LTH 425

STATUS OF TWIST-2 OPERATOR DIMENSIONS AT $O(1/N_f)^*$

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We review the computation of the anomalous dimensions of the twist-2 unpolarized operators in the large N_f expansion. Results are discussed for the predominantly gluonic singlet operator and the $O(1/N_f)$ part of the 3-loop splitting function is given.

1 Introduction

A current problem in multiloop perturbation theory is the construction of the 3-loop terms of the twist-2 operator dimensions which appear in the operator product expansion used in deep inelastic scattering. The calculation of the $\overline{\text{MS}}$ coefficients as a function of the momentum fraction x or equally the operator moment n, is necessary to perform the full 2-loop coefficient function evolution using the renormalization group. Currently the full 2-loop results as a function of n are known for the twist-2 flavour non-singlet and singlet, unpolarized and polarized operators. [1-4] At three loops exact results are known for the first four even moments for the unpolarized case and in addition for the non-singlet $n = 10$ moment.[5] However, the full result as a function of n has yet to be determined. As a first step in this direction, Matiounine et al have recently computed the finite parts of all the 2-loop diagrams for the twist-2 operators.[7] These are required as they will give contributions at 3-loops when multiplied by 1-loop counterterms. Aside from the perturbative expansion one can gain insight into the structure of the dimensions in other approximations. For instance, a low-x analysis can also be performed.[8]

Another expansion technique which has been applied to this problem is the $1/N_f$ method where N_f is the number of quarks. In this reordering of perturbation theory, where chains of quark bubbles form the dominant contribution, one can probe the perturbative structure beyond currently known orders. In particular results as a function of n can be provided for the 3-loop coefficients at $O(1/N_f)$ as $N_f \to \infty$ as well as at all higher loops.[9,10] These have been

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important in verifying the correctness of the results in [5,6] in the region of overlap. Currently the $1/N_f$ method has been applied to the twist-2 non-singlet and singlet fermionic operators. More recently the anomalous dimension of the outstanding singlet gluonic operator has been given in [11], which we focus on here.

2 Formalism

In standard notation the basic twist-2 unpolarized singlet operators are,

$$\begin{aligned}\mathcal{O}_q^{\mu_1 \ldots \mu_n} &= i^{n-1} \mathcal{S} \bar{\psi}^I \gamma^{\mu_1} D^{\mu_2} \ldots D^{\mu_n} \psi^I - \text{trace terms} \\ \mathcal{O}_g^{\mu_1 \ldots \mu_n} &= \tfrac{1}{2} i^{n-2} \mathcal{S} \, \mathrm{tr} \, G^{a\,\mu_1 \nu} D^{\mu_2} \ldots D^{\mu_{n-1}} G^{a}{}_{\nu}{}^{\mu_n} - \text{trace terms}\end{aligned} \tag{1}$$

As the operators $\mathcal{O}_q$ and $\mathcal{O}_g$ have the same canonical dimension they will mix under renormalization.[1] Hence one needs to introduce a mixing matrix, $\gamma_{ij}(a)$, of anomalous dimensions. The N_f dependence in the perturbative expansion of each entry in $\gamma_{ij}(a)$ is not the same. For example, with $\tilde{N}_f = T(R) N_f$

$$\begin{aligned}\gamma_{qq}(a) &= a_1 a + (a_{21}\tilde{N}_f + a_{22})a^2 + (a_{31}\tilde{N}_f^2 + a_{32}\tilde{N}_f + a_{33})a^3 + O(a^4) \\ \gamma_{gq}(a) &= b_1 a + (b_{21}\tilde{N}_f + b_{22})a^2 + (b_{31}\tilde{N}_f^2 + b_{32}\tilde{N}_f + b_{33})a^3 + O(a^4) \\ \gamma_{qg}(a) &= c_1 \tilde{N}_f a + c_2 \tilde{N}_f a^2 + (c_{31}\tilde{N}_f^2 + c_{32}\tilde{N}_f + c_{33})a^3 + O(a^4) \\ \gamma_{gg}(a) &= (d_{11}\tilde{N}_f + d_{12})a + (d_{21}\tilde{N}_f + d_{22})a^2 \\ &\quad + (d_{31}\tilde{N}_f^2 + d_{32}\tilde{N}_f + d_{33})a^3 + O(a^4)\end{aligned} \tag{2}$$

In the $1/N_f$ approach,[9,10,11], one computes sets of these coefficients by considering QCD at its non-trivial d-dimensional fixed point and studies the scaling behaviour of the appropriate Green's function there. For the present problem the resulting exponents give the eigen-anomalous dimensions of $\gamma_{ij}(a)$ at criticality. In terms of the perturbative coefficients the $O(1/N_f)$ eigenoperator dimensions involve the combinations $(a_{l1} - b_{l1}c_1/d_{11})$ for $\mathcal{O}_q$ and $(d_{l1} + b_{l1}c_1/d_{11})$ for $\mathcal{O}_g$ at the l-th loop.

3 Results

The application to QCD of the basic formalism developed in [12] for simple scalar theories yields an expression for the dimension of $\mathcal{O}_g$ at $O(1/N_f)$ as a function of n and the space-time dimension d.[11] The full all orders expression is given explicitly in [11]. Its ϵ-expansion, where $d = 4 - 2\epsilon$, agrees with all previous perturbative calculations.[1,2,3,6] At 3-loops the numerical values of the leading order gluonic eigen-operator coefficients are given in Table 1 and these can

Table 1: Numerical values of the coefficients of $[d_{31} \; + \; b_{31} c_1 / d_{11}]$.

n	$C_2(R)$ coefficient	$C_2(G)$ coefficient
2	-11.1769547325	-17.415637860
4	-6.1986353909	-12.475078189
6	-5.1270609536	-12.665273968
8	-4.8386758731	-13.094409108
10	-4.7463824737	-13.507443429
12	-4.7180193885	-13.876218903
14	-4.7136211552	-14.202839253
16	-4.7187348148	-14.493720966
18	-4.7275177819	-14.754952547
20	-4.7374464091	-14.991545068
22	-4.7473994042	-15.207488437
24	-4.7568866098	-15.405947998

be compared with the exact coefficients given in [11]. Clearly the modulus of these coefficients increases slowly with the moment. Another feature of the results is that since b_{31} depends only on the colour Casimir $C_2(R)$, then the ϵ^3 coefficient of $C_2(G)$ in the ϵ-expansion of gluonic eigen-dimension gives the exact 3-loop dependence of d_{31} as a function of n. Hence we can determine the x-dependence of the gluonic DGLAP splitting function which is proportional to $C_2(G)$. Using the Mellin transform we deduce

$$
\begin{aligned}
P_{gg}^{\text{3-loop}}(x, C_2(G)) = - \frac{1}{54} \Big[& 87 \delta(1-x) \; + \; (304 + 172x + 208x^2) \ln x \\
& - 48(1+x) \ln^2 x \; + \; 32 \; - \; \frac{32}{[1-x]_+} \\
& + 192(1+x)(\psi'(1) - \mathrm{Li}_2(x)) \\
& + \frac{4(1-x)}{x}(52 + 19x + 52x^2) \ln(1-x) \\
& + \frac{4(1-x)}{3x}(236 + 47x + 236x^2) \Big]
\end{aligned}
\tag{3}
$$

where $\mathrm{Li}_2(x)$ is the dilogarithm function and $\psi(x)$ is the derivative of the logarithm of the Euler Γ-function.

4 Discussion

The provision of the gluonic operator dimension in [11] now completes the $O(1/N_f)$ examination of the twist-2 unpolarized operator dimensions. More recently the same calculation has been completed for the polarized operators.[13] Future calculations in this area would involve computing the anomalous dimensions to the next order, $O(1/N_f^2)$. The starting point for this would be the non-singlet sector.

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