Journal Title
XX(X):1–7
©The Author(s) 2016
Reprints and permission:
sagepub.co.uk/journalsPermissions.nav
DOI: 10.1177/ToBeAssigned
www.sagepub.com/
SAGE

Lubrication of journal bearings by shear thinning lubricants using different constitutive models

Sako Azeez [1] and Volfango Bertola[2]

Abstract

The lubrication of hydrodynamic journal bearings using shear thinning fluids is investigated analytically by comparing three different constitutive models. Whilst journal bearings have been in use for a long time, they are still fundamental components of the most advanced mechanical systems, and will remain so for a long time to come. In particular, the modified Reynolds equations for the power-law (Ostwald-de Waele), the Carreau and the Cross models were derived based on the perturbation method. The three models were used to calculate the pressure distribution and the load carrying capacity, and their results were compared and discussed. It is shown that at high shear rates (i.e., high shaft speeds) the Carreau and Cross models, which better describe the rheology of shear-thinning fluids, yield higher magnitudes of the pressure than the power-law model, while at low shear rates the three models are in better agreement.

Keywords

Journal bearings, non-Newtonian lubrication, Carreau, Cross, Power Law, Shear thinning

Introduction

Journal bearings have been used for hundreds of years, since early engineering applications in mills and printing presses (1). Nowadays journal bearings are still used in many advanced applications such as turbomachinery, aircraft engines, machine tools and automotive applications (2). The shaft supporting system is very important for the performance of industrial rotary machines. Hydrodynamic bearings are able to carry the load under severe conditions, providing stabilizing support for the rotating shaft when subjected to unexpected dynamic forces. In addition, slider bearings have a very long service life (3). The operating conditions of journal bearings are often extreme; for instance, in cars journal bearings operate under the varying load conditions of the crankshaft and the high temperature of the engine for thousands of miles. While all applications require maximum durability and efficiency, in critical applications such as power plants the journal bearings of steam turbines must have 100 percent reliability (4).

Currently, hydrodynamic bearings are exposed to severe operational conditions such as heavy load and high speeds, leading to increasing temperatures of the lubricating film as a result of viscous friction. It is well known that the rise in lubricant temperature leads to decreased viscosity, thus it is necessary to improve the lubricants viscosity index by adding high-molecular weight polymers to prevent viscosity change with temperature. These additives make the lubricant behave as non-Newtonian shear-thinning fluids (5). Similarly, non-Newtonian behavior is also observed in lubricants laden with dirt particles and debris (6). The original Reynolds equation, which describes the pressure distribution in the lubricating film, assumes the lubricant is a Newtonian fluid, therefore it cannot be used when the lubricant exhibits non-Newtonian characteristics (4). Whilst the equation could be coupled with one of the shear-thinning constitutive models updating at each step the viscosity distribution in the journal bearing gap, this approach is computationally expensive and, depending on the fluid constitutive equation, convergence may be very slow or even show stability issues. Consequently, it is preferable to derive a modified Reynolds equation for non-Newtonian lubricants. Non-Newtonian lubricants are represented by a variety of constitutive models, some of which consist of purely empirical relationships obtained by data fitting, while others are derived from some theoretical basis (7). The simplest type of non-Newtonian flow behaviour occurs when the viscosity coefficient is a monomial function of the shear velocity gradient (power law, or Ostwald-De Waele model):

$$\eta = K\dot{\gamma}^{n-1} \tag{1}$$

where the consistency coefficient, K, and the power-law index, n, are empirical constants. The power-law index is indicative of the shear-thinning ($n < 1$) or shear-thickening ($n > 1$) behaviour of the fluid, whereas for $n = 1$ the Newtonian behaviour is retrieved. The consistency coefficient K represents the fluid viscosity when the shear rate, $\dot{\gamma}$, is low, and it is equal to Newtonian viscosity

[1]Laboratory of Technical Physics, School of Engineering, University of Liverpool, Liverpool, United Kingdom L69 3GH.
Email: sako_ngc@yahoo.com
[2]Senior Lecturer, Laboratory of Technical Physics, School of Engineering, University of Liverpool, Liverpool, United Kingdom L69 3GH.
Email: Volfango.Bertola@liverpool.ac.uk

Corresponding author:
Volfango Bertola, University of Liverpool, Laboratory of Technical Physics, Brownlow Hill, Liverpool, Merseyside, L69 3GH, UK.
Email: Volfango.Bertola@liverpool.ac.uk

for ($n = 1$) (8). Many lubricants such as silicone and polymer solutions can be modelled by a power law (9). The Ostwald-De Waele equation implies that viscosity will change indefinitely for any values of the shear rate. In other words, in the case of shear-thinning fluid ($n < 1$) viscosity tends to grow unlimited for ($\dot{\gamma} \to 0$), and to vanish for ($\dot{\gamma} \to \infty$); in these limits, the power law model fails to describe the behaviour of real fluids accurately. When the fluid behavior has a significant deviation at high and low shear strain from power law model, it is necessary to take into account the viscosities at very low and very high shear strain, such as in the Cross model (10):

$$\frac{\eta - \eta_\infty}{\eta_o - \eta_\infty} = \frac{1}{[1 + (\lambda\dot{\gamma})^2]^{\frac{1-n}{2}}} \tag{2}$$

and the Bird-Carreau-Yasuda model (11; 12; 13; 14), which was initially developed to describe shear-thinning observed in many viscoelastic (i.e. time-dependent) flows:

$$\frac{\eta - \eta_\infty}{\eta_o - \eta_\infty} = \frac{1}{[1 + (\lambda\dot{\gamma})^2]^n} \tag{3}$$

where η_o is the viscosity at low shear rate, η_∞ is the viscosity at high shear rate, n is a fitting parameter with a default value $n = 2$, and λ is equal to the Cross time constant. The Carreau model is more realistic as it takes into account the viscosity at minimum and maximum strain rate, while in power law equation the viscosity can change for any value of strain rate (8); , when $\lambda = 0$ then the Newtonian behavior is retrieved, if ($\eta >> \eta_\infty$) and ($\eta << \eta_o$), then Eq. (3) is reduced to power law model. (7).

In this paper, the derivation of modified Reynolds equation obtained for the simple power law model is revisited and extended for the Carreau-Yasuda and the Cross models. Then, the modified Reynolds equation for the three models is solved to obtain the pressure distribution and the load carrying capacity for a model journal bearing.

Problem formulation

In this section the derivation of modified Reynolds equation for power law fluids is reviewed, then the derivation is extended for Carreau and Cross models. The journal bearing layout is shown in Fig. (1), where y is the axis along the film thickness, z is the coordinate across the film, r is the journal bearings bore radius, and ($x = r\theta$) is the circumferential direction.

The velocities in the x and y directions are u and v respectively. The local film thickness, h, is a function of the angle, θ, and it can be found from (15):

$$h(\theta) = c(1 - \epsilon\, cos\theta) \tag{4}$$

where c is the mean film thickness (i.e. the radial clearance between the shaft and the journal bearing), and ϵ is the eccentricity ratio, which is the ratio between the shaft eccentricity e and the mean film thickness c.

Modified Reynolds equation for Power Law fluids

Dien and Elrod (16) derived a modified Reynolds equation for lubrication with non-Newtonian power law fluids in

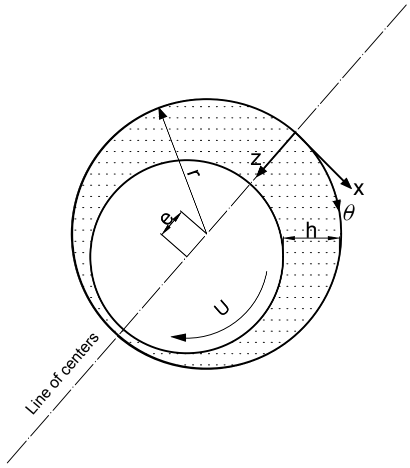

Figure 1. Journal bearing layout

journal bearings by using regular perturbation method to express the velocity field and pressure. They used a simplified momentum equation, which is applicable for laminar flow neglecting the inertial force of the fluid. In the lubrication theory the velocity derivative components are responsible for all the shear deformation of the fluid, so that the linear momentum conservation equations can be written as:

$$\frac{\partial}{\partial z}\eta\frac{\partial u}{\partial z} = \frac{\partial p}{\partial x} \tag{5}$$

$$\frac{\partial}{\partial z}\eta\frac{\partial v}{\partial z} = \frac{\partial p}{\partial y} \tag{6}$$

where p is the pressure. In generalized Newtonian flows, which include the shear-thinning behaviour, viscosity is dependent only on the second invariant of strain rate tensor (8). Then, the constitutive equation writes:

$$\eta = \eta(I) \tag{7}$$

where the second invariant of strain tensor, I, is given by:

$$I = \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 \tag{8}$$

In Newtonian fluids, velocities and their derivatives changes linearly with pressure gradient, while in non-Newtonian fluids it assumed that the strain rate is generated by surface velocity. Thus this approximation is more accurate for Couette-dominated non-Newtonian fluids. The pressure gradient is expressed as follow:

$$\nabla p = \delta \nabla \bar{p} \tag{9}$$

where δ is small non-dimensional amplitude and $\bar{p}$ is the reference pressure used for expansion. By using regular perturbation, the dependent variables can be expanded in the terms of δ . The expansion needs to be accurate to first order, then ($\delta^2 = 0$), then the velocity terms are as follows:

$$u = u_o + \delta u_1 \tag{10}$$

$$v = v_o + \delta v_1 \tag{11}$$

where u_o and v_o are the zero-order velocities in x and y directions respectively, while u_1 and v_1 are the first-order velocities in x and y directions respectively.

By substituting Eq.(10) and Eq.(11) into Eq.(8) the second invariant can be rewritten as:

$$I = I_o + \delta I_1 \tag{12}$$

where I_o and I_1 are zero-order and first-order strain tensors respectively. Replacing Eq.(12) in Eq.(7), and using the Taylor series expansion up to first order, viscosity can be written as:

$$\eta = \eta(I_o + \delta I_1 + ...) = \eta(I_o) + \delta \left(\frac{\partial \eta}{\partial I}\right)_{I=I_o} I_1 \tag{13}$$

where:

$$\bar{\eta_o} = \eta(I_o), \bar{\eta_1} = \left(\frac{\partial \eta}{\partial I}\right)_{I=I_o} I_1 \tag{14}$$

where $\bar{\eta_o}$ and $\bar{\eta_1}$ are the zero-order shear rate and first-order shear rate viscosities respectively. The zero-order differential equations are found by substituting the zero-order terms from Eq.(10) and Eq.(11) into Eq.(5) and Eq.(6) then:

$$\frac{\partial}{\partial z}\bar{\eta_o}\frac{\partial u_o}{\partial z} = 0 \tag{15}$$

$$\frac{\partial}{\partial z}\bar{\eta_o}\frac{\partial v_o}{\partial z} = 0 \tag{16}$$

Integrating the zero-order equations result:

$$\bar{\eta_o}\frac{\partial u_0}{\partial z} = C_1 \tag{17}$$

$$\bar{\eta_o}\frac{\partial v_0}{\partial z} = B_1 \tag{18}$$

where C_1 and B_1 are arbitrary integral constants. Squaring Eq.(17) and Eq.(18), then adding them together yields:

$$\bar{\eta_o}^2\left[\left(\frac{\partial u_o}{\partial z}\right)^2 + \left(\frac{\partial v_0}{\partial z}\right)^2\right] = \bar{\eta_o} I_o = C_1 + B_1 \tag{19}$$

As $\bar{\eta_0}$ is dependent only on I_o, then it concluded from Eq.(19) that $\bar{\eta_o}$ and I_o are constants. To find the zero-order velocities, the equations Eq.(17) and (18) are integrated and the following boundary conditions for journal bearing velocities are applied (16):

$$\begin{aligned} u &= 0 \quad at \quad z = 0 \quad , u = U \quad at \quad z = h \\ v &= 0 \quad at \quad z = 0 \quad , v = V \quad at \quad z = h \end{aligned} \tag{20}$$

where U and V are the surface velocities in x and y directions respectively. Then the zero-order velocities are found:

$$u_o = U\frac{z}{h} \tag{21}$$

$$v_o = V\frac{z}{h} \tag{22}$$

To find the first order-velocities, the viscosity Eq.(13) and the velocities Eq.(10) and Eq.(11) up to the first-order replaced into Eq.(5) and Eq.(6):

$$\frac{\partial}{\partial z}\bar{\eta_o}\frac{\partial u_1}{\partial z} + \frac{\partial}{\partial z}\bar{\eta_1}\frac{\partial u_o}{\partial z} = \frac{\partial \bar{p}}{\partial x} \tag{23}$$

$$\frac{\partial}{\partial z}\bar{\eta_o}\frac{\partial v_1}{\partial z} + \frac{\partial}{\partial z}\bar{\eta_1}\frac{\partial v_o}{\partial z} = \frac{\partial \bar{p}}{\partial y} \tag{24}$$

Replacing Eq.(21) and Eq.(22) into Eq.(23) and Eq.(24) results:

$$\frac{\partial^2 u_1}{\partial z^2} = \frac{1}{\bar{\eta_o}}\frac{\partial \bar{p}}{\partial x} - \frac{U}{h}\left(\frac{U}{h}\frac{\partial \bar{p}}{\partial x} + \frac{V}{h}\frac{\partial \bar{p}}{\partial y}\right)\frac{2\frac{\partial \eta}{\partial I}}{\bar{\eta_o}^2(1 + \frac{\partial \ln \eta}{\partial \ln(I^{\frac{1}{2}})})} \tag{25}$$

$$\frac{\partial^2 v_1}{\partial z^2} = \frac{1}{\bar{\eta_o}}\frac{\partial \bar{p}}{\partial y} - \frac{V}{h}\left(\frac{U}{h}\frac{\partial \bar{p}}{\partial x} + \frac{V}{h}\frac{\partial \bar{p}}{\partial y}\right)\frac{2\frac{\partial \eta}{\partial I}}{\bar{\eta_o}^2(1 + \frac{\partial \ln \eta}{\partial \ln(I^{\frac{1}{2}})})} \tag{26}$$

Integrating Eq.(25) and Eq.(26) twice and applying the boundary conditions, the first-order velocities are found as follow:

$$u_1 = \frac{z(z-h)}{2}\frac{\partial^2 u_1}{\partial z^2} \tag{27}$$

and

$$v_1 = \frac{z(z-h)}{2}\frac{\partial^2 v_1}{\partial z^2} \tag{28}$$

The velocities u and v are found by substituting zero-order velocities Eq.(21) and Eq.(22), and first-order velocities Eq.(27) and Eq.(28) into Eq.(10) and Eq.(11). To find the mass flux, $\dot{m}$, the velocities are integrated as follow :

$$\dot{m}_x = \int_0^h \rho\left(U\frac{z}{h} + \delta\frac{(z^2 - zh)}{2}\frac{\partial^2 u_1}{\partial z^2}\right).dz \tag{29}$$

similarly, for y-direction:

$$\dot{m}_y = \int_0^h \rho\left(V\frac{z}{h} + \delta\frac{(z^2 - zh)}{2}\frac{\partial^2 v_1}{\partial z^2}\right).dz \tag{30}$$

where ρ is the fluids' density while $\dot{m}_x$ and $\dot{m}_y$ are the mass flux in x and y directions respectively. Replacing $\frac{\partial^2 u_1}{\partial z^2}$ and $\frac{\partial^2 v_1}{\partial z^2}$ by using Eq.(25) and (26), then adding the mass flux in both directions, the total mass flux is found:

$$\begin{aligned} \dot{m} = \frac{\rho U h}{2} + \frac{\rho V h}{2} - \delta\left(\frac{\rho h^3}{12\bar{\eta_o}}\right) \\ \left[\left(\frac{\partial \bar{p}}{\partial x} - \frac{U}{h}\left(\frac{U}{h}\frac{\partial \bar{p}}{\partial x} + \frac{V}{h}\frac{\partial \bar{p}}{\partial y}\right)\frac{2\frac{\partial \eta}{\partial I}}{(\bar{\eta_o} + 2\frac{\partial \eta}{\partial I}I_o)}\right)\right. \\ \left. + \left(\frac{\partial \bar{p}}{\partial y} - \frac{V}{h}\left(\frac{U}{h}\frac{\partial \bar{p}}{\partial x} + \frac{V}{h}\frac{\partial \bar{p}}{\partial y}\right)\frac{2\frac{\partial \eta}{\partial I}}{(\bar{\eta_o} + 2\frac{\partial \eta}{\partial I}I_o)}\right)\right] \end{aligned} \tag{31}$$

By using the following vector expressions:

$$S = U + V \tag{32}$$

and

$$\hat{s} = \frac{U + V}{\sqrt{U^2 + V^2}} \tag{33}$$

where $\hat{s}$ is direction vector for the velocity and S is the surface velocity magnitude Eq.(31) is simplified to the following form:

$$\dot{m} = \frac{\rho S h}{2} - \frac{\rho h^3}{12\bar{\eta_o}}\left[\nabla p - \hat{s}\big(\hat{s}.\nabla p\big)\left(\frac{1}{1 + \frac{\partial \ln I^{1/2}}{\partial \ln \eta}}\right)\right] \tag{34}$$

The term $\frac{\partial \ln I^{1/2}}{\partial \ln \eta}$ in Eq.(34) represents the slope of the flow curve, viscosity as a function of the shear rate, when plotted in logarithmic scale. This term is convenient for power law fluids. As the line has obvious slope as shown in Fig.(2), but for Carreau and Cross models the relationship between (log I vs. log η) cannot be represented by this term as the curve has two plateaus as shown in Fig. (2). So, for the two latter models this term must be replaced in order to take into account the viscosities at both lower and higher extreme viscosities limits.

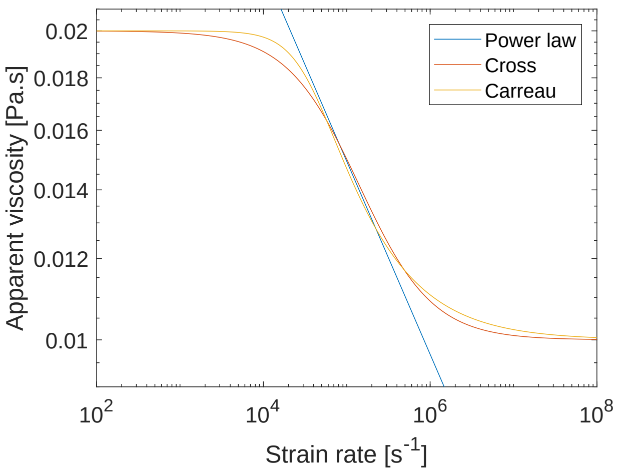

Figure 2. Log(η) vs. log ($\dot{\gamma}$) for (10W50) oil modelled by: Power law., Carreau and Cross.

For power-law fluids, this term can be easily calculated as:

$$\frac{\partial \ln I^{\frac{1}{2}}}{\partial \ln \eta} = \frac{1}{n - 1} \tag{35}$$

Replacing Eq.(35) into Eq.(34), the mass flux in x-direction is found as:

$$\frac{\partial \dot{m}}{\partial x} = \frac{\partial}{\partial x}\frac{\rho U h}{2} - \frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\bar{\eta_o}}\right)\frac{1}{n}\frac{\partial p}{\partial x} \tag{36}$$

and in y-direction, setting $(V = 0)$ since the journal bearing rotates in circumferential direction only:

$$\frac{\partial \dot{m}}{\partial y} = -\frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\bar{\eta_o}}\right)\frac{\partial p}{\partial y} = 0 \tag{37}$$

By equating the total mass flux (in the x and in the y direction) to zero, the modified Reynolds equation for lubrication of journal bearings with power law fluids is obtained:

$$\frac{\partial}{\partial x}\left[\frac{\rho h^3}{12\bar{\eta_o}}\frac{1}{n}\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{\rho h^3}{12\bar{\eta_o}}\frac{\partial p}{\partial y}\right] = \frac{\partial}{\partial x}\left(\frac{\rho U h}{2}\right) \tag{38}$$

The $\bar{\eta_o}$ is a function of I_o, taking into account that the velocity in y-direction is equal to zero, then from Eq.(1) and Eq.(21):

$$\bar{\eta_o} = K\left(\frac{U}{h}\right)^{n-1} \tag{39}$$

Substituting Eq.(39) into Eq.(38) assuming constant density, and replacing $x = r\theta$, the modified Reynolds equation for power law fluids can be written as:

$$\frac{1}{r^2}\frac{\partial}{\partial \theta}\left[\frac{h^{2+n}}{n}\frac{\partial p}{\partial \theta}\right] + \frac{\partial}{\partial y}\left[h^{2+n}\frac{\partial p}{\partial y}\right] = \frac{6KU^n}{r}\frac{\partial h}{\partial x} \tag{40}$$

Modified Reynolds equation for Carreau and Cross models

For the Carreau and the Cross models, a modification in Eq.(34) is done by using the identities $(d \ln I^{\frac{1}{2}} = \frac{dI}{2I})$ and $(d \ln \eta = \frac{d\eta}{\eta})$, resulting:

$$\dot{m} = \frac{\rho S h}{2} - \frac{\rho h^3}{12\bar{\eta_o}}\left[\nabla p - \hat{s}\big(\hat{s}.\nabla p\big)\left(\frac{2\frac{\partial \eta}{\partial I}}{\frac{\bar{\eta_o}}{I_o} + 2\frac{\partial \eta}{\partial I}}\right)\right] \tag{41}$$

By equating the total mass flux (in the x and the y direction) to zero:

$$\frac{\partial \dot{m}_x}{\partial x} + \frac{\partial \dot{m}_y}{\partial y} = 0 \tag{42}$$

From Eq.(41), one obtains in the x direction:

$$\frac{\partial \dot{m}}{\partial x} = \frac{\partial}{\partial x}\frac{\rho U h}{2} - \frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\bar{\eta_o}}\right)\frac{1}{n}\frac{\partial p}{\partial x} \tag{43}$$

and in the y-direction:

$$\frac{\partial \dot{m}}{\partial y} = \frac{\partial}{\partial y}\frac{\rho V h}{2} - \frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\bar{\eta_o}}\right)\frac{\partial p}{\partial y} \tag{44}$$

Substituting Eq.(43) and Eq.(44) into Eq.(42), and replacing $(x = r\theta)$ and $(V = 0)$,the modified Reynolds equation for the Carreau and the Cross models can be obtained as:

$$\frac{1}{r^2}\frac{\partial}{\partial \theta}\left[\frac{h^3}{12\eta}\frac{\partial p}{\partial \theta}\right] + \frac{\partial}{\partial y}\left[\frac{h^3}{12\bar{\eta_o}}\frac{\partial p}{\partial y}\right] = \frac{U}{2r}\frac{\partial h}{\partial \theta} \tag{45}$$

where η and $\bar{\eta_o}$ are the viscosity and the zero-order shear rate viscosity, respectively, defined as follows:

$$\bar{\eta_o} = \eta(I)_{I=I_o} \tag{46}$$

$$\eta = \bar{\eta_o} + 2I_o\left(\frac{\partial\eta}{\partial I}\right)_{I=I_o} \tag{47}$$

For the Carreau model (Eq. 3), viscosity can be expressed as:

$$\eta = \eta_\infty + (\eta_o - \eta_\infty)(1+\lambda^2\dot{\gamma}^2)^{\frac{n-1}{2}} \tag{48}$$

Since $\dot{\gamma}^2 = I^{\frac{1}{2}}$, and the zero-order velocities are found in Eq.(21) and Eq.(22), substituting these velocities into Eq.(8), and setting the velocity in y-direction equal to zero yields:

$$I_o^{\frac{1}{2}} = \frac{U}{h} \tag{49}$$

Then, for the Carreau model $\bar{\eta_o}$ and η can be found by substituting Eq. (49) into Eq.(48) and using Eq.(46) and Eq.(47) yields:

$$\bar{\eta_o} = \eta_\infty + (\eta_o - \eta_\infty)\left[1+\left(\lambda\frac{U}{h}\right)^2\right]^{\frac{n-1}{2}} \tag{50}$$

$$\eta = \eta_\infty + (\eta_o - \eta_\infty)\left[1+\lambda^2\frac{U}{h}\right]^{\frac{n-1}{2}} + 2\frac{U}{h}\lambda^2\left(\frac{n-1}{2}\right)(\eta_o - \eta_\infty)\left[1+\lambda^2\frac{U}{h}\right]^{\frac{n-3}{2}} \tag{51}$$

By substituting Eq.(50) and Eq.(51) into Eq.(45), the modified Reynolds equation for the Carreau model can be obtained. Finally, for the Cross model the viscosity in Eq.(2) can be expressed as:

$$\eta = \eta_\infty + (\eta_o - \eta_\infty)\left[1+(\lambda\dot{\gamma})^n\right]^{-1} \tag{52}$$

Then,

$$\bar{\eta_o} = \eta_\infty + (\eta_o - \eta_\infty)\left[1+\left(\lambda\frac{U}{h}\right)^n\right]^{-1} \tag{53}$$

$$\eta = \eta_\infty + (\eta_o - \eta_\infty)\left[1+\left(\lambda\frac{U}{h}\right)^n\right]^{-1} + 2\frac{U}{h}\frac{(\eta_\infty - \eta_o)}{2}\frac{n\,\lambda^n\left(\frac{U}{h}\right)^{\frac{n-2}{2}}}{\left[1+\lambda^n\left(\frac{U}{h}\right)^{\frac{n}{2}}\right]^2} \tag{54}$$

The modified Reynolds equation for the Cross model is obtained by substituting Eq.(53) and Eq.(54) into Eq.(45).

Results and discussion

The full modified Reynolds equation was solved numerically using the finite difference method (FDM), where the differential terms in the Reynolds equation were replaced by linear approximations of the function values at grid nodes on the bearing surface. The Reynolds boundary conditions for cavitation were used to obtain the equation solution, which states that(15):

$$\begin{aligned} &p = 0 && at \quad \theta = 0 \\ &p = 0,\ \frac{\partial p}{\partial\theta} = 0 && at \quad \theta = \theta^* \\ &p = 0 && at \quad y = 0, L \end{aligned} \tag{55}$$

where, L, is the journal bearings length and, θ^*, is the angle when the pressure starts to be negative.

The pressure is positive in the region ($0 < \theta < \theta^*$) and zero in ($\theta^* < \theta < 2\pi$). The θ^* can be found by iteration. In order to compare the solutions obtained using the three models (Power law, Carreau-Yasuda and Cross), the bearing dimensions were obtained from a manufacturer's catalogue (17) as follows:

Radius = 0.050 m
Radial Clearance = 30 μm
Length = 0.100 m

The calculation was carried out using the SAE 10W50 oil properties. This oil behaves as a shear-thinning non-Newtonian fluid, and it can be modelled with any of the three constitutive equations considered in the present work using the parameters listed in Table 1, which were obtained by curve fitting (18).

Table 1. 10W50 oil parameters for non-Newtonian models

Carreau	Cross	Power law
η_o=0.02	η_o=0.02	K=0.2
η_∞=0.01	η_∞=0.01	n=0.812
λ=3e-6	λ =1e-6	
n=0.341	n=1	

Using these parameters, the modified Reynolds equations Eq.(40) and Eq.(45) were solved numerically with a MATLAB code to obtain the pressure distribution and the load carrying capacity. The calculation was repeated several times for different shaft speeds and shaft eccentricity ratios.

The maximum pressure in the journal bearing is shown in Fig.(3). This plot shows that at low speeds the maximum pressures generated in the journal bearing are almost identical for the three models considered. However, increasing the speed and the eccentricity ratio the Carreau and the Cross models yield higher pressures with respect to the power law model. For instance, the pressure distribution in the journal bearing for $\epsilon = 0.9$ and length to diameter ratio ($L/D = 1$) at 10,000 rpm is shown in Fig.(4). The pressure distribution for the Carreau and the Cross models are almost identical while the power law model yields a lower pressure distribution, because at higher rotational speeds and eccentricity ratios the shear rate is very high therefore the Carreau and the Cross models behave as Newtonian fluids with constant viscosity equal to (η_∞), while the power law model continues to behave as a non-Newtonian shear-thinning fluid, therefore its viscosity decreases as the shear rate is increased.

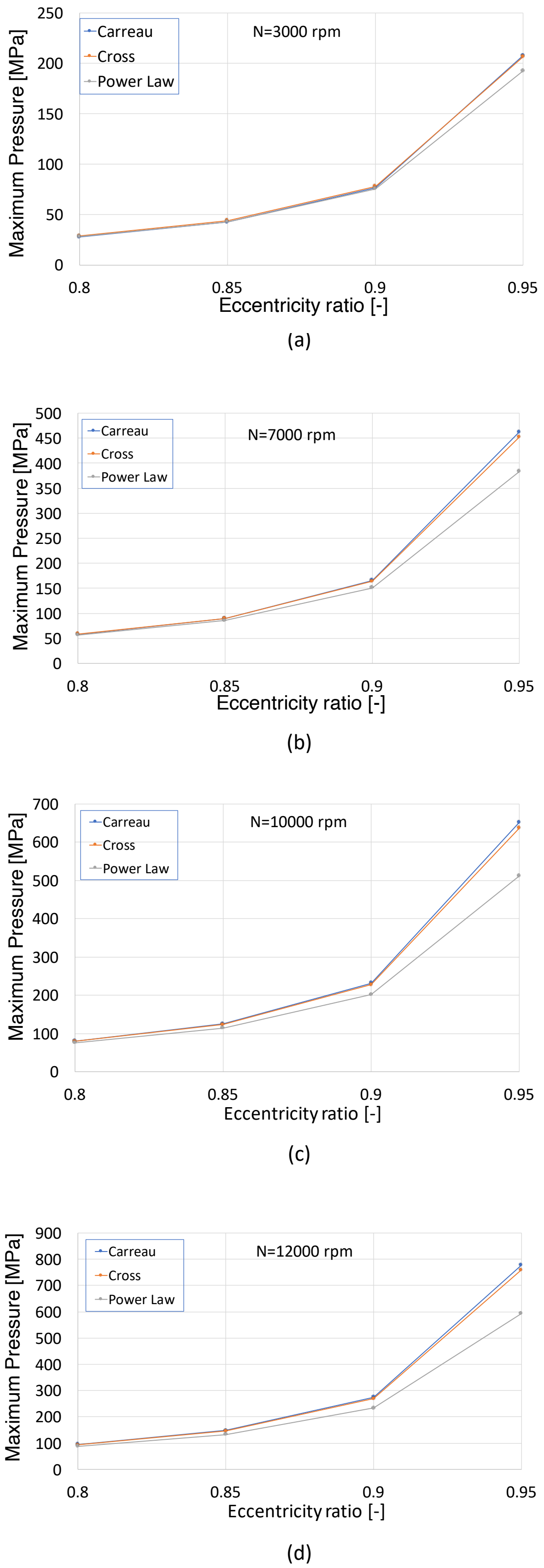

Figure 3. Maximum generated pressures for journal bearing modelled by Power Law, Carreau and Cross models for different shaft speeds (N) and eccentric ratios: (a) N=3000 rpm (b) N=7000 rpm (c) N=10,000 rpm (d) N=12000 rpm.

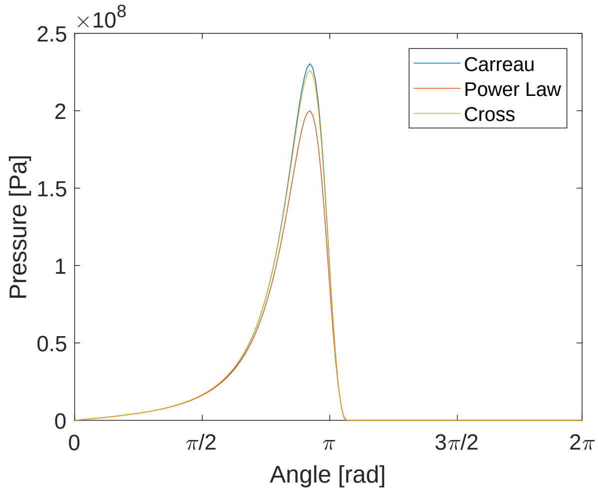

Figure 4. Pressure distribution in journal bearing with (ϵ=0.9, speed =10,000 rpm and L/D=1)

Since the load carrying capacity for journal bearings is the integration of the generated pressure over the bearing surface area, then its values follows similar trends to those shown in Fig.(3). Consequently, the load carrying capacity of the journal bearing is also underestimated by the power law model at high shear rates, as shown in Fig.(5).

Conclusions

The modified Reynolds equation accounting for the non-Newtonian shear-thinning flow behaviour was studied for three well-known constitutive models: the power law (Ostwald-de Waele), the Carreaus-Yasuda, and the Cross model, using the perturbation method. The resulting modified Reynolds equations were solved numerically using the finite difference method.

Results showed that non-Newtonian shear-thinning lubricants cannot be modelled accurately by the power law model at high shaft speeds (i.e. high shear rates), where this model underestimates the pressure distribution and the load carrying capacity, whereas the Cross and Carreau-Yasuda models yield more realistic results. At low shaft speeds (i.e. low shear rates) all the three models are in good agreement.

References

[1] Hannon WM and Braun MJ. Hydrodynamic journal bearing history. *Encyclopedia of Tribology* 2013; : 1729–1736.

[2] Javorova J and Angelova J. On the modified reynolds equation for journal bearings in a case of non-newtonian rabinowitsch fluid model. In *MATEC Web of Conferences*, volume 145. EDP Sciences, p. 03007.

[3] Faria MTC. On the hydrodynamic long journal bearing theory. In *Proceedings Of The World Congress On Engineering*, volume 2.

[4] Shigley JE, Mischke CR and Budynas RG. *mechanical engineering design*. New York ;London : McGraw-Hill Higher Education, 2004., 2004.

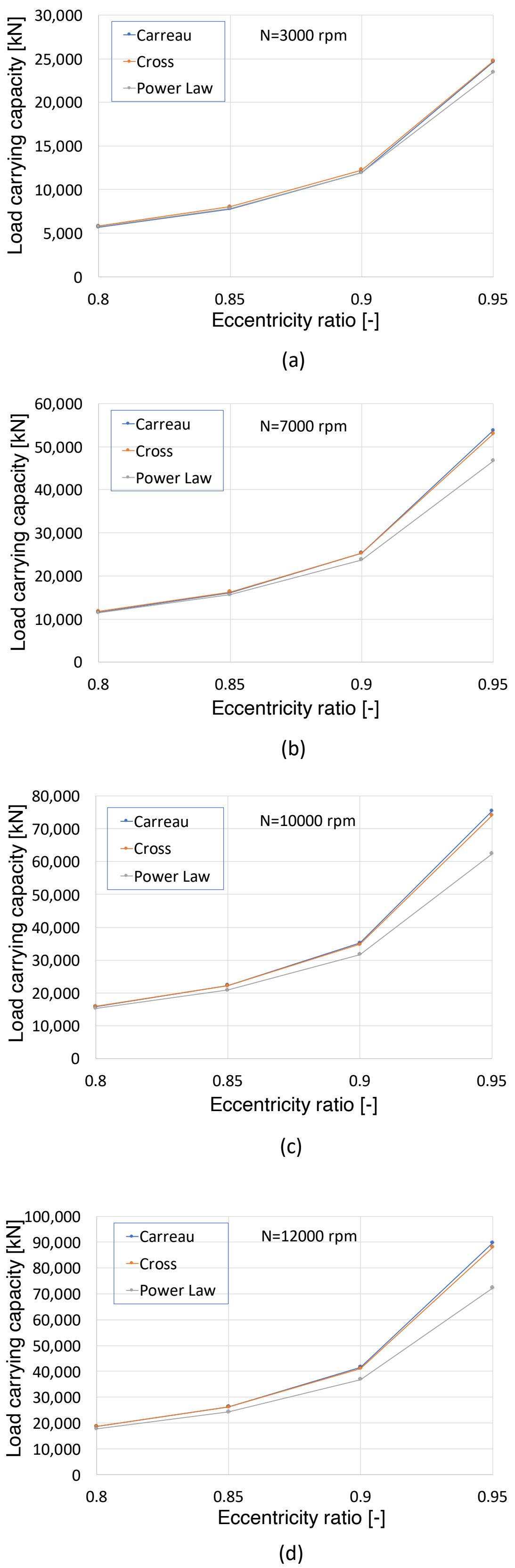

Figure 5. Load carrying capacity for journal bearing modelled by Power Law, Carreau and Cross models for different shaft speeds (N) and eccentric ratios: (a) N=3000 rpm (b) N=7000 rpm (c) N=10,000 rpm (d) N=12000 rpm.

[5] Wada S and Hayashi H. Hydrodynamic lubrication of journal bearings by pseudo-plastic lubricants: part 1, theoretical studies. *Bulletin of JSME* 1971; 14(69): 268–278.

[6] Singh C and Sinha P. Non-newtonian squeeze films in journal bearings. *Wear* 1981; 70(3): 311–319.

[7] Chhabra RP and Richardson JF. *Non-Newtonian flow and applied rheology: engineering applications*. Butterworth-Heinemann, 2011.

[8] Bertola V. Constitutive models of complex fluids. In *Transport Phenomena in Complex Fluids*. Springer, 2020. pp. 27–47.

[9] Prasad D, Panda S and Subrahmanyam S. Power law fluid film lubrication of journal bearing with squeezing and temperature effects. In *Proceedings of International Conference on Advances in Tribology and Engineering Systems*. Springer, pp. 73–84.

[10] Cross MM. Rheology of non-newtonian fluids: a new flow equation for pseudoplastic systems. *Journal of colloid science* 1965; 20(5): 417–437.

[11] Bird RB and Carreau PJ. A nonlinear viscoelastic model for polymer solutions and melts—i. *Chemical Engineering Science* 1968; 23(5): 427–434.

[12] Carreau PJ. Rheological equations from molecular network theories. *Transactions of the Society of Rheology* 1972; 16(1): 99–127.

[13] Yasuda K, Armstrong R and Cohen R. Shear flow properties of concentrated solutions of linear and star branched polystyrenes. *Rheologica Acta* 1981; 20(2): 163–178.

[14] Yamaguchi H. *Engineering fluid mechanics*, volume 85. Springer Science & Business Media, 2008.

[15] Someya T, Mitsui J, Esaki J et al. *Journal-bearing databook*. Springer Science & Business Media, 2013.

[16] Dien I and Elrod H. A generalized steady-state reynolds equation for non-newtonian fluids, with application to journal bearings. *Journal of Lubrication Technology* 1983; 105(3): 385–390.

[17] Kingsbury. *Fixed profile journal bearing catalogue*, 2018. See also URL `https://www.kingsbury.com/pdf/catalog-FPJ.pdf`.

[18] Tamai G. *Experimental study of engine oil film thickness dependence on liner location, oil properties and operating conditions*. Master's Thesis, Massachusetts Institute of Technology, 1995.