Probabilistic Analysis Methodology for the Thermal Protection System at the Conceptual Design

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Abstract:

This paper presents a probabilistic analysis methodology for non-ablative Thermal Protection System (TPS) of spacecraft at the conceptual design stage. The probabilistic analysis focuses on uncertainty characterization and uncertainty in failure prediction. TPS selection and sizing using sequential quadratic programming design optimization are first performed to provide the nominal values of the distribution parameters for uncertainty parameters such as the allowable temperature limits and thickness of TPS materials. Multi-inputs and multi-outputs support vector machines are utilized to approximate the thermal responses when failure modes are constructing, which dramatically reduces computational effort. Generalized Subset Simulation is used to estimate the failure probabilities at all nodes with a single simulation run, which further reduces the computational burden. The proposed methodology is applied to a lifting body vehicle model and a spacecraft model for conceptual design. Difficulties encountered and the performance of the method are investigated.

Keywords: Non-ablative, thermal protection system, conceptual design, failure prediction.

Introduction

One of the greatest challenges in conceptual design of the reusable spacecraft is determining the non-ablative Thermal Protection System (TPS) to protect the spacecraft from severe aerodynamic heating during its reentry into the atmosphere at hypersonic speeds. To provide capability for design and analysis process of TPS in the conceptual design stage of the spacecraft, McGuire et al. (2004), Chen et al. (2006), Bradford and Olds (2006), Coward and Olds (2000, 1999, and 2001) have developed automated TPS design tools. Those include TPSsizer, Hypersonic Aerodynamics/Aerothermodynamics for TPS, Sentry and Thermal Calculation Analysis Tool, based on aerodynamic/aeroheating analysis and TPS selection/sizing.

Uncertainties in geometry, loads and material properties, however, make the design process computationally intensive. Early attempts on TPS uncertainty analysis rely on expert experience assigning uncertainty level to the prediction of aeroheating and the evaluation in TPS sizing (Gnoffo et al. 1999). Although relatively rigorous trials on uncertainty assessment later were made, they are still limited by the computational and experimental burden for complex systems, such as non-liner or high-dimensional ones. These designs allocate risk implicitly in the choice of safety factors. More specifically, the traditional conservative ideas of design techniques always consider all the uncertainty with the help of enough safety margins rather than accurate analyses. However, they are becoming more and more inapplicable as the rapid development of the relevant techniques in aerospace and increasing of the mission requirements.

Thanks to the advent modern simulation techniques and computer technology, Monte Carlo (MC) based probabilistic analysis methods have received increasing attentions from researchers who generally focused on two objectives. One is the ablative TPS. Dec and Mitcheltree (2002) combined MC method with three degree-of-freedom trajectory calculation and a distributed heating environment prediction including turbulence effects with a material response calculation. A relationship between TPS sizing margins and failure probability was established through a Charring Material Thermal Response and Ablation Program (CMA). Inspired by MC, researchers from NASA Ames Research Center have carried out many relevant studies, such as simulating one-dimensional material response at stagnation point using Fully Implicit Ablation and Thermal (FIAT) response code (Chen and Milos 1996) and constructing the relationship between TPS thickness margins and the probability that can maintain the temperature of TPS material within specified limits (Chen et al. 2006). They have conducted TPS probabilistic analysis on Titan Atmospheric Entry, Mars Exploration Rover and the wing leading edge of X-37 (Chen et al. 2006,Deepak et al. 2004,Wright et al. 2007a,Wright et al. 2007b). Their work also included a series of probabilistic analyses on aeroheating (Bose et al. 2006,Sepka and Wright 2011,Wright et al. 2007a). Chen et.al (2006) pointed out that FIAT is more robust than CMA and so it was more suitable for automated MC analysis with a large amount of calculations. A typical MC based nonlinear TPS probabilistic analysis requires hundreds, thousands or even millions of computational fluid dynamics and/or material response analyses to statistically ensure required accuracy. Such a task is beyond the capacity of existing codes and impractical for the computational resources. To some extents, parallel processing can alleviate the huge burden of calculation and make this task possible. Tobin and Dec (2015) determined two different stages of MC analyses for the TPS probabilistic sizing of a hypersonic inflatable aerodynamic decelerator. The first stage was to test the inflatable thermal response model. The second stage was to reduce the error of the prediction for this model. Compared to the traditional root sum squared method for calculating margins, a lower design scheme of TPS with the estimation for failure probability of overheating was finally provided.

The other objective of Monte Carlo based probabilistic analysis methods focuses on the detailed design of the integrated TPS (ITPS). Most studies along this direction come from a research group at University of Florida. Ravishankar (2011) performed finite element analysis for ITPS using Abaqus software and reduced the computational burden by responses surface method. A Separable MC method (Smarslok et al. 2006,Smarslok 2009) was adopted for probabilistic analysis and Bootstrapping resampling technique was utilized to improve the accuracy of failure probability estimate. Matsumura, Haftka and Sankar (2011) and Villanueva (2013) proposed a method for estimating the failure probability during the design stage that considered the influence from future processes, such as tests and redesigns. Villanueva (2013) demonstrated that redesign following future test can reduce the failure probability by orders of magnitude. Additionally, Bayesian inference was also applied in the uncertainty reduction of model via testing.

In this paper, we are interested in the probabilistic analysis for non-ablative TPS of reusable spacecraft during the conceptual design stage. This is because the conceptual design of TPS determines the most cost of its whole life and sensitive to uncertainties. To achieve lower cost and risk, designers or decision-makers generally appeal probabilistic analysis methods. Several challenges are encountered. The main challenge comes from the huge computational burden associated with probabilistic analysis method, e.g., MC based method. Some questions that need to be answered include which probabilistic analysis method to use, what kind of output to produce, and how to carry out an efficient probabilistic analysis of a TPS. This study addresses these challenges through the development of a sampling-based methodology. In this methodology, the failure probabilities at all nodes which are used to discretize the investigated TPS are the preferable outputs of the probabilistic analysis. A Multi-inputs and Multi-outputs Support Vector Machine (MIMO-SVM) (Xu et al. 2013,Xu et al. 2014) is presented to replace the expensive physics simulation models of the failure modes at all nodes which have been specified for geometric modeling, aerodynamic analysis and aeroheating analysis. Since the built MIMO-SVM surrogate is still a system of multiple Limit State Functions (LSFs), evaluation on it by most available reliability methods except the direct MC requires repeated implementations, which often leads to a computational issue. Even MC based probabilistic analysis also suffers from the huge computational effort. Hence, the Generalized Subset Simulation (GSS) method (Li et al. 2015) is used to estimate the failure probabilities at all nodes with a single simulation run, which further reduces the computational burden. The developed methodology is generic and applicable to the probabilistic analysis of both single layer and stack-up TPS. Two numerical examples are used to illustrate the performance of the proposed methodology.

The remaining sections of this paper are organized as follow. The second section shows how to prepare a deterministic design of TPS including TPS selection and sizing. The third section gives the procedure of the proposed probabilistic methodology. Next, two application examples including a lifting body vehicle model and a spacecraft model are considered in the fifth section to demonstrate the performance of the proposed methodology. Finally, conclusions are given in the last section.

Preparation of the deterministic design

Thermal Analysis Methodology

Consider the following one-dimensional unsteady heat conduction equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (\alpha = k/c_p\rho) \tag{1}$$

where k, c_p and ρ are the thermal conductivity, the specific heat and the density of the material, respectively. T means temperature. t and x denote time and thickness, respectively.

At the top surface of the TPS material where $x = 0$, the boundary condition satisfies the energy balance relationship

$$q_{conv} - \varepsilon\sigma T_s^4 + k\frac{\mathrm{d}T}{\mathrm{d}x} = 0,\ \ x = 0 \tag{2}$$

where q_{conv} is the convection from the flow field, i.e., heat flux. T_s denotes temperature at the top surface of the TPS material and ε is the emissivity of the material. That is, all the quantities including convection from the flow field, radiation from the heated surface, and conduction absorbed by the TPS material are summed to equal zero in order to preserve the conservation of energy. While at $x = Lt$, a conservative boundary condition is employed assuming that there is an adiabatic wall at the back face of the material

$$\frac{\mathrm{d}T}{\mathrm{d}x} = 0,\ \ x = Lt \tag{3}$$

In practice, the temperature shifting of an adiabatic wall is incapable of fully modeling the heat capacitance of the cold structure that physically exists behind the TPS material. However, the required methods for evaluating the level of coupling between the cold structures and the TPS are not within the scope of conceptual design (Cowart and Olds 2000,Cowart and Olds 1999,Olds and Cowart 2001).

For stack-up where multiple materials are layered together, it is assumed that perfect contact exists and Equation (4) gives the interface condition for the heat transfer between the materials

$$k_{i-1}\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)_{i-1} = k_i\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)_i \tag{4}$$

where i denotes the i-th layer. Assume that no kinetic reactions occur in the boundary layer. Therefore, chemical equilibrium exists while thermal equilibrium does not. Moreover, all material properties remain constant throughout the analysis. Meanwhile, temperature-dependent material properties are not incorporated (Olds and Cowart 2001).

In view of Equations (1), (2), (3) and (4), three different types of finite difference discretization were adopted for obtaining the system of equations, which are required to solve for the in-depth temperature profile in the material as function of time. Specifically, a one-sided forward implicit difference scheme, a one-sided implicit backward finite difference scheme and a simple implicit central finite difference scheme are used to discretize Equations (3) or (4), (2) and (1), respectively. The corresponding accuracy is on the order of $O(\Delta t,\Delta x)$, $O(\Delta t,\Delta x)$ and $O(\Delta t,\Delta x^2)$, respectively. The system of equations is iteratively solved using the Newton-Raphson method.

TPS Sizing

In the conceptual design of TPS, materials selection and sizing are the main and vital processes. Each TPS stack-up candidate is analyzed through heat transfer analysis in conjunction with an optimization process on the thickness of the TPS material (Bradford and Olds 2006). The outer loop of TPS sizing is an optimization process of the thickness while the inner loop is a heat transfer analysis. The optimizer will adjust the thickness of one material in the TPS stack-up, in order to meet temperature limits.

This study considers three classical types of TPS materials, namely, Reinforced Carbon-Carbon (RCC), High-temperature Reusable Surface Insulation (HRSI) tiles, and Felt Reusable Surface Insulation (FRSI) . HRSI is made of coated Li-900 Silica ceramics and FRSI uses white Nomex felt blankets. Table 1 gives the properties of these materials. Note that k, c_p, and ε are the corresponding properties at 300 K, which is the initial temperature of the TPS.

Consider a lifting body vehicle (Fig. 1) as an illustration example. Totally 9,122 nodes and 18,240 triangle elements are allocated on the surface of the lifting body vehicle for aerodynamic, aeroheating and heating transfer analysis. The maximum values of the input heat flux are used to divide the whole surface of the lifting body vehicle into three regions. Material 1, 2 and 3 denote the densified Nomex, RCC and Li-900, respectively. It should be pointed out that the boundaries of materials should be rounded off during the detailed design stage so that it is convenient for manufacture and assembly. After one TPS stack-up is determined at each node, the TPS sizing process is implemented to minimize the weight of the TPS stack-up subjected to the allowable temperature limits. This optimization problem at a node is formulated as:

$$\begin{aligned} &\min\ W(\boldsymbol{x}) = \sum_{i=1}^{m} x_i \rho_i \quad (l_i \le x_i \le r_i) \\ &s.t.\ \ T_{i,\mathrm{top}}^{\max}(\boldsymbol{x}) - T_{i,\mathrm{limit}} \le 0, i = 1, 2, \ldots, m \\ &\qquad T_{i,\mathrm{back}}^{\max}(\boldsymbol{x}) - T_{i+1,\mathrm{limit}} \le 0, i = 1, 2, \ldots, m-1 \\ &\qquad T_{i,\mathrm{back}}^{\max}(\boldsymbol{x}) - T_{\mathrm{back}} \le 0, i = m \end{aligned} \tag{5}$$

where the objective function W is the weight (actually system mass) of the TPS stack-up; m denotes the number of layers in the stack-up; ρ_i and x_i are the material density and thickness in the i-th layer; $T_{i,\mathrm{top}}^{\max}$ and $T_{i,\mathrm{back}}^{\max}$ denote the maximum temperature of the top surface and the back face of the i-th material; T_{back} is the allowable temperature of the cold structure. For simplicity, a single layer of the TPS stack-up is considered in this study as the optimization processes are identical for the single-layer and multiple-layer TPS. Equation (3) is chosen as the boundary condition at the back face. In detail, the back face temperatures limits of the three TPS materials are 1585K, 1250K and 550K, respectively.

Probabilistic model of TPS

Uncertainty Modeling

Errors in modeling and simulation, manufacturing imperfections, variations in material properties, geometric dimensions, and loading conditions can bring in uncertainties which are generally modeled by random variables in engineering community. The conceptual design of TPS in this paper considers uncertain parameters like geometry, material properties and loading conditions. The primary geometry parameters are the thickness t of the TPS material, which are obtained from deterministic design optimization. Note that the value of the thermal load, i.e., heat flux at all the time instants are obtained from an interpolation based on 24 pre-observed heat fluxes. Those heat fluxes are calculated through aeroheating analysis at 24 interpolation points selected on the reentry trajectory of the spacecraft. Specifically, the 24 interpolation points are chosen as Point 4 to Point 27 from the Space Transportation System-1 (STS-1) reentry trajectory data (Olds and Cowart 2001).

Uncertainties in loading conditions are modeled in terms of 24 random heat fluxes. Uncertainties in material properties include the allowable temperature limits at both the top surface and the back face of the material layer. We denote the allowable temperature limits as $T_{\text{top}}^{\text{limit}}$ and $T_{\text{back}}^{\text{limit}}$ for the top surface and the back face, respectively. The input random parameters are shown in Table 2, where COV means the Coefficient of Variation. It can be seen that the problem has 27 random inputs at each node on the whole surface of TPS. A truncated normal distribution is used to described the uncertainties within the allowable temperature limits. It is expressed as

$$f\left(x;\mu,\sigma,x^{U},x^{L}\right)=\frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{x^{U}-\mu}{\sigma}\right)-\Phi\left(\frac{x^{L}-\mu}{\sigma}\right)} \tag{6}$$

where $\phi(\cdot)$ is the probability density function of the standard normal distribution and $\Phi(\cdot)$ is the cumulative distribution function. The definition domain is $\boldsymbol{\Omega}=\{x\text{: } x^{L}\leqslant x \leqslant x^{U}\}$. x^{L} and x^{U} are the lower and the upper boundaries for the variable, respectively. The mean μ is located at the centre of the definition domain, the standard deviation of the artificial distribution σ is chosen as $(x^{U} - x^{L})/6$ according to the three sigma limits.

Failure modes

Two failure modes are defined for a node in this study. One is the maximum temperature at the top surface of the material layer exceeding the corresponding allowable temperature limit. The other is the maximum temperature at the back face exceeding the corresponding allowable temperature limit. The LSFs at each node can be expressed as

$$\begin{gathered} g_{j1}(t,q_i,T_{\text{top}}^{\text{limit}})=T_{\text{top}}^{\text{limit}}-T_{\text{top}}^{\max}\left(t,q_i\right) \\ g_{j2}(t,q_i,T_{\text{back}}^{\text{limit}})=T_{\text{back}}^{\text{limit}}-T_{\text{back}}^{\max}\left(t,q_i\right) \\ (i=1,2,\ldots,m;\ j=1,2,\ldots,nn) \end{gathered} \tag{7}$$

where *nn* is the total number of nodes. Thus, there are totally 2 × *nn* failure probabilities needed to be estimated. For each node, $T_{\text{top}}^{\max}\left(t,q_i\right)$ and $T_{\text{back}}^{\max}\left(t,q_i\right)$ are obtained from the thermal analysis mentioned in second section. Obviously, the computational burden of probabilistic analysis for the whole TPS is huge since it involves a large number of LSFs, where repeated calculations are required when variance reduction technique, such as subset simulation method is used.

Proposed methodology

Overview of procedure

Fig. 2 presents the procedure of the proposed probabilistic analysis methodology, which consists of three modules. Module 1 is a deterministic TPS selecting and sizing process, which has been given in the second section. The optimization problem in TPS sizing is solved by Sequential Quadratic Programming (SQP) strategy. The distributional parameters of the random inputs are based on the analysis results obtained from TPS selecting and sizing at all nodes. Those results include the heat loading, i.e., the heat flux, the thickness of the material, and material properties (specifically, the allowable temperature limit) at each node. In Module 2, MIMO-SVM is used to build up two approximated models for each region and construct the two failure modes for each node in the third subsection. In Module 3, GSS is adopted to estimate all the failure probabilities at all the nodes with one simulation run.

MIMO-SVM

Since the number of nodes is commonly very large in practice, MIMO-SVM (Xu et al. 2013,Xu et al. 2014) surrogate models are adopted to approximate the multiple LSFs and dramatically reduce the computational effort.

For a system with *md* outputs, the training data set *S* is

$$S=\left\{\left(\boldsymbol{xx}_i,\boldsymbol{y}_i\right)_{i=1}^{l}\right\},\ \boldsymbol{xx}_i\in R^{nd},\ \boldsymbol{y}_i\in R^{md},\quad i=1,2,\cdots,l \tag{8}$$

where *nd* is the dimension of inputs, *l* is the training sample size, $\boldsymbol{xx}_i$ is the input parameters, $\boldsymbol{y}_i$ is the scalar output, respectively. The control parameter $\boldsymbol{w}_i$ in SVM regression expression is divided as $\boldsymbol{w}_i=\boldsymbol{w}_0+\boldsymbol{v}_i$ (Arora et al. 1998). If the output quantities are very different to each other, the mean vector $\boldsymbol{w}_0$ is relatively small, otherwise the vectors $\boldsymbol{v}_i$ is small. That is, $\boldsymbol{w}_0$ reflects the similarity among the output quantities, while $\boldsymbol{v}_i$ embodies the speciality of the *i*-th output quantity.

The regression parameters $\boldsymbol{w}_0$, $\boldsymbol{v}_i$ and b_i are obtained simultaneously by minimizing the following objective function with constraints

$$\begin{aligned}&\min\ \frac{1}{2}\boldsymbol{w}_0^{\mathrm{T}}\boldsymbol{w}_0+\frac{1}{2}\frac{\lambda}{md}\sum_{i=1}^{md}\boldsymbol{v}_i^{\mathrm{T}}\boldsymbol{v}_i+c\sum_{i=1}^{md}\boldsymbol{\xi}_i^{\mathrm{T}}\boldsymbol{\xi}_i\\&s.t.\ \ \boldsymbol{y}_i=\mathbf{Z}_i^{\mathrm{T}}\left(\boldsymbol{w}_0+\boldsymbol{v}_i\right)+b_i\,\mathbf{1}_l+\boldsymbol{\xi}_i\qquad\left(i=1,2,\cdots,md\right)\end{aligned}\tag{9}$$

where λ and c are two positive real regularized parameters, which are used to control the balance between variance and basis of the fitting. They can be selected with a 10-fold cross-validation (Xu et al. 2013,Xu et al. 2014). $\mathbf{1}_l$ =(1, 1, ⋯, 1)$^{\mathrm{T}}\in R^l$; $\mathbf{Z}=(\phi(\boldsymbol{x}_{i,1}), \phi(\boldsymbol{x}_{i,2}), \cdots, \phi(\boldsymbol{x}_{i,l}))$; $\boldsymbol{\xi}_i=(\xi_{i,1}, \xi_{i,2}, \cdots, \xi_{i,l})^{\mathrm{T}}$; and $\sum_{i=1}^{md}\boldsymbol{\xi}_i^{\mathrm{T}}\boldsymbol{\xi}_i$ is a quadratic loss function.

The optimization problem in Eq.(9) is formulated via the following Lagrange function

$$\begin{aligned}L\left(\boldsymbol{w}_0,\boldsymbol{v}_i,\boldsymbol{b},\boldsymbol{\xi}_i,\boldsymbol{\alpha}_i\right)=\frac{1}{2}\boldsymbol{w}_0^{\mathrm{T}}\boldsymbol{w}_0+\frac{1}{2}\frac{\lambda}{md}\sum_{i=1}^{md}\boldsymbol{v}_i^{\mathrm{T}}\boldsymbol{v}_i+c\sum_{i=1}^{md}\boldsymbol{\xi}_i^{\mathrm{T}}\boldsymbol{\xi}_i-\\\sum_{i=1}^{md}\boldsymbol{\alpha}_i^{\mathrm{T}}\left(\mathbf{Z}_i^{\mathrm{T}}\left(\boldsymbol{w}_0+\boldsymbol{v}_i\right)+b_i\,\mathbf{1}_l+\boldsymbol{\xi}_i-\boldsymbol{y}_i\right)\left(i=1,2,\cdots,md\right)\end{aligned}\tag{10}$$

where $\boldsymbol{\alpha}_i=(\alpha_{i,1}, \alpha_{i,2},\cdots,\alpha_{i,l})^{\mathrm{T}}$ are the Lagrange multipliers. The Karush-Kuhn-Tucker conditions for Eq.(10) are given by

$$\begin{aligned}&\frac{\partial L}{\partial\boldsymbol{w}_0}=0\,,\frac{\partial L}{\partial\boldsymbol{v}_i}=0,\frac{\partial L}{\partial b_i}=0\,,\frac{\partial L}{\partial\boldsymbol{\xi}_i}=0\,,\\&\frac{\partial L}{\partial\boldsymbol{\alpha}_i}=0\ \left(i=1,2,\cdots,md\right)\end{aligned}\tag{11}$$

This leads to the following linear equations

$$\begin{cases}\boldsymbol{w}_0=\mathbf{Z}\boldsymbol{\alpha},\ \ \boldsymbol{v}_i=\dfrac{md}{\lambda}\mathbf{Z}_i\boldsymbol{\alpha}_i,\\\displaystyle\sum_{i=1}^{m}\boldsymbol{\alpha}_i=0,\ \ \boldsymbol{\alpha}_i=2c\boldsymbol{\xi}_i,\qquad i=1,2,\cdots,md\\\boldsymbol{y}_i=\mathbf{Z}_i^{\mathrm{T}}\left(\boldsymbol{w}_0+\boldsymbol{v}_i\right)+b_i\,\mathbf{1}_l+\boldsymbol{\xi}_i\end{cases}\tag{12}$$

where $\mathbf{Z}=(\mathbf{Z}_1, \mathbf{Z}_2,\cdots, \mathbf{Z}_{md})$, and $\boldsymbol{\alpha}=(\boldsymbol{\alpha}_1^{\mathrm{T}}, \boldsymbol{\alpha}_2^{\mathrm{T}},\cdots, \boldsymbol{\alpha}_{md}^{\mathrm{T}})^{\mathrm{T}}$. Furtherly, the following matrix equation is formulated

$$\begin{bmatrix} \mathbf{0}_{md\times md} & \mathbf{O}^{\mathrm{T}} \\ \mathbf{O} & \mathbf{H} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{b} \\ \boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{md} \\ \mathbf{y} \end{bmatrix} \tag{13}$$

where $\mathbf{O}$=($\mathbf{1}_{l1}$, $\mathbf{1}_{l2}$,⋯, $\mathbf{1}_{lmd}$) is a block diagonal matrix. The positive definite matrix H=$\mathbf{Z}^{\mathrm{T}}\mathbf{Z}$+(1/2$c$)$\frac{1}{2c}$$\mathbf{I}_l$+($md/\lambda$)$\mathbf{B}$. $\mathbf{I}_l$ is a unitary matrix. $\mathbf{B}$=(K_1, K_2,⋯, K_{md}) is a block diagonal matrix, in which the i-th element satisfies $K_i=\mathbf{Z}_i^{\mathrm{T}}\mathbf{Z}_i$. Supposed that the solution to Eq.(13) are $\boldsymbol{\alpha}^*=(\boldsymbol{\alpha}_1^{*\mathrm{T}}, \boldsymbol{\alpha}_2^{*\mathrm{T}},\cdots, \boldsymbol{\alpha}_{md}^{*\mathrm{T}})^{\mathrm{T}}$ and $\mathbf{b}^*=(b_1^*,b_2^*,\cdots,b_{md}^*)^{\mathrm{T}}$, where $\boldsymbol{\alpha}_i^*=(\alpha_{i,1}^*, \alpha_{i,2}^*,\cdots, \alpha_{i,l}^*)^{\mathrm{T}}$. Then, the regression functions is expressed as

$$\begin{aligned} f_i\left(\boldsymbol{xx}\right) &= \phi\left(\boldsymbol{xx}\right)^{\mathrm{T}}\left(w_0^* + v_i^*\right) + b_i^* \\ &= \phi\left(\boldsymbol{xx}\right)^{\mathrm{T}}\left(\mathbf{Z}\boldsymbol{\alpha}^* + \frac{md}{\lambda}\mathbf{Z}_i\boldsymbol{\alpha}_i^*\right) + b_i^* \\ &= \sum_{i'=1}^{md}\sum_{j=1}^{l}\alpha_{i',j}^* K\left(\boldsymbol{xx}_{i',j}, \boldsymbol{xx}\right) + \frac{md}{\lambda}\sum_{j=1}^{l}\alpha_{i',j}^* K\left(\boldsymbol{xx}_{i,j}, \boldsymbol{xx}\right) + b_i^* \\ i &= 1, 2, \cdots, md \end{aligned} \tag{14}$$

where $\phi(\cdot)$ is the specified kernel function. More details about the MIMO-SVM algorithm can be referred to the references (Xu et al. 2013,Xu et al. 2014).

To train the MIMO-SVM models, 50, 100 and 50 supporting points were generated by Sobol's sequence in Region 1 (FRSI), Region 2 (HRSI), and Region 3 (RCC) for the lifting body vehicle, respectively. For each region, multi-inputs of each MIMO-SVM generally consist of the material properties including the density, the thermal conductivity, the specific heat and the emissivity of the material, the thickness of the material and 24 pre-observed heat flux values throughout the trajectory. The material properties at each region are the same while the thickness of the material and 24 pre-observed heat flux values vary for different nodes. Accordingly, the MIMO-SVM model at each region has 25 input variables. The multi-outputs of each MIMO-SVM are the approximated values of $T_{\mathrm{top}}^{\max}\left(t, q_i\right)$ and $T_{\mathrm{back}}^{\max}\left(t, q_i\right)$ for each node within a region, respectively. The responses of LSFs g_{j1} and g_{j2} at a node are obtained from the corresponding surrogate model. For the lifting body vehicle, the number of nodes for three regions are 1987, 6344, and 791, respectively.

To check the accuracy of the MIMO-SVM surrogates, the computational results based on the original models are generated from the TPS analysis at all nodes as references. The relative error for a LSF is calculated as

$$\frac{\left|\text{result from thermal analysis in the orignal model - result from MIMO-SVM}\right|}{\text{result from thermal analysis in the orignal model}} \times 100\% \tag{15}$$

The largest relative error at all nodes is 48.70%. For most of the nodes (8,460 nodes), the relative errors are less than 10%. There are 253 nodes where the relative errors are larger than 30%. In order to guarantee the accuracy of the probabilistic analysis, the original model at these 253 nodes are employed instead of building up the surrogate models for them (such as Node 1464).

Table 3 provides a clear view upon the accuracy of the surrogates used in three regions. More specifically, Sobol's sequence was used to generate five quasi-Monte Carlo points as observations in each region. The computational results from the original models for two LSFs at each of these nodes were calculated from the TPS analysis as references.

Generalized Subset Simulation

For a non-ablative TPS, system reliability method may provide a failure probability from a global perspective of the system. However, a single failure probability is incapable of reflecting the information at each component of the system. A system may have several weak points rather than one. Some of the weak points (such as those in the nose areas) are known based on prior engineering experiences while the others are unknown and some can be very potential. In order to support a robust and accurate design process, a comprehensive probability assessment method which can estimate all the failure probabilities for all the components of a system is very necessary. In this study, we suggest estimating the failure probabilities at nodes of the whole TPS. Consequently, a contour of failure probabilities can be obtained to provide sufficient information for designers, analysts and decision-makers.

In Module 3, the recently developed Generalized Subset Simulation (GSS) (Li et al. 2015) is used for estimating the failure probabilities at all nodes simultaneously. Compared to the original SS, GSS can utilize the correlation information among multiple LSFs of interest by constructing a unified intermediate event for each simulation level. There are indeed correlations among all LSFs in the TPS problem because they are defined for the same system and further share the common group of input random variables. Furthermore, the correlations include also the one between two LSFs, i.e., g_{j1} and $g_{j2,}$ at a node, since the temperatures of at the top surface and back face for a node are obtained from the same thermal analysis. To some extents, the correlations also exist in all LSFs at the top surface of all the nodes using the same material because they are calculated from the same system model, as well as those at the back face. These kinds of correlations together provide a possibility of efficiently solving the TPS problem through GSS.

GSS constructs a unified intermediate event, i.e., the union of the intermediate events for all LSFs concerned to resolve the sorting difficulty arising in the original SS for multiple LSFs. The union is viewed as a single driving event, which enables the simulation procedure to simultaneously estimate all the failure probabilities of the multiple LSFs.

For a problem with n LSFs (n≥2), the unified intermediate failure event F_i is defined as

$$\begin{aligned} F_i &= F_i^{(1)} \cup F_i^{(2)} \cup \cdots \cup F_i^{(n)} \\ &= \{g^{(1)} \le bb_i^{(1)}\} \cup \{g^{(2)} \le bb_i^{(2)}\} \\ &\quad \cup \cdots \cup \{g^{(n)} \le bb_i^{(n)}\} \end{aligned} \tag{16}$$

where $F_i^{(j)}, j = 1, \ldots, n$ are the intermediate event identified by the original SS, and the superscript (j) indicates the j-th LSF $g^{(j)}$. It satisfies n=2 × nn for this TPS problem according to Equation (7). The subscript i denotes the i-th simulation level, and $bb_i^{(j)}$ denotes the j-th LSF's threshold at the i-th simulation level. The conditional probability $\mathrm{P}\left(F_i \middle| F_{i-1}\right)$ is then written as

$$\begin{aligned} &\mathrm{P}\left(F_i \middle| F_{i-1}\right) \\ &= \mathrm{P}\left(F_i^{(1)} \cup F_i^{(2)} \cup \cdots \cup F_i^{(n)} \middle| F_{i-1}\right) \\ &= \sum_{j=1}^{n} \mathrm{P}(F_i^{(j)} \big| F_{i-1}) - \sum_{1 \le j < k \le n} \mathrm{P}(F_i^{(j)} F_i^{(k)} \big| F_{i-1}) + \sum_{1 \le j < k < l \le n} \mathrm{P}(F_i^{(j)} F_i^{(k)} F_i^{(l)} \big| F_{i-1}) + \\ &\quad \cdots + (-1)^{n-1} \mathrm{P}(F_i^{(1)} F_i^{(2)} \cdots F_i^{(n)} \big| F_{i-1}) \end{aligned} \tag{17}$$

As shown in Equation (17), the value of the conditional probability $\mathrm{P}(F_i|F_{i-1})$ depends on the correlation level among all the intermediate events of LSFs concerned, which constitute the i-th union. From the theoretical point of view, the conditional probability has a limit interval $p_0 \le \mathrm{P}(F_i \big| F_{i-1}) \le \min\left\{np_0, 1\right\}$, however, it never reaches the upper limit due to correlation, which allows GSS to have an acceptable efficiency.

The failure probability associated to the j-th target event $F^{(j)} (j = 1, 2, \cdots n)$ is calculated as

$$\begin{aligned} \mathrm{P}_F^{(j)} &= \mathrm{P}\left(F^{(j)}{}_{u_j} \middle| F_{u_j - 1}\right) \mathrm{P}\left(F_{u_j - 1} \middle| F_{u_j - 2}\right) \\ &\quad \cdots \mathrm{P}(F_2 \big| F_1) \mathrm{P}(F_1) \quad (j = 1, \ldots, n) \end{aligned} \tag{18}$$

where u_j denotes the required number of simulation levels to reach $F^{(j)}$. Technically, the estimator of $\mathrm{P}_F^{(j)}$ by GSS, i.e., $\overline{\mathrm{P}}_F^{(j)}$, can be estimated as

$$\mathrm{P}_F^{(j)} \approx \overline{\mathrm{P}}_F^{(j)} = \frac{N_{F^{(j)}_{u_j}}}{N} \times \frac{N_{F_k}}{N} \times \cdots \times \frac{N_{F_2}}{N} \times \frac{N_{F_1}}{N} \quad (j=1,\ldots,n) \tag{19}$$

where $N_{F^{(j)}_{u_j}}$ denotes the number of samples that finally satisfies the *j*-th target event in the u_j -th simulation level. The number of samples falling within F_k is counted and is denoted as N_{F_k} $(k=1,\ldots,u_j-1)$.

More Details of the fundamental principle and implementation procedure of GSS can be found in Ref. (Li et al. 2015).

Results and Discussions

The proposed methodology was applied to estimate the failure probabilities of two TPS models, including a lifting body vehicle model and a spacecraft model. During the implementation of GSS, the size of samples *N* and the conditional probability p_0 are set as 500 and 0.2. In consideration of the underlying random mechanism, 30 independent GSS runs were operated to statistically provide the mean values of failure probabilities, the mean size of samples required (N_T) and unit COVs (Δ). According to Ref. (Au et al. 2007), unit COV Δ (Δ= COV × $N_\mathrm{T}^{-1/2}$) measures the efficiency of the algorithm. For each region of each example, a Monte Carlo simulation was also tried at five nodes, which were randomly selected by Sobol's sequence, in order to examine the effectiveness of the proposed method. However, the maximum total number of samples used in MC is merely up to 10^6 due to the complexity of the examples.

The proposed method is coded in Matlab environment and computations of GSS are performed on a desktop PC with Intel CORE i7-3770 CPU @ 3.40 GHz and 16GB RAM. Computations of MC are performed with 12 cores on a cluster.

Example 1: The lifting body vehicle

Consider the lifting body vehicle mentioned in second section as the first example (Fig. 1). As stated before, the deterministic optimization process (TPS sizing) was operated to obtain the thickness of the TPS material at each node as the mean value of one input uncertainty. MIMO-SVM surrogates were used to approximate the two failure modes at each node, in order to dramatically reduce the calculations. The accuracy of the MIMO-SVM surrogates for this problem has been already discussed in second section.

Fig. 4 and Fig. 5 present the failure probabilities at all nodes on the top surface and the back face of TPS materials for the lifting body vehicle, respectively. The symbol Pfs denotes failure probabilities at the top surface and Pfb denotes those at the back face. For the top surface, failure probabilities at 8,865 out of 9,122 nodes are less than 2.57×10^{-4}, as shown in the dark and light blue regions in Fig. 4. There are still 238 nodes where the failure probabilities are larger than 2.57×10^{-4} as they are shown with the green and red regions in Fig. 4. For the worst case, the failure probabilities at 113 nodes are around 0.0028 (red regions in Fig. 4). Moreover, the maximum failure probability is 0.029 at Node 3457 (marked in the first figure in Fig. 4). For the back face, however, there are 229 nodes where the failure probabilities are larger than 10^{-4}. The failure probabilities at 164 out of 229 nodes are larger than 10^{-3}, as displayed in the green and red regions in Fig. 5. The worst case happens at Node 4310 (It is marked on the first figure in Fig. 5) with a failure probability of 0.030. Fig.3 presents the heating history and temperature history of the deterministic design at Node 4310, in order to give a better understanding on the risk of the worst case. qconv denotes the convection from the flow field, i.e., heat flux; qrad-top denotes the thermal radiation from the surface; qcond denotes the thermal conduction within the material. They are consistent with the first, second and third item in Equation (2), respectively. Note that the maximum temperature (at point P) at the backface of the material (RCC) is 1574K, which is very close to the allowable temperature limit of the backface (1585K). It indicates why a high risk happens at this node while considering the defined uncertainty inputs.

According to these failure probabilities obtained in the conceptual design stage, designers can adjust the design process in conceptual design or arrange the design process of detailed design. For instance, a redesign including TPS selecting and sizing can be considered later in the early stage of detailed design within regions where failure probabilities are comparatively large. The design process that assigns safety factors into the design parameters is one common way to address this problem. However, these traditional design methods are based on experiences and engineering judgment, sometimes resulting in overly conservative designs in some respect but yet potentially inadequate in other aspects. Reliability-Based Design Optimization (RBDO) combines two major considerations in structural design, i.e., reliability considerations and design optimization into a single framework. It should be noted that both the conservative design methods and RBDO process are beyond the scope of this paper and will not be discussed.

In total, average 5345 samples were used in the proposed methodology for estimating failure probabilities. Obviously, the proposed methodology is much more efficient than MC for this problem. Table 4 lists the statistical performance based on 30 independent runs of the proposed methodology and a MC simulation at five randomly selected nodes in each region (each node is marked in Fig. 4 and Fig. 5). Due to the huge amount of calculations required on all the nodes, we only used 10^6 samples (It took 240 hours by the cluster for this problem). Consequently, those results from MC which are larger than 10^{-4} can be regarded as reasonable references only. It also indicates that the proposed methodology is applicable for the probabilistic analysis of TPS where small failure probabilities cannot be estimated by direct MC because of the massive computational burden. The unit COV of MC is calculated as

$$\Delta = \sqrt{\frac{1-\overline{\mathrm{P}_f}}{\overline{\mathrm{P}_f} N_T}} \times \sqrt{N_T} = \sqrt{\frac{1-\overline{\mathrm{P}_f}}{\overline{\mathrm{P}_f}}} \tag{20}$$

where $\overline{\mathrm{P}_f}$ denotes the estimate of failure probability and N is the sample size. From the view of unit COV (Au et al. 2007), the proposed method is more efficient than MC.

Example 2: The Spacecraft model

The second example considers a spacecraft model (Fig. 6). Totally 96,392 nodes and 192,780 triangle elements were defined to cover the surface of the spacecraft through meshing process for aerodynamic analysis. Similar to Example 1, three TPS materials including white Nomex felt blankets in FRSI (Material 1), coated Li-900 Silica ceramics in HRSI (Material 2) and RCC (Material 3) were selected for three different regions on the surface of the spacecraft. The boundaries between different material regions should be round off later in detailed design stage as well. For simplicity, we still model the TPS with one layer.

The numbers of the nodes for the three regions are 33,648, 58,162, and 4,582, respectively. Then, the three MIMO-SVMs in this example employ 150, 200 and 50 supporting points generated by Sobol' Sequence for Region 1 (FRSI), Region 2 (HRSI), and Region 3 (RCC), respectively. For each region in this example, the surrogate models are consistent with those in Example 1. The responses and failure probabilities of LSFs g_{j1} and g_{j2} at each node, therefore, can be obtained easily.

For most of the nodes (89,623 out of 96,392 nodes), the relative errors are less than 10%. Even though the largest relative error among all the LSFs is 88.23%, the number of nodes where the relative errors are larger than 30% is only 3625 (3.76% of the total). Similar to Example 1, the original models at these 3625 nodes were used rather than the surrogate models, for the sake of ensuring accuracy. It can be found that the surrogate models for Example 2 are less accurate than those for Example 1.

Table 5 gives a clear illustration on the accuracy of the surrogate models we used in three regions by randomly selecting some supporting points (marked in Fig. 8 and Fig. 9) using Sobol's sequence. As a contrast to the surrogate models, the computational results from the original models are calculated directly from the TPS analysis.

Fig. 8 and Fig. 9 present the failure probabilities at all nodes on the top surface and the back surface for the spacecraft. On the top surface, the failure probabilities at almost 74391 nodes are less than 10^{-4}, as shown in the dark blue and light blue regions in Fig. 8. There are 22001 nodes where the failure probabilities are larger than 10^{-4}. In addition, the maximum failure probability is 0.008 at Node 83670 (It is marked in the first figure in Fig. 8). On the back face, however, there are a fraction of nodes (7833) where the failure probabilities are larger than 0.010, as shown in the green and red regions in Fig. 9. The worst cases occur at 2379 out of 7833 nodes with the failure probabilities around 0.015, as shown in the red regions in Fig. 9. The maximum failure probability is 0.019 at Node 8759 (marked in first figure in Fig. 9). Also, Fig .7 presents the heating history and temperature history of the deterministic design at Node 4310. It should be mentioned that the maximum temperature (at point P) at the backface of the material (RCC) is 1578K, which is very close to the allowable temperature limit of the backface (1585K). It also explains why a high risk happens at the backface at Node 8759 while considering the defined uncertainty inputs. As it is pointed out in Example 1, a redesign is needed to be taken into consideration at these nodes within the whole red region in the subsequent detailed design.

It is obvious that the proposed methodology is much more efficient than MC since only 5786 samples were required for the evaluation of all the failure probabilities. Table 6 gives the statistical performance based on 30 independent runs of the proposed methodology and a MC simulation for the spacecraft model at five randomly selected nodes in each region. Again, the unit COV shows that that the proposed method is more efficient than MC.

According to the results of probabilistic analysis, failures most likely occur in the nose, the leading edges of the wings and the empennages. This observation is consistent with the engineering experience that the TPS materials in these regions usually bear the severest heat loads. As we know, a system reliability only reflects the most severe failure while the probability of each component can quantify every detailed failure of the whole system. The proposed probabilistic analysis methodology provides the failure information of every single node, rather than a single failure probability of the whole system. It will benefit the redesign processes in the conceptual design loop and detailed design stage.

Conclusions

A probabilistic analysis methodology is proposed for the thermal protection system. The current study focuses on the conceptual design stage. In the proposed methodology, multi-inputs and multi-outputs support vector machines are utilized to approximate the thermal responses for failure modes at all nodes. The Generalized Subset Simulation is used for the probabilistic analysis on the non-ablative thermal protection system. Two application examples including a lifting body vehicle model and a spacecraft model have been used to demonstrate the performance of the proposed method. It has been tested that MIMO-SVM is accurate enough for engineering design and can dramatically reduce the computational burden. Estimating all the failure probabilities of the failure modes of TPS with a single run of GSS is significantly more efficient than direct Monte Carlo, as evidenced from the lower value of unit COVs. The proposed methodology has provided an alternative probabilistic analysis procedure for TPS conceptual design.

Based on the observation of large failure probabilities for the two examples in conceptual design stage, redesigns can subsequently be taken into considerations in the detailed design stage. It will be helpful to combine reliability consideration together with design optimization, i.e., using reliability based design optimization techniques. The surrogate models adopted in this paper were trained with a fixed number of samples. Future work will involve adaptively updating strategy for constructing surrogate model to improve their confidence.

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Nomenclature

T = temperature

k = thermal conductivity

c_p = the specific heat

ε = emissivity

ρ = density

q = heat flux

m = number of layers in the thermal protection system stack-up

Lt = thickness of the thermal protection system stack-up

t = thickness of the thermal protection system layer (Only one layer in this paper)

$T_{i,\text{top}}^{\max}$ = max temperature of the top surface of the *i*-th material.

$T_{i,\text{back}}^{\max}$ = max temperature of the back face of the *i*-th material.

T_{back} = allowable temperature of the cold structure surface of the spacecraft

COV = coefficient of variance

Pfs = failure probability at each node on the top surface of thermal protection system materials

Pfb = failure probability at each node on the back face of thermal protection system materials

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Figure Captions list

Fig. 1 TPS material distribution of the lifting body vehicle

Fig. 2 Flowchart of the whole procedure of the TPS uncertainty analysis

Fig. 3 Heating history and Temperature history of deterministic design at Node 4031 of the lifting body vehicle

Fig. 4 Failure probabilities on the top surface of TPS layer for the lifting body vehicle

Fig. 5 Failure probabilities on the back face of TPS layer for the lifting body vehicle

Fig. 6 TPS material distribution of the spacecraft

Fig. 7 Heating history and Temperature history of deterministic design at Node 8759 of the spacecraft

Fig. 8 Failure probabilities on the top surface of TPS layer for the spacecraft

Fig. 9 Failure probabilities on the back face of TPS layer for the spacecraft

Table 1 Material properties

Material	Allowable temperature (K)	ρ (kg/m^3)	k (W/m-K)	c_p (kJ/kg-K)	ε
Densified Nomex Felt	717	86.508	0.01488	1.32	0.8
Li-900	1497	144.18	0.07	0.708	0.8
RCC	1900	1580	4.3	0.77	0.79

Table 2 Variation of the input random variables

Parameters	Nominal value	Distributional parameters	Distribution
q_i (i=1,…,24)	Input value from aeroheating analysis	10%(COV)	Normal
t	Deterministic optimum	10%(COV)	Normal
$T_{\text{top}}^{\text{limit}}$ in RCC	μ=1900K	σ=40K/6	Truncated Normal
$T_{\text{back}}^{\text{limit}}$ in RCC	μ=1585K	σ=40K/6	Truncated Normal
$T_{\text{top}}^{\text{limit}}$ in HRSI	μ=1497K	σ=40K/6	Truncated Normal
$T_{\text{back}}^{\text{limit}}$ in HRSI	μ=1250K	σ=40K/6	Truncated Normal
$T_{\text{top}}^{\text{limit}}$ in FRSI	μ=717K	σ=40K/6	Truncated Normal
$T_{\text{back}}^{\text{limit}}$ in FRSI	μ=550K	σ=40K/6	Truncated Normal

Table 3 Relative errors of the surrogate models for the lifting body vehicle

Node number		g_1			g_2		
		Original model (K)	Surrogates (K)	Relative error (%)	Original model (K)	Surrogates (K)	Relative error (%)
Region 1	1464	387.2	457.1	18.03	300.0	430.4	43.48
	1907	1079.9	1060.9	1.76	1067.0	1034.8	3.01
	2606	467.9	491.4	5.02	522.2	521.2	0.21
	3014	558.7	555.7	0.53	585.3	576.3	1.54
	5072	570.1	560.2	1.73	586.6	576.1	1.79
Region 2	2222	1242.1	1242.1	0.00	1248.4	1248.4	0.00
	3184	1501.6	1501.5	0.01	1445.8	1445.9	0.00
	4403	1496.4	1497.0	0.04	1467.3	1467.4	0.01
	5931	508.5	525.1	3.26	472.2	497.9	5.45
	7197	590.3	580.5	1.65	583.6	581.5	0.35
Region 3	3977	1699.6	1701.3	0.10	1635.3	1635.8	0.03
	4064	1537.6	1539.2	0.10	1554.3	1552.6	0.11
	4310	1839.1	1859.9	1.76	1925.8	1862.1	3.31
	7905	1563.5	1563.0	0.03	1541.9	1541.4	0.04
	8469	1569.3	1569.6	0.02	1573.7	1574.6	0.06

Table 4 Statistical Performance for the lifting body vehicle at some selected nodes

Nodes number		g_1				g_2			
		P_{fs}(surface)		unit COV		P_{fb} (back face)		unit COV	
		GSS	MC	GSS	MC	GSS	MC	GSS	MC
Region 1	1464	4.39×10^{-6}	0	17.8	-	4.39×10^{-6}	0	17.8	-
	1907	4.39×10^{-6}	0	17.8	-	4.39×10^{-6}	0	17.8	-
	2606	4.39×10^{-6}	0	17.8	-	4.39×10^{-6}	0	17.8	-
	3014	4.39×10^{-6}	0	17.8	-	3.73×10^{-7}	0	21.0	-
	5072	4.39×10^{-6}	0	17.8	-	1.24×10^{-7}	0	16.5	-
Region 2	2222	4.39×10^{-6}	0	17.8	-	4.39×10^{-6}	0	17.8	-
	3184	4.39×10^{-6}	0	17.8	-	4.39×10^{-6}	0	17.8	-
	4403	4.39×10^{-6}	0	17.8	-	3.41×10^{-4}	7.39×10^{-4}	10.7	36.8
	5931	4.39×10^{-6}	0	17.8	-	4.39×10^{-6}	0	17.8	-
	7197	4.39×10^{-6}	0	17.8	-	4.39×10^{-6}	0	17.8	-
Region 3	3977	4.39×10^{-6}	0	17.8	-	4.39×10^{-6}	0	17.8	-
	4064	4.39×10^{-6}	0	17.8	-	4.39×10^{-6}	0	17.8	-
	4310	4.39×10^{-6}	0	17.8	-	1.85×10^{-4}	4.33×10^{-4}	9.4	48.0
	7905	4.39×10^{-6}	0	17.8	-	2.54×10^{-5}	1.03×10^{-5}	15.4	311.6
	8469	1.85×10^{-4}	3.31×10^{-4}	11.6	55.0	1.40×10^{-5}	0	8.26	-

Table 5 Relative errors of the surrogate models for the spacecraft

Nodes number		g_1			g_2		
		Original model (K)	Surrogate model (K)	Relative error (%)	Original model (K)	Surrogate model (K)	Relative error (%)
Region 1	3217	1557.2	1582.1	1.59	1537.0	1577.0	2.60
	4244	1852.8	1846.8	0.33	1813.4	1815.7	0.13
	10907	1284.9	1285.1	0.02	1471.9	1471.3	0.04
	22157	652.5	652.5	0.00	657.1	656.1	0.14
	28672	595.5	597.8	0.38	599.0	600.7	0.29
Region 2	5560	1116.8	1118.5	0. 15	1105.5	1107.2	0.16
	7335	1349.0	1348.5	0.04	1364.2	1364.3	0.00
	18853	737.6	739.3	0.24	734.6	736.2	0.22
	38299	1154.3	1154.4	0.01	1154.6	1154.7	0.00
	49561	582.7	585.8	0.54	584.7	586.6	0.33
Region 3	438	1552.7	1551.4	0.08	1556.1	1554.5	0.10
	578	1585.5	1584.8	0.04	1594.1	1595.7	0.10
	1486	1539.5	1544.6	0.33	1735.5	1734.7	0.05
	3018	1842.7	1841.7	0.05	1854.6	1853.4	0.06
	3905	2038.2	2035.6	0.12	2195.4	1976.0	9.99

Table 6 Statistical performance for the spacecraft at some nodes

Nodes number		g_1				g_2			
		P_{fs}(surface)		unit COV		P_{fb}(back face)		unit COV	
		GSS	MC	GSS	MC	GSS	MC	GSS	MC
Region 1	3217	3.11×10^{-3}	3.71×10^{-3}	11.2	16.4	2.75×10^{-6}	0	29.2	-
	4244	8.13×10^{-5}	0	23.8	-	2.56×10^{-3}	4.12×10^{-3}	8.5	15.5
	10907	6.70×10^{-3}	6.33×10^{-3}	9.4	12.5	0.018	0.018	5.4	7.4
	22157	2.75×10^{-6}	0	29.2	-	2.75×10^{-6}	0	29.2	-
	28672	2.75×10^{-6}	0	29.2	-	2.75×10^{-6}	0	29.2	-
Region 2	5560	2.32×10^{-3}	1.50×10^{-3}	13.8	25.8	2.75×10^{-6}	0	29.2	-
	7335	2.75×10^{-6}	0	29.2	-	2.75×10^{-6}	0	29.2	-
	18853	2.54×10^{-5}	7.78×10^{-5}	27.2	113	5.38×10^{-5}	7.14×10^{-5}	16.4	118
	38299	2.75×10^{-6}	0	29.2	-	2.75×10^{-6}	0	29.2	-
	49561	2.75×10^{-6}	0	29.2	-	2.75×10^{-6}	0	29.2	-
Region 3	438	1.07×10^{-3}	2.25×10^{-3}	9.6	21.1	3.46×10^{-4}	6.37×10^{-4}	16.4	39.6
	578	3.12×10^{-3}	4.37×10^{-3}	8.5	15.1	5.61×10^{-3}	7.38×10^{-3}	16.4	24.6
	1486	2.75×10^{-6}	0	29.2	-	2.75×10^{-6}	0	29.2	-
	3018	2.35×10^{-3}	2.25×10^{-3}	11.6	21.1	2.75×10^{-6}	0	29.2	-
	3905	2.75×10^{-6}	0	29.2	-	2.75×10^{-6}	0	29.2	-