

# Limbus misrepresentation in parametric eye models

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Short Title: Limbus misrepresentation in eye models

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## Abstract

**Purpose:** To assess the axial, radial and tangential limbus position misrepresentation when parametric models are used to represent the cornea and the sclera.

**Methods:** This retrospective study included 135 subjects aged 22 to 65 years (36.5 mean  $\pm$ 9.8 STD), 71 females and 64 males. Topography measurements were taken using an Eye Surface Profiler topographer and processed by a custom-built MATLAB code. Eye surfaces were freed from edge-effect artefacts and fitted to spherical, conic and biconic models.

**Results:** When comparing the radial position of the limbus, average errors of  $-0.83\pm 0.19\text{mm}$ ,  $-0.76\pm 0.20\text{mm}$  and  $-0.69\pm 0.20\text{mm}$  were observed within the right eye population for the spherical, conic and biconic models fitted up to 5mm. For the same fitting radius, the average fitting errors were  $-0.86\pm 0.23\text{mm}$ ,  $-0.78\pm 0.23\text{mm}$  and  $-0.73\pm 0.23\text{mm}$  for the spherical, conic and biconic models respectively within the left eye population. For the whole cornea fit, the average errors were  $-0.27\pm 0.12\text{mm}$  and  $-0.28\pm 0.13\text{mm}$  for the spherical models,  $0.02\pm 0.29\text{mm}$  and  $-0.05\pm 0.27\text{mm}$  for the conic models, and  $-0.22\pm 0.16\text{mm}$  and  $0.24\pm 0.17\text{mm}$  for the biconic models in the right and left eye populations respectively.

**Conclusions:** Through the use of spherical, conic and biconic parametric modelling methods, the eye's limbus is being mislocated. Additionally, it is evident that the magnitude of fitting error associated with the sclera may be propagating through the other components of the eye. This suggests that a corneal nonparametric model may be necessary to improve the representation of the limbus.

48 **Introduction**

49 The tunic of the human eyeball consists of two main components, the cornea and the sclera, that meet each  
50 other at the limbus. [1] The cornea provides nearly 73 % of the eye's refractive power, [2, 3] however, the  
51 sclera is the main load-bearing structure of the eye and forms about 85% of the surface of the eye globe. [4]  
52 Although the limbus, the Latin word for 'border', is usually known as the corneal boundary where the cornea  
53 and the sclera are connected, it has been defined in several different ways according to the specific  
54 application. It can be defined as the junction between the cornea and the sclera from a microscopic point of  
55 view, [5] the border between the opaque sclera and the transparent cornea in terms of transparency, [6] and,  
56 when considering eye topography, it is the contour where there is a change in the curvature at the junction  
57 of eye's two main structures. [7]

58 Clinically, the location of the limbus is important in the field of customisable soft contact lens fitting, as it is  
59 used when estimating the lens's overall diameter. It is also essential in mini scleral and scleral contact lens  
60 fittings where avoidance of contact with limbal stem cells is crucial. Clinical measurement of the limbus's  
61 location is often inaccurate and contact lens fitters tend to use the horizontal visible iris diameter (HVID) as  
62 an approximation. When used for modelling purposes, this approximation can yield significantly different  
63 geometries, thus affecting accuracy. This is primarily due to the fact that the white-to-white distance in the  
64 human eye is significantly greater in the nasal-temporal direction than in the superior-inferior direction. The  
65 human limbus diameter is not thought to vary significantly in these directions. [8] As most of the non-  
66 disposables soft contact lenses are being fitted through empirical fitting rules that depend highly on the  
67 limbus's position, the true limbus diameter would be far more informative than the HVID. Additionally, it is  
68 also an essential parameter in the design and manufacture of single diameter disposable lens.

69 Many techniques have been developed for the parametric modelling of the eye. [9] The first recognised  
70 technique for modelling the corneal surface was fairly simplistic. This approach involved computing the  
71 Cartesian coordinates (x,y,z) of the corneal surface as if it were a perfect sphere with radius  $R$ , Eq 1. It was  
72 soon realised that spherical models were predicting higher spherical aberration and oblique astigmatism than  
73 were experimentally measured. [10, 11]

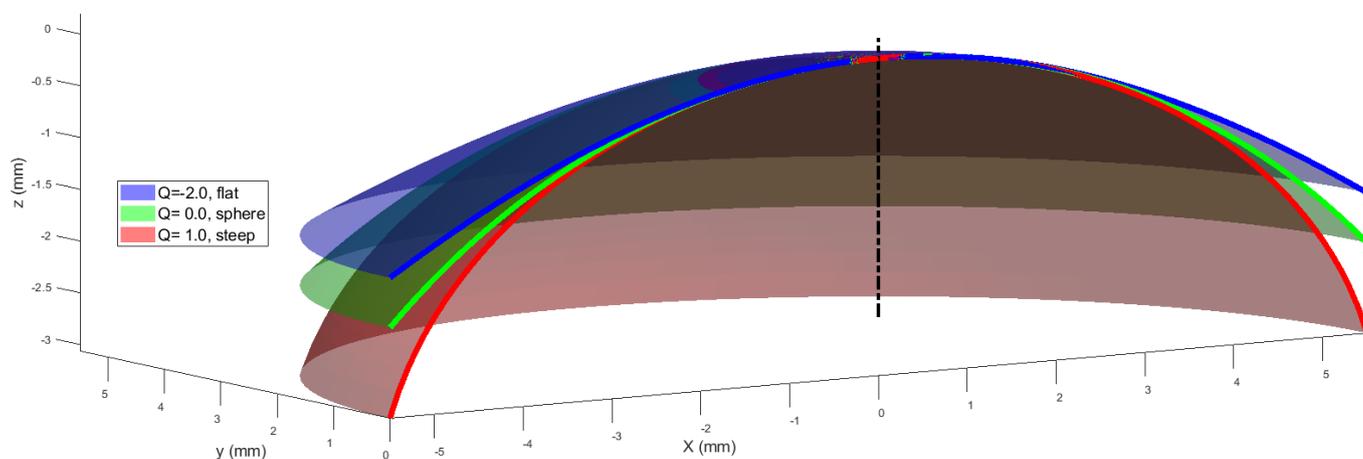
$$z = \sqrt{R^2 - (x^2 + y^2)} - R \qquad \text{Eq 1}$$

74 In an attempt to eliminate the spherical aberration, a conic surface model was introduced, Eq 2. This model  
 75 extended the abilities of the spherical equation by adding an asphericity coefficient  $Q$  to control the curvature  
 76 of the corneal surface. When the asphericity  $Q$  is set to zero, Eq 2 is reduced to Eq 1 and the fitted corneal  
 77 surface becomes spherical.

$$z = \frac{\sqrt{R^2 - (x^2 + y^2)(Q + 1)} - R}{Q + 1} \quad \text{Eq 2}$$

78 The asphericity factor  $Q$  is synonymous to the overall curvature of the cornea, with positive values leading to  
 79 increased steepness and negative values inducing a flattening effect, Fig 1.

80



81

82 Fig 1: Corneal surface models generated for differing values of asphericity.

83 Both the spherical and conical models assume complete rotational symmetry and ignore the effects of  
 84 astigmatism. Astigmatism occurs as a result of a phenomenon known as toricity, whereby the curvature of  
 85 the nasal-temporal meridian is flatter than the superior-inferior corneal meridian. [12-14] A revision of the  
 86 conic model led to the inclusion of toricity to form what is known as a biconic surface. [15] The mathematical  
 87 expression that defines this model is provided in Eq 3. This model bifurcated the radius of corneal curvature  
 88 into two separate values  $R_x$  and  $R_y$  representing corneal radii in the two principal directions; nasal-temporal  
 89 (N-T) and superior-inferior (S-I) respectively. Two asphericity coefficients  $Q_x$  and  $Q_y$  were also included in an  
 90 attempt to control the steepness of the cornea along with X-axis (N-T) and Y-axis (S-I) directions  
 91 independently.

$$z = \frac{-\left(\frac{x^2}{R_x} + \frac{y^2}{R_y}\right)}{1 + \sqrt{1 + (1 + Q_x)R_x^2 x^2 - (1 + Q_y)R_y^2 y^2}} \quad \text{Eq 3}$$

92 Additionally, a biconic eye model is often rotated around its nominal axis of rotational symmetry (Z-axis) to  
 93 account for the axis of the cylindrical power of astigmatism. Rotation about the two principal directions (X and  
 94 Y) can also be utilised to compensate for the difference between the visual and optical axis when measuring  
 95 corneal topography. [9, 16] Due to the large number of variables included in the biconic model, its equation  
 96 normally allows for a higher degree of surface control compared to the conic model (Eq 3).

97 Corneal conic asphericity has been reported in varied ranges down to -0.82 [17-34] (Table 1), however,  
 98 biconic asphericity was reported down to -0.28 [34, 35] depending on the algorithm used and the sample  
 99 size. For example, Ying provided corneal radius of 7.83mm [12] through a conic model with varying  
 100 asphericity Q=-0.18:-0.3 because of astigmatism skewed distribution, [36] however, when an alternative  
 101 method was introduced using corneal tangential radius of curvature instead of sagittal radius, Q was reported  
 102 in a range of -0.33 to 0.12 and varied between corneal principal meridians. [37]

103 Values of asphericity presented in Table 1 were only validated against the central corneal zone as measuring  
 104 the peripheral corneal zone and the anterior sclera simultaneously has not been possible until recently. The  
 105 use of an eye topographer to characterise the corneoscleral shape in vivo was not possible until the past few  
 106 years as most of the topographers were not able to measure the area of the eye that covers the limbus and  
 107 part of the sclera. [38] The situation has changed recently, and some newly developed topographers are able  
 108 to do this by capturing the cornea and the exposed portion of the sclera, such as the Pentacam Cornea  
 109 Scleral Profile (CSP) optional software and the Eye Surface Profiler (ESP). The ESP can cover up to a 20mm  
 110 diameter of the eye. [39] This development in the instrumentation capabilities motivated the authors to  
 111 investigate and parametrically characterise the corneoscleral zone using these recently developed  
 112 topographers that can provide the anterior eye surface up to a few millimetres beyond the limbus. [8] On one  
 113 hand, as most of the corneal topographers are covering up to 5mm of the corneal surface only, this study  
 114 looked at the difference between fitting an eye parametric model to the 5mm central corneal radius only and  
 115 fitting it to the whole corneal surface. On the other hand, as most of the eye's parametric models were using  
 116 parameters that were fitted to the central cornea, using a modern ESP surface profiler, this study is

117 investigating how these limitations in the longstanding parametric techniques misrepresented the limbus  
 118 diameter, position and how this misrepresentation can be compensated for.

119

120 Table 1: Summary of the anterior corneal radius and asphericity as reported in the literature

| Author             | Year | Number of eyes                              | Model   | Radius (mm)  | Asphericity  |
|--------------------|------|---|---------|--|--|
| Mainstone [17]     | 1998 | 35  | conic   | R= 7.36:8.54   | Q= -0.332±0.297  |
| Holladay [18]      | 1999 | 14  | conic   | Not reported   | Q= -0.16±0.12  |
| Douthwaite [19]    | 1999 | 98  | conic   | R= 7.78:7.93   | Q= 0.13:0.5  |
| Holmes-Higgin [20] | 1999 | 25  | conic   | R= 7.5:9.0   | Q= -0.01:-1.44   |
| Budak [21]         | 1999 | 287   | conic   | R= 6.7:9.4   | Q= -0.03±0.23  |
| Dubbelman [22]     | 2002 | 83  | conic   | R= 7.87±0.27   | Q= -0.82±0.18  |
| Langenbucher [40]  | 2002 | 50<br>30 keratoconic<br>20 fuchs' dystrophy | biconic | not reported   | Q <sub>x</sub> = -0.42±0.21<br>Q <sub>y</sub> = -0.53±0.13 |
| Cuesta [23]        | 2003 | 92  | conic   | not reported   | Q= -0.26±0.18  |
| Hersh[41]          | 2003 | 11  | conic   | R= 6.50:6.75   | Q= -0.17±0.14  |
| Manns [24]         | 2004 | 24  | conic   | R= 10.15±1.39  | Q= -0.64±1.85  |
| Somani [25]        | 2004 | 278   | conic   | not reported   | Q= -0.60:0.0   |
| Llorente [26]      | 2004 | 46<br>24 Myopic<br>22 Hyperopic             | conic   | R= 7.86±0.37<br>R= 7.97±0.30                             | Q= -0.10±0.23<br>Q= -0.20±0.17                             |
| Davis [27]         | 2005 | 643<br>Myopes<br>Emmeropes<br>Hyperopes     | conic   | R= 7.53±0.27<br>R= 7.54±0.24<br>R= 7.57±0.23             | Q= -0.32±0.10<br>Q= -0.35±0.10<br>Q= -0.36±0.10            |
| Priest [42]        | 2005 | 151   | biconic | R <sub>x</sub> = 7.76±0.24<br>R <sub>y</sub> = 7.60±0.28 | Q <sub>x</sub> = -0.24±0.31<br>Q <sub>y</sub> = -0.11±0.32 |
| Dubbelman [28]     | 2006 | 114   | conic   | R= 7.79±0.025  | Q= -0.41±0.26  |
| Mastropasqua [43]  | 2006 | 74  | biconic | R <sub>x</sub> = 6.87±0.26<br>R <sub>y</sub> = 6.56±0.36 | Q <sub>x</sub> = -0.18±0.12<br>Q <sub>y</sub> = -0.21±0.13 |
| Navarro [44]       | 2006 | 123   | biconic | R <sub>x</sub> = 7.63<br>R <sub>y</sub> = 7.40           | Q <sub>x</sub> = -0.465<br>Q <sub>y</sub> = -0.481         |

|                       |      |   |         |  |  |
|-----------------------|------|---|---------|--|--|
| González-Méijome [29] | 2007 | 36  | conic   | R= 7.8   | Q = -0.39±0.11   |
| Nieto-Bona [30]       | 2009 | 118<br>30 Emmetropic<br>56 Myopic<br>32 Hyperopic | conic   | R= 7.72±0.05<br>R= 7.40±0.03<br>R= 7.80±0.04   | Q= -0.32±0.03<br>Q= -0.36±0.02<br>Q= -0.37±0.03  |
| Piñero [31]           | 2010 | 71  | conic   | R= 7.89±0.31   | Q= -0.29±0.09  |
| Bottos [32]           | 2011 | 209   | conic   | not reported   | Q= -0.27±0.12  |
| Zhang [33]            | 2011 | 1052  | conic   | R= 7.80±0.25   | Q= -0.30±0.12  |
| Ortiz [45]            | 2012 | 3   | biconic | R <sub>x</sub> = 7.40 ± 0.07<br>R <sub>y</sub> = 7.53 ± 0.03   | Q <sub>x</sub> = 0.10±0.01<br>Q <sub>y</sub> = -0.19±0.05  |
| Navarro [35]          | 2013 | 407   | biconic | R <sub>x</sub> = 7.01:8.44<br>R <sub>y</sub> = 6.85:8.31   | Q <sub>x</sub> = -0.77:-0.10<br>Q <sub>y</sub> = -0.81:-0.09   |
| Bao [34]              | 2013 | 684<br>342 Right<br>342 Left                      | biconic | R <sub>x</sub> = 7.81±0.25<br>R <sub>y</sub> = 7.62±0.28<br>R <sub>x</sub> = 7.81±0.26<br>R <sub>y</sub> = 7.62±0.27 | Q <sub>x</sub> = -0.27±0.09<br>Q <sub>y</sub> = -0.28±0.12<br>Q <sub>x</sub> = -0.27±0.08<br>Q <sub>y</sub> = -0.27±0.12 |

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## 123 Materials and Methodology

### 124 Participants

125 Although no participant had been recruited specially for this study, this record review study was conducted  
126 according to the tenets of the Declaration of Helsinki and was approved by the IRB (Institutional Review  
127 Board) and Human Ethics Committee of the Federal University of São Paulo (UNIFESP, SP, Brazil). The  
128 study utilised a collection of secondary data where healthy eyes were selected to be processed. According  
129 to the University of Liverpool's Policy on Research Ethics, ethical approval was unnecessary for secondary  
130 analysis of fully anonymised data. The study included 125 subjects aged 22 to 65 years (36.5 mean  $\pm$ 9.8  
131 STD), 66 females and 59 males. Participants suffering from ocular diseases or having a history of trauma or  
132 ocular surgery were excluded. The data were collected and anonymised at Brighten Optix Corporation in  
133 Taipei, Taiwan where participants (most of them soft contact lenses wearers) were told not to wear soft  
134 contact lens for two weeks before the topography measurement, and those who were wearing rigid gas-  
135 permeable (RGP) contact lens were asked not to wear them for four weeks before the scan.

136 The ESP measurement technique involves using Moire fringes reflected from the surface of the tear film.  
137 Fixation was established by asking the subject to observe the red-cross target on the faceplate of the  
138 instrument while this was viewed by the clinician on the computer monitor. The alignment was achieved by  
139 identifying the centre point of two corneal images of lights originating from the instrument. The red-cross was  
140 then aligned with this central point and a reading initiated. Once this had been done, the subject was directed  
141 to sit back and one unpreserved lubricating drop (Lubrisitol, 1mg/mL sodium hyaluronate) was instilled into  
142 the lower fornix. This was followed by the application of fluorescein in the upper and lower fornix to maximise  
143 coverage. The subject was directed to blink a couple of times, and the level of coverage was then checked  
144 visually before proceeding further. The subject was instructed to open their eyelids as wide as possible while  
145 a measurement was being taken. This was to ensure good data coverage beyond the limbal zone. The  
146 measurement was then repeated twice and the best scan, in terms of coverage range and quality as assessed  
147 by the ESP software, was used. Participants' eye shape clinical parameters are reported in Table 2 as  
148 measured by the ESP built-in interface (see S1 for full details).

150 Table 2: Participants' eye shape clinical parameters as measured by the ESP

|                 | Corneal shape parameters (M $\pm$ STD) |                       |                  |                  | Simulated keratometry (Sim-K) parameters (M $\pm$ STD) |                  |                  |                   |
|-----------------|--|-----------------------|------------------|------------------|--|------------------|------------------|-------------------|
|                 | HVID (mm)                              | Astigmatism (Dioptre) | Axis (Degree)    | Sphere (Dioptre) | Astigmatism (Dioptre)                                  | Axis (Degree)    | Flat Radius (mm) | Steep Radius (mm) |
| Right eyes (OD) | 11.9 $\pm$ 0.4                         | -1.6 $\pm$ 0.7        | 99.5 $\pm$ 43.2  | 43.1 $\pm$ 1.7   | -2.5 $\pm$ 1.1   | 100.4 $\pm$ 43.5 | 8.3 $\pm$ 0.4    | 7.9 $\pm$ 0.3     |
| Left eyes (OS)  | 11.9 $\pm$ 0.4                         | -1.9 $\pm$ 0.8        | 100.8 $\pm$ 42.5 | 43.1 $\pm$ 1.7   | -2.9 $\pm$ 1.2   | 95.9 $\pm$ 44.1  | 8.4 $\pm$ 0.4    | 7.9 $\pm$ 0.4     |

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## Data collection and processing

153 The data was exported from the ESP software in MATLAB (MathWorks, Natick, USA) binary data container  
154 format (\*.mat). The eye surface data was processed by custom-built MATLAB codes entirely independent  
155 from the built-in ESP software digital signal processing (DSP) algorithms. Firstly, each eye surface was freed  
156 of edge-effect artefacts following Abass's three-dimensional non-parametric method [46], Fig 2a,b. Three-  
157 dimensional surfaces were fitted to the ESP topographical data using the spherical, conic and biconic  
158 techniques outlined in the introduction. In each case, the sclera was modelled as a sphere with its radius  
159 calculated as the best fit radius by the least-squares method. [47] The spherical corneal models were  
160 generated by deducing the radius and the centre that was necessary to fit the ESP data to a perfect sphere,  
161 and then using this value to calculate the modelled surface.

162 The fitting of the conic and biconic surfaces introduced an added complexity. In order to gain values of the  
163 required variables, the Levenberg-Marquardt nonlinear least-squares algorithm (LMA) was utilised. [48] LMA  
164 is a robust technique for solving nonlinear curve fitting problems by searching for a solution that minimises  
165 the fitting error. [49] The algorithm involves defining an objective function based on either the conic or biconic  
166 models and then deducing the values of asphericity and corneal radius that minimise the root mean square  
167 (RMS) error between the eye surface and the fitted surfaces, [50] Fig 2c,d. As the algorithm requires an initial  
168 guess of the parameters, the initial estimates of the corneal radius of curvature and asphericity used in the  
169 conic fitting process were 7.8mm and 0.0 respectively. [51] The biconic model required initial values of radii  
170 of curvature in the two principal directions  $R_x$ ,  $R_y$  and the angle of rotation about the Z-axis ( $\theta_z$ ) that is required

171 to account for astigmatism and the asphericity in the two principal directions. The two radii were given initial  
 172 guess values of 7.8mm and all other variables were initially set to zero.

173 Following the calculation of the aforementioned variables, the surfaces were generated using the spherical,  
 174 conic and biconic surface definitions. An additional rotation about the Z-axis was included for the biconic  
 175 surfaces and achieved by multiplying the Cartesian coordinates, for each point on the surface, by the matrix  
 176 for rotation about the Z-axis [52]. For a rotation angle of  $\theta_z$ , this matrix is given by:

$$W_z(\theta_z) = \begin{bmatrix} \cos \theta_z & -\sin \theta_z & 0 \\ \sin \theta_z & \cos \theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Eq 4}$$

177 Following the elemental rotation rule, the rotated coordinates of the corneal surface  $X_r$ ,  $Y_r$  and  $Z_r$  were  
 178 calculated as

$$\begin{bmatrix} x_{r1} & x_{r2} & x_{r3} & \dots & x_{rn} \\ y_{r1} & y_{r2} & y_{r3} & \dots & y_{rn} \\ z_{r1} & z_{r2} & z_{r3} & \dots & z_{rn} \end{bmatrix} = W_z(\theta_z) * \begin{bmatrix} x_1 & x_2 & x_3 & \dots & x_n \\ y_1 & y_2 & y_3 & \dots & y_n \\ z_1 & z_2 & z_3 & \dots & z_n \end{bmatrix} \quad \text{Eq 5}$$

179 Before moving to the next processing stage, the origin position of each levelled eye's surface was shifted to  
 180 the highest point of the limbus-levelled eye surface (apex). The process of fitting the sclera to the spherical  
 181 surface was accompanied by the calculation of the scleral radius and centre offset, relative to the corneal  
 182 apex, Fig 2e. Each of the surfaces was plotted and compared directly to the surface outputted by the ESP  
 183 through different geometrical observations. These observations were the radius of curvature, the positioning  
 184 of both the detected limbus and the fitted one, Fig 2a,f, and the angle that the corneal surface makes at the  
 185 limbus ( $\alpha$ ), Fig 3. The radii of curvature values were fairly simple to compute through the coordinates that  
 186 define the surface, however, deducing the geometrical properties of the limbus required an alternative  
 187 approach. To compute the limbal tangent angle  $\alpha$ , Fig 3, the tangent of each measured eye surface was  
 188 computed, for each meridian through the first derivative of the surface's z-values in the polar coordinate  
 189 system ( $z_p$ ), with respect to the instantaneous radius ( $r$ ) at the same meridian around  $360^\circ$  (Eq.5). The tangent  
 190 angle at the limbus position was then determined by applying a piecewise Cubic Hermite interpolating  
 191 polynomial at the limbal position of each meridian.

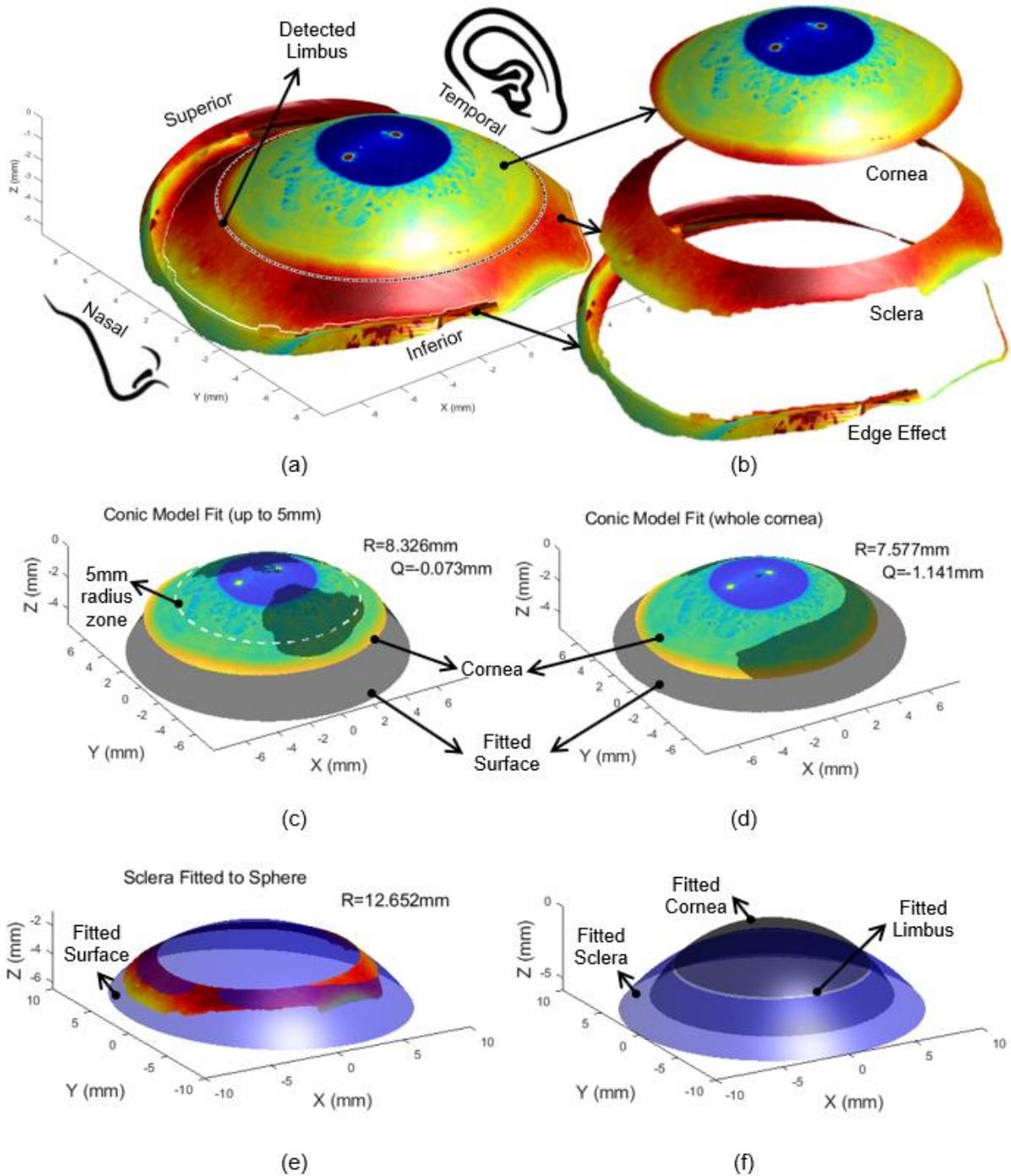
$$\alpha_t = \tan^{-1} \frac{dz_p}{dr} \quad \text{Eq.5}$$

192 The position of the detected limbus on the corneal surface was identified through the use of a three-  
 193 dimensional non-parametric method for limbus detection introduced by Abass et al. in 2018 [8]. Briefly, the

194 algorithm is based on the fact that the cornea and the sclera have different curvatures [53] and the limbus is  
195 the boundary where the corneal curvature changes to match that of the sclera. [54] Therefore, the position  
196 of the limbus was detected by locating the turning point of the raw elevation 2<sup>nd</sup> derivative at each meridian.  
197 This limbal position is referred to as the detected limbus position in this manuscript. The intersection of the  
198 fitted corneal and scleral surfaces did not necessarily occur in the same radial or axial position as the positions  
199 measured experimentally. The difference between the fitted and actual limbal positions was calculated and  
200 presented in this study.

201 For each of the geometrical properties, the associated fitting errors were calculated by taking the actual value  
202 and subtracting it from the value present in the fitted model. Using this approach allowed for the  
203 understanding of whether the fitting technique was over or underestimating the geometry.

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Fig 2: Left eye of a 25 year old male subject where the eye's front image was projected on the surface for displaying purpose (a) as scanned by the ESP, where the limbus was detected in three-dimensions [8] (b) after being processed to split cornea, sclera and the edge effect artefacts following Abass's method. Two edge detection strategies were used simultaneously to cut the edge of the eye's surface data at the border between the authentic eye surface and the artificial boundaries which result from interference of tears, eyelid edges or lashes [46] (c) corneal conic fit up to 5mm, (d) whole corneal conic fit, (e) sclera fitted to a sphere, (f) Fitted limbus as constructed by the intersection of the fitted surfaces representing the cornea and the sclera.

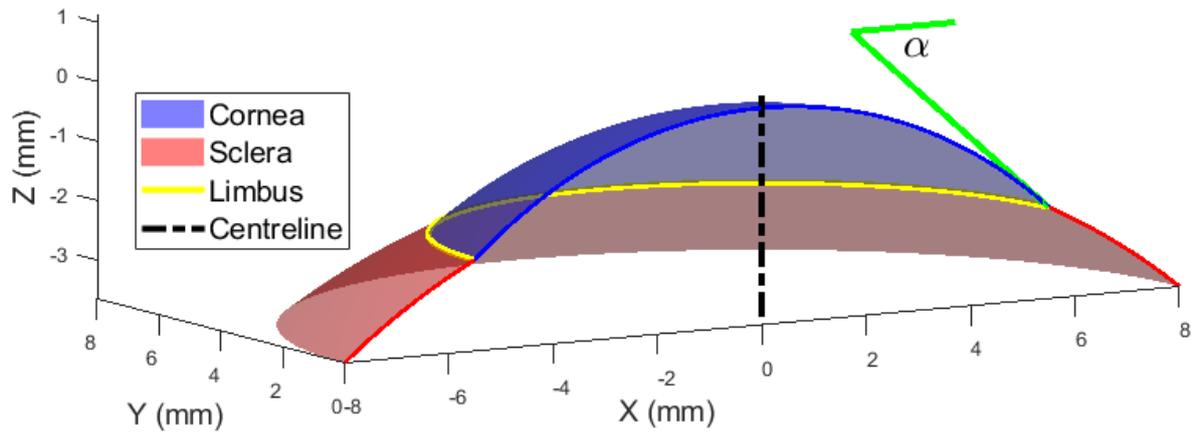


Fig 3: Graphical representation of the angle that the corneal surface makes at the point of connection to the limbus. The tangent angle at the limbus is represented by  $\alpha$ .

### Example of finite element modelling

In order to demonstrate the effect of misrepresenting the limbus area in finite element modelling, the left eye of a 25 year old male subject, shown in Fig 2, was modelled using two different techniques. The first model was built by using the geometry of the eye as measured and extracted from the ESP, however the second was constructed using the parametric values obtained by fitting the central 5mm radius of the cornea to a conic model ( $R= 8.3258\text{mm}$ ,  $Q= -0.0729$ ). In both cases, the sclera was fitted by minimising the RMS error in the axial direction to 12.652mm sphere after separating the scleral portion of the topography data, Fig 2b. Central corneal thickness (CCT) was taken as 0.55mm [55] which increased to 0.70mm and 0.56mm at the peripheral corneal zone and scleral equatorial ring respectively. [56, 57] Additionally, at the eye posterior pole the thickness was taken as 0.84mm. [58] Eight-node first-order continuum solid hybrid brick elements 'C3D8H' were used in two layers of elements to build the eye model with an aspect ratio close to one in ABAQUS (Dassault Systèmes, Vélizy-Villacoublay, France) finite element software package licenced to the University of Liverpool, UK. The in-vivo human eye globe topography is measured whilst the eye is stressed due to the intraocular pressure (IOP) hence, cannot be used for modelling without pre-processing. [59] To achieve the stress-free geometry that is needed for modelling purposes, eye globe models were initially built with the inflated dimensions, then the stress-free version of each model was calculated separately by following the iteration-based method presented in [60]. In each case the stress-free model was computed by

235 considering an IOP of 15mmHg and a maximal node position error less than  $10^{-4}$ mm. Once the stress-free  
 236 models were determined, they were loaded up to IOP=15mmHg while the equatorial nodes were bounded in  
 237 the axial direction and the Von Mises [61] stress distribution was monitored through ABAQUS colour-scale.  
 238 In order to keep the focus on the geometrical effect only, single 1<sup>st</sup> order (N=1) Ogden hyperelastic material  
 239 model was used for the whole eye globe where the material parameters were set to average values of  $\mu_1=0.07$   
 240 MPa and  $\alpha_1=110.836$  based on our previous knowledge. [62-64] The Ogden constitutive strain energy  
 241 equation can be expressed as

$$U = \sum_{i=1}^N \frac{2\mu_i}{\alpha_i^2} (\bar{\lambda}_1^{\alpha_i} + \bar{\lambda}_2^{\alpha_i} + \bar{\lambda}_3^{\alpha_i} - 3) \quad \text{Eq.6 [65]}$$

242 Where  $U$  is the strain energy;  $\mu_i$  and  $\alpha_i$ , are material parameters;  $\bar{\lambda}_i$  are the deviatoric principal stretches in  
 243 principal directions (i=1,2 and 3). Accordingly, the Von Mises stress  $\sigma_v$  can be expressed as

$$\sigma_v = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} \quad \text{Eq.7 [66]}$$

244 where  $\sigma_j$  are the principal stresses (j=1,2 and 3).

## 245 Statistical Analysis

246 The results were subjected to statistical analysis through the use of the MATLAB Statistics and Machine  
 247 Learning Toolbox. A significance level of 5% was set and the probability of the null hypothesis (p-value) was  
 248 computed using a paired-sample t-test. [67] This calculation was carried out on pairs of data sets to ensure  
 249 that the observed effects were not occurring as a result of sampling error. Due to the choice of significance  
 250 level, the observed effects were deemed significant if they achieved a p-value lower than 0.05. Additionally,  
 251 root mean square fitting errors, RMS, were computed using the z coordinates of the clinical and modelled  
 252 data as

$$RMS = \sqrt{\frac{1}{N} \sum_{i=1}^N (Z_{i_{clinical}} - Z_{i_{model}})^2} \quad \text{Eq.8}$$

253 where N is the number of points used in the RMS calculation.

## 255 Results

### 256 Corneal Radius of Curvature

257 The radii of curvature measured from spherical, conical and biconical fitted corneas are shown in Fig 4. A  
258 representation of the accuracy of these fitting techniques is also presented as the RMS error between the  
259 radii produced by fitting and the corresponding truly calculated values. For an up to 5mm fit, the spherical  
260 corneas produced RMS errors of  $0.19\pm 0.06\text{mm}$  and  $0.2\pm 0.06\text{mm}$  in the right and left eye populations  
261 respectively. As would be expected, this error is reduced considerably when the geometry is modelled using  
262 measurements from the whole cornea. This approach yields an RMS error  $0.12\pm 0.03\text{mm}$  in both the right  
263 and left population, for the spherical corneas.

264 The addition of the asphericity factor in the conic fitting procedure leads to a further reduction in the  
265 associated error. When this approach is considered, RMS readings of  $0.16\pm 0.05\text{mm}$  and  $0.06\pm 0.02\text{mm}$  were  
266 observed, for both the right and left eye populations, using an up to 5mm fit and whole cornea fit respectively.  
267 The biconic approach, with an up to 5mm fit, yielded RMS fitting errors  $0.13\pm 0.05\text{mm}$  and  $0.13\pm 0.04\text{mm}$  in  
268 the right and left eye populations respectively. When the biconic models were fitted to the whole cornea, the  
269 RMS fitting error fell to  $0.06\pm 0.02\text{mm}$  and  $0.05\pm 0.02\text{mm}$  in the right and left eye samples respectively.

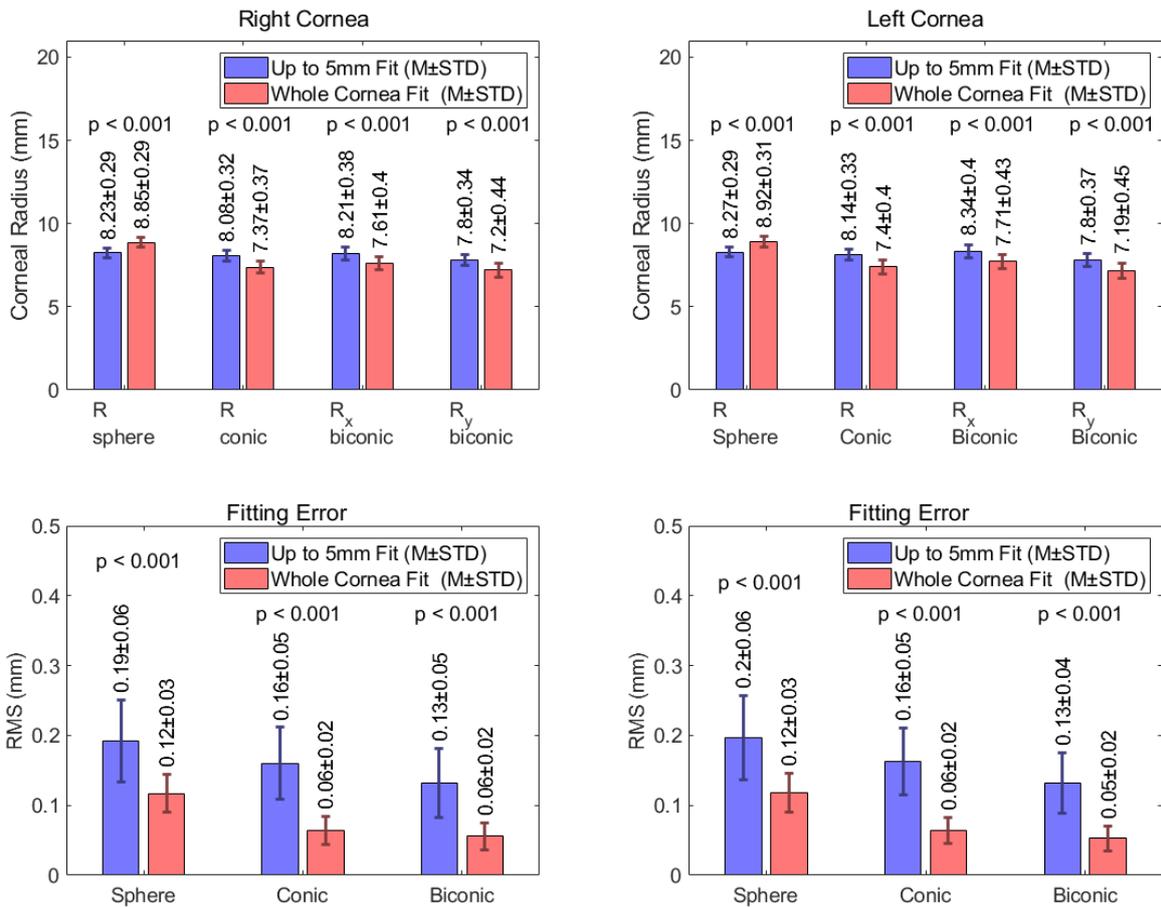
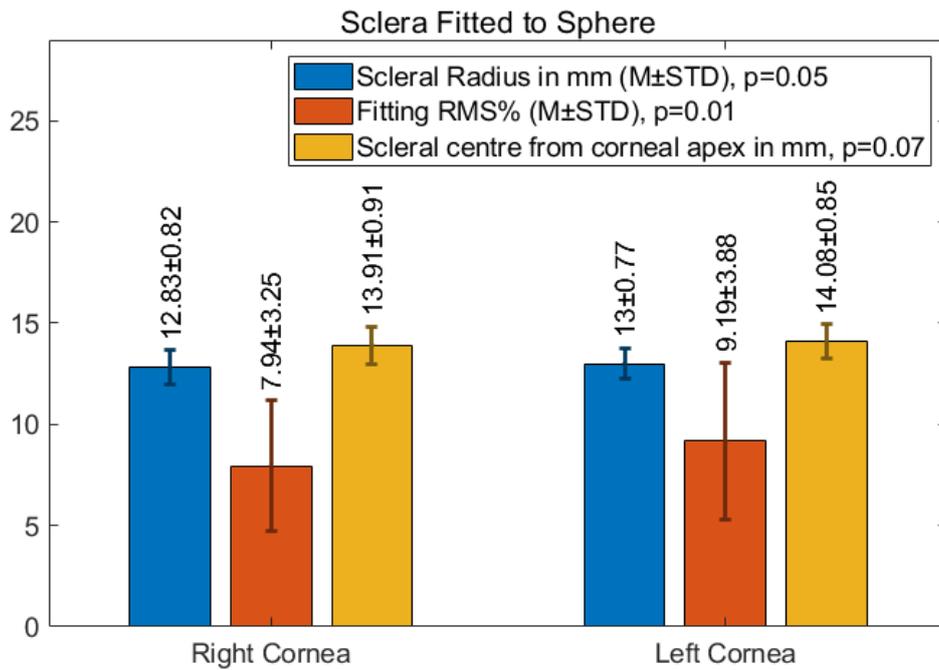


Fig 4: Corneal radius of curvature values measured from corneal models produced using spherical, conic and biconic fitting procedures. Fitting errors representing the RMS error between the values produced by the model and those obtained clinically.

## Scleral Fitting

The mean values of scleral radius and central offset and the overall RMS scleral fitting error are presented in Fig 5. The average scleral radius for the fitted surface was  $12.83 \pm 0.82$ mm and  $13.00 \pm 0.85$ mm in the right and left eye populations respectively. As would be expected, the offset of the sclera's centre with respect to the corneal apex was greater than the radius. The magnitude of the offset was  $13.91 \pm 0.91$ mm in the right eye population and  $14.08 \pm 0.85$ mm in the left. The error associated with the scleral fitting was statistically consistent across both eye ( $p=0.01$ ) and took values of  $0.0794 \pm 0.0325$  and  $0.0919 \pm 0.0388$  in the right and left eye populations respectively.



282

283 Fig 5: Mean scleral radius, distance from the scleral centre to the corneal apex and fitting error (RMS) for  
 284 fitting the sclera to a sphere.

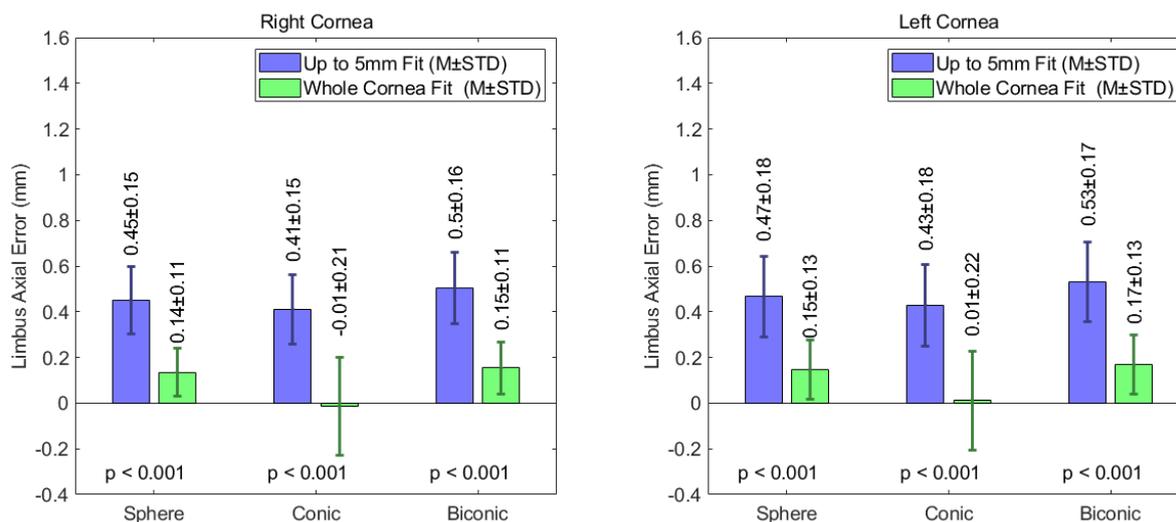
285

## 286 Limbal Positioning

287 The fitting errors for the limbus's axial position, introduced through the spherical, conic and biconic fitting  
 288 models, are presented in Fig 6. This data demonstrates that the limbus's axial position is being consistently  
 289 overestimated. A spherical fit, of up to 5mm, introduced axial position errors of  $0.45\pm 0.15\text{mm}$  and  
 290  $0.47\pm 0.18\text{mm}$  in the right and left cornea populations respectively. The adoption of a conical approach, with  
 291 the same fitting radius, yielded limbal axial position errors of  $0.41\pm 0.15\text{mm}$  for the right eye samples and  
 292  $0.43\pm 0.18\text{mm}$  for the left. The increase in complexity associated with the biconic fitting procedure did not lead  
 293 to an increase in accuracy when considering the positioning of the limbus. For an up to 5mm fitting radius,  
 294 the RMS limbus axial position error was  $0.50\pm 0.16\text{mm}$  and  $0.53\pm 0.17\text{mm}$  for the right and left eyes  
 295 respectively.

296 The extension of the fitting radius to the whole cornea led to a general decrease in the fitting error. When  
 297 fitted to the entire cornea, the spherical models produced axial position errors of  $0.14\pm 0.11\text{mm}$  and  
 298  $0.15\pm 0.13\text{mm}$  in the right and left eyes respectively. The RMS fitting error was almost eliminated when the

299 conic models were extended to a whole cornea fit. In right and left eye populations, the RMS limbus axial  
 300 position errors were  $-0.01 \pm 0.21 \text{mm}$  and  $0.01 \pm 0.22 \text{mm}$  respectively. As was observed for the 5mm fit, the  
 301 whole cornea biconic models were the least accurate when considering the elevation of the limbus. The RMS  
 302 limbus axial position errors for these models were  $0.15 \pm 0.11 \text{mm}$  and  $0.17 \pm 0.13 \text{mm}$  for the right and left  
 303 corneas respectively.



304

305 Fig 6: Limbus axial position error of fitted limbus compared to detected limbus when spherical, conic and  
 306 biconic fitting modelling techniques.

307 The accuracy of the limbal positioning was also considered by comparing the radial positions of the limbus  
 308 in the numerically generated models to the experimentally measured data. The fitting errors associated with  
 309 this comparison are represented graphically in Fig 7, and indicate that the radial positioning of the limbus in  
 310 the fitted model is consistently smaller than the actual readings. For an up to 5mm fit, the models in which  
 311 the cornea was modelled as a sphere produced RMS limbal radial position errors of  $-0.83 \pm 0.19 \text{mm}$  and  $-$   
 312  $0.86 \pm 0.23 \text{mm}$  for the right and left eye populations respectively. For the same fitting radius, this error was  
 313 reduced when a conic modelling technique was adopted. This approach yielded RMS limbus radial position  
 314 errors of  $-0.76 \pm 0.20 \text{mm}$  in the right eye population and  $-0.78 \pm 0.23 \text{mm}$  in the left. The radial position error  
 315 was reduced further when a biconic approach was considered. For an up to 5mm fit, the spheroid biconic  
 316 corneas produced RMS errors of  $-0.69 \pm 0.2 \text{mm}$  and  $-0.73 \pm 0.23 \text{mm}$  for right and left corneas respectively.

317 As noted previously, the use of whole a cornea fit vastly improves accuracy when considering the positioning  
 318 of the limbus. When adopting this radius of fit, the spherical and biconic corneas produced similar magnitudes  
 319 of limbus radial position error. The limbus radial position errors for the spherical and biconic corneas were  $-$

0.27±0.12mm and -0.22±0.16mm for the right corneas, and -0.28±0.13mm and -0.24±0.17mm respectively in the left cornea population. These errors are considerably higher than in the conic models, where the radial fitting error values were -0.02±0.29mm and -0.05±0.27mm in the right and left corneas respectively.

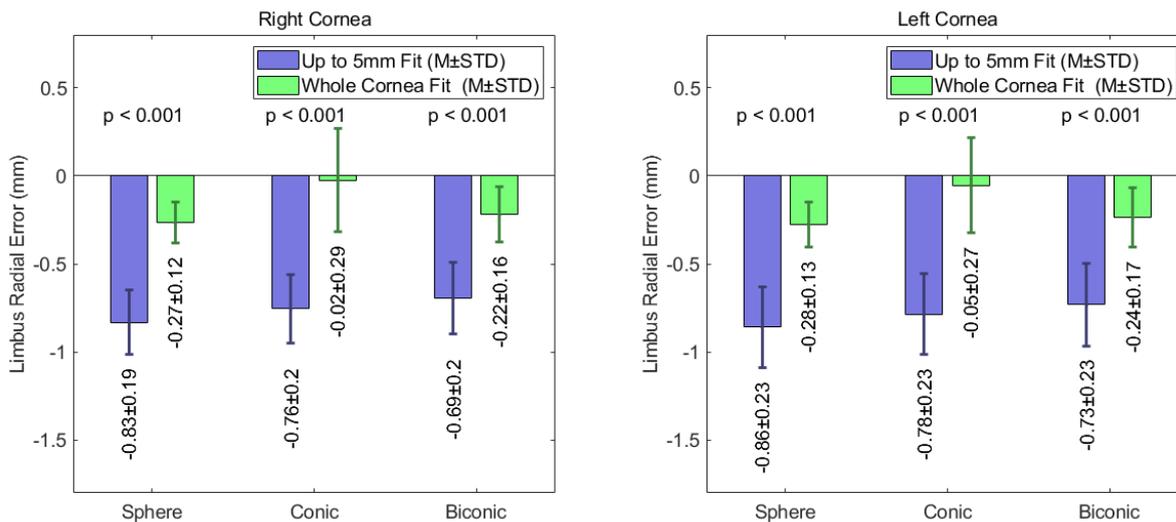


Fig 7: Limbus radial position error of fitted limbus compared to detected limbus when spherical, conic and biconic fitting modelling techniques limbus, for spherical, conic and biconic modelling techniques

## Limbal Angle

The results generated by measuring the angle at the limbal-corneal connection are presented in Fig 8. In the data generated using a fit with a radius of up to 5mm, the right corneas yielded mean angles of 46.26±2.34°, 45.24±2.61° and 44.65±2.55° for the spherical, conic and biconic fits respectively. The mean angles observed in the left corneas were 45.95±2.50°, 45.03±2.75°, 44.48±2.65° for the spherical, conical and biconical fitting techniques respectively. Despite the general consistency in these values, they each differ significantly to the experimentally measured values of 36.54±1.77° in the population of right eyes and 36.20±1.75° in the left eye population. When a whole cornea fitting approach is utilised, an improvement is observed in the conical data, with average limbal angles of 40.93±2.38° and 40.54±2.58° for the right and left eyes respectively, and the biconical data, with average right and left limbal angles of 41.19±2.10° and 40.65±2.36° respectively. Despite this, improvements were not achieved when the spherical fitting approach was utilised for the whole cornea. In this case, the average limbal angle increased to 47.37±2.29° in the right eyes and 47.06±2.37° in the left eyes, when compared to the up to 5mm fit (see S2 for full details).

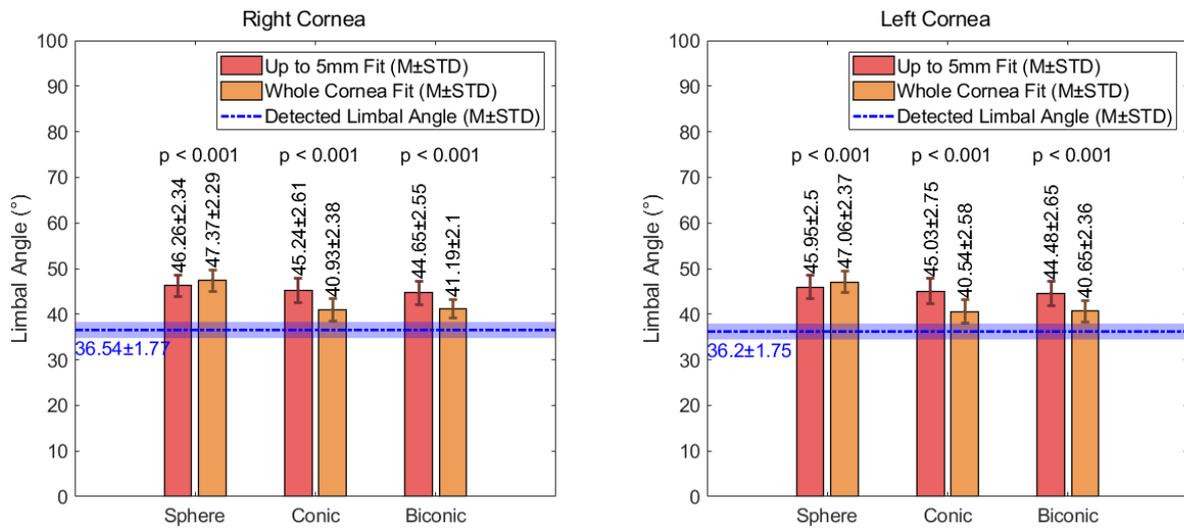
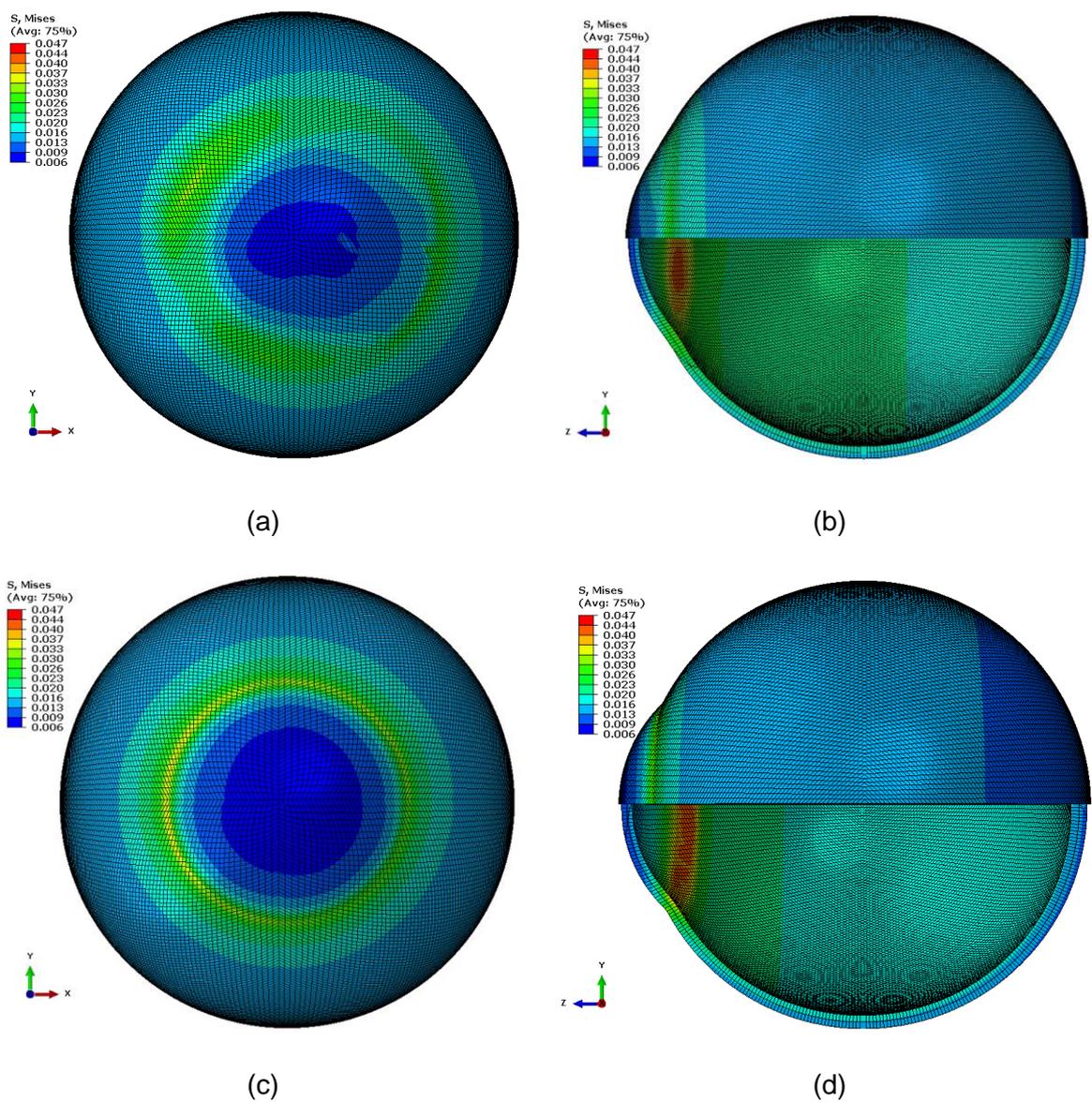


Fig 8: Observed values of the angle at the limbus for corneal surfaces fitted using spherical, conic and biconic techniques.

### Stress Concentration Effect

The distribution of the Von Mises stress for the example investigated in this study showed a smooth stress distribution when the eye globe was modelled with the actual anterior geometry as measured by the ESP, Fig 9a,b. Despite this, the Von Mises stress distribution showed a clear stress concentration ring around the limbus when the anterior geometry constructed using paramedic values obtained by fitting the central 5mm radius of the cornea to a conic model with  $R= 8.3258\text{mm}$  and  $Q= -0.0729$ , Fig 9c,d. As both models were built using a single set of material parameters, it could be deduced that the geometry of models was causing this stress concentration (Abaqus odb files for the clinical limbus representation as measured by the ESP and the conic model representation are available as S3, S4 respectively).



360 Fig 9: Von Mises stress distribution in MPa of two finite element models for the same subject plotted in Fig  
 361 2, (a,b) with the corneal geometry as measured by the ESP, (c,d) with corneal geometry constructed using  
 362 the paramedic values obtained by fitting the central 5mm radius of the cornea to a conic model with R=  
 363 8.3258mm and Q= -0.0729.

364 **Discussion**

365 In this study, the geometrical errors induced by spherical, conic and biconic fitting techniques are quantified  
 366 for a population of 270 healthy corneas. This paper focuses specifically on the effect of using these  
 367 techniques on the representation of the limbus. Results were achieved by measuring the eye topography  
 368 using the ESP device, fitting the surface to the aforementioned parametric models and computing the

369 geometrical differences between the models and the surfaces provided by the ESP. As the ESP is capable  
370 of measuring corneal topography at varying distances from the centre of the cornea, the effect of fitting the  
371 models to surfaces with radii of up to 5mm and those fitted to the whole cornea were also considered.

372 The errors associated with the corneal radius of curvature demonstrate that, in all cases, if the whole corneal  
373 surface is used for fitting purposes, the accuracy of the results is improved significantly ( $p < 0.001$  in all cases).  
374 It is also evident from the up to 5mm fit data, that a biconic approach should be utilised to minimise the  
375 associated fitting error.

376 Inter-eye differences were observed in a few ocular parameters. For instance, ocular tilt during the fixation  
377 process tends to be higher in the right eye. [16] Regarding the axial length, Mahroo, et al have observed in  
378 a large twin cohort that right eyes were slightly but significantly longer than left eyes; and that this difference  
379 was reverted for left-eye dominants, the minority of that cohort. [68] In the present study, a slightly but  
380 significantly lower RMS fitting error was observed in right eyes. Further investigations are necessary to better  
381 understand this apparent more regularity in the right side, such as considering the eye dominance and tilt.

382 Unlike Tan et al [69] who suggested a sum of the squared orthogonalized residuals (SSRo) nondimensional  
383 metric for quantifying the limbus, this study presents dimensional assess for the axial, radial and tangential  
384 limbus position misrepresentation when parametric models are used to represent the anterior eye. The  
385 presented evaluation is modelling-friendly as it describes physical numerical values and their units of  
386 measurement.

387 The limbus is an area that incurs highly concentrated stresses when conducting finite element analysis. [50]  
388 A possible source of this concentration is the abrupt change in curvature at the limbus that is directly related  
389 to the limbal angle. When modelling the limbus, this angle is often too large and therefore does not represent  
390 reality. These issues indicate that an accurate representation of limbus geometry is vital for both the stability  
391 and accuracy of ocular models.

392 When performing the finite element analysis, one of the major issues was the poor quality of the modelled  
393 geometry, which led to instabilities in the analysis. The sharp edges that are introduced when considering a  
394 false representation of the limbus are a source of numerical singularity. At these locations the analysis  
395 algorithm can fail to predict a sensible result, even with a fine mesh. Whilst finite element modellers tend to  
396 resolve these artificial stress concentrations by replacing sharp edges with fillets and chamfers this leads to

397 a significant misrepresentation when considering the ocular globe. Many of the ocular models that are based  
398 on the parametric representation of the eye were exposed to this side effect. [70-73]

399 The values of the limbal tangent angle, calculated for the different modelling techniques, reinforce the  
400 suggestion that increasing the area of the cornea used for fitting purposes provides a reduction in the induced  
401 fitting error ( $p < 0.001$ , Table 3). It is also evident that, for both fitting sizes, the accuracy of the conic approach  
402 matches that of the biconic ( $p < 0.05$  in all cases). This suggests that the biconic technique may not always  
403 be necessary to achieve the required accuracy. The data also indicates that if the most accurate approach is  
404 utilised, whereby a whole corneal surface is fitted using a conic or biconic technique, a consistent correction  
405 factor of  $-4^\circ$  is sufficient to provide a significantly improved angle for modelling purposes. For all other  
406 approaches, a correction factor of  $-9^\circ$  will provide an improved representation of the limbus.

Table 3: Values of p-value calculated through significance testing, a p-value &lt; 0.05 indicates significance

**Corneal Radius of Curvature RMS**

Up to 5mm Fit

|         | Sphere    | Conic     | Biconic   |
|---------|-----------|-----------|-----------|
| Sphere  |           | p < 0.001 | p < 0.001 |
| Conic   | p < 0.001 |           | p < 0.001 |
| Biconic | p < 0.001 | p < 0.001 |           |

Whole Cornea Fit

|         | Sphere    | Conic     | Biconic   |
|---------|-----------|-----------|-----------|
| Sphere  |           | p < 0.001 | p < 0.001 |
| Conic   | p < 0.001 |           | p < 0.001 |
| Biconic | p < 0.001 | p < 0.001 |           |

**Limbal Tangent Angle ( $\alpha$ )**

Up to 5mm Fit

|         | Actual    | Sphere    | Conic     | Biconic   |
|---------|-----------|-----------|-----------|-----------|
| Actual  |           | p < 0.001 | p < 0.001 | p < 0.001 |
| Sphere  | p < 0.001 |           | p < 0.001 | p < 0.001 |
| Conic   | p < 0.001 | p < 0.001 |           | p < 0.001 |
| Biconic | p < 0.001 | p < 0.001 | p < 0.001 |           |

Whole Cornea Fit

|         | Actual    | Sphere    | Conic     | Biconic   |
|---------|-----------|-----------|-----------|-----------|
| Actual  |           | p < 0.001 | p < 0.001 | p < 0.001 |
| Sphere  | p < 0.001 |           | p < 0.001 | p < 0.001 |
| Conic   | p < 0.001 | p < 0.001 |           | p = 0.006 |
| Biconic | p < 0.001 | p < 0.001 | p = 0.004 |           |

### Limbal Axial Position Error

Up to 5mm Fit

|         | Sphere    | Conic     | Biconic   |
|---------|-----------|-----------|-----------|
| Sphere  |           | p < 0.001 | p < 0.001 |
| Conic   | p < 0.001 |           | p < 0.001 |
| Biconic | p < 0.001 | p < 0.001 |           |

Whole Cornea Fit

|         | Sphere    | Conic     | Biconic   |
|---------|-----------|-----------|-----------|
| Sphere  |           | p < 0.001 | p = 0.002 |
| Conic   | p < 0.001 |           | p < 0.001 |
| Biconic | p = 0.005 | p < 0.001 |           |

### Limbal Radial Position Error

Up to 5mm Fit

|         | Sphere    | Conic     | Biconic   |
|---------|-----------|-----------|-----------|
| Sphere  |           | p < 0.001 | p < 0.001 |
| Conic   | p < 0.001 |           | p < 0.001 |
| Biconic | p < 0.001 | p < 0.001 |           |

Whole Cornea Fit

|         | Sphere    | Conic     | Biconic   |
|---------|-----------|-----------|-----------|
| Sphere  |           | p < 0.001 | p < 0.001 |
| Conic   | p < 0.001 |           | p < 0.001 |
| Biconic | p < 0.001 | p < 0.001 |           |

409 The positioning of the limbus and its associated fitting error have also been presented in this paper by  
410 considering both its axial and radial position. The data representing the elevation of the limbus (axial position)  
411 demonstrate strikingly different behaviour to the previously considered geometric properties. This is  
412 represented by the consistently higher fitting error produced by the biconic models when compared to the  
413 other fitting techniques. This discontinuity is not amended by increasing the area of the cornea that is used  
414 for fitting. This suggests that the use of a biconic modelling technique may hinder the modelling of the limbus  
415 greatly. In the measurements gained from the two fitting areas, the conic approach provided the most  
416 accurate values of limbal elevation and, if used solely in conjunction with a whole cornea fit, the conic fitting  
417 technique can reduce the fitting error to magnitudes that can be considered negligible. For both fitting radii,  
418 it was generally observed that the fitted models were overestimating the overall limbal elevation. A correction  
419 factor is not necessary if a conic model is utilised with the higher fitting radius, however if the model is  
420 spherical or biconic, a constant correction factor of -0.15mm can be applied if used in conjunction with the  
421 whole cornea fit. If a smaller fitting radius is used, a correction factor of -0.45mm is suitable for each of the  
422 modelling techniques.

423 The fitting errors that correspond to the limbus's radial position indicate that, in all cases, the radial position  
424 of the limbus in the fitted models was lower than the actual measurements. For the up to 5mm fit, the spherical  
425 and biconic techniques produce the largest and smallest fitting errors respectively. This relation is not  
426 observed when the models are fitted to the whole cornea and, as was present in the limbal elevation data,  
427 the conic technique produces fitting errors that are considerably lower than those produced using spherical  
428 or biconical methods ( $p < 0.001$  in both cases). A correction factor of +0.75mm will lead to significantly lower  
429 fitting errors for each technique when using an up to 5mm fit. For an extended fit, the conic model does not  
430 require correction, however adding 0.25mm to the limbal radial position will vastly improve the accuracy of  
431 the spherical and biconic modelling techniques.

432 As the limbus and sclera are directly connected and share some geometrical properties, accurate scleral  
433 representation is vital for valid and reliable limbal modelling. The errors associated with the scleral fitting  
434 process indicate that despite their consistency, their magnitudes are relatively high when compared to the  
435 other fitting methods. As the sclera is the largest component of the eye, the size of this error will propagate  
436 through any of the other model components, including the limbus. This indicates that a possible method of  
437 mitigating the limbal fitting errors may be to modify the scleral fitting process. An alternative to the sphere

438 fitting technique is to fit the sclera to a polynomial function. [74] Although this method will reduce the RMS  
439 fitting error around the limbus, it will never provide a realistic realisation of the entire ocular globe. This is due  
440 to the inherent nature of the function and the fact that the in-vivo topography can only be measured across a  
441 limited portion of the eye, thus making extrapolation beyond the measured region impossible. For this reason,  
442 when considering a whole eye model, the use of any fitting surface other than a sphere will be problematic.  
443 Although it could be viewed as a limitation, the sphere fitting technique is the most commonly used when  
444 producing whole-eye models. As this paper is presenting the current misrepresentation of the limbus,  
445 considering the error induced by this technique was key to highlighting the issues involved in modelling the  
446 ocular globe through current methods.

447 Additionally, the presented study did not consider the eye's posterior surface and was limited to the anterior  
448 surface only. Unlike Scheimpflug or optical coherence tomography (OCT) based measuring devices, the ESP  
449 only takes measurements of the anterior surface of the cornea. This was not deemed to be a disadvantage  
450 as it is not uncommon for studies to exclude the posterior surface. [75-77] This is mainly due to the relatively  
451 small contribution of the posterior surface that provides only -5.9 D of the eye's refractive power compared  
452 to 48.9 D from the anterior surface. [78] In addition to this, the measurement of the posterior surface is often  
453 deemed relatively unreliable. [28, 79-82]

454 The data presented in this paper suggests that the limbus is not being represented accurately through current  
455 modelling techniques. However, for the purpose of modelling, the most accurate method of representing the  
456 limbus is to use corneal topography data from the whole cornea in conjunction with the conic modelling  
457 technique. In doing so, the fitting errors can be minimised, and any residual errors can be accounted for by  
458 using correction factors based on the data provided. This will allow for a more accurate representation of the  
459 limbus, paramount for the fitting of mini-scleral and scleral contact lenses, and the customisation of the soft  
460 contact lenses. All types of lenses that can restore visual acuity in patients with irregular corneas, and address  
461 ocular pathologies such as keratoconus and ocular surface disease. [83, 84]

462 Additionally, it is evident that scleral geometry may be problematic when considering a whole eye model.  
463 Despite this, other modelling techniques are not currently feasible for generating realistic representations of  
464 the entire ocular globe. Advances in scleral modelling techniques will, therefore, be vital for the improvement

465 of limbal representation. These developments will greatly enhance accuracy, which in turn will lead to  
466 improvements in the validity of ocular models.

467

468

## 469 Financial Disclosure

470 None of the authors have financial disclosures.

471

## 472 Declaration of interest

473 The authors report no conflicts of interest.

474

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713 **Supporting Information**

714 S1 Table. Clinical data as measured by the ESP.

715 S2 Table. Results dataset.

716 S3 Dataset. Clinical limbus representation as measured by the ESP.

717 S4 Dataset. Modelled limbus representation as generated by the conic model.