Combating Greenwashers in Emerging Markets: A Game-Theoretical Exploration of Firms, Customers and Government Regulations

Abstract

The sustainability boom in developing countries has caused the emergence of greenwashing behaviors that are impeding sustainable progress in the third world. To combat greenwashing behaviors in emerging markets, this paper proposes a game-theory model that studies the competitive pricing strategies of green incumbents facing competition from greenwashing entrants in emerging markets. The research considers customer behaviors, greenwashing threats, and the different implications of anti-greenwashing government enforcement strategies. The results reveal that greenwashing acts are not always performed with negative intent, but benefit both green incumbents and total customer surplus if the market greenness gap is small. In addition, a loose enforcement level contributes to higher social welfare in emerging markets.

Keywords: Greenwash; Green-brown competition; Game theory; Anti-greenwash enforcement

1. Introduction

Since the turn of the millennium, companies in emerging markets have experienced a surge of interest in applying green management systems or creating green corporate branding and images. According to a study by Nielsen (2014), the propensity to buy socially and environmentally responsible brands online is apparent in the Asia-Pacific (64%), Latin America (63%), the and Middle East/Africa (63%). This is because the global sustainability movement has placed significant pressure on such as those in the aforementioned regions to keep up with developed economies in terms of their environmental stewardship. Moreover, because of rapid mass urbanization, developing countries face higher resource depletion and environmental pollution problems. Hence, governments and societies in emerging markets are proactively pushing their green agendas (Jamali et al., 2017). For example, by 2008, China had become the largest producer of clean technologies in financial terms; these technologies accounted for 1.4% of its gross domestic product (ADB, 2012). Further, more than 80 institutions (including government bodies, non-government organizations (NGOs), and international companies) in emerging markets (such as Vietnam, Saudi Arabia, and Indonesia) have joined the green development coalition supported by Chinas 'One Belt, One Road' initiative to push for the development of product standards and the implementation of green development (Hou, 2018). By 2015, over 95% of Egyptians were living in urban areas, increasing the need for sustainable consumption (Hegazy et al., 2017). Driven by community demands to go green and by the need for effective cost management (Kassaye, 2001), companies are increasingly claiming their products and services to be "*eco-friendly*", "*sustainable*", and "green". In return, emerging economies are benefiting from improved operating environments and local markets as well as boosts to the development of environmental technologies (Zadek, 2011). Apart from the advantages this has brought to the exploitation of natural resources (e.g. less carbon emission, renewable energy etc.), green growth has also helped emerging markets depart from conventional business practices, which are costly and unrealistic in developing countries. According to OECD (2012), emerging markets are the most vulnerable to climate change and tend to be more dependent on natural resources than advanced economies; therefore, they are less likely to achieve long-run success based on traditional economic growth patterns. Sveen (2013) has also reported that in nearly 70% of identified cases, emerging markets were leading global innovation projects because of demands for sustainability.

However, claiming to be green does not necessarily guarantee that a company's products or services are genuinely green. Consequently, greenwashing has become a significant problem and is a side effect of the greening market (Parguel et al., 2015). Greenwashing has been defined as "disinformation disseminated by an organization so as to present an environmentally responsible public image" Soanes & Stevenson (2004). In practice, this means that companies are deceptive in their advertising and exaggerate the greenness of their products. This leads to unfair market competition between green and "brown" (greenwashing) firms; consumer green trust, purchase intentions, and word of mouth being negatively influenced (Chen & Chang, 2013); and the possibility that the entire green market experience may fail, and that overall social welfare may deteriorate. Despite the short-term gains of greenwashing, it will lead to a "lose-lose" situation in the long term. Over the past decade, a large number of greenwashing cases in emerging markets have been reported, despite the boom in genuine sustainability initiatives. Fernando et al. (2014) have shown that 51.7% of company claims in India were greenwashed, and most of these (37.7%) were ambiguous. A Canadian-based environmental market agency investigated the Brazilian green market, finding that 82% of brands were guilty of greenwashing (Moodie, 2015). Bao Steel Group, the largest steelmaker in China, reportedly caused enormous pollution by emitting large amounts of heavy metals and other chemicals into the environment in early 2013, despite their ambitious claims of heavy investment in green and sustainable initiatives since 2006 (Marquis & Chen, 2013). Despite its prevalence in emerging markets, greenwashing is not unique to developing countries firms in developed countries have also greenwashed their products. For example, Volkswagen's greenwashing scandal revealed that emission cheating is a common practice for

many giants in the car industry, including BMW, Ford, and Mercedes-Benz (Lane, 2016). In 2019, the NGO Truth in Advertising published a list of well-known companies, all in developed economies, accused of greenwashing (Truthinadvertising.org, 2019). However, greenwashing in emerging markets significantly differs from that in developed countries because of the different characteristics of these two types of economy; different approaches are thus required to address greenwashing in developed and developing economies.

- First, unlike developed markets, which are stable, the competition environment in emerging markets is heavier and more volatile. As a result, compared with brown firms in developed countries, brown firms in developing countries are significantly bigger threats to genuinely green firms. According to Joekes & Evans (2008), unfair competition in developing countries places companies in stronger competitive positions because of the volatility of the rapidly expanding market and the lack of long-term dominators. The green sector in developing countries is still relatively young, and green firms have less financial strength to compete with brown firms, who have better cost advantages. Therefore, a theoretical decision-making support model (e.g., for pricing) that can map the complexities and provide advice using parameterized solutions may explicitly support green firms to remain competitive in an uncertain market. Moreover, a theoretical method may provide governments and policymakers with holistic guidance for optimizing the entire market.
- Second, customers in emerging markets are much more easily deceived than those in developed markets. Green and sustainability concepts have long been established in developed countries; hence, customers willing to pay green premiums are more knowledgeable than those in emerging markets (Panjaitan & Sutapa, 2010). Therefore, the time it takes for customers in emerging markets to perceive the differences between green and greenwashed offerings has created opportunities for greenwashers. This has resulted in reduced benefits for customers, who may be unconsciously harming their health by opting for products or services from brown firms. The delay in the market awareness of greenwashing may lead to a drop in customer trust in the whole green sector.
- Finally, the lack of government enforcement in preventing greenwashing activities in developing countries means that greenwashing is easier to conduct and less likely to be penalized. As Delmas & Burbano (2011) have indicated, most developed nations have dedicated regulatory bodies (such as the Advertising Standards Authority in the United Kingdom, the Australian Competition and Consumer Commission in Australia, and the Canadian Standards Association in Canada) that discourage the use of vague claims and misleading advertising, while most

developing countries have no such regulatory bodies. Hence, green developments in emerging markets are further hampered by the easy entry of brown firms.

With respect to these special characteristics (delayed customer awareness of greenwashing and weak government enforcement), it is essential to provide theoretical support to green companies to manage the challenges presented by brown firms. The rest of this paper is organized as follows. Section 2 reviews the relevant literature and identifies the research gaps. Section 3 explores the underlying problems in more detail and formulates them mathematically. Section 4 presents the competition outcomes of the game. Section 5 discusses price and profit, customer surplus, and social welfare. Section 6 concludes the paper.

2. Related literature

With respect to the proposed research goal, studies related to green behaviors and sustainability are reviewed first. Next, in light of the research context, the literature on greenwashing is discussed. Finally, by critiquing various research methods and settings, the novelties of this paper are identified and summarized as the research objectives.

Following the recent trend in promoting green management in developing economies, extensive research has been conducted on corporate social responsibility, sustainability, and green concepts in emerging market business operations (Tang & Zhou, 2012; Shou et al., 2016; Choi & Luo, 2019). As shown by various researchers (Tang, 2018; Hsu et al., 2016), globalization and stakeholders' increasing awareness of sustainability are the key drivers pushing emerging markets to be both profitable and environmentally sustainable. However, various constraints arising from the nature of emerging markets mean that the green movement cannot be a simple repetition of the green movement in developed countries. To tackle low productivity in emerging markets, Bai et al. (2017) have developed a novel path model and promoted green information technology (IT) systems to guide mining industry organizations and managers in developing countries in green supply chain management planning and investment decisions. Similarly, Khuntia et al. (2018) have explored the value of computerized information systems in enhancing environmental sustainability in emerging markets, finding a positive relationship between the implementation of green IT and the reduction of energy consumption via IT equipment.

Another stream of research has focused on strategic supply chain relationships, investigating how green agendas and sustainability have influenced sourcing, distributing, and selling strategies in developing economies. Tang (2018) has simultaneously highlighted the key constraints and existing opportunities for companies in developing countries to participate in socially responsible supply chains. Including people living in poverty in supply chain operations may vield mutual benefits for both companies and society. With respect to supply, Tong et al. (2018) have explored how and why sustainable operations are implemented in developing countries; Karnani (2007) has showed the importance of assisting suppliers in emerging markets promoting business sustainability and boosting the consumer market; and Vachani & Smith (2008) have claimed that sustainable operations create more job opportunities and effectively improve the downstream market. Although they have all focused on different topics, the aforementioned studies have all attempted to leverage opportunities to promote sustainable development in emerging markets. By contrast, some authors have addressed issues that impede the growth of sustainability in emerging markets. Jayanti & Gowda (2014) have pointed out that the high demand for economic development in emerging markets has hindered the progress of sustainability; their study focuses on the best means of finding a balance between economic growth and sustainability in the Indian business context. Choi & Luo (2019) have proposed an analytical model to investigate the influence of poor data quality in emerging economies on firm profit and social welfare. Ahmed et al. (2019) have found a positive relationship between product quality and environmental practices, showing uncertain product quality in emerging markets. Shou et al. (2016) have identified that information transparency and legal enforceability in supply chains often leads to contract ineffectiveness in emerging markets, further harming overall social welfare.

Although greenwashing behavior is evident in emerging markets, the abovementioned papers have barely considered it. Its existence significantly hampers the growth of sustainability in emerging markets by harming genuine green firms and causing a deterioration in social welfare (see examples in the first section). Hence, in a practical sense, it is important to fill this gap and provide insights that are applicable to the real world. Moreover, filling this gap may open up further research opportunities. For example, a study by Bai et al. (2017) shows that greenwashers may use green IT systems inappropriately; thus, green organizations should consider this in their implementation strategies. Choi & Luo (2019) could further extend their research on how poor data quality influences green-brown competition in emerging economies.

Empirical research on the impacts of greenwashing is extensive. Nyilasy et al. (2014) have evaluated the effects of deceptive green advertising and corporate environmental performance on customer reactions, finding that green efforts can easily backfire when they do not align with firms' actual environmental performance. Blome et al. (2017) have studied the effects of supply chain leadership and ethical incentives on suppliers' greenwashing activities, finding that leadership styles strongly affect such activities. However, as Lyon & Montgomery (2015) and Marquis et al. (2016) have demonstrated, more theories and models are needed to study greenwashing mechanisms and evaluate their effects on corporate performance and social welfare. While empirical research on greenwashing is desirable, methodological limitations may limit the applicability of research outcomes in three ways. First, although empirical research can help identify the interactions between various factors, it is difficult to advise practitioners on explicit ways to achieve optimization. Second, empirical research may not be sufficient for identifying the sensitivities of different factors; in turn, some managerial insights may be overlooked. Finally, it is difficult for empirical research to study dynamic processes when the topic under investigation is associated with multiple stages and stakeholders. As a result, practitioners lack decision-making support when external elements are altered or overall objectives change. In addition, the aforementioned characteristics of emerging markets further constrain the applicability of reviewed papers. For example, Tang (2018) has argued that consumers in developing countries have comparatively lower education levels and limited access to information and technology. Therefore, varying levels of awareness of green concepts in diverse markets may result in different levels of greenwashing and different challenges associated with managing it. Moreover, the nature of competition in emerging economies (Joekes & Evans, 2008) makes it difficult for empirical research to capture dynamic details and their corresponding influences.

Therefore, this paper aims to provide theoretical support to green companies that can help them address the challenges presented by brown firms by considering relevant emerging market features. A mathematical approach may better fit this goal by quantifying all dynamic details in one picture and supporting decision-making processes with quantified solutions. Mathematical approaches such as sensitivity analysis or scenario analysis can yield results that differ from those of empirical studies. This enhances accuracy, especially when the research context is complex and dynamic.

Research on green competition is lacking. The existing literature includes studies on pricing strategies along with green investments or advertising decisions (Yakita & Yamauchi, 2011; Su et al., 2012; Liu et al., 2012; Du et al., 2019; Dong et al., 2019; Dai & Zhang, 2017; Liu & Chen, 2019); how product greenness influences competition in the supply chain both upstream and downstream (Zhang et al., 2015; Ma et al., 2018); and green and brown product development strategies, with consideration given to different constraints (Yenipazarli & Vakharia, 2015a,b; Wei et al., 2019; Hong et al., 2018; Ma et al., 2017). Studies on sustainability-related topics, such as green firms' competitive decisions and greenness designs, are often based on the assumption that firms always behave honestly and give customers true information about product greenness. Unfortunately, this is never the case when greenwashing activities occur in emerging markets. Instead, green-based competition in our research setting was much more complex because greenwashers deliberately concealed real product information and exaggerated product greenness to attract customers. Additionally, various features of emerging markets (such as longer customer green learning curves and weaker government enforcement environments) have heightened the complexity of the issues under investigation, but the relevant existing research is limited in addressing such issues.

To the best of our knowledge, the most similar studies to ours are from Lyon & Maxwell (2011) and Wu et al. (2020). Lyon & Maxwell (2011) have established an analytical model to study the interactions between firms' greenwashing behaviors and NGOs' punitive strategies. However, the authors do not include the challenges and dynamics of emerging markets in their model. Lee et al. (2018) further complemented the their study by addressing the effects of competitive forces on firms' greenwashing intentions. By contrast, our research aims offer guidelines to incumbent firms in developing economies to help them better defend themselves against brown entrants who practice greenwashing activities. It also considers various government anti-greenwashing enforcement (AGE) levels, which Lee et al. (2018) have missed. Wu et al. (2020) studied the game between profit optimization-driven firms and genuine socially responsible firms with respect to different information transparency levels. Their study reveals how information transparency results in greenwashing as well as how such greenwashing affects a firm's strategies and the social welfare. Both the positive and negative aspects of greenwashing are identified subsequently. Comparatively, our research also explores how a firm's strategies and the social welfare can be affected by greenwashing related competition, but our research context differentiates our studies from theirs in two ways. Firstly, our study includes the dynamic customer learning process. This is not considered in their study, but firms' strategies ought to be adjusted when customers gradually build up their knowledge about green products. Secondly, our studies considers the influence of government enforcement level and tries to help policy makers from government side to better handle greenwashing behaviour.

In summary, the investigation of greenwashing activities in the emerging markets context sets our research apart from existing studies. Out study adds greenwashing research to the academic agenda of sustainability in emerging markets and contributes a new methodological problem formulation to existing greenwashing studies. Accordingly, green competition, greenwashing activities, customers' green learning curves, and government regulatory designs are jointly considered in the following model formulation section.

3. Model formulation

3.1. Notations

The notations throughout the paper are presented in Table 1.

Table 1: Notations						
Decision Variables						
p_i	Incumbent's selling price in the two periods.					
p_e	Entrant's selling price in the second period.					
	Other Notations					
k	Superscript, $k \in \{G, N\}$, where $k = G$ means entrant adopts greenwashing					
n	strategy; $k = N$ means entrant adopts nongreenwashing strategy.					
δ	Greenness gap of entrant's and incumbent's product.					
Ι	Government's anti-greenwash enforcement level.					
G_i	Greenness level of incumbent's product.					
G_e	Greenness level of entrant's product.					
v	Customers' valuation for unit product greenness, normalized to 1.					
x	Customer's type, where $x \in [0, 1]$.					
t	Customers' unit preference misfit cost.					
C_p	Penalty cost of greenwashing.					
$U_i^1(x)$	x type customers' first-period utility towards incumbent's products.					
$U^{2k}(x)$	x type customers' second-period utility towards incumbent's products un-					
$O_i(x)$	der strategy k , where $k = \{G, N\}$.					
$U^{2k}(r)$	x type customers' second-period utility towards entrant's products under					
$\mathcal{O}_{e}(x)$	strategy k, where $k = \{G, N\}$.					
d_i^1	Incumbent's demand in the first period.					
d_i^{2k}	Incumbent's second-period demand under strategy k, where $k = \{G, N\}$.					
d_e^{2k}	Entrant's second-period demand under strategy k, where $k = \{G, N\}$.					
π_i	Incumbent's profit.					
π_e^k	Entrant's profit under strategy k , where $k = \{G, N\}$.					
\bar{E}	The initial unit product environmental impact when greenness is zero.					
λ	Coefficient parameter for the unit environmental impact.					
μ	Unit anti-greenwash enforcement cost.					

3.2. Firms and customers

In accordance with the reality that green and brown firms often coexist in emerging markets, we consider a two-period incumbent-entrant model. In Period 1, the market consists of a monopolistic green firm (firm i); in Period 2, a brown firm (firm e) enters the market and competes with the green firm. The two firms offer products in the same category and are situated at the endpoints of a unit customer interval or linear city. Without loss of generality, we assume the green firm locates at x = 0 and the brown firm locates at x = 1. We denote G_i and G_e as the incumbent's and entrant's product greenness, and the condition $G_i > G_e$ should be satisfied. Note that, as a green firm, the incumbent has no incentive to be deceptive. However, the brown entrant can choose to greenwash by exaggerating its product greenness from G_e to G_i to mislead customers. To ease the expositions, a parameter is defined as

$$\delta = \frac{G_i - G_e}{G_i} \in (0, 1) \tag{1}$$

in the following analysis¹. It denotes the greenness gap of the two firms as well as the entrant's greenwashing tendency.

Scenario N: the brown firm does not go greenwashing. Customers' gross valuation is assumed to be homogeneous and positively related to products' greenness level. Therefore, the gross utility for the incumbent and the entrant can be modelled respectively as $G_i v$ and $G_e v$, where v is the green valuation or green satisfaction generated by unit greenness. Meanwhile, customers are heterogeneous in their preference for the two firms' products². A consumer at preference location x incurs a misfit cost tx when purchasing from firm i and a cost of t(1 - x) when purchasing from firm e. In the cost functions, parameter t stands for each customer's unit distance transportation cost (disutility) caused by preference misfit.

Considering the selling prices for the incumbent (p_i) and the entrant (p_e) (see in Cai & Chen, 2011; Zhang et al., 2019b; Chen et al., 2018), customers' utility functions can be formulated as

Scenario N
$$\begin{cases} U_i^1(x) = U_i^2(x) = G_i v - tx - p_i, & x \in [0, 1]. \\ U_e^{2N}(x) = G_e v - t(1 - x) - p_e^N, & x \in [0, 1]. \end{cases}$$
(2)

Scenario G: the brown firm chooses to go greenwashing. As mentioned in section 1, customers are not knowledgeable enough to distinguish between the "true" green products from the "greenwashed"

¹The parameter δ characterizes the greenness gap between the two firms' products. Here, $\delta \to 0$ means the two firms' products have the same level of greenness (i.e., $G_i = G_e$), while $\delta \to 1$ means the entrant's product greenness reaches zero (i.e., $G_e = 0$).

 $^{^{2}}$ The two firms' products can be referred to as clothing by different brands that is produced using similar materials (e.g., cotton), but comes in different sizes, colors, or packages.

ones in emerging markets, and this is especially the case for newly launched products. However, in accordance with general practice, we assume that, by using green products, customers will become more knowledgeable and less likely to be deceived by greenwashers (the *learning effect*). This concept is similar to that proposed by Zhang et al. (2019a) that people will be more experienced after doing the same thing for a certain period. That is, more experienced buyers will gain the ability to tell the difference between green products and greenwashed ones. We take the organic food industry as an example. Some properties (e.g., taste, appearance, etc.) of organic food are highly significant compared with conventional products (non-organic food); customers can discern these properties and use them to evaluate the actual product greenness level (Kirchheimer, 2012).

To model this *learning effect*, we start with the Period 1 problem. Customers whose valuation is positive (i.e., $U_i^1(x) \ge 0$) will purchase product *i* in Period 1; otherwise no purchase occurs. We define $\Gamma_i^1 = \{x : U_i^1(x) \ge 0\}$ and $\Gamma_{-i}^1 = \{x : U_i^1(x) \le 0\}$ as the sets of customers who purchase and do not purchase product *i* in Period 1, respectively. Therefore, at the end of Period 1, customers can be divided into two groups: the *experienced* ones (in set Γ_i^1) and the *inexperienced* ones (in set Γ_{-i}^1). We assume that experienced customers will never be misled by greenwashers, but inexperienced customers are easy targets. Experienced customers are capable of knowing if products are greenwashed, as they are familiar with the characteristics of genuine green products. By contrast, this is much harder for inexperienced customers. Therefore, when greenwashing occurs, we assume that only inexperienced buyers are affected.

In Period 2, considering customers' greenness learning effect, all customers' utility toward the incumbent's products and experienced customers' utility toward the entrant's products remains unchanged. However, the inexperienced customers' gross valuation of the entrant's products changes from $G_e v$ to $G_i v$ when the entrant goes greenwashing.

Therefore, customers' utilities towards the two firms' products can be formulated as

Scenario
$$G \begin{cases} U_i^1(x) = U_i^2(x) = G_i v - tx - p_i, & x \in [0, 1]. \\ U_e^{2G}(x) = \begin{cases} G_e v - t(1 - x) - p_e^G, & x \in \Gamma_i^1. \\ G_i v - t(1 - x) - p_e^G, & x \in \Gamma_{-i}^1. \end{cases}$$
(3)

3.3. Government's anti-greenwash enforcement

Government AGE is modelled as a penalty for greenwashers. Under such enforcement, penalty costs are imposed on greenwashers (including reputational damage, monetary loss, etc.) when governments detect their greenwashing activities. We assume that the brown firm's lump sum penalty cost is positively correlated with both a government's AGE level and its exaggerated greenness, which is formulated as

$$C_p = \beta I_0 \left(\delta G_i\right)^2. \tag{4}$$

In Eq.(4), parameter I_0 measures the unit penalty cost and parameter $\beta \in [0, 1]$ denotes the probability that a government will detect its greenwashing activities. As mentioned in section 1, in emerging markets, both penalty cost and governmental greenwashing-detection efforts are not as high as they are in developed countries. That is, I_0 and β are both relatively low in emerging markets. We use I to denote the expected unit penalty cost of βI_0 in the following analysis. The parameter is regarded as a proxy of a government's AGE level. Lower I means the government's efforts and willingness to fight against greenwashing acts are weak (which corresponds with the situation in emerging markets), while higher I denotes a stronger enforcement level (in developed countries) to combat greenwashing acts.

In addition, the cost function increases convexly in the exaggerated greenness δG_i of the brown firm's products. Therefore, the brown firm suffers more from greenwashing when the exaggerated greenness is relatively high; otherwise, customers and governments are more likely to tolerate the greenwashing activities when the greenness levels of the two products are close.

Without loss of generality, the value of v is normalized to 1 in the following analysis. Changing the value of v will not generate new insights in our paper.

3.4. Firms' Problems

For a certain type of brown firm (i.e., greenwasher or non-greenwasher), customers will make their purchase decisions to maximize their utilities. Customers in the set $\Gamma_i^{2k} = \{x : U_i^2 \ge U_e^{2k}, U_i^2 \ge 0\}, k = \{G, N\}$ will purchase product *i*; customers in the set $\Gamma_e^{2k} = \{x : U_e^{2k} \ge U_i^2, U_e^{2k} \ge 0\}, k = \{G, N\}$ will purchase product *e*; otherwise, customers will not buy. The demand functions in periods 1 and 2 for the two products can be formulated as

$$d_{i}^{1} = \int_{x \in \Gamma_{i}^{1}} dx, \qquad d_{i}^{2k} = \int_{x \in \Gamma_{i}^{2k}} dx, \qquad d_{e}^{2k} = \int_{x \in \Gamma_{e}^{2k}} dx, \tag{5}$$

in which $k = \{G, N\}$.

The profit functions for the two firms can be expressed as

$$\begin{cases} \pi_i(p_i) = p_i \left(d_i^1 + d_i^{2k} \right), & k = \{G, N\}, \\ \pi_e(p_e^k) = p_e^k d_e^{2k} - C_p \cdot \mathbb{1}_k, & k = \{G, N\}, \end{cases}$$
(6)

In the indicator $\mathbb{1}_k \in \{0,1\}$, $\mathbb{1}_k = 0$ when k = N and $\mathbb{1}_k = 1$ when k = G. To ensure both firms'

demands in Period 2 are positive, we assume the incumbent's product greenness level is not too high -that is, $G_i \leq 3t$. In addition, to avoid trivial cases in which the market is always partially covered and horizontal competition never happens, we also assume that the greenness of the incumbent's product is not too low -that is, $G_i \geq t$.

3.5. Decision Sequences

In this paper, the incumbent is assumed to be a Stackelberg leader, while the entrant acts as a follower. The products are assumed to be non-durable. Each customer can only purchase and consume at most one unit of firm i's or firm e's product in each period. In the main model, we assume that the incumbent sets the same price in the two periods and the price is preannounced in Period 1. This assumption allows us to model customers' learning effect and to model the entrant's strategic decisions.

The specific game sequence is explained below and shown in Figure 1.

- Step 1: In Period 1, the incumbent sets its optimal price of p_i based on the anticipation of competition from the entrant and its type. Then, customers make their purchasing decisions in Period 1. After purchasing, customers are categorized as one of two types: experienced or inexperienced.
- Step 2: After acknowledging the incumbent's selling price and customers' market knowledge (i.e., experienced or inexperienced), the entrant strategically makes decisions about whether (k = G) or not (k = N) to proceed with the greenwashing initiative.
- Step 3: After k is determined, the entrant formulates the optimal selling price p_e^k . Customers make purchasing decisions to maximize their utilities.

4. Model analysis

In this section, we derive the pricing equilibrium when the incumbent faces competition from a greenwashing entrant. Based on the game sequence, we solve this model with backward induction.

4.1. Problem in Step 3 : The entrant's pricing decisions

We start with the problem in step 3, which is the entrant's pricing decisions based on whether it is a greenwasher.



Figure 1: The game sequence.

4.1.1. The entrant is a greenwasher

When the entrant is a greenwasher, inexperienced customers will be deceived and make purchasing decisions based on the belief that the entrant's products are green. However, experienced customers will understand the true greenness of the entrant's products (denoted G_e), even if the entrant distorts its greenness information.

The first case we analyzed was one in which the market was fully covered by the incumbent in Period 1 (i.e., $p_i \leq G_i - t$) (see Figure 2(a)). In this case, all customers will be experienced because they will all have purchased and used green products during Period 1. For different values of the parameters G_i , δ , and p_i (incumbent's price), the entrant has three strategies from which to choose:

- Strategy G1: Incumbent Out (G-IO). When $p_e^G \leq p_i t \delta G_i$, the entrant will drive the incumbent out of the market because all customers' utility toward product e is higher than that of product i in Period 2; therefore, they are never motivated to choose product i.
- Strategy **G2**: Competition (**G-C**). When $p_i t \delta G_i \leq p_e^G \leq p_i + t \delta G_i$, the entrant will compete with the incumbent and both firms can have positive market shares.
- Strategy G3: Entry Blocked (G-EB). When p_e^G ≥ p_i+t-δG_i, the entrant is unable to penetrate the market because all customers' utility toward product e is lower than that of product i. That is, none of the customers will choose product e.



Figure 2: Customers' utility for (a) Scenario G and (b) Scenario N

Using simple algebra to combine the three strategies under different conditions, the entrant's profit function in Eq.(6) can be re-expressed as follows:

$$G(p_e^G|p_i) = \begin{cases} p_e^G - C_p, & p_e^G \leqslant p_i - t - \delta G_i. \\ \frac{p_e^G(t - \delta G_i + p_i - p_e^G)}{2t} - C_n, & p_i - t - \delta G_i \leqslant p_e^G \leqslant p_i + t - \delta G_i. \end{cases}$$
(G-IO) (7)

$$\pi_{e}^{G}(p_{e}^{G}|p_{i}) = \begin{cases} \frac{p_{e}^{G}(t-\delta G_{i}+p_{i}-p_{e}^{G})}{2t} - C_{p}, & p_{i}-t-\delta G_{i} \leqslant p_{e}^{G} \leqslant p_{i}+t-\delta G_{i}. & (\mathbf{G-C}) \\ 0 - C_{p} & p_{e}^{G} \geqslant p_{i}+t-\delta G_{i}. & (\mathbf{G-EB}) \end{cases}$$
(7)

To maximize this function, we obtain the following Lemma.

Lemma 1. When the entrant chooses to go greenwashing and the market is fully covered by the incumbent in period 1, i.e., $p_i \leq G_i - t$, the entrant's optimal response is

$$p_e^{G*} = \begin{cases} \frac{t - \delta G_i + p_i}{2}, & \max(\delta G_i - t, 0) \leqslant p_i \leqslant G_i - t. & (\boldsymbol{G-C}) \\ 0, & p_i \leqslant \max(\delta G_i - t, 0). & (\boldsymbol{G-EB}) \end{cases}$$
(8)

Then, we turn to the case when the market is *partially covered* by the incumbent in Period 1 (i.e., $p_i \ge G_i - t$) (see Figure 2(a)). For different values of parameters G_i , t, δ , and incumbent's price p_i , the entrant has four strategies from which to choose:

- Strategy G4: Incumbent Out (G-IO). When $p_e^G \leq p_i t \delta G_i$, the entrant drives the incumbent out of the market.
- Strategy G5: Competition (G-C). When $p_i t \delta G_i \leq p_e^G \leq (2 \delta)G_i t p_i$, the entrant directly competes with the incumbent and cannibalizes the incumbent's market share.
- Strategy **G6**: Accommodation (**G-A**). When $(2 \delta)G_i t p_i \leq p_e^G \leq 2G_i t p_i$, the entrant touches with the incumbent, and the market is fully covered by the two firms.
- Strategy G7: Separation (G-S). When $p_e^G \ge 2G_i t p_i$, the entrant separates with the incumbent and the market is not fully covered.

The entrant's problem is to maximize the profit function below.

$$\begin{cases} p_e^G - C_p, & p_e^G \leqslant p_i - t - \delta G_i. \\ \frac{p_e^G(t - \delta G_i + p_i - p_e^G)}{2} - C_p, & p_i - t - \delta G_i \leqslant p_i^G \leqslant (2 - \delta)G_i - t - p_i. \end{cases}$$
(G-IO)

$$\begin{cases} \frac{2t}{e}(p_e^G|p_i) = \begin{cases} \frac{2t}{t} & C_p, \quad p_i \in C \ \text{odd}_i \in p_e \in (2-0)G_i - t - p_i. \quad (\mathbf{C} \cdot \mathbf{C}) \\ \frac{p_e^G(t - G_i + p_i)}{t} - C_p, & (2 - \delta)G_i - t - p_i \leqslant p_e^G \leqslant 2G_i - t - p_i. \quad (\mathbf{G} \cdot \mathbf{A}) \end{cases}$$
(9)

$$\begin{pmatrix}
\frac{p_e^G(G_i - p_e^G)}{t} - C_p, \quad p_e^G \ge 2G_i - t - p_i.
\end{cases}$$
(G-S)

Maximizing Eq.(9), we obtain Lemma 2.

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Lemma 2. When the entrant chooses to go greenwashing and the market is is partially covered by the incumbent in period 1, i.e., $p_i \ge G_i - t$, the entrant's optimal response is

$$p_{e}^{G*} = \begin{cases} \frac{G_{i}}{2}, & p_{i} \geq \frac{3G_{i}}{2} - t. & (G-S) \\ 2G_{i} - t - p_{i}, & \frac{12 + \delta - 2\sqrt{6\delta - 2\delta^{2}}}{9}G_{i} - t \leq p_{i} \leq \frac{3G_{i}}{2} - t. & (G-A) \\ \frac{t - \delta G_{i} + p_{i}}{2}, & G_{i} - t \leq p_{i} \leq \frac{12 + \delta - 2\sqrt{6\delta - 2\delta^{2}}}{9}G_{i} - t. & (G-C) \end{cases}$$
(10)

Lemma 1 and 2 show the entrant's best responses when the incumbent's price (p_i) changes. It is interesting to note that several regions exist in which the entrant will select different entry strategies and apply different selling prices accordingly. First, when p_i is relatively high (i.e., $p_i \ge \frac{3}{2}G_i - t$) and the incumbent's market coverage is small, the entrant will be separated from the incumbent and establish a relatively low price. In this condition, both firms will act as monopolists in Period 2. Second, when both p_i and the incumbent's market coverage are moderate $(\frac{12+\delta-2\sqrt{6\delta-2\delta^2}}{9}G_i - t \leq p_i \leq \frac{3G_i}{2} - t)$, the incumbent will accommodate the entrant and they will share the total market. In this condition, the entrant will sacrifice some of its market share by setting a relatively high price to avoid direct competition with the incumbent. That is, direct competition does not exist in this region. Last, when p_i is relatively small $(G_i - t \leq p_i \leq \frac{12+\delta-2\sqrt{6\delta-2\delta^2}}{9}G_i - t)$, the incumbent is aggressive, and the residual market share is small, the entrant must compete with the incumbent for more of the market share and may face fierce competition from the incumbent. In this condition, their prices will change in the same direction.

The results shown in Lemma 1 and 2 help us better understand how the incumbent's pricing decisions interplay with the entrant's pricing and entry decisions. Combining the results presented in Lemma 1 and 2, we obtain the entrant's optimal profit under greenwashing as

$$\begin{cases} \frac{G_i^2}{4t} - C_p, & p_i \geqslant \frac{3G_i}{2} - t. \\ \frac{(2G_i - t - p_i)(t - G_i + p_i)}{2} & q_i \geqslant \frac{12 + \delta}{2} \frac{2\sqrt{6\delta - 2\delta^2}}{2} q_i \qquad (3C_i) \end{cases}$$

$$\pi_{e}^{G*} = \begin{cases} \frac{(1-\delta G_{i}+p_{i})^{2}}{t} - C_{p}, & \frac{12+\delta - 2\sqrt{6\delta - 2\delta^{2}}}{9}G_{i} - t \leqslant p_{i} \leqslant \frac{6G_{i}}{2} - t. & (\mathbf{G-A}) \\ \frac{(t-\delta G_{i}+p_{i})^{2}}{8t} - C_{p}, & \max(\delta G_{i} - t, 0) \leqslant p_{i} \leqslant \frac{12+\delta - 2\sqrt{6\delta - 2\delta^{2}}}{9}G_{i} - t. & (\mathbf{G-C}) \\ 0 - C_{p}, & p_{i} \leqslant \max(\delta G_{i} - t, 0). & (\mathbf{G-EB}) \end{cases}$$
(11)

4.1.2. The entrant is not a greenwasher

When the entrant is not a greenwasher, both the "experienced" and "inexperienced" customers will know the real greenness of product e. We show customers' utility change with x under different conditions in Figure 2(b). We first analyze the case when the market is *fully covered* in Period 1 (i.e., $p_i \leq G_i - t$). In this case, the problem formulation is the same as that in *Scenario G*. It is straightforward that the optimal price is

$$p_e^{N*} = p_e^{G*} = \begin{cases} \frac{t - \delta G_i + p_i}{2}, & \max(\delta G_i - t, 0) \leqslant p_i \leqslant G_i - t. & (\mathbf{N-C}) \\ 0, & p_i \leqslant \max(\delta G_i - t, 0). & (\mathbf{N-EB}) \end{cases}$$
(12)

Then, we analyze the case when the market is *partially covered* by the incumbent in Period 1 (i.e., $p_i \ge G_i - t$). For different values of parameters G_i , δ , and incumbent's price p_i , the entrant has three strategies from which to choose:

- Strategy N4: Incumbent Out (N-IO). When $p_e^N \leq p_i t \delta G_i$, the entrant will drive the incumbent out of the market.
- Strategy N5: Competition (N-C). When $p_i t \delta G_i \leq p_e^N \leq (2 \delta)G_i t p_i$, the entrant will

compete with the incumbent and cannibalize the incumbent's market share.

• Strategy N6: Separation (N-S). When $p_e^N \ge 2G_i - t - \delta - p_i$, the entrant will separate with the incumbent and the market is not fully covered.

The entrant's profit function in Eq.(6) can be re-expressed as follows:

$$\left(\frac{p_e^N\left((1-\delta)G_i - p_e^N\right)}{t}, \quad p_e^N \ge (2-\delta)G_i - t - p_i.\right)$$
(N-S)

$$\pi_{e}^{N}(p_{e}^{N}|p_{i}) = \begin{cases} \frac{p_{e}^{N}(t-\delta G_{i}+p_{i}-p_{e}^{N})}{2t}, & p_{i}-t-\delta G_{i} \leqslant p_{e}^{N} \leqslant (2-\delta)G_{i}-t-p_{i}. \quad (\mathbf{N-A}) \\ p_{e}^{N}, & p_{e}^{N} \leqslant p_{i}-t-\delta G_{i}. \quad (\mathbf{N-IO}) \end{cases}$$
(13)

Lemma 3. When the entrant chooses not to greenwashing and the market is partially covered by the incumbent in period 1, i.e., $p_i \ge G_i - t$, the entrant's optimal response is

$$p_{e}^{N*} = \begin{cases} \frac{(1-\delta)G_{i}}{2}, & p_{i} \geq \frac{(3-\delta)G_{i}}{2} - t. & (N-S) \\ (2-\delta)G_{i} - t - p_{i}, & \frac{(4-\delta)G_{i}}{3} - t \leq p_{i} \leq \frac{(3-\delta)G_{i}}{2} - t. & (N-A) \\ \frac{t-\delta G_{i} + p_{i}}{2}, & G_{i} - t \leq p_{i} \leq \frac{(4-\delta)G_{i}}{3} - t. & (N-IO) \end{cases}$$
(14)

From Eq.(12) and Lemma 3, we can see that the entrant's market entry decisions are also highly dependent on the incumbent's price. As discussed in the greenwashing model, there are also three market entry strategies from which the entrant can choose. However, when comparing the results from the two models, we find that the size of the competition region is larger in the greenwashing model, indicating that the entrant will be more likely to compete with the incumbent. This is because when the entrant is non-deceptive, customers' perceived quality of its product will be lower, and the selling price will drop. In this condition, an accommodating strategy will result in a sharper decline in profit. To protect its profitability, the entrant must compete with the incumbent for a larger market share.

Combining the results in Eq.(12) and Lemma 3, we summarize the entrant's optimal profit as

$$\pi_{e}^{N*} = \begin{cases} \frac{(1-\delta)^{2}G_{i}^{2}}{4t}, & p_{i} \geq \frac{(3-\delta)G_{i}}{2} - t. & (\mathbf{N-S}) \\ \frac{((2-\delta)G_{i} - t - p_{i})(t - G_{i} + p_{i})}{t}, & \frac{(4-\delta)G_{i}}{3} - t \leq p_{i} \leq \frac{(3-\delta)G_{i}}{2} - t. & (\mathbf{N-A}) \\ \frac{(t-\delta G_{i} + p_{i})^{2}}{8t}, & \max(\delta G_{i} - t, 0) \leq p_{i} \leq \frac{(4-\delta)G_{i}}{3} - t. & (\mathbf{N-C}) \\ 0, & p_{i} \leq \max(\delta G_{i} - t, 0). & (\mathbf{N-EB}) \end{cases}$$
(15)

4.2. Problem in Step 2: Entrant's decision to greenwash or not

Comparing the optimal profits when the entrant is greenwashing in Eq.(11) with not greenwashing in Eq.(15), we obtain the entrant's optimal decision w.r.t., incumbent's selling price p_i , and parameters I, G_i, δ , and t. By defining

$$\Lambda = \begin{cases} \Lambda_1 := (1 + \widehat{\Delta})G_i - t, & I \in \left[0, \frac{1 - \delta}{3t\delta}\right].\\ \Lambda_2 := (1 + It\delta)G_i - t, & I \in \left[\frac{1 - \delta}{3t\delta}, \frac{1 - \delta}{2t\delta}\right]\\ \Lambda_3 := (1 + \widetilde{\Delta})G_i - t, & I \in \left[\frac{1 - \delta}{2t\delta}, \frac{2 - \delta}{4t\delta}\right] \end{cases}$$

in which, $\widehat{\Delta} = \frac{3+\delta-2\sqrt{6\delta-2\delta^2-18It\delta^2}}{9}$ and $\widetilde{\Delta} = \frac{1-\sqrt{2\delta-\delta^2-4It\delta^2}}{2}$, we present the entrant's optimal green-washing and market entry strategies in the following proposition.

Proposition 1. (Entrant's Response)

The entrant will

(a) go greenwashing when $p_i \ge \Lambda$ and $I \le \frac{2-\delta}{4t\delta}$. Specifically,

- (1) if $p_i \ge \frac{3G_i}{2} t$, it will be separated from the incumbent (**G-S**);
- (2) if $p_i \leq \frac{3G_i}{2} t$, it will accommodate with the incumbent (**G-A**).
- (b) not go greenwashing when $(p_i \leqslant \Lambda \text{ and } I \leqslant \frac{2-\delta}{4t\delta})$ or $I \geqslant \frac{2-\delta}{4t\delta}$. Specifically,
 - (1) if $p_i \ge \frac{(3-\delta)G_i}{2} t$, it will be separated from the incumbent (**N-S**);
 - (2) if $\frac{(4-\delta)G_i}{3} t \leq p_i \leq \frac{(3-\delta)G_i}{2} t$, it will accommodate with the incumbent (**N-A**);
 - (3) if $p_i \leq \frac{(4-\delta)G_i}{3} t$, it will compete with the incumbent (**N-C**).

Proposition 1 and Figure 3 illustrate the entrant's optimal pricing and market entry strategies when p_i is exogenously given. First, when the AGE level I is relatively low and the incumbent's price is relatively high, the entrant will be likely to engage in greenwashing. This conforms to the notion that lower penalties will encourage entrants to engage in greenwashing activities to increase their profits by deceiving inexperienced customers. Additionally, with a relatively high p_i , the market coverage of the incumbent firm will become smaller, meaning that more customers will be inexperienced in Period 2. Under this condition, greenwashing to gain a green valuation may deceive a higher number of inexperienced customers. Nevertheless, the market expansion of greenwashers has no effect on experienced customers. As Lemma 1 and 2 demonstrate, when the two firms are separated from or accommodate each other, the entrant will never engage in greenwashing and thus will never compete with the incumbent under a greenwashing strategy.

When AGE level I is high, or incumbent's selling price p_i is relatively low, the entrant will not tend to engage in greenwashing activities. In this case, the entrant faces the following three entry



Figure 3: Entrant's responsive decisions. Note: $\widehat{\Delta}_0 = \widehat{\Delta}|_{I=0}$.

modes: 1) it will choose to be separated from the incumbent when p_i is high (in Region N-S); 2) it will accommodate the incumbent when p_i is moderate (in Region N-A); 3) it is highly likely to compete against the incumbent when p_i is very low (in Region N-C). This makes the current case fully different from the greenwashing case. The thresholds in Figure 3 will move downward if: 1) the products from each side (the entrant and the incumbent) are more highly differentiated (i.e., with higher t); or 2) the entrant has a high degree of greenness (i.e., higher δ). In either case, the area of greenwashing region (i.e., Region N-S +Region N-A) will expand, implying that when the entrant's product is more horizontally differentiated from the incumbent's product or if the entrant's greenwashing degree is relatively low, it is more likely to be a greenwasher with the given p_i .

4.3. Problem in Step 1: The incumbent's pricing decision

Continuing with the backward induction, we then study the incumbent's optimal decisions. By anticipating the entrant's responsive price and market entry decisions (shown in Proposition 1), the incumbent will choose its selling price to maximize its profits. In regions G-A, G-S, N-A, and N-S, it can be seen that the entry of a brown firm will have no impact on the incumbent's market share or profit with a fixed p_i . However, in regions N-C, the incumbent's market share will shrink in Period 2 because of competition from the entrant. Based on the results from Proposition 1, we express the incumbent's profit function as follows:

$$\pi_{i}(p_{i}) = \begin{cases} \frac{2p_{i}(G_{i}-p_{i})}{t}, & p_{i} \ge \min(\Lambda_{1}, \frac{(4-\delta)G_{i}}{3}-t). \\ p_{i} \cdot \min\left(\frac{G_{i}-p_{i}}{t}, 1\right) + \frac{p_{i}(3t+\delta G_{i}-p_{i})}{4t}, & \max(\delta G_{i}-t, 0) \le p_{i} \le \min(\Lambda_{1}, \frac{(4-\delta)G_{i}}{3}-t). \end{cases}$$
(a)

Subproblem (16a) represents the case in which the entrant's entry does not affect the incumbent's market share. This occurs when the incumbent's selling price is so high that its market coverage is relatively small. However, in Subproblem (16b), when the incumbent's price becomes relatively lower, the incumbent's market coverage increases, and the entrant must compete with the incumbent. Then, in Subproblem (16c), when the incumbent sets an extremely low selling price, the entrant never has a chance to enter the market. This subproblem only exists when the entrant's greenwashing degree is very high ($\delta \geq \frac{t}{G_i}$) or the two products are much more differentiated (i.e., $G_i \geq \frac{t}{\delta}$). With Eq.(16), the incumbent's decision is to choose an optimal p_i that maximizes the total profit function.

The following lemma characterizes the equilibrium results.

Lemma 4. With the entry threat of a brown firm who can choose to go greenwashing, the equilibrium results are presented in Table 2.

The conditions in Table 2 are defined as follows:

$$\begin{cases} \Omega^{G-A1} &:= \left\{ \left(\frac{1}{G_i}, \delta\right) : \rho_1(\delta, I, t) \leqslant \frac{1}{G_i} \leqslant \frac{1}{t} \right\}, \\ \Omega^{G-A2} &:= \left\{ \left(\frac{1}{G_i}, \delta\right) : \rho_2(\delta, I, t) \leqslant \frac{1}{G_i} \leqslant \rho_1(\delta, I, t), \delta \leqslant (1+3It)^{-1} \right\}, \\ \Omega^{N-S} &:= \left\{ \left(\frac{1}{G_i}, \delta\right) : \frac{2-\delta}{2t} \leqslant \frac{1}{G_i} \leqslant \min\left[\rho_1(\delta, I, t), \frac{1}{t}\right] \right\}, \\ \Omega^{N-A1} &:= \left\{ \left(\frac{1}{G_i}, \delta\right) : \frac{5-2\delta}{6t} \leqslant \frac{1}{G_i} \leqslant \min\left[\frac{2-\delta}{2t}, \rho_1(\delta, I, t)\right] \right\}, \\ \Omega^{N-A2} &:= \left\{ \left(\frac{1}{G_i}, \delta\right) : \frac{28-13\delta}{39t} \leqslant \frac{1}{G_i} \leqslant \frac{5-2\delta}{6t}, \delta \geqslant (1+3It)^{-1} \right\}, \\ \Omega^{N-C1} &:= \left\{ \left(\frac{1}{G_i}, \delta\right) : \frac{6-\delta}{13t} \leqslant \frac{1}{G_i} \leqslant \min\left[\rho_2(\delta, I, t), \frac{28-13\delta}{39t}\right] \right\}, \\ \Omega^{N-C2} &:= \left\{ \left(\frac{1}{G_i}, \delta\right) : \frac{1}{3t} \leqslant \frac{1}{G_i} \leqslant \min\left[\rho_2(\delta, I, t), \frac{6-\delta}{13t}\right] \right\}. \end{cases}$$

in which, functions $\rho_1(\delta, I, t)$ and $\rho_2(\delta, I, t)$ are defined in Table B6 in Appendix B.

The incumbent's and entrant's decisions are summarized in Proposition 2.

Proposition 2. (Equilibrium Strategies)

(a) When $(\frac{1}{G_i}, \delta)$ satisfies conditions Ω^{G-A1} or Ω^{G-A2} , the incumbent's optimal price will incentivize the entrant to be a greenwasher and to accommodate the incumbent. Specifically,

:		Incumbent's Decision	su		Entrant's Dec	isions
Condition	p_i^*	$(d_i^{1\ast},d_i^{2\ast})$	π^*_i	p_e^*	d_e^{2*}	π_e^*
Ω^{G-A1}	$\frac{G_i}{2}$	$ig(rac{G_i}{2t}, rac{G_i}{2t}ig)$	$\frac{G_{i,i}^2}{2t}$	$rac{3G_i}{2}-t$	$1-rac{G_i}{2t}$	$\frac{(3G_i-2t)(2t-G_i)}{4t} - C_p$
Ω^{G-A2}	$(1+\widehat{\Delta})G_i-t$	$\left(rac{t-\widehat{\Delta}G_i}{t},rac{t-\widehat{\Delta}G_i}{t} ight)$	$\frac{2((1\!+\!\widehat{\Delta})G_i\!-\!t)(t\!-\!\widehat{\Delta}G_i)}{t}$	$(1-\widehat{\Delta})G_i$	$\frac{\widehat{\Delta}G_i}{t}$	$\frac{(1-\hat{\Delta})\hat{\Delta}G_i^2}{t}-C_p$
Ω^{N-S}	$rac{G_i}{2}$	$ig(rac{G_i}{2t}, rac{G_i}{2t}ig)$	$\frac{G_{i,i}^2}{2t}$	$\frac{(1\!-\!\delta)G_i}{2}$	$\frac{(1\!-\!\delta)G_i}{2t}$	$\frac{(1-\delta)^2 G_i^2}{4t}$
Ω^{N-A1}	$\frac{G_i}{2}$	$ig(rac{G_i}{2t}, rac{G_i}{2t}ig)$	$\frac{G_{i,i}^2}{2t}$	$\frac{(3-2\delta)G_i-2t}{2}$	$1-rac{G_i}{2t}$	$\frac{((3-2\delta)G_i-2t)(2t-G_i)}{4t}$
Ω^{N-A2}	$\frac{(4-\delta)G_i-3t}{3}$	$\left(\frac{3t-(1-\delta)G_i}{3t}, \frac{3t-(1-\delta)G_i}{3t}\right)$	$\frac{2(3t+(\delta-1)G_i)((4-\delta)G_i-3t))}{9t}$	$\frac{2(1\!-\!\delta)G_i}{3}$	$\frac{(1-\delta)G_i}{3t}$	$\frac{2(1-\delta)^2G_i^2}{9t}$
Ω^{N-C1}	$\frac{(4+\delta)G_i+3t}{10}$	$\left(\frac{(6-\delta)G_i - 3t}{10t}, \frac{27t + (9\delta - 4)G_i}{40t}\right)$	$\frac{\left((4+\delta)G_i+3t\right)^2}{80t}$	$\frac{13t + (4-9\delta)G_i}{20}$	$\frac{13t + (4-9\delta)G_i}{40t}$	$\frac{\left(13t + (4 - 9\delta)G_i\right)^2}{800t}$
Ω^{N-C2}	$G_i - t$	$\left(1,rac{4t+\left(\delta-1 ight)G_{i}}{4t} ight)$	$\frac{(8t+(\delta-1)G_i)(G_i-t)}{4t}$	$\frac{(1\!-\!\delta)G_i}{2}$	$\frac{(1\!-\!\delta)G_i}{4t}$	$\frac{(1-\delta)^2 G_i^2}{8t}$

Table 2: Equilibrium results for the main model

- (1) under condition Ω^{G-A1} , the entrant does not affect the incumbent's decisions;
- (2) under condition Ω^{G-A2} , the entrant hurts the incumbent's profit.

(b) When $(\frac{1}{G_i}, \delta)$ satisfies conditions Ω^{N-S} , Ω^{N-A1} , Ω^{N-A2} , Ω^{N-C1} , or Ω^{N-C2} , the incumbent's optimal price will prevent the entrant from being a greenwasher. Specifically,

- (1) under conditions Ω^{N-S} and Ω^{N-A1} , the entrant does not affect the incumbent profit;
- (2) otherwise, the entrant hurts the incumbent's profit.



(a) Decision zones when $I = \frac{1}{2}$ and t = 1.



(b) Impacts of I on greenwash region when t = 1.

Figure 4: Decision zones shaped by δ , I and $\frac{1}{G_i}$.

Proposition 2 highlights the important role that parameters G_i and δ play in both firms' pricing decisions and the entrant's greenwashing and market entry strategies. In particular, if a product's greenness level is relatively low or the greenness gap is relatively small (i.e., $\left(\frac{1}{G_i}, \delta\right) \in \{\Omega^{D-A1} \cup \Omega^{D-A2}\}$), the entrant will choose to engage in greenwashing, which the incumbent will tolerate. Figure 4(a) clearly illustrates the above results.

The reasons for these results are twofold. First, for a fixed δ , when G_i is small, the incumbent has less market expanding ability and the market coverage shrinks in Period 1. Consequently, inexperienced customers will be less able to assess the true greenness of the entrant's products if the entrant is a greenwasher. Instead, they will perceive the entrant's products to be as green as those of the incumbent. Hence, customers will be willing to pay more for greenwashed products, establishing a higher product price than that of non-greenwashed products. Thus, when G_i is relatively low, the entrant will be more motivated to deceive customers. Consequently, the incumbent can manipulate its product prices to prevent deception by the entrant, but this will be harmful to its overall profits and sales margin.

Second, for a fixed G_i , we find that lower values of δ will encourage entrants to engage in greenwashing. Although a high δ may enhance a greenwashing entrant's sales revenue, it will also generate a higher penalty cost. To balance this trade-off, the entrant will never engage in greenwashing when δ is high. Moreover, the incumbent will be much less competitive against the entrant if the entrant is greenwashing under a high δ . This may further harm its profit when greenwashing is tolerated. When the incumbent acknowledges this, a high δ value will lead the incumbent to be more likely to compromise its sales margin in exchange for higher market coverage. As a result, the entrant will be prevented from greenwashing. Overall, it will be easier for the entrant to engage in greenwashing behaviors when the incumbent's greenness level is low and the greenness gap between the two parties is small.

Figure 4(a) illustrates that, when the incumbent's greenness level (G_i) is low and the entrant's potential greenwashing degree (δ) is high (in regions N-S), the market will be partially covered and the two firms will be separated from each other. Under this condition, greenwashing will have no impact on the incumbent's decisions. When G_i is moderate (see regions G-A1, G-A2, N-A1 and N-A2), the entrant will accommodate the incumbent. In these regions, direct competition will be avoided, but the market will be fully covered. In the accommodation regions, a relatively high greenness level (in regions G-A2 and N-A2) will push the incumbent to be more conservative and intentionally set a higher selling price. By doing this, the incumbent will remain with a smaller market share and avoid direct competition with the entrant. Conversely, when the greenness level is low (in Region G-A1 and N-A1), the entrant's greenwashing will have no impact on the incumbent's decisions. Last, when the greenness level is high (in Region N-C1 and N-C2), direct competition between the two firms will be inevitable, and greenwashing activities will be prevented because of the wide greenness gap between the two companies.

Figure 4(b) shows the effects of the government's AGE level I on the entrant's optimal greenwashing or non-greenwashing regions. It conforms to the intuition that a stricter enforcement environment will weaken the entrant's motivation to greenwash. Specifically, by using simple algebra to analyze the boundaries of regions G-A1 and G-A2, we find that when enforcement is strict (i.e., I = 10), the area of the greenwashing region shrinks to nearly zero, and greenwashing by the entrant is unlikely to be an option. Conversely, when the enforcement level is low (i.e., $I = \frac{1}{10}$), the area of the greenwashing region expands. However, the non-greenwashing region still exists, indicating that even when greenwashing activities cost almost nothing, it is still possible that the entrant will choose not to greenwash under certain conditions.

5. Discussions

5.1. Prices and profits

To study the impacts of the brown firm's entry threat to the incumbent, we compare the main model with Model NE (see Appendix A). The results are shown in Proposition 3.

Proposition 3. (Impacts of brown firm's entry on incumbent's profit)

Considering the brown firm's entry threat,

(a) when $(\frac{1}{G_i}, \delta)$ satisfies conditions Ω^{G-A1} , Ω^{N-S} , and Ω^{N-A1} , such entry does not affect the incumbent's price decision and optimal profit;

(b) otherwise, such entry incents the incumbent to raise its price, but it harms the profit.

Proposition 3 shows that when G_i is relatively small, the incumbent's initial market share will also be small. The entrant will not have sufficient competitive power to cannibalize the incumbent's market share. Therefore, there will be no direct competition between the two firms, and the incumbent will remain in a monopolist position. However, when G_i is relatively high, the incumbent's initial market share will be relatively high as well. Thus, the entrant can easily cannibalize the incumbent's market share with little loss of its sales margin. That is, a higher G_i will induce intense competition and harm the incumbent's profits.

To study the impact of the entrant's greenwashing activities, we compare the main model with Model NGE (see Appendix A). The results are shown in Proposition 4.

Proposition 4. (Impacts of greenwashing on equilibrium prices and profits)

In the presence of greenwashing acts,

(a) the incumbent will raise its price and gain higher profit in Region Ω^{G-A2} . Otherwise, greenwashing has no impact on the incumbent's pricing decisions and profit.

(b) the entrant will raise its price to gain higher profit in both regions Ω^{G-A1} and Ω^{G-A2} . Otherwise, greenwashing has no impact on the entrant's pricing decisions and profit.

Proposition 4 shows that when the entrant chooses to be a greenwasher, both firms may raise their selling prices in the greenwashing regions. While it is reasonable for the entrant to raise its prices to avoid direct competition with the incumbent, it is counterintuitive for an incumbent to also raise its price when facing a powerful entrant. This can be explained as follows: when two products are differentiated horizontally, greenwashed products can increase customers' perceived value of brown products. In turn, the increased selling price generates more profits from the entrant's market share, which also protects the entrant from direct competition with the incumbent. Following the same logic, the incumbent may not attempt to directly compete with the entrant by also raising its price. This will cause a drop in the incumbent's market share, but this may be offset by the absence of potential competition-related loss. In summary, Proposition 4 illustrates that in a horizontally differentiated market, the entrant's greenwashing activities may ease competition between the two firms, thus benefiting both firms under some conditions.

Next, we investigate the impacts of δ on both firm's optimal prices, demands, and profits in the main model. The results are presented in Table 3.

Conditions	Ω^{D-A1}	Ω^{D-A2}	Ω^{N-S}	Ω^{N-A1}	Ω^{N-A2}	Ω^{N-C1}	Ω^{N-C2}
p_i^*	_	$\downarrow \left(if \delta \leqslant \widetilde{\delta} ight), \uparrow \left(if \delta \geqslant \widetilde{\delta} ight)$	_	_	\downarrow	↑	_
p_e^*	_	$\uparrow \left(if \delta \leqslant \widetilde{\delta} ight) , \downarrow \left(if \delta \geqslant \widetilde{\delta} ight)$	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow
$(d_i^{1\ast},d_i^{2\ast})$	(-, -)	$(\uparrow,\uparrow)\left(if \ \delta \leqslant \widetilde{\delta}\right), (\downarrow,\downarrow)\left(if \ \delta \geqslant \widetilde{\delta}\right)$	(-, -)	(-, -)	(\uparrow,\uparrow)	(\downarrow,\uparrow)	$(-,\uparrow)$
d_e^{2*}	_	$\downarrow \left(if \delta \leqslant \widetilde{\delta} ight), \uparrow \left(if \delta \geqslant \widetilde{\delta} ight)$	\downarrow	_	\downarrow	\downarrow	\downarrow
π_i^*	_	$\uparrow \left(if \delta \leqslant \widetilde{\delta} ight) , \downarrow \left(if \delta \geqslant \widetilde{\delta} ight)$	_	_	¢	↑	¢
π_e^*	Ļ	$\downarrow \left(if \ \delta \leqslant \min\left(\widehat{\delta}, (1+3It)^{-1}\right) \right),$ $\uparrow (Otherwise)$	Ļ	Ļ	Ļ	Ļ	Ļ

Table 3: Sensitivity results of parameter δ

Note: 1) Symbols ' \uparrow ', '-' and ' \downarrow ' indicate the underlying parameter is increasing, unchanged and decreasing with δ respectively.

Proposition 5. (Impacts of δ on profits)

In Region Ω^{G-A2} , (a) when $\delta \ge \widetilde{\delta}$, the incumbent's profit decreases in δ ; otherwise, it increases in δ , (b) when $I \ge \frac{1}{t}$ and $\delta \ge \widehat{\delta}$, the entrant's profit increases in δ ; otherwise, it decreases in δ , where $\widetilde{\delta} = \frac{1}{2(9It+1)} \left(3 - \frac{1}{\sqrt{8It+1}}\right)$ and $\widehat{\delta} = \frac{1}{2(9It+1)} \left(3 + \frac{2\sqrt{2}}{\sqrt{It+1}}\right)$.

Intuitively, when the entrant's greenness level drops (i.e., δ rises), the competition between the two firms will be less intensive, and the incumbent will enjoy a better business environment. However, because of the entrant's greenwashing activities in this model, the incumbent's profits may be negatively influenced if the greenness gap between the two firms widens. This paper defines this phenomenon

as the *Greenness-Premium Trap*, in which the incumbent's greenness advantage will not contribute to its competitiveness but will, conversely, cause a drop in its profits. Note that the trap only exists in Region Ω^{G-A2} , in which the two firms accommodate each other and parameter δ is relatively high. This indicates to green firms that when faced with a potential greenwasher, increasing their greenness is not always a better choice. To enjoy their greenness premium, green firms should invest in enhancing their greenness and escaping the trap.



Figure 5: Sensitivity results of δ when I = 4, t = 1 and $G_i = \frac{5}{3}$.

To better illustrate the results from propositions 5(a) and (b), we present a numerical example in Figure 5 in which parameter values are set as I = 4, t = 1, and $G_i = \frac{5}{3}$. In the greenwashing region in Figure 5(a), the incumbent's equilibrium price is convex for δ , while the entrant's is concave for δ . Moreover, Figure 5(b) shows that the incumbent's equilibrium demand is concave for δ , while the entrant's is convex in δ . It can be seen that when δ is relatively low (i.e., $\delta \leq \tilde{\delta}$), the rise in δ will make the incumbent more aggressive by reducing its price for market share expansion. As a result, its profit will increase with δ in this region. However, when δ becomes relatively high (i.e., $\delta \geq \tilde{\delta}$), the entrant's greenwashing penalty cost will rise, and it will be less motivated to engage in greenwashing and targets the production and sales of genuine products. To avoid such competition, the incumbent will remain conservative in its market activities. As a result, the entrant will enjoy a steady profit increase along with the rise in δ . Such an increase in profit may even offset the increased penalty costs from the entrant's greenwashing activities.

Interestingly, we also find that a rise in δ may even benefit the entrant when $\delta \ge \hat{\delta}$ and $I \ge \frac{1}{t}$ (see Figure 5(c)). As explained above, when δ is relatively high, the entrant's market share will rise as the incumbent's desire for competition is reduced. Next, we investigate the impact of government enforcement level I on the two firms' profits.

Proposition 6. (Impacts of I on profits)

In the presence of greenwashing,

(a) In Region Ω^{G-A1} , the incumbent's profit is not affected by I, while the entrant's profit decreases in I;

- (b) In Region Ω^{G-A2} , the incumbent's profit decreases, while the entrant's profit increases in I;
- (c) Otherwise, parameter I has no impacts on both firms' profits.



Figure 6: Profits change w.r.t. δ and I when $G_i = \frac{5}{3}$ and t = 1.

Proposition 6 presents another interesting result. Numerical test results are shown in Figure 6,

where $G_i = \frac{5}{3}$ and t = 1. This shows that when a government's AGE is stricter (i.e., a higher value of I), the incumbent's profit will decrease while the entrant's profit will increase. This will only occur when a green firm tolerates a brown firm's greenwashing behaviors. Increasing I will intensify market competition and subsequently harm a green firm's profit. This also implies that a government's AGE will be ineffective in protecting incumbent green firms' benefits. By contrast, incumbent green firms will embrace weak enforcement policies, even though their brown rivals may engage in greenwashing.

5.2. Customer surplus

In this subsection, we compare the customer surplus (CS) in the main model with that in Model NEand Model NGE, respectively, to study the impacts of brown firm's entry and brown firm's greenwashing acts on CS. Based on the equilibrium results presented in Table 2, Table A5, and Eq.(A2), CS for the three models can be calculated, and these are presented in Appendix B. Expressions for $K_1 - K_4$ and $J_1 - J_3$ are presented in Table C6 in Appendix C.

Proposition 7. (Main model v. Model NE)

(a) In the non-greenwashing regions, two thresholds $(K_1 \text{ and } K_2)$ exist, and when $K_1 \leq \frac{1}{G_i} \leq K_2$, the entry of an imitator hurts CS; otherwise, the entry of a brown firm benefits CS.

(b) In the greenwashing regions, two thresholds $(K_3 \text{ and } K_4)$ exist, and when $\frac{1}{G_i} \ge \max\{K_3, K_4\}$, the entry of an imitator benefits CS; otherwise, the entry of a brown firm hurts CS.

Proposition 7 presents the effects of a brown firm's entry on CS. A numerical example is depicted in Figure 7(a), and shows that customers do not always benefit from a new entrant. In the nongreenwashing regions, the entry of a non-deceptive brown firm with a moderate G_i (i.e., $K_1 \leq \frac{1}{G_i} \leq K_2$) will reduce total CS. However, for both high or low levels of G_i , the entry of a brown firm will benefit CS.

The reasons for this are twofold. First, when G_i is high (i.e., $\frac{1}{G_i} \leq K_1$), the entry of a brown firm results in fierce competition between the two firms. In contrast with a monopoly, the incumbent will lower its selling price to compete with the brown firm during selling periods. Therefore, total surplus increases as a greater quantity of low-price products are supplied to the market. Second, when G_i is low (i.e., $\frac{1}{G_i} \geq K_2$ in regions Ω^{N-S} and Ω^{N-A1}), the entry of a brown firm has no impact on the incumbent firm or its buyers. However, the total surplus may still increase if more customers switch to purchasing from that brown firm. By contrast, a moderate G_i will lead to a more conservative role for the incumbent with a higher selling price by mitigating direct competition with the brown firm. Although this will not change the market share, it will decrease CS because customers are paying more for products. Next, we turn to the greenwashing region. Unlike the results seen in the non-greenwashing region, only low levels of G_i and δ will benefit CS, while a higher G_i or higher δ may harm CS. This is because when G_i or δ is high, customers paying high prices for brown products from the greenwashing entrant will not be able to enjoy the expected green valuation. Hence, the total CS will also fall.



Figure 7: Comparison of customer surplus when t = 1 and $I = \frac{3}{2}$. Note: In the shadowed (blank) area, entry of a greenwasher leads to higher (lower) customer surplus.

Proposition 8. (Main model v. Model NGE)

(a) In the non-greenwashing regions, three thresholds (J₁, J₂, and J₃) exist, and when max{J₂, J₃} ≤ ¹/_{G_i} ≤ J₁, greenwash benefits CS; otherwise, greenwash hurts CS.
(b) In the non-greenwashing regions, greenwashing has no impact on CS.

Proposition 8 reveals the effects of the entrant's greenwashing activities on CS. As shown in Figure 7(b), greenwashing has no natural impact on firm profit or CS in the non-greenwashing region. However, it is interesting that in the greenwashing region, when a brown firm chooses to greenwash and the incumbent tolerates this act, a region exists in which greenwashing benefits CS. A similar result that firms' greenwashing acts may generate welfare benefits is also presented in Wu et al. (2020). In their paper, they have provided sufficient explanations for this counter-intuitive phenomenon, - that is

moderate CSR information transparency will induce a profit maximizer to overinvest in CSR, thus increase welfare benefits. However, in our models, the result can be explained in another way. With algebraic analysis, the shaded region in Figure 7(b) indicates that greenwashing behavior expands the incumbent's market coverage but shrinks the entrant's market coverage because more people will purchase from the incumbent for better product value. Eventually, the total CS will increase due to the CS gain from the incumbent, which will outweigh the CS loss from the entrant.

5.3. Social welfare implications

The preceding analysis suggests that firm profitability and CS are affected by the regulator's AGE level. In this section, we investigate the impacts of AGE on total social welfare. By engaging in greenwashing behaviors and exaggerating their true greenness level, brown firms may expand their market coverage. This results in the emergence of more brown products in the market, causing a stronger environmental impact. Hence, it is crucial to establish appropriate enforcement levels to impede the consumption of brown products and protect the environment. In the following section, we incorporate environmental impact and social welfare in the whole market for two periods, before moving onto the problem of maximizing total social welfare by adjusting the enforcement level.

(1) Environmental Impacts (ENV): Since the environmental impact of brown products is more severe than that of green products, following Cachon (2014), the unit environmental impact (in monetary terms) of green and brown products is modelled respectively as $(\bar{E} - \lambda G_i)$ and $(\bar{E} - \lambda G_e)$. In the formulation, the unit environmental impact for each product decreases with respect to product greenness. To ensure the positivity of unit environmental impact, \bar{E} is assumed to be sufficiently large. Combining the results presented in Table 2, under a certain level of I, the total environmental impact can be formulated as

$$ENV = (\bar{E} - \lambda G_i)(d_i^1 + d_i^2) + (\bar{E} - \lambda G_e)d_e^2.$$

$$\tag{17}$$

(2) Government's Expenses (**EXP**): The expenses to conduct the government's enforcement regulation as

$$EXP = \frac{\mu I^2}{2},\tag{18}$$

which is convexly increasing in the enforcement level I. This formulation indicates that the marginal effect of unit expense is decreasing.

(3) Government's Finance Income (GFI): The penalty costs for brown firms' greenwashing acts

are also a source of government finance income, which can be modelled as

$$GFI = \begin{cases} I(\delta G_i)^2, & \left(\frac{1}{G_i}, \delta\right) \in \left\{\Omega^{G-A1} \cup \Omega^{G-A2}\right\} \\ 0, & Otherwise. \end{cases}$$
(19)

Finally, Social welfare (**SW**) consists of π_i , π_e , CS, ENV, GFI and EXP, which can be formulated as

$$SW = f_1(\pi_i + \pi_e) + f_2(CS + GFI - ENV) - EXP.$$
 (20)

In this equation, we introduce two weight parameters. f_1 denotes the weight of economic value and f_2 denotes the weight of social value. The two parameters satisfy $f_1 + f_2 = 2$ and $0 \leq f_1, f_2 \leq 2$. In existing studies, economic value and social value are treated as equal factors in social welfare (i.e., $f_1 = f_2 = 1$) (Yenipazarli, 2016). However, in practice, we observe that many governments in emerging markets concentrate more on economic growth than on social value (Uken, 2013; Guo & Ma, 2008). That is, when creating regulations, governments in emerging markets tend to allocate more weight to economic value than to social value (i.e., $f_1 \geq 1$). In developed countries, the opposite is true -governments are oriented more toward social value (i.e., $f_2 \geq 1$). Therefore, different values of f_1 and f_2 will result in different regulator target functions, eventually leading to different regulations.

The government's objective is to maximize social welfare by setting an optimal I. Attributable to its complexity, there is no analytical expression of the optimal I. Therefore, we conduct numerical analysis to study the government's optimal anti-greenwashing regulation design. The values of the parameters are set as $\delta = 0.1$, $G_i = 5/3$, $\bar{E} = 1$, $\lambda = 0.5$, t = 1, $\mu = 0.01$, and $f_1 = 1.2$ in Figure 8. The sensitivity analysis results are shown in Table 4.

Observation 1. In the greenwashing region, social welfare is concave in I; however, in other regions, social welfare is decreasing in I.

Figure 8 illustrates an interesting result: with an increase of I, the entrant will move from the greenwashing region to the non-greenwashing region. This means that when a government imposes stricter enforcement, the entrant is less likely to engage in greenwashing. We also notice that SW does not change monotonically with I; rather, it first increases and then decreases in I, illustrating the existence of an optimal point ($I^* = 1.383$) that achieves maximal social welfare ($SW^* = 0.7344$). However, in the non-greenwashing region, social welfare always decreases with I, showing no need for a higher enforcement level.

Observation 2. (a) The government's optimal anti-greenwashing enforcement level is concave in δ , while monotonically decreasing in G_i and f_1 .



Figure 8: Social welfare changing with I.

(b) The maximum social welfare is monotonically decreasing in δ , while increasing in G_i .

		Table 4. Impacts of parameters of and G _i on the optimal 1 and SW					
f_1	G_i	$\delta = 0.05$	$\delta = 0.10$	$\delta = 0.15$	$\delta = 0.20$	$\delta = 0.25$	$\delta = 0.30$
1.4	8/5	(0.238, 1.4247)	(1.044, 1.3676)	(1.130, 1.3222)	(0.841, 1.2408)	(0.679, 1.2300)	(0.598, 1.2192)
	5/3	(0.000, 1.6164)	(0.519, 1.5636)	(0.747, 1.5148)	(0.690, 1.4701)	(0.588, 1.4277)	(0.473, 1.3811)
	26/15	(0.000, 1.8036)	(0.022, 1.7608)	(0.361, 1.7118)	(0.430, 1.6661)	(0.404, 1.6234)	(0.358, 1.5833)
	8/5	(0.448, 0.7854)	(1.383, 0.7344)	(1.355, 0.6981)	(1.062, 0.6618)	(0.832, 0.6256)	(0.665, 0.5905)
1.2	5/3	(0.192, 0.9653)	(0.875, 0.9153)	(0.987, 0.8749)	(0.825, 0.8375)	(0.663, 0.8016)	(0.536, 0.7675)
	26/15	(0.000, 1.1416)	(0.402, 1.0969)	(0.614, 1.0553)	(0.573, 1.0175)	(0.483, 0.9823)	(0.399, 0.9494)
1.0	8/5	(0.674, 0.1466)	(1.891, 0.1043)	(1.678, 0.0792)	(1.236, 0.0483)	(0.934, 0.0160)	(0.723, -0.0154)
	5/3	(0.472, 0.3149)	(1.367, 0.2703)	(1.332, 0.2396)	(1.019, 0.2085)	(0.775, 0.1777)	(0.598, 0.1486)
	26/15	(0.262, 0.4800)	(0.907, 0.4365)	(0.975, 0.4031)	(0.781, 0.3723)	(0.601, 0.3433)	(0.462, 0.3165)
0.8	8/5	(0.919, -0.4916)	(2.876, -0.5162)	(1.846, -0.5305)	(1.321, -0.5587)	(0.999, -0.5891)	(0.778, -0.6185)
	5/3	(0.779, -0.3345)	(2.257, -0.3680)	(1.788, -0.3846)	(1.289, -0.4119)	(0.952, -0.4407)	(0.701, -0.4676)
	26/15	(0.630, -0.1804)	(1.700, -0.2173)	(1.553, -0.2391)	(1.109, -0.2650)	(0.793, -0.2908)	(0.570, -0.3140)
0.6	8/5	(1.189, -1.1292)	(2.877, -1.1520)	(1.847, -1.1844)	(1.322, -1.2289)	(1.000, -1.2741)	(0.779,-1.3147)
	5/3	(1.120, -0.9829)	(2.758, -0.9954)	(1.789, -1.0336)	(1.290, -1.0819)	(0.984,-1.1302)	(0.775, -1.1760)
	26/15	(1.044, -0.8392)	(2.611, -0.8311)	(1.716, -0.8756)	(1.247 - 0.9279)	(0.958, -0.9794)	(0.760, -1.028)

Table 4: Impacts of parameters δ and G_i on the optimal I and SW

Table 4 and Figure 9 illustrate the impacts of parameters δ , G_i . and f_1 on the optimal I and the maximum SW.

First, it can be seen that the optimal I is concave in δ with a fixed G_i . This means that with either a significant low or high δ , the government should exert a low enforcement level. This is because when δ is low, the entrant's greenness level will be high, and, in turn, greenwashing activities will have greater effect on CS as well as on the environment. Therefore, a higher enforcement level will be



Figure 9: Sensitivity analysis of parameters f_1 , G_i and δ

unnecessary. Conversely, when δ becomes high, the incumbent will voluntarily combat the entrant's greenness behaviors to protect its own profitability. Under this condition, it is also necessary for the government to set a high enforcement level. However, for a moderate value of δ , because the incumbent has less incentive to combat greenwashers, the government must set a higher enforcement level to protect the environment and CS. Therefore, the optimal I is concave in δ .

Second, intuitively, an optimal I will monotonically decrease with G_i . This indicates that when the incumbent's product greenness is high, the government should impose a less strict enforcement level. Table 4 and Figure 9 also shows that the optimal SW always decreases with G_i and increases with δ , which conforms to intuition as well.

Third, it is interesting that governments tend to have a lower enforcement level with a higher f_1 . This is because a stricter enforcement level may restrict a firm's economic benefits (see Figure 6), even though it may improve CS and reduce environmental impacts. Considering the economycentric attitude in emerging markets, the results explain why greenwashing activities are prevalent in developing countries, but are seldom countered by their governments.

6. Conclusions

In an era of global sustainable development, greenwashing activities remain an issue for both industry and governments, especially in emerging markets. To deal with existing greenwashing issues in emerging markets, a novel two-period game model was established, with consideration given to governments' AGE regulations. In the game model, the incumbent acted as the Stackelberg leader, while the entrant acted as the follower. Using backward induction, the entrant's decisions about greenwashing, market entry mode, and selling price in Period 2, and the incumbent's decisions on selling price in periods 1 and 2, were obtained. The effects of the entrant's greenwashing activities and the model's critical parameters (including the incumbent's greenness level, the entrant's potential greenwashing degree, and the government's AGE level) on prices, profits, and CS were explicitly discussed. In addition, how governments could maximize social welfare by making optimal decisions on AGE levels was also explored.

The main insights are summarized as follows:

- First, when a green firm's greenness level and an entrant's greenwashing degree are both relatively low, it is counterintuitive that the green firm would tolerate the brown firm's greenwashing activities. That is, an entrant's greenwashing acts may benefit the incumbent to some extent. This is because in a horizontal competitive environment, a non-greenwashing entrant can only threaten the incumbent's market share using a low-price strategy. In this sense, accepting a certain degree of greenwashing may be beneficial to an incumbent's overall profitability. However, in other regions, greenwashing is not beneficial at all and can be prevented by the incumbent through strategic pricing mechanisms.
- Second, it was observed that in the presence of greenwashing activities, the total CS may rise in a small region. It was shown that when the greenness gap between green and brown firms is small and the green firm's greenness is moderate, the entrant's greenwashing behavior eases competition and benefits the incumbent with a larger market share. Consequently, the total CS increases because the rise of the incumbent's CS outweighs other losses. Nevertheless, greenwashing has no benefits to CS outside this small region.
- Third, we investigated governments' regulatory decisions to optimize social welfare with respect to their different preferences for economic or social value. The numerical examples show that the optimal enforcement level in the greenness gap is concave. This means that governments should only impose a strict enforcement policy for moderate greenness gap levels, while strict policies are less necessary for narrow or wide greenness gaps. Meanwhile, sensitivity analysis indicate that economically focused governments in emerging markets prefer loose enforcement levels compared with governments in developed markets. Our results partially explain why greenwashing activities are prevalent in developing regions and why governments take fewer actions in developing contexts.

In contrast with the existing literature, this paper extends the research on green-brown competition, government regulation, and incumbent-entrant gaming by incorporating firms' greenwashing activities,

thus providing meaningful managerial insights for industry and governments to achieve both economic and environmental sustainability.

In addition to its contributions, this study also had some limitations. First, only two firms were considered. This provides the opportunity to extend the model horizontally with multiple entrants or vertically with upstream suppliers. Second, this model assumes that a firm's greenness is predetermined, whereas in reality, greenness levels may be random and stochastic. Third, future empirical studies may potentially reveal more insights and open up additional research opportunities.

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Appendix A. Special cases

We provide two special cases, enabling us to study the impacts of the brown firm's entry threat and the brown firm's greenwashing act by comparing them with the base model. In the first case, the green firm is a monopoly without a rival's entry threat (Model NE). In the second case, the green firm faces competition from a non-deceptive brown firm (Model NGE).

Model NE: We start with the benchmark, where there is no threat of an entrant. In both periods, customers will buy the incumbent's products if and only if $U_i^1(x) \ge 0$ and $U_i^2(x) \ge 0$. Therefore, customers located within the interval of $x \in \left[0, \min\left(\frac{G_i - p_i}{t}, 1\right)\right]$ will purchase the products; otherwise, there is no purchase. The demands in the two periods can be expressed as $d_i^1 = d_i^2 = \min\left(\frac{G_i - p_i}{t}, 1\right)$. The total profit function is

$$\pi_i = 2p_i \cdot \min\left(\frac{G_i - p_i}{t}, 1\right). \tag{A.1}$$

By maximizing the incumbent's profit function, we obtain the optimal price, demand, and profit and the corresponding CS as follows

$$(p_i^*, d_i^{1*}, d_i^{2*}, \pi_i^*) = \begin{cases} (G_i - t, 1, 1, 2G_i - 2t), & G_i \in [2t, 3t].\\ \left(\frac{G_i}{2}, \frac{G_i}{2t}, \frac{G_i}{2t}, \frac{G_i^2}{2t}\right), & G_i \in [t, 2t]. \end{cases}$$
(A.2)

As Eq.(A.2) shows, when the greenness level is relatively high (i.e., $2t \leq G_i \leq 3t$), the market is fully covered in both periods. As the greenness level drops, the market becomes partially covered, and the firm sets a lower price. This monopoly case indicates that product greenness level plays an important role in the determination of selling prices, market coverage, and profit.

Model NGE: We next analyze the benchmark where the entrant is a non-deceptive brown firm. Compared with the main model, the entrant's Period 2 decision is always k = N. The solution of the game is quite similar to the main model; therefore, we omit the proof for the equilibrium results. By defining five regions as follows, we present the equilibrium results in Table A.5.

$$\begin{cases} \Phi^{N-S} & := \left\{ \left(\frac{1}{G_i}, \delta\right) : \frac{1}{G_i} \geqslant \frac{2-\delta}{2t} \right\}, \\ \Phi^{N-A1} & := \left\{ \left(\frac{1}{G_i}, \delta\right) : \frac{5-2\delta}{6t} \leqslant \frac{1}{G_i} \leqslant \frac{2-\delta}{2t} \right\}, \\ \Phi^{N-A2} & := \left\{ \left(\frac{1}{G_i}, \delta\right) : \frac{28-13\delta}{39t} \leqslant \frac{1}{G_i} \leqslant \frac{5-2\delta}{6t} \right\}, \\ \Phi^{N-C1} & := \left\{ \left(\frac{1}{G_i}, \delta\right) : \frac{6-\delta}{13t} \leqslant \frac{1}{G_i} \leqslant \frac{28-13\delta}{39t} \right\}, \\ \Phi^{N-C2} & := \left\{ \left(\frac{1}{G_i}, \delta\right) : \frac{1}{3t} \leqslant \frac{1}{G_i} \leqslant \frac{6-\delta}{13t} \right\}. \end{cases}$$

Table A.5 presents the incumbent's and entrant's optimal decisions and corresponding profits. As it shows, when the greenness level is relatively high, the two firms will compete directly; when the greenness level is moderate, the two firms will accommodate each other; when the greenness level is very low, the market is not fully covered and the two firms are separated, with no competition involved.

a 111		Incumbent's Decisio	Entrant's Decisions			
Condition	p_i^*	(d_i^{1*}, d_i^{2*})	π_i^*	p_e^*	d_{e}^{2*}	π_e^*
Φ^{N-S}	$\frac{G_i}{2}$	$\left(rac{G_i}{2t}, rac{G_i}{2t} ight)$	$\frac{G_i^2}{2t}$	$\frac{(1-\delta)G_i}{2}$	$\frac{(1-\delta)G_i}{2t}$	$\frac{(1-\delta)^2 G_i^2}{4t}$
Φ^{N-A1}	$\frac{G_i}{2}$	$\left(rac{G_i}{2t}, rac{G_i}{2t} ight)$	$\frac{G_i^2}{2t}$	$\tfrac{(3-2\delta)G_i-2t}{2}$	$1 - \frac{G_i}{2t}$	$\tfrac{((3-2\delta)G_i-2t)(2t-G_i)}{4t}$
Φ^{N-A2}	$\tfrac{(4-\delta)G_i-3t}{3}$	$\left(\frac{3t - (1 - \delta)G_i}{3t}, \frac{3t - (1 - \delta)G_i}{3t} \right)$	$\tfrac{2(3t+(\delta-1)G_i)((4-\delta)G_i-3t))}{9t}$	$\frac{2(1-\delta)G_i}{3}$	$\frac{(1-\delta)G_i}{3t}$	$\tfrac{2(1-\delta)^2G_i^2}{9t}$
Φ^{N-C1}	$\tfrac{(4+\delta)G_i+3t}{10}$	$\left(\tfrac{(6-\delta)G_i-3t}{10t}, \tfrac{27t+(9\delta-4)G_i}{40t} \right)$	$\tfrac{((4+\delta)G_i+3t)^2}{80t}$	$\tfrac{13t+(4-9\delta)G_i}{20}$	$\tfrac{13t+(4-9\delta)G_i}{40t}$	$\tfrac{(13t + (4 - 9\delta)G_i)^2}{800t}$
Φ^{N-C2}	$G_i - t$	$\left(1, \frac{4t + (\delta - 1)G_i}{4t}\right)$	$\tfrac{(8t+(\delta-1)G_i)(G_i-t)}{4t}$	$\frac{(1-\delta)G_i}{2}$	$\frac{(1-\delta)G_i}{4t}$	$\frac{(1-\delta)^2 G_i^2}{8t}$

Table A.5: Equilibrium results for Model NGE

Appendix B. Computing results for CS in the main model, Model NE, and Model NGE

(1) In the main model and Model NGE, CS can be calculated with the following function.

$$CS = \underbrace{\int_{0}^{d_{i}^{1*}} U_{i}^{1}(x|p_{i}^{*})dx}_{\text{Buy from incumbent in period 1}} + \underbrace{\int_{0}^{d_{i}^{2*}} U_{i}^{2}(x|p_{i}^{*})dx}_{\text{Buy from entrant in period 2}} + \underbrace{\int_{1-d_{e}^{2*}}^{1} U_{e}^{N2}(x|p_{e}^{*})dx}_{\text{Buy from entrant in period 2}}$$
(B.1)

CS in main model and Model NGE can be expressed respectively as

$$Main \ model: \ CS = \begin{cases} \frac{3}{8t}G_i^2 + \frac{\delta-t}{2t}G_i + \frac{t}{2} - \delta, & \left(\frac{1}{G_i}, \delta\right) \in \Omega^{G-A1} \\ \frac{3\hat{\Delta}^2}{2t}G_i^2 - \frac{(2t+\delta)\hat{\Delta}}{t}G_i + t, & \left(\frac{1}{G_i}, \delta\right) \in \Omega^{N-A2} \\ \frac{\delta^2 - 2\delta + 3}{8t}G_i^2, & \left(\frac{1}{G_i}, \delta\right) \in \Omega^{N-S} \\ \frac{3}{8t}G_i^2 - \frac{1}{2}G_i + \frac{t}{2}, & \left(\frac{1}{G_i}, \delta\right) \in \Omega^{N-A1} \\ \frac{(1-\delta)^2}{6t}G_i^2 - \frac{2(1-\delta)}{3}G_i + t, & \left(\frac{1}{G_i}, \delta\right) \in \Omega^{N-A2} \\ \frac{89\delta^2 - 96\delta + 232}{1600t}G_i^2 - \frac{388 - 173\delta}{800}G_i - \frac{1039}{1600}t, & \left(\frac{1}{G_i}, \delta\right) \in \Omega^{N-C1} \\ \frac{(1-\delta)^2}{16t}G_i^2 + t, & \left(\frac{1}{G_i}, \delta\right) \in \Omega^{N-C2} \end{cases}$$

$$Model \ NGE: \ CS = \begin{cases} \frac{\delta^2 - 2\delta + 3}{8t}G_i^2, & \left(\frac{1}{G_i}, \delta\right) \in \Phi^{N-S} \\ \frac{3}{8t}G_i^2 - \frac{1}{2}G_i + \frac{t}{2}, & \left(\frac{1}{G_i}, \delta\right) \in \Phi^{N-A1} \\ \frac{(1-\delta)^2}{6t}G_i^2 - \frac{2(1-\delta)}{3}G_i + t, & \left(\frac{1}{G_i}, \delta\right) \in \Phi^{N-A2} \end{cases}$$

$$(B.3)$$

$$\frac{89\delta^2 - 96\delta + 232}{1600t}G_i^2 + \frac{388 - 173\delta}{800}G_i - \frac{1039}{1600}t, & \left(\frac{1}{G_i}, \delta\right) \in \Phi^{N-A2} \end{cases}$$

(2) In Model $N\!E\!,\,\mathrm{CS}$ can be calculated with the next function.

$$CS = \underbrace{\int_{0}^{d_{i}^{1*}} U_{i}^{1}(x|p_{i}^{*})dx}_{0} + \underbrace{\int_{0}^{d_{i}^{2*}} U_{i}^{2}(x|p_{i}^{*})dx}_{0}$$
(B.4)

Buy from incumbent in period 1 \quad Buy from entrant in period 2

CS in Model $N\!E$ can be expressed respectively as

$$Model \quad NE: \quad CS = \begin{cases} t, & G_i \in [2t, 3t] \\ \frac{G_i^2}{4t}, & G_i \in [t, 2t] \end{cases}$$
(B.5)

Appendix C. Definitions of thresholds

The thresholds in the paper are presented in Table C.6.

$$\begin{split} & \text{Table C.6: Definitions of thresholds} \\ \rho_1(\delta, I, t) &= \begin{cases} \rho_{11}(\delta, I, t) := \frac{1+2i\delta}{2t}, & \delta \leqslant (1+3It)^{-1} \\ \rho_{12}(\delta, I, t) := \frac{1+2I\delta}{2t}, & (1+3It)^{-1} \leqslant \delta \leqslant (1+2It)^{-1} \\ \rho_{13}(\delta, I, t) := \frac{1+2i\delta}{2t}, & \delta \geqslant (1+2It)^{-1} \\ \rho_{2}(\delta, I, t) &= \begin{cases} \rho_{21}(\delta, I, t) := \frac{1}{t} \frac{\delta^2 + 8\delta + 160\hat{\Delta}^2 + 160\hat{\Delta} + 16}{16\hat{\Delta} - 3\hat{\Delta} + 40\sqrt{(1-\delta-3\hat{\Delta})(\delta+3\hat{\Delta} + 12)/10+68}}, & \rho_{21}(\delta, I, t) \geqslant \frac{6-\delta}{13} \\ \rho_{22}(\delta, I, t) := \frac{1}{t} \frac{\delta^2 + 8\hat{\Delta} + \delta - 1}{\delta^2 + 8\delta + 16\hat{\Delta} - 1}, & \rho_{22}(\delta, I, t) \leqslant \frac{6-\delta}{13} \\ \rho_{22}(\delta, I, t) := \frac{1}{t} \frac{\delta^2 + 8\hat{\Delta} + \delta - 1}{\delta + 16\hat{\Delta} - 1}, & \rho_{22}(\delta, I, t) \leqslant \frac{6-\delta}{13} \\ K_1 &= \begin{cases} \frac{89\delta^2 - 168\delta - 96}{t(173\delta + 20\sqrt{306\delta^2} - 772\delta + 25\overline{4} - 388)}, & \delta \leqslant 0.097 \\ \frac{89\delta^2 - 168\delta + 304}{t(173\delta + 20\sqrt{662\delta^2} - 1444\delta + 2382 - 388)}, & \delta \geqslant 0.097 \end{cases} \\ K_2 &= \frac{1+4\delta - 2\delta^2}{2t(2\delta - \sqrt{7+4\delta} - 2\delta^2 - 2)} \\ K_3 &= \frac{1}{2t - 4\delta} \\ K_4 &= \frac{6\hat{\Delta}^2 - 1}{2(\sqrt{4}(\delta + 2)^2t^2 + 12\delta(3 - 2\hat{\Delta}^2t^2 + t^2)} \\ J_1 &= \frac{6\hat{\Delta}^2 - 1}{2(\sqrt{4}(\delta + 2)^2t^2 + 12\delta(3 - 2\delta^2 + 2\delta)t + 9\delta^2 + 4\delta t - 3\delta - t)} \\ J_2 &= \frac{9\hat{\Delta}^2 - \delta^2 + 2\delta - 1}{6\hat{\Delta} - 4t + 4\delta t + 12\hat{\Delta}t} \\ J_3 &= \frac{2400\hat{\Delta}^2 - 89\delta^2 + 96\delta - 232}{(388 - 173\delta + 1600\hat{\Delta})t + 800\delta\hat{\Delta} - 2\sqrt{2}} \sqrt{\frac{8000\delta^2 \hat{\Delta}^2 + (-34600\delta + 32000\hat{\Delta} + 77600)\delta\hat{\Delta}t}{+(33100\delta^2 - 69200\delta\hat{\Delta} - 48449\delta - 471700\hat{\Delta}^2 + 155200\hat{\Delta} + 95349)t^2}} \end{cases}$$

Appendix D. Proofs

Appendix D.1. Lemma 1

Proof. When $p_i \leq G_i - t$, the problem is to maximize the piecewise function as follows.

$$\pi_{e}^{G}(p_{e}^{G}|p_{i}) = \begin{cases} p_{e}^{G} - C_{p}, & p_{e}^{G} \leq p_{i} - t - \delta G_{i}. & (\mathbf{G-IO}) \\ \frac{p_{e}^{G}(t - \delta G_{i} + p_{i} - p_{e}^{G})}{2t} - C_{p}, & p_{i} - t - \delta G_{i} \leq p_{e}^{G} \leq p_{i} + t - \delta G_{i}. & (\mathbf{G-C}) \\ 0 - C_{p} & p_{e}^{G} \geq p_{i} + t - \delta G_{i}. & (\mathbf{G-EB}) \end{cases}$$
(D.1)

• For scenario G-IO, $\frac{\partial \pi_e^G}{\partial p_e} > 0$ is always hold. Therefore, in region $p_e^G \leq p_i - t - \delta G_i$, the optimal decision is $p_e^{G*} = p_i - t - \delta G_i$, the maximum profit is

$$\pi_e^{G*} = p_i - t - \delta G_i - C_p \tag{D.2}$$

• For scenario G-C, solving $\frac{\partial \pi_e^G}{\partial p_e^G} = \frac{t - \delta G_i + p_i - 2p_e^G}{2} = 0$, we obtain $p_e^{G*} = \frac{t - \delta G_i + p_i}{2}$. Since p_e^G is positive, it is also required that $p_i \ge max(\delta G_i - t, 0)$, otherwise $p_e^{G*} = 0$. The profit can be expressed as

$$\pi_{e}^{G*} = \begin{cases} \frac{(t-\delta G_{i}+p_{i})^{2}}{8t} - C_{p}, & \max(\delta G_{i}-t,0) \leqslant p_{i} \leqslant G_{i}-t. \\ 0 - C_{p}, & p_{i} \leqslant \max(\delta G_{i}-t,0). \end{cases}$$
(D.3)

Comparing the optimal profit in the three scenarios, we find that when $p_i \leq G_i - t$, Scenario G-C dominates the other two scenarios. The maximum price decision is presented in Lemma 1.

Appendix D.2. Lemma 2-3

Proof. Proof of Lemma 2-3 is similar to that of Lemma 1. Therefore, it is omitted.

Appendix D.3. Proposition 1

Proof. The results presented in Proposition 1 can be obtained by comparing the entrant's profit in Eq.(11) and Eq.(15). As shown in Figure D.10, the whole region can be divided into five parts.



Figure D.10: Comparison of entrants' profit under greenwash and no greenwash.

- In the first part (i.e., $p_i \leq \frac{12+\delta-2\sqrt{6\delta-2\delta^2}}{9}G_i t$), comparing the profits of scenarios G-C and N-C, we find that the profit of N-C is always higher than that of G-C.
- In the second part (i.e., $\frac{12+\delta-2\sqrt{6\delta-2\delta^2}}{9}G_i t \leq p_i \leq \frac{(4-\delta)G_i}{3} t$), comparing the profits of scenarios G-A and N-C, we find that (1) when $p_i \geq (1 + \frac{3+\delta-2\sqrt{6\delta-2\delta^2-18It\delta^2}}{9})G_i t$ and $I \leq \frac{1-\delta}{3\delta t}$, the profit of G-A is higher than that of N-C; (2) otherwise, the profit of N-C is higher than that of G-A.
- In the third part (i.e., $\frac{(4-\delta)G_i}{3} t \leq p_i \leq \frac{(3-\delta)G_i}{2} t$), comparing the profits of scenarios G-A and N-A, we find that (1) when $I \leq \frac{1-\delta}{3\delta t}$, or when $\frac{1-\delta}{3\delta t} \leq I \leq \frac{1-\delta}{2\delta t}$ and $p_i \geq (1 + It\delta)G_i t$, the profit of G-A is higher than that of N-A; (2) otherwise, the profit of N-A is higher than that of G-A.
- In the fourth part (i.e., $\frac{(3-\delta)G_i}{2} t \leq p_i \leq \frac{3G_i}{2} t$), comparing the profits of scenarios G-A and N-S, we find that (1) when $I \leq \frac{1-\delta}{2\delta t}$, or when $\frac{1-\delta}{2\delta t} \leq I \leq \frac{2-\delta}{4\delta t}$ and $p_i \geq (1 + \frac{1-\sqrt{2\delta-\delta^2-4It\delta^2}}{2})G_i t$, the profit of G-A is higher than that of N-S; (2) otherwise, the profit of N-S is higher than that of G-A.

• In the fifth part (i.e., $p_i \ge \frac{3G_i}{2} - t$), comparing the profits of scenarios G-S and N-S, we find that when $I \le \frac{2-\delta}{4\delta t}$, the profit of G-S is higher than that of N-S; (2) otherwise, the profit of N-S is higher than that of G-S.

Based on the above results, the entrant's response is summarized in Proposition 1. $\hfill \Box$

Appendix D.4. Lemma 4

Proof. The problem is to maximize the incumbent's profit function of

$$\pi_{i}(p_{i}) = \begin{cases} \frac{2p_{i}(G_{i}-p_{i})}{t}, & p_{i} \ge \min(\Lambda_{1}, \frac{(4-\delta)G_{i}}{3}-t). & (a) \\ p_{i} \cdot \min\left(\frac{G_{i}-p_{i}}{t}, 1\right) + \frac{p_{i}(3t+\delta G_{i}-p_{i})}{4t}, & \max(\delta G_{i}-t, 0) \le p_{i} \le \min(\Lambda_{1}, \frac{(4-\delta)G_{i}}{3}-t). & (b) & (D.4) \\ 2p_{i}, & p_{i} \le \max(\delta G_{i}-t, 0). & (c) \end{cases}$$

• When $\delta \leq \frac{1}{1+3It}$, the condition $\Lambda_1 \leq \frac{(4-\delta)G_i}{3} - t$ always holds. Therefore, the problem is simplified to

$$\pi_{i}(p_{i}) = \begin{cases} \frac{2p_{i}(G_{i}-p_{i})}{t}, & p_{i} \ge \Lambda_{1}. \\ p_{i} \cdot \min\left(\frac{G_{i}-p_{i}}{t}, 1\right) + \frac{p_{i}(3t+\delta G_{i}-p_{i})}{4t}, & \max(\delta G_{i}-t, 0) \le p_{i} \le \Lambda_{1}. \end{cases} (b) \\ 2p_{i}, & p_{i} \le \max(\delta G_{i}-t, 0). \end{cases} (c)$$

i) For subproblem (a), the optimal price and profit can be calculated as

$$(p_i^*, \pi_i^*) = \begin{cases} (\frac{G_i}{2}, \frac{G_i^2}{2t}), & \frac{1}{G_i} \ge \frac{1}{t}(\frac{1}{2} + \widehat{\Delta}).\\ (\Lambda_1, \frac{2\Lambda_1(G_i - \Lambda_1)}{t}), & \frac{1}{G_i} \le \frac{1}{t}(\frac{1}{2} + \widehat{\Delta}) \end{cases}$$
(D.6)

ii) For subproblem (b), the optimal price and profit can be calculated as

$$(p_i^*, \pi_i^*) = \begin{cases} (\Lambda_1, \frac{\Lambda_1(G_i - \Lambda_1)}{t} + \frac{\Lambda_1(3t + \delta G_i - \Lambda_1)}{4t}), & \frac{1}{G_i} \ge \frac{10}{13t}(\frac{6 - \delta}{10} + \widehat{\Delta}).\\ (\frac{(4 + \delta)G_i + 3t}{10}, \frac{((4 + \delta)G_i + 3t)^2}{80t}), & \frac{6 - \delta}{13t} \le \frac{1}{G_i} \le \frac{10}{13t}(\frac{6 - \delta}{10} + \widehat{\Delta}).\\ (G_i - t, \frac{(8t + (\delta - 1)G_i)(G_i - t)}{4t}), & \frac{1}{G_i} \le \frac{6 - \delta}{13t} \end{cases}$$
(D.7)

iii) For subproblem (c), the optimal price and profit can be calculated as

$$(p_i^*, \pi_i^*) = (max(\delta G_i - t, 0), 2 * max(\delta G_i - t, 0)).$$
(D.8)

Comparing the optimal profits in the three subproblems, the optimal decisions of this function can be

expressed as

$$(p_i^*, \pi_i^*) = \begin{cases} (\frac{G_i}{2}, \frac{G_i^2}{2t}), & \frac{1}{G_i} \ge \frac{1}{t}(\frac{1}{2} + \widehat{\Delta}). \\ (\Lambda_1, \frac{2\Lambda_1(G_i - \Lambda_1)}{t}), & \rho_{21}(\delta, I, t) \le \frac{1}{G_i} \le \frac{1}{t}(\frac{1}{2} + \widehat{\Delta}) \\ (\frac{(4+\delta)G_i + 3t}{10}, \frac{((4+\delta)G_i + 3t)^2}{80t}), & \frac{6-\delta}{13t} \le \frac{1}{G_i} \le \rho_2(\delta, I, t). \\ (G_i - t, \frac{(8t + (\delta - 1)G_i)(G_i - t)}{4t}), & \frac{1}{G_i} \le \min(\frac{6-\delta}{13t}, \rho_2(\delta, I, t)) \end{cases}$$
(D.9)

• When $\delta \ge \frac{1}{1+3It}$, the condition $\Lambda_1 \ge \frac{(4-\delta)G_i}{3} - t$ always holds. Therefore, the problem is simplified to

$$\pi_{i}(p_{i}) = \begin{cases} \frac{2p_{i}(G_{i}-p_{i})}{t}, & p_{i} \geq \frac{(4-\delta)G_{i}}{3} - t. & (a) \\ p_{i} \cdot \min\left(\frac{G_{i}-p_{i}}{t}, 1\right) + \frac{p_{i}(3t+\delta G_{i}-p_{i})}{4t}, & \max(\delta G_{i}-t, 0) \leq p_{i} \leq \frac{(4-\delta)G_{i}}{3} - t. & (b) \\ 2p_{i}, & p_{i} \leq \max(\delta G_{i}-t, 0). & (c) \end{cases}$$

Following a similar solution procedure, the optimal price and profit can be expressed as

$$(p_{i}^{*}, \pi_{i}^{*}) = \begin{cases} \left(\frac{G_{i}}{2}, \frac{G_{i}^{2}}{2t}\right), & \frac{1}{G_{i}} \ge \frac{5-2\delta}{6t} \\ \left(\frac{(4-\delta)G_{i}}{3} - t, \frac{2(3t+(\delta-1)G_{i})((4-\delta)G_{i}-3t)}{2t}\right), & \frac{28-13\delta}{39t} \le \frac{1}{G_{i}} \le \frac{5-2\delta}{6t} \\ \left(\frac{(4+\delta)G_{i}+3t}{10}, \frac{((4+\delta)G_{i}+3t)^{2}}{80t}\right), & \frac{6-\delta}{13t} \le \frac{1}{G_{i}} \le \frac{28-13\delta}{39t} \\ \left(G_{i} - t, \frac{(8t+(\delta-1)G_{i})(G_{i}-t)}{4t}\right), & \frac{1}{G_{i}} \le \frac{6-\delta}{13t} \end{cases}$$
(D.11)

Substituting the results in (D.9) and (D.11) into the entrant's responsive functions, we obtain the final results in Table 2. $\hfill \square$

Appendix D.5. Proposition 2-3

Proof. The results in Proposition 2 can be obtained by comparing the incumbent's optimal price and profit in the main model (see Table 2) with that in Model NE (see Eq.(A.2)).

The results in Proposition 3 can be obtained by comparing the incumbent's optimal price and profit in the main model (see Table 2) with that in Model NGE (see Tab.A.5).

Appendix D.6. Proposition 4

Proof. In region Ω^{G-A2} , the incumbent and the entrant's profits are

$$\pi_i^* = \frac{2((1+\hat{\Delta})G_i - t)(t-\hat{\Delta}G_i)}{t}, \quad \pi_e^* = \frac{(1-\hat{\Delta})\hat{\Delta}G_i^2}{t} - I(\delta G_i)^2.$$
(D.12)

The first and second derivatives w.r.t. parameter δ for the incumbent's profit function are as follows

$$\frac{\partial \pi_i^*}{\partial \delta} = -\frac{2G_i(G_i - 2t + 2G_i\widehat{\Delta})}{t} \frac{\partial \widehat{\Delta}}{\partial \delta},
= -\frac{2G_i(G_i - 2t + 2G_i\widehat{\Delta})}{9t} \left(1 + \frac{-6 + 4\delta(1 + 9It)}{\sqrt{6\delta - 2\delta^2 - 18It\delta^2}}\right)$$
(D.13)

$$\frac{\partial^2 \pi_i^*}{\partial \delta^2} = -\frac{2G_i(G_i - 2t + 2G_i\widehat{\Delta})}{\sqrt{2}t(3\delta - \delta^2 - 9It\delta^2)^{\frac{3}{2}}} - \frac{4G_i^2}{81t} \left(1 + \frac{-6 + 4\delta(1 + 9It)}{\sqrt{6\delta - 2\delta^2 - 18It\delta^2}}\right)^2 \tag{D.14}$$

Because in region Ω^{G-A2} , G_i satisfies $\frac{1}{G_i} \leq \frac{1+2\widehat{\Delta}}{2t}$, the first part of Eq.(D.14) is negative. Also, it is easy to verify that the second part is negative. Therefore, $\frac{\partial^2 \pi_i^*}{\partial \delta^2} < 0$, which means the incumbent's profit is convex in δ . Solving the first order condition, i.e., $\frac{\partial \pi_i^*}{\partial \delta} = 0$, we have

$$\widetilde{\delta} = \frac{1}{2(9It+1)} \left(3 - \frac{1}{\sqrt{8It+1}} \right). \tag{D.15}$$

The first and second derivatives w.r.t. parameter δ for the entrant's profit function are as follows

$$\frac{\partial \pi_e^*}{\partial \delta} = \frac{(1-2\widehat{\Delta})G_i^2}{t} \frac{\partial \widehat{\Delta}}{\partial \delta} - 2IG_i^2 \delta
= \frac{(1-2\widehat{\Delta})G_i^2}{9t} \left(1 + \frac{-6 + 4\delta(1+9It)}{\sqrt{6\delta - 2\delta^2 - 18It\delta^2}}\right) - 2IG_i^2 \delta$$
(D.16)

$$\frac{\partial^2 \pi_e^*}{\partial \delta^2} = \frac{(1-2\widehat{\Delta})G_i^2}{\sqrt{2}t(3\delta - \delta^2 - 9It\delta^2)^{\frac{3}{2}}} - \frac{2G_i^2}{81t} \left(1 + \frac{-6 + 4\delta(1+9It)}{\sqrt{6\delta - 2\delta^2 - 18It\delta^2}}\right)^2 - 2IG_i^2 \tag{D.17}$$

Solving the first order condition, i.e., $\frac{\partial \pi_e^*}{\partial \delta} = 0$, we have

$$\widehat{\delta} = \frac{1}{2(9It+1)} \left(3 + \frac{2\sqrt{2}}{\sqrt{It+1}} \right) \tag{D.18}$$

It is difficult to prove the positivity of the second order derivtive. However, by substituting $\hat{\delta}$ into Eq.(D.17), we find that when $I \ge \frac{1}{t}$, $\frac{\partial^2 \pi_e^*}{\partial \delta^2}|_{\delta=\hat{\delta}} > 0$ holds. Therefore, the function is concave in δ at point $\delta = \hat{\delta}$.

Appendix D.7. Proposition 5

Proof. In region Ω^{G-A1} , we have

$$\frac{\partial \pi_i^*}{\partial I} = 0. \tag{D.19}$$

$$\frac{\partial \pi_e^*}{\partial I} = -(\delta G_i)^2 < 0. \tag{D.20}$$

In region Ω^{G-A2} , we have

$$\frac{\partial \pi_i^*}{\partial I} = -\frac{2G_i(G_i - 2t + 2G_i\widehat{\Delta})}{t} \frac{\sqrt{2}\delta^2 t}{\sqrt{3\delta - \delta^2 - 9It\delta}} < 0 \tag{D.21}$$

$$\frac{\partial \pi_e^*}{\partial I} = \frac{(1-2\Delta)G_i^2}{t} \frac{\sqrt{2\delta^2 t}}{\sqrt{3\delta - \delta^2 - 9It\delta}} - (\delta G_i)^2
= \frac{\delta^2 G_i}{9\sqrt{6\delta - 2\delta^2 - 18It\delta}} (3 - 2\delta - 5\sqrt{6\delta - 2\delta^2 - 18It\delta}).$$
(D.22)

For $\delta \leq \frac{1}{1+3It}$, $3 - 2\delta - 5\sqrt{6\delta - 2\delta^2 - 18It\delta} > 0$ holds. Therefore, $\frac{\partial \pi_e^*}{\partial I} > 0$.

Appendix D.8. Proposition 6-7

Proof. The results can be obtained by comparing the CS for the main model (See Eq.(B.2)) and Model NE (See Eq.(B.5)).

The results can be obtained by comparing the CS for the main model (See Eq.(B.2)) and Model NGE (See Eq.(B.3)).

Appendix E. Extension: Customer group updating

In the main model, we assume that the two firms are facing the same group of customers with the same learning effect after consuming products in Period 1. Then, the customer group needs to be updated, as the old customers may be replaced by new ones at a later time. To characterize this effect, we define a customer group updating rate as $\gamma \in (0, 1)$. Specifically, we assume that after staying in the market for one period, γ amount of customers will leave the market, and the rest $(1 - \gamma)$ will stay in the market to become experienced customers. At the beginning of Period 2, another γ amount of inexperienced customers will enter the market. Note that $\gamma \to 0$ means the customer group updating rate is zero and the problem has already been studied in the manuscript; while $\gamma \to 1$ means all the Period 1 customers will leave the market and be replaced by newcomers in Period 2. To ease our analysis, we assume that the numbers of customers who leave the market and those who come into the market are equal, which ensures that the updating rate is steady and the market size does not change.

In the following analysis, we vary the level of customer group updating rate (γ) to investigate its impacts on the two firms' competitive behaviors and equilibrium outcomes. We use superscript u to denote firms' pricing decisions in the presence of customer updating. For customers who stay in the market $(1 - \gamma)$, their learning effect exists and the entrant's profit functions in response to the incumbent's pricing decisions are the same as those in the basic model (see section 4.1). For customers who enter the market in Period 2, (γ) , their learning effect is inactive. That is, the newcomers cannot detect the brown firm's greenwashing activities. The detailed proof for the entrant's responsive profit functions are omitted for expositional simplicity. We only show part of the results as follows:

(1) Entrant's responsive profit functions and conditions

When the brown entrant chooses to greenwash, the result is

$$\pi_{e}^{G*} = \begin{cases} \frac{G_{i}^{2}}{4t} - C_{p}, & p_{i} \geqslant \frac{3G_{i}}{2} - t. & (\mathbf{G}-\mathbf{S}, \mathbf{G}^{u}-\mathbf{S}) \\ \frac{(2G_{i}-t-p_{i})(t-G_{i}+p_{i})}{t} - C_{p}, & \frac{4}{3}G_{i} - t \leqslant p_{i} \leqslant \frac{3G_{i}}{2} - t. & (\mathbf{G}-\mathbf{A}, \mathbf{G}^{u}-\mathbf{A}) \\ (1-\gamma)\frac{(2G_{i}-t-p_{i})(t-G_{i}+p_{i})}{t} + \gamma\frac{(t+p_{i})^{2}}{8t} - C_{p}, & \frac{12+\delta-2\sqrt{6\delta-2\delta^{2}}}{9}G_{i} - t \leqslant p_{i} \leqslant \frac{4}{3}G_{i} - t. & (\mathbf{G}-\mathbf{A}, \mathbf{G}^{u}-\mathbf{C}) \\ (1-\gamma)\frac{(t-\delta G_{i}+p_{i})^{2}}{8t} + \gamma\frac{(t+p_{i})^{2}}{8t} - C_{p}, & 0 \leqslant p_{i} \leqslant \frac{12+\delta-2\sqrt{6\delta-2\delta^{2}}}{9}G_{i} - t. & (\mathbf{G}-\mathbf{C}, \mathbf{G}^{u}-\mathbf{C}) \\ (\mathbf{E}-1) \end{cases}$$

When the brown entrant choose not to greenwash, the result is

$$\pi_{e}^{N*} = \begin{cases} \frac{(1-\delta)^{2}G_{i}^{2}}{4t}, & p_{i} \geqslant \frac{(3-\delta)G_{i}}{2} - t. & (\mathbf{N-S,N^{u}-S}) \\ \frac{((2-\delta)G_{i} - t - p_{i})(t - G_{i} + p_{i})}{t}, & \frac{(4-\delta)G_{i}}{3} - t \leqslant p_{i} \leqslant \frac{(3-\delta)G_{i}}{2} - t. & (\mathbf{N-A,N^{u}-A}) \\ \frac{(t-\delta G_{i} + p_{i})^{2}}{8t}, & \max(\delta G_{i} - t, 0) \leqslant p_{i} \leqslant \frac{(4-\delta)G_{i}}{3} - t. & (\mathbf{N-C,N^{u}-C}) \end{cases}$$
(E.2)

Define

$$\begin{cases} \Lambda_1^u = \frac{3-\sqrt{(2-\delta-4I\delta t)\delta}}{2}G_i - t, \\ \Lambda_2^u = (1+I\delta t)G_i, \\ \Lambda_3^u = \frac{-4\delta+12\gamma+2\sqrt{2\delta(2\delta-3\gamma+9I\delta\gamma t)}}{9\gamma}G_i - t, \\ \Lambda_4^u = \frac{4}{3}G_i + \frac{\delta-\sqrt{\delta((9\gamma-8)\delta+24(1-3I\delta t)(1-\gamma))}}{9(1-\gamma)}G_i - t, \\ \Lambda_5^u = \frac{\gamma+8It}{2\gamma}\delta G_i - t. \end{cases}$$

By comparing the profits, Figure E.11 shows the entrant's optimal entry strategy. We can see that the same combinations of p_i and I may result in different equilibriums for the new and repeat buyers. For example, in Region III, the incumbent's pricing will incentivize the entrant to go greenwashing and compete for new buyers $(G^u - C)$, and it will stop the entrant moving into the incumbent's market share for repeat buyers. Comparing this to Figure 2, under the consideration of customer group updating, we can see that the competition scenario exists when the entrant chooses to go greenwashing.

(2) The incumbent's problem

Considering the entrant's responses, the incumbent will optimally set its selling prices w.r.t. different values of G_i , δ , and I.

$$\pi_{i}(p_{i}) = \begin{cases} \frac{2p_{i}(G_{i}-p_{i})}{t}, & (p_{i},I) \in I, II, V, VI. \\ (1-\gamma)\frac{2p_{i}(G_{i}-p_{i})}{t} + \gamma \left(\frac{p_{i}(G_{i}-p_{i})}{t} + \frac{p_{i}(3t-p_{i})}{4t}\right), & (p_{i},I) \in III. \\ p_{i} \cdot min\left(\frac{G_{i}-p_{i}}{t},1\right) + \frac{p_{i}(3t+\delta G_{i}-p_{i})}{4t}, & (p_{i},I) \in VII. \\ p_{i} \cdot min\left(\frac{G_{i}-p_{i}}{t},1\right) + (1-\gamma)\frac{p_{i}(3t+\delta G_{i}-p_{i})}{4t} + \gamma \frac{p_{i}(3t-p_{i})}{4t}, & (p_{i},I) \in IV. \end{cases}$$
(E.3)

(3)Numerical results

Note that when $\gamma \to 0$, the problem equals the main model. Because of the problem's complexity, it impossible to obtain the analytical solutions. Therefore, numerical tests are conducted to illustrate some important results. In the numerical tests, the exogenous parameters are set as $G_i = \frac{5}{3}$, t = 1, I = 4, $\gamma = \{0.1, 0.2, 0.3\}$, and $\delta = \{0, 0.01, 0.02, ..., 0.25\}$. The impacts of γ on both firms' profits and equilibrium strategies are shown in Figure E.12. Comparing the results with those in the main model (See Figure 5(c)), our key findings still stand. This means that the assumption about customers' repeat purchasing does not influence the main results. Interestingly, we find that the incumbent's and the entrant's profits are decreasing in parameter γ in the "greenwash" region. This indicates that when the customer updating rate is low, more customers will be experienced in Period 2. This will intensify competition between the two firms, thus leading to a "lose-lose" outcome.



Figure E.11: Entrant's responsive decisions.



Figure E.12: Impacts of γ and δ on profits and equilibrium strategies.