to the system in Section 3.2.3, the importance of various of CCFs can be computed, see Figure 13. All kinds of CCF events have crucial importance in the system except component with type 4. The reason is that there are 2 hot-spare components for the main component type 4. Even when a common cause leads to a two-components failure event, one spare component remains functioning to keep the system working properly. The ranking of CCF importance for different types of components is approximately

$$RI_{CCF}(Type2) > RI_{CCF}(Type3) > RI_{CCF}(Type1) > RI_{CCF}(Type5)$$

$$\approx RI_{CCF}(Type6) > RI_{CCF}(Type4_2) > RI_{CCF}(Type4_3)$$
(32)

By contrast, it is obvious that the ranking of CCFs importance for different types of CCCGs is almost the same as the component's importance ranking. Figure 14 represents the comparison of various CCFs' importance with and without cause information. Under all information we have gotten, failure causes information has unconscious effect on importance of CCFs, but has relative obvious influence on importance of components. However, the uncertainty of CCFs could be reduced with rational use of all kinds of information.



Figure 13 Importance of various CCFs to system without cause information



Figure 14 Comparison of CCFs' importance with and without cause information

To reflect the computing efficiency of the comprehensive method in this paper, the Bayesian network method[50] is used as a comparison method. To avoid the impact of different factors on computation time, these two methods are both coded by Matlab R2016b 64-bit (win64), and running on the same computer under all the same operating conditions. The computing time of system survival function by this comprehensive met9hod is 5.86s, while the time consuming by Bayesian network is 18.48s. Besides, for the calculation time of various CCF events importance by the proposed method and Bayesian network are 17.37s and 27.49s, respectively. From this, we can conclude that this comprehensive method is less time consuming and has relatively high efficiency.

5. Conclusion

This paper proposed an effective method for reliability evaluation of redundant system based on survival signature and α factor model. CCF events are modelled by the α factor parameter method which is only focused on the occurrence frequencies of CCF events. A Gamma factor is introduced to quantify the risk-significance of a set of possible common causes. The α -decomposition method and partial α factors are developed to determine the CCF triggering ability of various coupling causes. However, due to the lack of available failure data, unknown uncertainties always exist in the CCF parameters evaluation process. A Bayesian inference method is adopted to reveal the combination of critical failure events, and the renewed global α factors for the whole system can be obtained with reduced uncertainty.

The system reliability and the component importance are formulated and analyzed by survival signature method. This paper special focus on the importance of various CCF events to system reliability, and give a definition of CCF importance based on relative joint importance of components. Finally, with a case, system reliability of an aerospace subsystem (the dual-axis pointing mechanism which is a commonly used satellite antenna control mechanism for communication satellite) is evaluated considering failure frequencies and coupling cause. The importance of components and CCF events is also investigated and ranked. The result shows that the reliability will be too optimistic compared with system under uncertainty and dependent assumption, which will lead to an overestimation of system reliability. The ranking of importance could help to perform maintenance actions of repairable systems, but for non-repairable systems such as aerospace systems, this result will also give rational guidance for the renewal design. However, the quantification of uncertainty in system hasn't been accurately measured, these imprecision and epistemic uncertainties can be expressed by different forms of survival function with optimization algorithm will be an avenue in future work.

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