October 2020

Type 0 $\mathbb{Z}_2 \times \mathbb{Z}_2$ Heterotic String Orbifolds and Misaligned Supersymmetry

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Abstract

The $\mathbb{Z}_2 \times \mathbb{Z}_2$ heterotic string orbifold yielded a large space of phenomenological three generation models and serves as a testing ground to explore how the Standard Model of particle physics may be incorporated in a theory of quantum gravity. In this paper we explore the existence of type 0 models in this class of string compactifications. We demonstrate the existence of type 0 $\mathbb{Z}_2 \times \mathbb{Z}_2$ heterotic string orbifolds, and show that there exist a large degree of redundancy in the space of GGSO projection coefficients when the type 0 restrictions are implemented. We explore the existence of such configurations in several constructions. The first correspond to essentially a unique configuration out of a priori 2^{21} discrete GGSO choices. We demonstrate this uniqueness analytically, as well as by the corresponding analysis of the partition function. A wider classification is performed in $\tilde{S}$–models and S–models, where the first class correspond to compactifications of a tachyonic ten dimensional heterotic string vacuum, whereas the second correspond to compactifications of the ten dimensional non–tachyonic $SO(16) \times SO(16)$. We show that the type 0 models in both cases contain physical tachyons at the free fermionic point in the moduli space. These vacua are therefore necessarily unstable, but may be instrumental in exploring the string dynamics in cosmological scenarios. we analyse the properties of the string one–loop amplitude. Naturally, these are divergent due to the existence of tachyonic states. We show that once the tachyonic states are removed by hand the amplitudes are finite and exhibit a form of misaligned supersymmetry.

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1 Introduction

The holy grail of contemporary theoretical physics is the synthesis of quantum mechanics and general relativity. String theory provides an arena to explore this synthesis within a perturbatively self–consistent framework. The important advantage of string theory is that its internal consistency conditions mandate the appearance of the gauge and matter structures that exist in the observed world, hence facilitating the development of a phenomenological approach. It seems likely that there exist numerous mathematical structures that can be used to developed consistent theories of quantum gravity, and that mathematical consistency alone will not be sufficient for the development of a physically relevant theory, *i.e.* one that is relevant for experimental observations. It is therefore sensible to construct string models that aim to reproduce the physical characteristics of the observed physical world. Such toy models can in turn be used to explore the mathematical structures underlying string theory and their possible relation to observable phenomena. Of particular interest is the relation of string vacua to their effective field theory limits. In this respect it should be noted that contemporary understanding of string theory is confined to its static limits. Dynamical questions in nature, such as the cosmological evolution in the early universe, are explored at present primarily in the effective field theory limit.

Given the vast space of potential string vacua, it makes further sense to study classes of models that exhibit certain common structures and that produce examples of quasi–realistic phenomenological models. While it is folly to expect at present any of the models to be fully realistic, it is feasible that some characteristics of a given class of models may be imprinted in the observable data. The task of string phenomenology is to identify such imprints in different classes of string compactifications, and how the experimental data may discern between them. One class of string compactification that have been explored in this spirit are the $\mathbb{Z}_2 \times \mathbb{Z}_2$ heterotic string orbifolds.

The $\mathbb{Z}_2 \times \mathbb{Z}_2$ heterotic string orbifolds have been studied in the free fermionic formulation of the heterotic string [1] since the late eighties. They produce a large space of quasi–realistic three generation models with different unbroken $SO(10)$ and E_6 subgroups [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Quasi–realistic $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold models have also been constructed by using the free bosonic formulation [13]. The two formulations provide complementary tools to explore the same physical spaces, and detailed dictionaries have been developed to translate models from one formulation to the other [14]. The $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds exhibit a rich symmetry structure, which has been the interest of numerous mathematical studies [15]. The rich symmetry structure of the $\mathbb{Z}_2$ orbifold may also be relevant for the Standard Model flavour problem [16, 17]. Indeed, it was demonstrated that quasi–realistic fermion mass and mixing spectrum may be obtained from the fermionic $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds [16]. The $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactifications have also been studied in other string limits [18].

While the majority of studies of phenomenological string models have been of $N = 1$ supersymmetric vacua, non–supersymmetric compactifications have been of increased interest over the past decade. The starting point for most of these non–supersymmetric constructions is the $SO(16) \times SO(16)$ heterotic string in ten dimensions [19, 20, 21, 22, 23, 24]. However, it is well known that string theory also admits solutions that are tachyonic in ten dimensions [19, 25, 26]. It was recently argued that these ten dimensional vacua may also serve as starting points for the constructions of viable phenomenological models, provided that the tachyonic modes are projected from the physical spectrum in the phenomenological four dimensional models [27, 28, 29]. Indeed, a tachyon free three generation Standard–like Model was constructed in this class [28], whereas in ref. [29] a broad classification of models with unbroken $SO(10)$ gauge group was presented including the analysis of their vacuum energy, and numerous models that satisfy the condition $n_b = n_f$, *i.e.* models with equal numbers of massless bosons and fermions.

In this paper, we extend the analysis of the aforementioned classification of this class of vacua. Our interest here is in the existence of models that do not contain any fermions at all. Such models are known in the literature as type 0 models and have been of interest in other string theory limits [30]. Such models are of particular interest to explore the boundaries of the space of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactifications. It is plausible that progress on some of the phenomenological issues in string theory, in particular in relation to the cosmological evolution and vacuum selection, will be obtained by improved understanding of these vacua. Moreover, it is likely that further insight can be achieved by exploring some of the features of these vacua in connection with the phenomenological string vacua. In this paper we therefore pursue this line of investigation. We present several type 0 models in this class. We further adapt the systematic classification method that was developed using the free fermionic rules [31, 32, 33, 7, 9, 10, 11, 12, 29] to this class of models. This requires careful analysis of tachyonic states that proliferate in these configurations. While we do not find any model which is completely free of tachyonic states, we present a model with a minimal set of tachyonic states. Similar to the analysis in ref. [24], we may explore the possibility that these tachyonic states become massive when the moduli is moved away from the free fermionic point. Another issue that we analyse in detail is the calculation of the vacuum energy and the finiteness properties of the string one–loop amplitude. Naturally, these are divergent due to the existence of tachyonic states. However, once the tachyonic states are removed by hand the amplitudes are finite and exhibit a form of misaligned supersymmetry.

Our paper is organised as follows: in Section 2 we present an explicit example of a type 0 model that does not contain any massless fermionic states. In Sections 2.1–2.3 we show that the type 0 constraints are in fact very restrictive and showing that for the chosen set of basis vectors the type 0 model is in fact unique. Hence, we demonstrate the existence of a huge redundancy in the space of Generalised GSO phases that span the models. We show that the type 0 model in this configuration does contain a tachyonic state that transform in the vector representation of the hidden sector gauge group. A question of interest that we undertake in the following sections is the existence of a tachyon free type 0 model. We investigate this question by adopting the free fermionic classification methodology to these models that enables the scan of large space of models and the extraction of models with specific characteristics. We explore the existence of tachyon free type 0 models in both $\tilde{S}$–models as well as S–models that utilise the SUSY generating basis vector, Our scan yields a null result, which suggests that such a tachyon free model may only exist away from the free fermionic point. In Section 5 we demonstrate the existence of a misalligned supersymmetry in the type 0 models that guarantee the finiteness of the one–loop amplitude, aside from the divergence due to the tachyonic states. Section 6 concludes our paper.

2 Type 0 $\mathbb{Z}_2 \times \mathbb{Z}_2$ Heterotic String Orbifold

In the free fermionic contruction [1] models are specified in terms of boundary condition basis vectors and one–loop Generalised GSO phases. Details of the formalism are to be found in the literature and are not repeated here. We adopt the conventional notation used in the free fermionic constructions [2, 3, 4, 5, 6, 31, 7, 8, 9, 10, 11, 12]. The first model that we present uses the $\overline{\text{NAHE}}$–set that was introduced in [27, 28]. In this set the basis vector S that generates spacetime supersymmetry in NAHE–based models [34] is augmented with four periodic right–moving fermions, which amounts to making the gravitinos massive. This introduces a general $S \to \tilde{S}$ map in the space of models that was discussed in detail in ref. [29]. The set of basis vectors is given by

$$\begin{aligned}
\mathbb{1} &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, w^{1,\dots,6} \mid \overline{y}^{1,\dots,6}, \overline{w}^{1,\dots,6}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^{1,2,3}, \overline{\phi}^{1,\dots,8}\},\\
\tilde{S} &= \{\psi^\mu, \chi^{1,\dots,6} \mid \overline{\phi}^{3,4,5,6}\},\\
b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \mid \overline{y}^{34}, \overline{y}^{56}, \overline{\eta}^1, \overline{\psi}^{1,\dots,5}\},\\
b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, w^{56} \mid \overline{y}^{12}, \overline{w}^{56}, \overline{\eta}^2, \overline{\psi}^{1,\dots,5}\},\\
b_3 &= \{\chi^{12}, \chi^{34}, w^{12}, w^{34} \mid \overline{w}^{12}, \overline{w}^{34}, \overline{\eta}^3, \overline{\psi}^{1,\dots,5}\},\\
z_1 &= \{\overline{\phi}^{1,\dots,4}\},\\
x &= \{\overline{\psi}^{1,\dots,5}, \overline{\eta}^{1,2,3}\}.
\end{aligned} \tag{2.1}$$

A model may then be specified through the assignment of modular invariant GGSO phases $C\left[\begin{smallmatrix} v_i \\ v_j \end{smallmatrix}\right]$ between the basis vectors. A type 0 configuration arises for the assignment

$$C\begin{bmatrix} v_i \\ v_j \end{bmatrix} = \begin{array}{c} \\ \mathbf{1} \\ \tilde{S} \\ b_1 \\ b_2 \\ b_3 \\ z_1 \\ x \end{array} \!\!\! \begin{array}{c} \begin{array}{ccccccc} \mathbf{1} & \tilde{S} & b_1 & b_2 & b_3 & z_1 & x \end{array} \\ \begin{pmatrix} 1 & 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & 1 & -1 & 1 \\ -1 & 1 & -1 & -1 & -1 & 1 & 1 \end{pmatrix} \end{array} \tag{2.2}$$

The gauge bosons arising from the Neveu–Schwarz (NS) sector produce the vector bosons of a $SO(10) \times U(1)^3 \times SO(4)^3 \times SU(2)^8$ gauge symmetry. Additional vector bosons may arise from the sectors in the z_1, $z_3 = \mathbf{1} + \tilde{S} + b_1 + b_2 + b_3 = \{\bar{\phi}^{1,2,7,8}\}$ and $z_4 = \mathbf{1} + \tilde{S} + b_1 + b_2 + b_3 + z_1 = \{\bar{\phi}^{3,4,7,8}\}$, which can affect the observable and the hidden gauge group factors or just the hidden, depending on the right-moving oscillator. A solely observable gauge enhancement may also arise from the sector x. In the above model, the hidden $SU(2)^8$ gauge symmetry is enhanced to $SO(16)$ by vector bosons arising in z_1, z_3 and z_4. The four dimensional gauge group is therefore

$$SO(10) \times U(1)^3 \times SO(4)^3 \times SO(16).$$

The NS–sector and the three sector above produce in total sixteen the tachyonic states that transform in the 16 representation of the hidden $SO(16)$ gauge symmetry. The vectorial fermionic sectors in the model are

$$\begin{aligned}
&\tilde{S};\\
&\tilde{S}+z_1;\\
&\tilde{S}+z_4\sim S+z_2,
\end{aligned}\tag{2.3}$$

while the massless fermionic spinorial sectors are

$$\begin{aligned}
&\tilde{S}+b_{1,2,3}+x;\\
&\tilde{S}+b_{1,2,3}+x+z_1;\\
&\tilde{S}+b_{1,2,3}+x+z_4;\\
&\tilde{S}+z_3\sim S+z_1+z_2;
\end{aligned}\tag{2.4}$$

where we defined $z_2=\{\bar{\phi}^{5,\cdots,8}\}$ and $S=\{\psi^{1,2},\chi^{1,\cdots,6}\}$, neither of which are basis vector combinations in the additive group. We note that the S–vector coincides with the supersymmetry generator in supersymmetric free fermionic models, as well as those that are compactifications of the ten dimensional $SO(16)\times SO(16)$ heterotic string. These definitions comply with the terminology used in the classification of the supersymmetric free fermionic heterotic string models. We emphasise that the absence of the S–vector from the additive group is the crucial feature of the $\tilde{S}$–models. As discussed in refs. [27, 28, 29] the absence of the S–vector is the characteristic property of vacua that descend from the tachyonic ten dimensional vacua. The massless states from all the fermionic sectors are projected out from the physical spectrum by the choice of GGSO phases in eq. (2.2). In addition to the NS–sector, sectors giving rise to spacetime massless bosonic states are

$$\begin{aligned}
&x;\\
&z_{1,3,4};\\
&b_{1,2,3};\\
&b_{1,2,3}+x;\\
&b_{1,2,3}+x+z_1;\\
&b_{1,2,3}+x+z_3;\\
&b_{1,2,3}+x+z_1+z_3.
\end{aligned}\tag{2.5}$$

They give rise to scalar spacetime bosons that transform in representations of the four dimensional gauge symmetry. They are of no particular interest here and we do not list their detail explicitly.

In Section 3 and 4 we perform a more general search for similar type 0 heterotic string models using the free fermionic classification methodology. In particular, we search for type 0 models without tachyons. Our search is conducted using the $\tilde{S}$ based models as well as models that use S. First, however, it is interesting to study a bit more closely the basis (2.1), and what additional constraints type 0 vacua may satisfy.

2.1 Analytic conditions on type 0 vacua

Since we are interested in the construction and analysis of type 0 vacua, we focus on the massless fermionic sectors and will seek to project them out, leaving only bosonic states at the massless level. The Hilbert space in the free fermionic construction is given by the collection of GGSO-projected states $|S_\xi\rangle$

$$\mathcal{H} = \bigoplus_{\xi\in\Xi} \prod_{i=1}^{k} \left\{ e^{i\pi v_i\cdot F_\xi} |S_\xi\rangle = \delta_\xi C \begin{bmatrix} \xi \\ v_i \end{bmatrix}^* |S_\xi\rangle \right\}. \tag{2.6}$$

For the type 0 case, this will only have contributions from sectors ξ in the additive space Ξ with bosonic spin statistic index $\delta_\xi = +1$ at the massless level.

For the basis (2.1), These definitions comply with the terminology used in the classification of the supersymmetric free fermionic heterotic string models. In particular, we note that the S–vector coincides with the supersymmetry generator in supersymmetric free fermionic models, as well as those that are compactifications of the ten dimensional $SO(16) \times SO(16)$ heterotic string which we analyse in Section 4.

However, since we want to find choices of GGSO phases for which the fermionic sectors may be projected out to leave type 0 vacua, it is worth exploring explicitly the analytic conditions on their projection. Another important part of the analysis will be to consider the presence of tachyonic sectors in our models which we turn to in Section 2.2. Due to our basis being relatively simple, writing down analytic conditions for generating type 0 models seems tractable *a priori* and we will see that, indeed, it is entirely solvable.

In order to project the fermionic massless sectors given in equations (2.3) and (2.4) we can write down analytic conditions from the GGSO projection equation for the existence of type 0 models. For the fermionic vectorial sectors (2.3)

$$\{\bar{y}/\bar{w}\}\,|\tilde{S}\rangle \text{ projected} \iff \left(1 - C\begin{bmatrix}\tilde{S}\\ x\end{bmatrix}\right)\left(1 - C\begin{bmatrix}\tilde{S}\\ z_3\end{bmatrix}\right) = 0 \tag{2.7}$$

$$\{\bar{\psi}^{1,\dots,5}/\bar{\eta}^{1,2,3}\}\,|\tilde{S}\rangle \text{ projected} \iff \left(1 + C\begin{bmatrix}\tilde{S}\\ x\end{bmatrix}\right)\left(1 - C\begin{bmatrix}\tilde{S}\\ z_3\end{bmatrix}\right) = 0 \tag{2.8}$$

$$\{\bar{\phi}^{1,2,7,8}\}\,|\tilde{S}\rangle \text{ projected} \iff \left(1 - C\begin{bmatrix}\tilde{S}\\ x\end{bmatrix}\right)\left(1 + C\begin{bmatrix}\tilde{S}\\ z_3\end{bmatrix}\right) = 0 \tag{2.9}$$

$$\{\bar{y}/\bar{w}\}\,|\tilde{S}+z_1\rangle \text{ projected} \iff \left(1 - C\begin{bmatrix}\tilde{S}+z_1\\ x\end{bmatrix}\right)\left(1 - C\begin{bmatrix}\tilde{S}+z_1\\ z_4\end{bmatrix}\right) = 0 \tag{2.10}$$

$$\{\bar{\psi}^{1,\dots,5}/\bar{\eta}^{1,2,3}\}\,|\tilde{S}+z_1\rangle \text{ projected} \iff \left(1 + C\begin{bmatrix}\tilde{S}+z_1\\ x\end{bmatrix}\right)\left(1 - C\begin{bmatrix}\tilde{S}+z_1\\ z_4\end{bmatrix}\right) = 0 \tag{2.11}$$

$$\{\bar{\phi}^{3,4,7,8}\}\,|\tilde{S}+z_1\rangle \text{ projected} \iff \left(1 - C\begin{bmatrix}\tilde{S}+z_1\\ x\end{bmatrix}\right)\left(1 + C\begin{bmatrix}\tilde{S}+z_1\\ z_4\end{bmatrix}\right) = 0 \tag{2.12}$$

$$\{\bar{y}/\bar{w}\}\,|\tilde{S}+z_4\rangle \text{ projected} \iff \left(1 - C\begin{bmatrix}\tilde{S}+z_4\\ x\end{bmatrix}\right)\left(1 - C\begin{bmatrix}\tilde{S}+z_4\\ z_1\end{bmatrix}\right) = 0 \tag{2.13}$$

$$\{\bar\psi^{1,\dots,5}/\bar\eta^{1,2,3}\}\,|\tilde S+z_4\rangle \text{ projected } \iff \left(1+C\begin{bmatrix}\tilde S+z_4\\ x\end{bmatrix}\right)\left(1-C\begin{bmatrix}\tilde S+z_4\\ z_1\end{bmatrix}\right)=0 \tag{2.14}$$

$$\{\bar\phi^{1,2,3,4}\}\,|\tilde S+z_4\rangle \text{ projected } \iff \left(1-C\begin{bmatrix}\tilde S+z_4\\ x\end{bmatrix}\right)\left(1+C\begin{bmatrix}\tilde S+z_4\\ z_1\end{bmatrix}\right)=0 \tag{2.15}$$

Using the ABK rules on equations (2.7), (2.8) and (2.9) we deduce that

$$C\begin{bmatrix}\tilde S\\ x\end{bmatrix}=1 \quad\text{and}\quad C\begin{bmatrix}\tilde S\\ b_1\end{bmatrix}C\begin{bmatrix}\tilde S\\ b_2\end{bmatrix}C\begin{bmatrix}\tilde S\\ b_3\end{bmatrix}=-1 \tag{2.16}$$

using these results and the ABK rules on equations (2.10), (2.11) and (2.12) gives the further results

$$C\begin{bmatrix}z_1\\ x\end{bmatrix}=1 \quad\text{and}\quad C\begin{bmatrix}z_1\\ b_1\end{bmatrix}C\begin{bmatrix}z_1\\ b_2\end{bmatrix}C\begin{bmatrix}z_1\\ b_3\end{bmatrix}=1. \tag{2.17}$$

Finally, the first bracket of equations (2.13), (2.14) and (2.15) implies the result

$$C\begin{bmatrix}x\\ 1\end{bmatrix}C\begin{bmatrix}x\\ b_1\end{bmatrix}C\begin{bmatrix}x\\ b_2\end{bmatrix}C\begin{bmatrix}x\\ b_3\end{bmatrix}=1. \tag{2.18}$$

Meanwhile, for the fermionic spinorial sectors (2.4) we have

$$\tilde S+b_i+x \text{ projected}$$
$$\iff \left(1-C\begin{bmatrix}\tilde S+b_i+x\\ b_j+b_k+x\end{bmatrix}\right)\left(1-C\begin{bmatrix}\tilde S+b_i+x\\ z_3\end{bmatrix}\right)=0 \tag{2.19}$$

$$\tilde S+b_i+z_1+x \text{ projected}$$
$$\iff \left(1-C\begin{bmatrix}\tilde S+b_i+z_1+x\\ b_j+b_k+x\end{bmatrix}\right)\left(1-C\begin{bmatrix}\tilde S+b_i+z_1+x\\ z_4\end{bmatrix}\right)=0 \tag{2.20}$$

$$\tilde S+b_i+z_4+x \text{ projected}$$
$$\iff \left(1-C\begin{bmatrix}\tilde S+b_i+z_4+x\\ b_j+b_k+x\end{bmatrix}\right)\left(1-C\begin{bmatrix}\tilde S+b_i+z_4+x\\ z_1\end{bmatrix}\right)=0 \tag{2.21}$$

$$\tilde S+z_3 \text{ projected}$$
$$\iff \left(1-C\begin{bmatrix}\tilde S+z_3\\ b_1+b_2+b_3+x\end{bmatrix}\right)\left(1-C\begin{bmatrix}\tilde S+z_3\\ x\end{bmatrix}\right)=0 \tag{2.22}$$

where $i\neq j\neq k=1,2,3$. Using results (2.16),(2.17) and (2.18) in equation (2.21) implies that

$$C\begin{bmatrix}z_1\\ b_1\end{bmatrix}=C\begin{bmatrix}z_1\\ b_2\end{bmatrix}=C\begin{bmatrix}z_1\\ b_3\end{bmatrix}=1 \tag{2.23}$$

and using results (2.16) and (2.18) in equation (2.19) allows us to deduce the results

$$C\begin{bmatrix}\tilde S\\ b_1\end{bmatrix}C\begin{bmatrix}b_1\\ b_2\end{bmatrix}C\begin{bmatrix}b_1\\ b_3\end{bmatrix}=-1,\quad C\begin{bmatrix}\tilde S\\ b_2\end{bmatrix}C\begin{bmatrix}b_2\\ b_1\end{bmatrix}C\begin{bmatrix}b_2\\ b_3\end{bmatrix}=-1\quad\text{and}\quad C\begin{bmatrix}\tilde S\\ b_3\end{bmatrix}C\begin{bmatrix}b_3\\ b_1\end{bmatrix}C\begin{bmatrix}b_3\\ b_2\end{bmatrix}=-1 \tag{2.24}$$

since (2.16) means there are only two independent equations here, we can use this result to fix two of the phases: $C\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, C\begin{bmatrix} b_1 \\ b_3 \end{bmatrix}$ and $C\begin{bmatrix} b_2 \\ b_3 \end{bmatrix}$.

Gathering together the results (2.16), (2.17), (2.18), (2.23) and (2.24) we find the following necessary and sufficient conditions on the projection of massless fermions within models derived from the basis (2.1)

$$
\begin{aligned}
C\begin{bmatrix} \tilde{S} \\ x \end{bmatrix} &= 1, \quad C\begin{bmatrix} z_1 \\ x \end{bmatrix} = 1, \quad C\begin{bmatrix} z_1 \\ b_1 \end{bmatrix} = C\begin{bmatrix} z_1 \\ b_2 \end{bmatrix} = C\begin{bmatrix} z_1 \\ b_3 \end{bmatrix} = 1 \\
C\begin{bmatrix} \tilde{S} \\ b_1 \end{bmatrix} &= -C\begin{bmatrix} \tilde{S} \\ b_2 \end{bmatrix} C\begin{bmatrix} \tilde{S} \\ b_3 \end{bmatrix} \\
C\begin{bmatrix} x \\ 1 \end{bmatrix} &= \ C\begin{bmatrix} x \\ b_1 \end{bmatrix} C\begin{bmatrix} x \\ b_2 \end{bmatrix} C\begin{bmatrix} x \\ b_3 \end{bmatrix} \\
C\begin{bmatrix} b_2 \\ b_3 \end{bmatrix} &= -C\begin{bmatrix} \tilde{S} \\ b_2 \end{bmatrix} C\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \\
C\begin{bmatrix} b_3 \\ b_1 \end{bmatrix} &= -C\begin{bmatrix} \tilde{S} \\ b_2 \end{bmatrix} C\begin{bmatrix} \tilde{S} \\ b_3 \end{bmatrix} C\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}.
\end{aligned}
\tag{2.25}
$$

These conditions mean that 9 of the 21 GGSO phases are fixed in order to obtain type 0 vacua. Hence, the number of possible type 0 models is reduced to $2^{12} = 4096$.

Armed with the conditions (2.25) we can look now at the bosonic sectors (2.5) and in fact prove that all these sectors must appear in all 4096 possible type 0 models. To prove this we can go through the projection conditions as we did above for the fermionic sectors. In particular, taking the sectors b_i, for $i = 1, 2, 3$ we get

$$
b_i \text{ survives} \iff C\begin{bmatrix} b_i \\ \tilde{S} + b_j + b_k \end{bmatrix} = 1 \quad \text{and} \quad C\begin{bmatrix} b_i \\ z_1 \end{bmatrix} = 1 \tag{2.26}
$$

these conditions coincide exactly with conditions (2.24) and (2.23), respectively, from the projection of fermions analysis above. A similar result can easily be found for the other spinorial bosonic sectors: $b_{1,2,3} + x + z_1, b_{1,2,3} + x + z_3$ and $b_{1,2,3} + x + z_4$.

For the bosonic vectorial sectors: $b_i + x$, $i = 1, 2, 3$ we have the conditions

$$
\{\bar{y}/\bar{w}/\bar{\psi}^{1,\dots,5}/\bar{\eta}^i/\bar{\phi}^{5,6}\} \, |b_i + x\rangle \text{ survives} \implies \frac{1}{4}\left(1 + C\begin{bmatrix} b_i + x \\ z_3 \end{bmatrix}\right)\left(1 + C\begin{bmatrix} b_i + x \\ z_1 \end{bmatrix}\right) = 1 \tag{2.27}
$$

$$
\{\bar{\phi}^{1,2}\} \, |b_i + x\rangle \text{ survives} \implies \frac{1}{4}\left(1 - C\begin{bmatrix} b_i + x \\ z_3 \end{bmatrix}\right)\left(1 - C\begin{bmatrix} b_i + x \\ z_1 \end{bmatrix}\right) = 1 \tag{2.28}
$$

$$
\{\bar{\phi}^{3,4}\} \, |b_i + x\rangle \text{ survives} \implies \frac{1}{4}\left(1 + C\begin{bmatrix} b_i + x \\ z_3 \end{bmatrix}\right)\left(1 - C\begin{bmatrix} b_i + x \\ z_1 \end{bmatrix}\right) = 1 \tag{2.29}
$$

$$
\{\bar{\phi}^{7,8}\} \, |b_i + x\rangle \text{ survives} \implies \frac{1}{4}\left(1 - C\begin{bmatrix} b_i + x \\ z_3 \end{bmatrix}\right)\left(1 + C\begin{bmatrix} b_i + x \\ z_1 \end{bmatrix}\right) = 1 \tag{2.30}
$$

the type 0 conditions (2.25) guarantee that $C\begin{bmatrix} b_i+x \\ z_3 \end{bmatrix} = 1$ and $C\begin{bmatrix} b_i+x \\ z_1 \end{bmatrix} = 1$ and thus the first case survives.

Finally let us show that the bosonic sector x survives. In order to make this sector massless there must be a left moving oscillator, which could make it a gauge boson if this oscillator is ψ^μ. However, since for type 0 models $C\left[\begin{smallmatrix} x \\ \tilde{S} \end{smallmatrix}\right] = 1$ only the states of the type $\{y^i/w^i\}_{1/2}\,|x\rangle$ can survive, which they must do due to $C\left[\begin{smallmatrix} x \\ z_1 \end{smallmatrix}\right] = C\left[\begin{smallmatrix} x \\ z_3 \end{smallmatrix}\right] = 1$.

For the z_1 massless sector the conditions for type 0 models necessitate that states of the type $\{y/w\}\{\bar{y}/\bar{w}\}\,|z_1\rangle$ and extra gauge bosons of the type $\{\psi^\mu\}\{\bar{\phi}^{5,6,7,8}\}\,|z_1\rangle$ survive. Similarly for the z_3 massless sector type 0 models must have states of the type $\{y/w\}\{\bar{y}/\bar{w}\}\,|z_3\rangle$ and extra gauge bosons of the type $\{\psi^\mu\}\{\bar{\phi}^{3,4,5,6}\}\,|z_3\rangle$. Finally, for the z_4 massless sector type 0 models must have states of the type $\{y/w\}\{\bar{y}/\bar{w}\}\,|z_4\rangle$ and extra gauge bosons of the type $\{\psi^\mu\}\{\bar{\phi}^{1,2,5,6}\}\,|z_4\rangle$. Therefore, all type 0 models derived from this basis (2.1) have a hidden sector enhancement of $SU(2)^4 \rightarrow SO(16)$.

This analysis tells us that all 4096 possible type 0 models contain all bosonic sectors 2.5 with the specific set of oscillators given above. In other words, their massless spectra are identical. Doing the counting of all the bosonic states can be shown to give 4264, which is thus the constant term in the q-expansion of the partition function in all 4096 cases. Having seen how restrictive the type 0 conditions 2.25 are at the massless level it makes sense to analyse what happens with the tachyonic sectors for our type 0 models.

2.2 Tachyon analysis for type 0 vacua

The tachyonic sectors for models derived from the basis (2.1) come from the sectors $|z_1\rangle$, $|z_3\rangle$ and $|z_4\rangle$ as well as the untwisted tachyon of the NS sector. We can immediately see that all vacua in this basis will contain an untwisted tachyon $\{\bar{\phi}^{5,6}\}\,|NS\rangle$. This can be seen as being related to the absence of $z_2 = \{\bar{\phi}^{5,6,7,8}\}$ in the basis which would allow for the projection of this tachyon since we would be equipped with the GGSO projection with the phase $C\left[\begin{smallmatrix} NS \\ z_2 \end{smallmatrix}\right] = 1$.

In regard to the tachyons from the sectors $|z_1\rangle$, $|z_3\rangle$ and $|z_4\rangle$, we see that the type 0 conditions (2.25) necessitate their presence. For example, all phases that could project the z_1 tachyon: $C\left[\begin{smallmatrix} z_1 \\ b_1 \end{smallmatrix}\right], C\left[\begin{smallmatrix} z_1 \\ b_2 \end{smallmatrix}\right], C\left[\begin{smallmatrix} z_1 \\ b_3 \end{smallmatrix}\right], C\left[\begin{smallmatrix} z_1 \\ x \end{smallmatrix}\right]$ are all equal to $+1$ and thus leave it in the Hilbert space. Therefore, we conclude that all type 0 in this construction contain the tachyons from the sectors $|z_1\rangle$, $|z_3\rangle$ and $|z_4\rangle$, along with the model-independent untwisted tachyon $\bar{\phi}^{5,6}\,|NS\rangle$.

2.3 Equivalence of type 0 models

Having shown that the massless spectrum and tachyonic sectors are identical for all the 4096 choices of GGSO phases consistent with the type 0 conditions (2.25), we might wonder whether these models are in fact identical at all massive levels. Calculating the partition function for all 4096 type 0 models proved that they indeed all have the same partition function

$$Z = 2\bar{q}^{-1} + 16q^{-1/2}\bar{q}^{-1/2} + 4264 + 45056q^{1/4}\bar{q}^{1/4} + \cdots \tag{2.31}$$

and thus there is only one type 0 model in our construction with degeneracy 4096. This is a good example of the non-uniqueness of the free fermionic construction definition of a model since the partition function (2.31) is invariant under the 12 phases: $C\left[\begin{smallmatrix} 1 \\ \tilde{S} \end{smallmatrix}\right]$, $C\left[\begin{smallmatrix} 1 \\ b_1 \end{smallmatrix}\right]$, $C\left[\begin{smallmatrix} 1 \\ b_2 \end{smallmatrix}\right]$, $C\left[\begin{smallmatrix} 1 \\ b_3 \end{smallmatrix}\right]$, $C\left[\begin{smallmatrix} 1 \\ z_1 \end{smallmatrix}\right]$, $C\left[\begin{smallmatrix} \tilde{S} \\ b_2 \end{smallmatrix}\right]$, $C\left[\begin{smallmatrix} \tilde{S} \\ b_3 \end{smallmatrix}\right]$, $C\left[\begin{smallmatrix} \tilde{S} \\ z_1 \end{smallmatrix}\right]$, $C\left[\begin{smallmatrix} b_1 \\ b_2 \end{smallmatrix}\right]$, $C\left[\begin{smallmatrix} x \\ b_1 \end{smallmatrix}\right]$, $C\left[\begin{smallmatrix} x \\ b_2 \end{smallmatrix}\right]$, $C\left[\begin{smallmatrix} x \\ b_3 \end{smallmatrix}\right]$. This result will ultimately be related to the many symmetries underlying models defined by the basis (2.1).

3 Classification of Type 0 $\tilde{S}$-Models

Having found that type 0, tachyonic models exist for the simple basis (2.1), we can consider a more general basis

$$
\begin{aligned}
\mathbb{1} &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, w^{1,\dots,6} \mid \overline{y}^{1,\dots,6}, \overline{w}^{1,\dots,6}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^{1,2,3}, \overline{\phi}^{1,\dots,8}\},\\
\tilde{S} &= \{\psi^\mu, \chi^{1,\dots,6} \mid \overline{\phi}^{3,4,5,6}\},\\
T_1 &= \{y^{1,2}, w^{1,2} \mid \overline{y}^{1,2}, \overline{w}^{1,2}\},\\
T_2 &= \{y^{3,4}, w^{3,4} \mid \overline{y}^{3,4}, \overline{w}^{3,4}\},\\
T_3 &= \{y^{5,6}, w^{5,6} \mid \overline{y}^{5,6}, \overline{w}^{5,6}\},\\
b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \mid \overline{y}^{34}, \overline{y}^{56}, \overline{\eta}^1, \overline{\psi}^{1,\dots,5}\},\\
b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, w^{56} \mid \overline{y}^{12}, \overline{w}^{56}, \overline{\eta}^2, \overline{\psi}^{1,\dots,5}\},\\
b_3 &= \{\chi^{12}, \chi^{34}, w^{12}, w^{34} \mid \overline{w}^{12}, \overline{w}^{34}, \overline{\eta}^3, \overline{\psi}^{1,\dots,5}\},\\
z_1 &= \{\overline{\phi}^{1,\dots,4}\},
\end{aligned}
\tag{3.1}
$$

where the T_i, $i = 1, 2, 3$ allow for internal symmetric shift in the compactified coordinates around the 3 tori. The only other difference to the basis (2.1) is that x is now a linear combination:

$$x = b_1 + b_2 + b_3 + T_1 + T_2 + T_3 \tag{3.2}$$

and we have the same combinations $z_3 = 1+\tilde{S}+b_1+b_2+b_3$ and $z_4 = z_3+z_1$. We can further note that the space of independent GGSO phase configuration is now $2^{36} \sim 6 \times 10^{10}$ for this basis.

The addition of the T_i's has some key consequences in relation to finding tachyon-free type 0 vacua. It multiplies the number of massless fermionic sectors and also increases the number of ways to project the (fermionic) sectors. Furthermore, we now have 15 tachyonic sectors: $z_1, z_3, z_4, T_i, z_1 + T_i, z_3 + T_i$ and $z_4 + T_i$, $i = 1, 2, 3$ rather than just the 3 for basis (2.1). We can notice that the model–independent NS tachyon $\{\bar{\phi}^{5,6}\}\,|NS\rangle$ remains in this construction so the minimal number of tachyons is to only have the NS tachyon.

3.1 Fermionic sector analysis

Using a similar methodology to Section 2.1, we wish to analyse the conditions on the projection of all fermionic sectors from these models. Due to increased size of the space of models from the added complexity of having of $T_{i=1,2,3}$ in the basis, we developed a computer algorithm to scan efficiently over the space of vacua and check for the absence of fermionic massless states.

We note that massless fermionic vectorials in these models arise from the sectors

$$
\begin{aligned}
&\tilde{S};\\
&\tilde{S} + z_1;\\
&\tilde{S} + z_4
\end{aligned}
\tag{3.3}
$$

and the massless fermionic spinorial sectors from

$$\begin{aligned} &\tilde{S} + b_i + b_j + T_k + pT_i + qT_j; \\ &\tilde{S} + b_i + b_j + z_1 + T_k + pT_i + qT_j; \\ &\tilde{S} + b_{1,2,3} + x + z_4; \\ &\tilde{S} + z_3. \end{aligned} \tag{3.4}$$

Our computer algorithm can then be further applied to analyse the tachyonic sectors arising in type 0 models. The results of this computerised scan are presented in the following section.

3.2 Results of classification

By implementing the projection conditions on the massless fermionic sectors (3.3) and (3.4) in a computer scan we can collect data for the number of fermionic states remaining in the Hilbert Space of a model and see how many are fermion-free and thus type 0. The distribution of the number of fermionic states for a scan of 10^7 is displayed in Figure 1. In this sample we find a total of 24508 which are free of fermionic states.

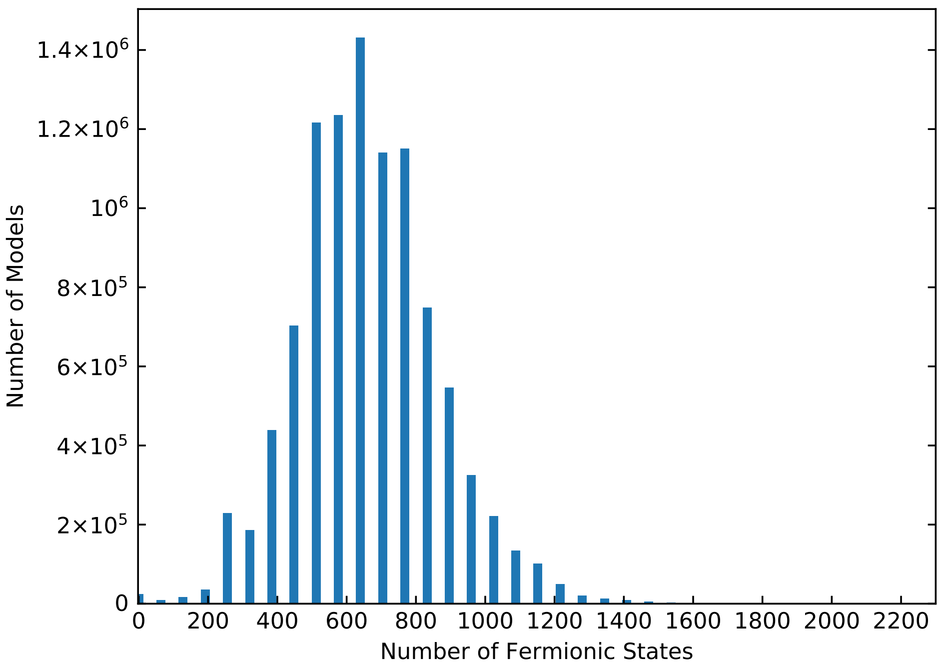

Figure 1: *Frequency plot for the number of fermionic states in a model from a sample of* 10^7 *randomly generated GGSO configurations.*

In order to gather a slightly larger sample of type 0 models in this basis we take a larger sample of 10^8 models which still does not take much computing time. From this sample, we find 245685 type 0 models which gives a probability $\sim 2.46 \times 10^{-3}$ for type 0 vacua in the total space. We now wish to classify these type 0 configurations according to which tachyonic sectors remain in their spectra (along with the model-independent untwisted tachyon), as shown in Table 1.

z_k Tachyon	$z_k + T_i$ Tachyon	$\{\bar{\lambda}^a\} \vert T_i\rangle$ Tachyon	Frequency
0	2	2	42773
1	2	1	33513
1	2	2	19402
1	0	2	17405
1	0	1	17140
1	1	2	12056
0	3	1	11996
3	0	1	7141
0	1	3	6044
3	0	2	5708
1	2	3	5575
1	2	0	5175
1	1	1	5170
1	4	2	5071
0	4	2	5017
0	0	2	4262
0	2	3	4253
1	4	1	4226
3	0	0	3827
0	3	2	3405
0	1	1	3389
1	4	0	3322
1	1	3	2625
1	3	3	2179
0	3	3	1774
0	4	3	1724
3	3	2	1713
1	3	2	1631
3	0	3	1529
3	3	3	1168
1	5	2	913
1	5	3	888
0	4	1	854
1	0	3	840
1	4	3	795
1	6	3	583
0	0	3	308
3	6	3	291

Table 1: *Number of tachyonic sectors for 245685 type 0 $\tilde{S}$-models, where $k = 1, 3, 4$, $i = 1, 2, 3$ and $\bar{\lambda}^a$ is any right-moving oscillator with NS boundary condition.*

These results clearly show that all type 0 models have both the model-independent untwisted tachyon and some combination of at least 2 twisted tachyonic sectors. One

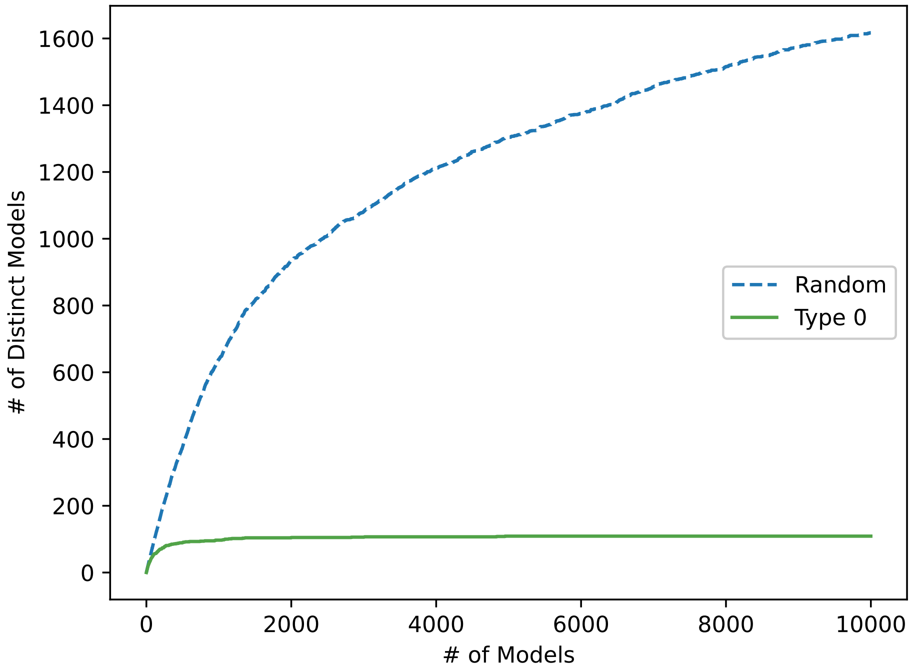

Figure 2: *The degeneracy of models for a random sample of models versus type 0 models for a sample of* 10^4 *models each. We see that the space of type 0 models is indeed highly degenerate.*

might wonder how general this result is since our sample size of 10^8 only covers about $1:687$ models in the total space of GGSO phase configurations. Recalling the 4096 degeneracy factor from the analysis of models in the basis (2.1), we can reasonably suppose that type 0 models are highly constrained and degenerate also in the current construction where the $T_{i=1,2,3}$ are incorporated in the basis.

To see this we took 10^4 type 0 models out from the 245685 total sample and calculated their partition functions and found a total of 109 distinct ones. In Figure 2 a comparison between the degeneracy of these 10^4 type 0 models and those of a random sample of 10^4 models is shown and the number of different type 0 models are seen to converge fast to just over 100, This shows, just as in the earlier case, that the subspace of type 0 vacua is highly symmetric. This result strongly justifies the generality of our results from the 10^8 sample for the tachyonic analysis and makes it highly likely that our 245685 type 0 models from the 10^8 sample captures all such unique models. In Section 5, we will further discuss the structure of these type 0 models from the point of view of the partition function and one-loop vacuum energy.

4 Classification of Type 0 S–Models

Having explored a space of $\tilde{S}$-models in the previous section we now wish to do the same analysis for models deriving from the ten-dimensional $SO(16)\times SO(16)$ non-supersymmetric heterotic string, which we will refer to as S-models since their basis contains the SUSY–

generating vector S. The precise basis we use is:

$$
\begin{aligned}
\mathbb{1} &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, w^{1,\dots,6} \mid \overline{y}^{1,\dots,6}, \overline{w}^{1,\dots,6}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^{1,2,3}, \overline{\phi}^{1,\dots,8}\},\\
S &= \{\psi^\mu, \chi^{1,\dots,6}\},\\
T_1 &= \{y^{1,2}, w^{1,2} \mid \overline{y}^{1,2}, \overline{w}^{1,2}\},\\
T_2 &= \{y^{3,4}, w^{3,4} \mid \overline{y}^{3,4}, \overline{w}^{3,4}\},\\
T_3 &= \{y^{5,6}, w^{5,6} \mid \overline{y}^{5,6}, \overline{w}^{5,6}\},\\
b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \mid \overline{y}^{34}, \overline{y}^{56}, \overline{\eta}^1, \overline{\psi}^{1,\dots,5}\},\\
b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} \mid \overline{y}^{12}, \overline{y}^{56}, \overline{\eta}^2, \overline{\psi}^{1,\dots,5}\},\\
z_1 &= \{\overline{\phi}^{1,\dots,4}\},\\
z_2 &= \{\overline{\phi}^{5,\dots,8}\},
\end{aligned}
\tag{4.1}
$$

which is in fact identical to that used in ref. [24] and the same as that used in the supersymmetric classifications of [31, 32, 33, 7, 9, 10, 11, 12, 29] up to the swap $T_{1,2,3} \to e_{1,\dots,6}$. As noted in these works, we will make regular use of the important linear combination x [35], which appears as the combination

$$x = 1 + S + \sum_{i=1}^{3} T_i + z_1 + z_2 \tag{4.2}$$

in this basis and we further have the combination $b_3 = b_1 + b_2 + x$ to give the generator of the third orbifold plane.

The untwisted gauge bosons in this construction generate a gauge group of $U(1)^6 \times SO(10) \times U(1)^3 \times SO(8)^2$ and the full space of models is again given by the combinations of modular invariant GGSO phase configurations $2^{36} \sim 6 \times 10^{10}$.

An important difference from the $\tilde{S}$-construction is that we do not have a model-independent tachyon as the NS tachyon is automatically projected. This leaves the door open for possible tachyon-free type 0 models.

Now we turn to the massless fermionic vectorial sectors which for our S-models arise from

$$
\begin{aligned}
&S\\
&S + z_1;\\
&S + z_2\\
&S + b_i + x + pT_j + qT_k
\end{aligned}
\tag{4.3}
$$

and the massless fermionic spinorial sectors from

$$
\begin{aligned}
&S + x\\
&S + z_1 + z_2\\
&S + b_i + pT_j + qT_k;\\
&S + b_i + z_1 + x + pT_j + qT_k;\\
&S + b_i + z_2 + x + pT_j + qT_k;
\end{aligned}
\tag{4.4}
$$

where $i \neq j \neq k \in \{1, 2, 3\}$ and $p, q \in \{0, 1\}$. We note that there are more fermionic sectors in this construction than in the $\tilde{S}$ case. In particular, the familiar $\mathbf{16}/\overline{\mathbf{16}}$ of

$SO(10)$ from the sectors $S + b_i + pT_j + qT_k$ and vectorial **10** of $SO(10)$ from the sectors $(S+)b_i + x + pT_j + qT_k$ arise in this construction. However, for the $\tilde{S}$-models they were deliberately chosen to be absent and there are in fact no spinorial fermionic sectors with non-trivial representation under the $SO(10)$ observable gauge group factor.

As in the case of the $\tilde{S}$-models we use a computer algorithm to scan for type 0 configurations where these fermionic sectors are projected out. The results are presented in the following section.

4.1 Results of classification

As in the case of the $\tilde{S}$-models we generate a distribution for the number of fermionic states across a sample of 10^7 randomly generated GGSO phase configurations. This is shown in Figure 3. Comparing this to Figure 1 for the $\tilde{S}$ case we see a more dense distribution with more possible values for the fermionic states.

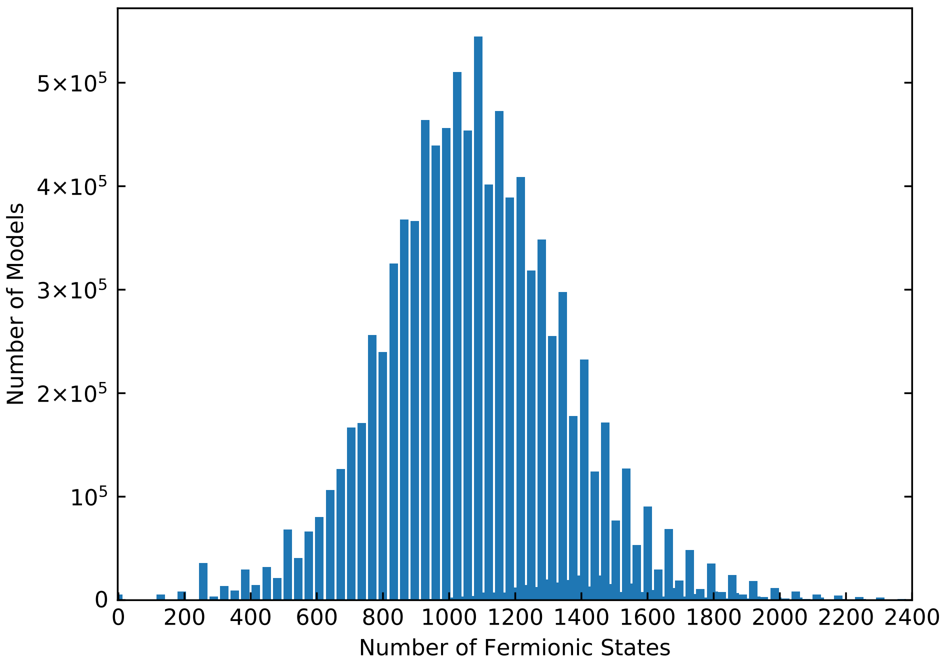

Figure 3: *Frequency plot for the number of fermionic states for S-models from a random sample of 10^7 GGSO configurations.*

As in the $\tilde{S}$ case we generate a larger sample of type 0 models by taking a scan of 10^8 GGSO configurations. From this scan we find 54590 type 0 models which is probability $\sim 5.46 \times 10^{-4}$ which makes them approximately 5 times rarer than in the $\tilde{S}$ case. The likely reason for this and main difference in general between the S case and $\tilde{S}$ case is the already mentioned fact that in the S-models we have more fermionic massless sectors to project.

Despite being rarer we already noted that we do not have any model-independent tachyons for these S-models which leaves the possibility of tachyon-free type 0 vacua open. The data for the numbers of tachyons is shown in Table 2 and we see again that no tachyon-free models exist and that the minimal number of tachyonic sectors is 2, which always includes at least 1 vectorial tachyon of the type $\{\bar{\lambda}^a\} |T_i\rangle$.

z_k Tachyon	$z_k + T_i$ Tachyon	$\{\bar{\lambda}^a\} \|T_i\rangle$ Tachyon	Frequency
1	1	2	11605
1	0	2	10471
1	1	1	4431
1	0	3	4388
2	0	2	4066
2	0	1	3749
1	0	1	3363
1	1	3	2384
1	2	2	1870
2	0	3	1509
1	2	3	1318
1	2	1	1080
2	2	1	871
2	2	2	538
0	1	3	488
0	1	2	454
0	2	1	299
1	3	1	291
1	3	2	290
0	0	2	236
1	3	3	189
0	4	3	151
0	2	3	151
0	2	2	135
0	0	3	135
2	4	1	128

Table 2: *Number of tachyonic sectors for 54860 type 0 S-models, where $k = 1, 2$ and $i = 1, 2, 3$.*

As in the case of the $\tilde{S}$-models, we observe a degeneracy in the space of type 0 models and from the sample of 54590 type 0 S-models, we found just 89 independent partition functions and the same convergence pattern as shown in Figure 2, therefore we can confidently claim that the lack of tachyon-free type 0 models is a generic result in this class of vacua. It is however worth remembering that our models are defined at a free fermionic point in moduli space and so it may be that such tachyonic instabilities may disappear when a model is translated into a bosonic language and defined away from the fermionic point. The process for doing this in the same basis as we are employing here for the S-models is detailed in ref. [24].

5 Misaligned Supersymmetry in Type 0 Models

The partition function of string models encapsulates all the information one knows about its structure, symmetries and spectrum. Thus to fully understand these type 0 models it is essential to get a handle on their partition function. The analysis of the partition function is particularly instrumental in non-supersymmetric constructions, since it gives a complementary tool to count the total number of massless states, and its integration over the fundamental domain corresponds to the cosmological constant.

The partition function for free fermionic theories is given by the integral

$$Z = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z_B \sum_{Sp.Str.} C\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \prod_f Z\begin{bmatrix}\alpha(f)\\ \beta(f)\end{bmatrix}, \tag{5.1}$$

where $d^2\tau/\tau_2^2$ is the modular invariant measure and Z_B is the bosonic contribution arising from the worldsheet bosons. The sum over spin structures represents the contributions from the worldsheet fermions. The integral is over the fundamental domain of the modular group

$$\mathcal{F} = \{\tau \in \mathbb{C} \,|\, |\tau|^2 > 1 \ \wedge \ |\tau_1| < 1/2\}.$$

This ensures that only physically inequivalent geometries are counted. The above expression (5.1) specifically represents the one-loop vacuum energy of our theory and so we may refer to it as the cosmological constant.

The practical way to perform this integral is as presented in [36, 29] using the expansion of the η and θ functions in terms of the modular parameter, or more conveniently in terms of $q \equiv e^{2\pi i\tau}$ and $\bar{q} \equiv e^{-2\pi i\bar{\tau}}$. This leads to a series expansion of the one-loop partition function

$$Z = \sum_{n.m} a_{mn} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3} q^m \bar{q}^n =: \sum_{m,n} a_{mn} I_{mn}. \tag{5.2}$$

Writing the partition function as above ensures that the a_{mn} physically represent the difference between bosonic and fermionic degrees of freedom at each mass level, i.e. $a_{mn} = N_b - N_f$. Further details on the q–expansions and the general calculation of these integrals can be found in Section 6 of [29]. As expected, on-shell tachyonic states, *i.e.* states with $m = n < 0$, have an infinite contribution. On the other hand off–shell tachyonic states may contribute a finite value to the partition function. It is also important to note that modular invariance only allows states with $m - n \in \mathbb{Z}$.

In theories with spacetime supersymmetry, it is ensured that the bosonic and fermionic degrees of freedom are exactly matched at each mass level. That is, we necessarily have that $a_{mn} = 0$ for all m and n, which in turn causes the vanishing of the cosmological constant as one expects. For our non-supersymmetric models, this level-by-level cancellation is not ensured and so such theories in general produce a non-zero value for Λ. It is, however, not obviously clear that they should produce finite partition functions. Such finiteness is however ensured by the mechanism of misaligned supersymmetry [37, 38].

As one expects, the degeneracy of states grows rapidly going up the infinite tower of massive states. This growth, in theory, could counteract the suppression received from the decreasing contributions from the integrals of (5.2) and cause divergences. The mechanism of misaligned supersymmetry, however, causes the states in the massive tower to oscillate between an excess of bosons and an excess of fermions. This behaviour is referred to as boson-fermion oscillation. Instead of cancelling level-by-level as in the

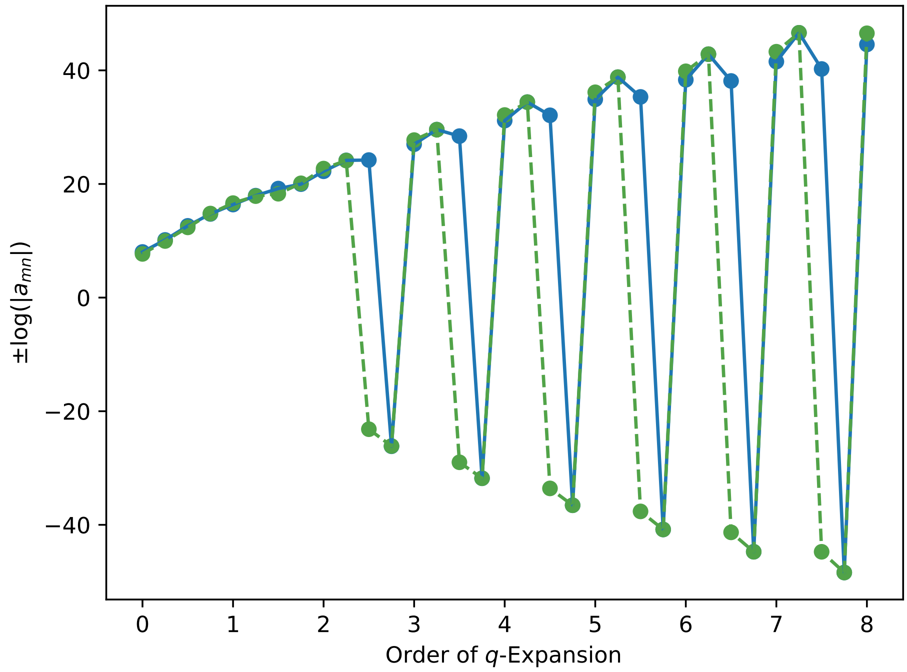

Figure 4: *The boson-fermion oscillation of misaligned supersymmetry for the on-shell states of two $\tilde{S}$ models to 8^{th} order in the q-expansion. The overall sign of* $\pm\log(|\, a_{mn}\,|)$ *is chosen according to the sign of* a_{mn}.

supersymmetric case, the cancellation is misaligned causing the oscillation meaning a large positive contribution is followed by an even larger negative contribution and so on. This mechanism ensures that the partition function of our non-supersymmetric models remains finite.

Such behaviour of the physical spectrum in heterotic theories is well documented in the literature [37, 38, 23, 29]. It however remains to see whether this mechanism is replicated for type 0 heterotic theories described in previous sections. We find that indeed both sets of type 0 models generated by (2.1) and (3.1) exhibit such misalignment of their on-shell massive tower of states. The misalignment pattern appears to have no correlation to whether a specific model has massless fermions or not.

As an example, for the $\tilde{S}$-models of Section 3 we observe the two general patterns shown in Figure 4, or in general a combination of these two. This is true whether or not the choice of GGSO coefficients project all massless fermions. The only observable difference we find for type 0 models from the partition function point of view is the larger value of a_{00}. This is of course fully expected due to the lack of fermions at the massless level. The misalignment pattern is mostly similar for the S–models of Section 4. It is important to note that, unlike for tachyon–free non–supersymmetric theories, this oscillatory behaviour does not result in a finite value for the cosmological constant due to the presence of the physical tachyons described in Sections 3 and 4.

6 Conclusions

The $\mathbb{Z}_2 \times \mathbb{Z}_2$ heterotic string orbifold produced a rich space of phenomenological three generation models and serves as a testing ground to explore how the Standard Model of particle physics may be incorporated in a theory of quantum gravity. The $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold also exhibits a rich symmetry structure which has been of interest from a purely mathematical point of view. It is of course of much further interest to explore the role of these mathematical structures in the phenomenological properties of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold models. In this paper we explored the existence of type 0 models in this class of string compactifications. Type 0 string vacua are those that do not contain any massless fermionic states and have been of interest in other string theory limits, *e.g.* the issue of tree level stability have been studied in the framework of type II orientifolds, whereas other authors have advocated that there is holographic duality of the type 0B string theory and four dimensional non–supersymmetric Yang–Mills theory [39]. In this paper we demonstrated the existence of type 0 $\mathbb{Z}_2 \times \mathbb{Z}_2$ heterotic string orbifolds. We showed that there exist a large degree of redundancy in the space of GGSO projection coefficients when the type 0 restrictions are implemented. We explored the existence of such configurations in several constructions. The one presented in Section 2 correspond to essentially a unique configuration out of a priori 2^{21} discrete GGSO choices. We demonstrated this uniqueness analytically in Section 2.1 as well as by the corresponding analysis of the partition function in Section 2.3. In Sections 3 and 4 we performed a wider classification in $\tilde{S}$–models and S–models, where the first class correspond to compactifications of a tachyonic ten dimensional heterotic string vacuum, whereas the second correspond to compactifications of the ten dimensional non–tachyonic $SO(16) \times SO(16)$. We showed that the type 0 models in both cases contain physical tachyons at the free fermionic point in the moduli space. These vacua are therefore necessarily unstable. we demonstrated the existence of a misalligned supersymmetry in the type 0 models that guarantee the finiteness of the one–loop amplitude, aside from the divergence due to the tachyonic states. Given their rather restrictive structure, the type 0 models may be instrumental in exploring the string dynamics in cosmological scenarios in the spirit of the analysis performed in ref. [24]. Of further interest will be to explore vacua with a minimal number of bosonic states. Whereas the string models always contain massless bosons from the untwisted sector, we may look for models in which the bosonic states from twisted sectors are projected out. Another possibility is to look for models in which the fermions from the observable sectors are balanced by scalars charged under the hidden sector.

Acknowledgments

The work of VGM is supported in part by EPSRC grant EP/R513271/1. The work of BP is supported in part by STFC grant ST/N504130/1.

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