Bayesian Model Updating of Reliability Parameters using Transitional Markov Chain Monte Carlo with Slice Sampling

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This research work presents a comparison of the performances between the Transitional Markov Chain Monte Carlo (TMCMC) and the TMCMC-Slice algorithm. Transitional Markov Chain Monte Carlo (TMCMC) algorithm is a popular approach in the estimation of epistemic parameters from Bayesian Inference. By sampling from a series of intermediate probability density functions, the sampler can generate samples from any target probability density functions. In the TMCMC algorithm, the Metropolis-Hastings sampling algorithm is adopted to generate samples from the intermediate probability density functions whilst in TMCMC-Slice algorithm, the Slice sampling algorithm is adopted to do so. In this work, the performance of the TMCMC-Slice over the TMCMC sampler is investigated for different number of samples. For this purpose, the two samplers are then adopted in a reliability parameter update of the Emergency Diesel Generator system that is employed in Daya Bay Nuclear Power Plant. The results show that while the TMCMC-Slice approach is able to produce slightly more precise estimates compared to the TMCMC approach, the computational time evolved in the case of the former was significantly greater compared to the latter. In addition, the Two-sample Kolmogorov–Smirnov test also provided sufficient evidence to reject the null hypothesis that the samples obtained from both techniques are from the same distribution at 5% level of significance.

Keywords: Nuclear Power Plant, Reliability, Bayesian Model Updating, Slice Sampling, Transitional Markov Chain Monte Carlo .

1. Introduction

Transitional Markov Chain Monte Carlo (TMCMC) has been shown to be a robust sampler to generate realisation from difficult distribution including multi-modal Probability Distribution Functions (PDFs), very peaked PDFs, and PDFs with flat manifold according to Ching and Chen (2007). TMCMC generates samples from simpler intermediate distributions. Currently, Metropolis-Hastings algorithm is adopted to perform re-sampling of samples via Markov Chain Monte Carlo. Here, we adopt the use of the TMCMC-Slice sampling algorithm proposed by Zhang and Yang (2014) to generate samples from the intermediate distributions and the performance as well as the results obtained from these two samplers are critically compared with different number of samples. Both renditions of the TMCMC samplers are applied on a Bayesian model update problem in the form of updating the reliability parameters for an Emergency Diesel Generator system of the Daya Bay Nuclear Power Plant. In the work by Zhang and Yang (2014), however, a similarity test between the distributions of the samples obtained using TMCMC and TMCMC-Slice sampling method was not addressed. As such, in this work, the similarity test will be done using the Two-sample Kolmogorov–Smirnov (KS) test at 5% level of significance for each sample size used. This is done to compare the distributions of the samples and to investigate if there is sufficient evidence to reject the null hypothesis that their respective samples follow the same distribution. Details to the Two-sample KS Test can be found in the literatures by Massey (1951), Miller (1956), and Marsaglia et al. (2003).

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1.1. *Bayesian Model Updating*

The concept of Bayesian model updating, introduced by Beck and Katafygiotis (1998), is based on the well-known Bayes' Rule conceptualised by Bayes (1958). Its mathematical formulation is as follows:

$$P(\boldsymbol{\theta}|\boldsymbol{D}) = \frac{P(\boldsymbol{D}|\boldsymbol{\theta}) \cdot P(\boldsymbol{\theta})}{P(\boldsymbol{D})} \tag{1}$$

whereby $\boldsymbol{\theta}$ represents the vector of parameters of interest, $\boldsymbol{D}$ represents the vector of observed data, $P(\boldsymbol{\theta})$ represents the Prior distribution function, $P(\boldsymbol{D}|\boldsymbol{\theta})$ represents the Likelihood function, $P(\boldsymbol{\theta}|\boldsymbol{D})$ represents the Posterior distribution, and $P(\boldsymbol{D})$ is the Evidence or the normalising constant to ensure that the Posterior distribution integrates to 1. Often, the denominator term, $P(\boldsymbol{D})$, is neglected as it is only a numerical constant. Thus, Equation (1) can be re-expressed as such:

$$P(\boldsymbol{\theta}|D) \propto P(D|\boldsymbol{\theta}) \cdot P(\boldsymbol{\theta}) \tag{2}$$

1.2. *Transitional Markov Chain Monte Carlo*

In addressing Bayesian model updating problems, the Posterior distribution, $P(\boldsymbol{\theta}|\boldsymbol{D})$, is usually known up to a normalising constant as suggested by Equation (2). In such cases, direct Monte Carlo simulations would not be able to perform sampling effectively. One way to sample from such distributions would be through the use of TMCMC method, a sampling technique proposed by Ching and Chen (2007). It is an iterative sampling algorithm which is used to obtain samples from complex Posterior distributions through a series of intermediate "Transitional" distributions, P_j, defined as such:

$$P_j(\boldsymbol{\theta}|\boldsymbol{D}) \propto P(\boldsymbol{D}|\boldsymbol{\theta})^{\beta_j} \cdot P(\boldsymbol{\theta}) \tag{3}$$

whereby j denotes the iteration number taking integer values from 0 to m (for $m > 0$), and β_j takes values such that $\beta_0 = 0 < \beta_1 < ... < \beta_{m-1} < \beta_m = 1$. This implies that the "Transitional" distribution, P_j, is such that it gradually transits from the Prior distribution initially to the Posterior distribution in the final iteration m.

The workings of the TMCMC algorithm can be summarized as such. Given N samples to be obtained from the Posterior distribution, for each iteration j, N samples are re-sampled from P_{j-1} via N independent single-step MCMC sampling. In essence, each of the N samples from P_{j-1}, in the previous iteration $j-1$, would serve as starting point of the MCMC chain and each MCMC chain would generate 1 sample out of it for P_j in the current iteration j. This procedure would repeat itself from $j = 1$ to $j = m$. According to Ching and Chen (2007), the MCMC sampling is done through the use of the Metropolis-Hastings algorithm whose tuning parameter is defined by the covariance matrix, $\boldsymbol{\Sigma}_j$:

$$\boldsymbol{\Sigma}_j = \beta^2 \sum_{k=1}^{N_j} \frac{\omega(\theta_{j,k})}{N_j \cdot S_j} \{\theta_{j,k} - \bar{\theta}_j\} \times \{\theta_{j,k} - \bar{\theta}_j\}^T \tag{4}$$

whereby

$$\bar{\theta}_j = \frac{\sum_{r=1}^{N} \omega(\theta_{r,j})\theta_{r,j}}{\sum_{r=1}^{N} \omega(\theta_{r,j})} \tag{5}$$

$$S_j = \frac{1}{N_j} \sum_{k=1}^{N_j} \omega(\theta_{j,k}) \tag{6}$$

Here, $\theta_{j,k}$ represents the k^{th} sample in the j^{th} iteration, $\omega(\theta_{j,k})$ denotes the statistical importance weight of $\theta_{j,k}$, S_j denotes the mean statistical importance weight in the j^{th} iteration, and β is the scaling factor of the tuning parameter whose optimum value is determined to be 0.2. The workings of the TMCMC algorithm is as follows:

- Step 1: At iteration $j = 0$, a sample set of size N is obtained from the Prior distribution, $P(\boldsymbol{\theta})$, via direct Monte-Carlo sampling to ensure that the obtained samples are evenly spread about the sample space. (Note: Direct Monte-Carlo sampling can be employed at this stage because the Prior distribution is usually a well-defined distribution function.)
- Step 2: A small increment, $\Delta\beta$, is added to β_j such that $\beta_{j+1} = \beta_j + \Delta\beta$. $\Delta\beta$ has to be small such that the Coefficient of Variation (COV) of $P(D|\boldsymbol{\theta})^{\beta_{j+1}-\beta_j}$ is close to 100%. This is done so as to ensure that the transition from P_j to P_{j+1} is gradual and smooth.
- Step 3: For each iteration j, the plausibility weight of the individual k^{th} sample, $\omega(\theta_{j,k})$, would be calculated via the following equation:

$$\omega(\theta_{j,k}) = P(\boldsymbol{D}|\theta_k)^{\beta_{j+1}-\beta_j} \tag{7}$$

- Step 4: In the same iteration j, the Resampling procedure is executed to obtain N samples from P_j to form the sample set for P_{j+1}. Resampling is performed such that the individual $\theta_{j+1,k}$ sample is sampled from $\theta_{j,k}$ with probability $\frac{\omega(\theta_{j,k})}{\sum_{k=1}^{N} \omega(\theta_{j,k})}$. This is achieved through the use of MCMC sampling via the Metropolis-Hastings algorithm with tuning parameter $\boldsymbol{\Sigma}_j$.

- Step 5: Repeat Steps (2) to (4) until iteration $j = m$ and obtain N samples from the Posterior: $P_m = P(\boldsymbol{\theta}|\boldsymbol{D})$.

For more information, the reader is referred to the literature by Ching and Chen (2007).

2. Research Methodology

For this research, we will be looking into the performance of the TMCMC-Slice sampling algorithm proposed by Zhang and Yang (2014) compared against the TMCMC sampling algorithm which adopts the Metropolis-Hastings algorithm to execute MCMC sampling.

The Metropolis-Hastings algorithm, devised by Hastings (1970), is a random-walk algorithm which provides a selection criteria to which samples are chosen during sampling. The samples are chosen through the use of a Proposal distribution, $q(\boldsymbol{\theta})$, to select proposed samples as seeds for the next sample of the Markov Chain. The choice of the Proposal distribution is usually such that it is symmetric such as the Normal distribution. The width of this Proposal distribution, σ_p, is also known as the Tuning parameter as the choice of σ_p would affect the acceptance rate of samples for the Markov Chain, thereby having an impact on the performance of the algorithm. As a guide, the choice of σ_p should be one which achieves an acceptance rate close to 0.234 as proposed by Roberts and Rosenthal (2001). In the context of TMCMC, this value of σ_p is the covariance matrix $\boldsymbol{\Sigma}_j$. A summary of the workings of the Metropolis-Hastings algorithm is as follows:

- Step 1: Starting from a random sample, θ_1, the symmetric Proposal distribution would then select the next random sample, θ_2. For example, if a Normal distribution is chosen as the Proposal distribution, θ_2 will be chosen randomly from that distribution with mean θ_1 and a defined value of σ_p assigned by the user.
- Step 2: Upon choosing θ_2, the value of the Posterior distribution at θ_2, $P(\theta_2|\boldsymbol{D})$, is calculated. This value is then compared to $P(\theta_1|\boldsymbol{D})$ by taking the ratio between these two quantities to determine the acceptance ratio α:

$$\alpha = \frac{P(\theta_2|\boldsymbol{D})}{P(\theta_1|\boldsymbol{D})} \tag{8}$$

- Step 3: A random number, r, is drawn from a Uniform distribution ranging between 0 and 1. If the value of α is greater than r, θ_2 will be accepted as the new sample and the process repeats from Step 1 with θ_2 as the seed. Otherwise, θ_2 will be rejected and Step 1 is repeated using θ_1 as the seed.

TMCMC-Slice sampling algorithm, on the other hand, adopts the Slice sampling technique to perform MCMC. Slice sampling, according to Neal (2003), is a method which stems from the assumption that a sample, θ_i, can be obtained through uniform sampling of the region under the curve representing the Posterior distribution. This is done by introducing an auxiliary variable, y, as seen in Figure 1 which will be explained subsequently. An auxiliary variable is one which does not exist in the model initially, but is introduced so as to facilitate the process of sampling. The workings of the Slice sampling method to obtain samples can be summarised as such:

- Step 1: Initiate the sampler by choosing an initial sample from the Posterior distribution, θ_0. From there the value of the Posterior distribution at θ_0 is evaluated, $P(\theta_0|\boldsymbol{D})$.
- Step 2: The auxiliary variable, y, is sampled from a uniformly between 0 and $P(\theta_0|\boldsymbol{D})$.
- Step 3: A horizontal line is drawn across the Posterior distribution as represented by the blue line in Figure 1 such that it cuts across the Posterior distribution function at $P(\boldsymbol{\theta}|\boldsymbol{D}) = y$. From there, a sample, θ_i, is obtained via uniform sampling across region of the line whereby $P(\boldsymbol{\theta}|\boldsymbol{D}) > y$.
- Step 4: Repeat Steps 1 to 3 for new values of θ_0 until sufficient samples are obtained.

From the workings described above, it can be see that one key advantage of the Slice sampling method is that it is highly automated in the sense that unlike the Metropolis-Hastings algorithm, there is no need for a Proposal distributions. This eradicates the need of a Tuning parameter thereby removing the need by the TMCMC algorithm to compute $\boldsymbol{\Sigma}_j$, thus, providing the motivation to observe the performance of the TMCMC algorithm when Slice sampling is used in the MCMC sampling step. To do so, we will apply this TMCMC-Slice sampling algorithm on a Bayesian model update problem by Zubair and Zhijian (2013) which will be discussed in Section 3.

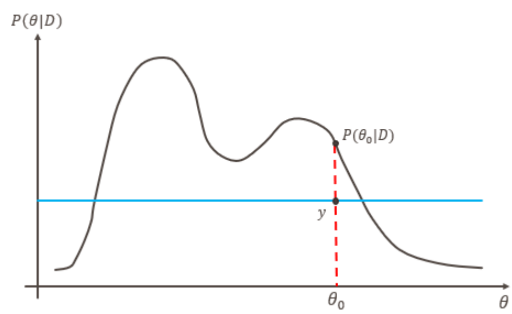

Fig. 1. An illustration of how Slice sampling is performed on the Posterior distribution, $P(\boldsymbol{\theta}|\boldsymbol{D})$.

3. Application Case Study: Daya Bay Nuclear Power Plant

3.1. *Description*

In the study by Zubair and Zhijian (2013), the Bayesian model updating framework is adopted to update the reliability parameters used to perform Living Probabilistic Safety Assessment (LPSA) on the EDG of Daya Bay NPP. The Daya Bay NPP consists of two 900 MW Pressurized Water Reactors, whose design originated from France, and has been in commercial operation within China since the year 1994. Each nuclear power generating units of diesel generator system is made up of two identical series A (LHP) components and two identical series B (LHQ) components. These diesel generator sets, together with their related auxiliary equipment, are installed in separate factories. In the event of a loss of electricity, the EDG would then supply 6.6 KV of power to both series A and B. Each diesel generator set consists of the following:

(i) Two diesel engines along with its related auxiliary equipment;
(ii) A generator along with the excitation and protection equipment;
(iii) Auxiliary systems: fuel system, lubricating system, engine cooling and preheating system, air starting system, combustion air and engine exhaust systems, ventilation systems, and diesel engine plant instrumentation measurement and control equipment.

This presents the need to update the reliability parameters for the EDG to predict failure so as to be able to perform predictive maintenance on the components. For this purpose, 4-years worth of operational data for the EDG has been collected and presented in the literature by Zubair and Zhijian (2013). Table 1 provides a summary of the EDG data that was obtained.

Table 1. Summary of EDG data obtained from literature by Zubair and Zhijian (2013).

Name	Data
Start of data collection	1-Jan-1997
End of data collection	31-Dec-2001
Total operation time (hours)	579.37
Number of starts	290
Number of operational failure(s)	1
Total number of demand failure(s)	7

Source: Equipment failure data is sampled from Experience Feedback System (EFS).

3.2. *Bayesian Model Updating of Parameters*

In their work, Zubair and Zhijian (2013) performed reliability parameter update for the following two parameters:

- Demand failure probability, θ
- Operational failure rate, λ

According to IAEA (1992), a failure is defined as the loss of an ability of a system to perform its required function. Thus, the demand failure probability can be interpreted as the probability of the system failing to commence operation when required while the operational failure rate can be interpreted as number of times the system fails to operate within the total operating time. The latter can also be understood as the rate of failure per unit operational time of the system.

The Bayesian set-up to estimate the demand failure probability, θ, is as follows. The Prior distribution of θ is set to follow a Beta distribution with shape parameters α and β. Its density function is expressed as:

$$P(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \cdot \theta^{\alpha-1} \cdot (1-\theta)^{\beta-1} \quad (9)$$

whereby $\alpha = 23$ and $\beta = 6717$ according to the literature. The Likelihood function is chosen to be a Binomial distribution, due to the data consisting of k failures in n demands, whose density function is expressed as:

$$P(X = k|\theta) = \frac{n!}{k!(n-k)!} \cdot \theta^k \cdot (1-\theta)^{n-k} \quad (10)$$

whereby X is a random variable denoting the number of failure, k is the number of failures, and n is the total demand. With reference to Table 1, $n = 290$ and $k = 7$.

Making use of the conjugate relation between the Prior distribution and the Likelihood function, the Posterior distribution can be determined analytically to follow a Beta distribution with shape parameters α_{post} and β_{post}:

$$P(\theta|X = k) \propto \theta^{\alpha_{post}-1} \cdot (1-\theta)^{\beta_{post}-1} \quad (11)$$

whereby $\alpha_{post} = k+\alpha$ and $\beta_{post} = n-k+\beta$. In the literature, α_{post} was determined to be 30 while β_{post} was determined to be 7000. From there, Zubair and Zhijian (2013) calculated the mean demand failure probability, θ_{mean}, to be 0.0043 per hour using the following equation to obtain the mean value from a Beta distribution:

$$\theta_{mean} = \frac{\alpha_{post}}{\alpha_{post}+\beta_{post}} \quad (12)$$

The Bayesian set-up to estimate the operational failure rate, λ, is as follows. The Prior distribution of λ is set to follow a Gamma distribution with shape and rate parameters μ and ν respectively. Its density function is expressed as:

$$P(\lambda) = \frac{\nu^{\mu}}{\Gamma(\mu)} \cdot \lambda^{\mu-1} \cdot exp^{-\lambda\nu} \tag{13}$$

whereby $\mu = 22$ and $\nu = 2920.63$ according to the literature. The Likelihood function is chosen to be a Poisson distribution whose density function is expressed as:

$$P(Y = j|\lambda) = \frac{exp^{-\lambda t} \cdot (\lambda t)^{j}}{j!} \tag{14}$$

whereby Y is a random variable denoting the operational failure time, j is the operational failure time, and t is the total operation time. With reference to Table 1, $j = 1$ and $t = 579.37$.

Making use of the conjugate relation between the Prior distribution and the Likelihood function, the Posterior distribution can be determined analytically to follow a Gamma distribution with shape and rate parameters $\mu_{post} = 23$ and $\nu_{post} = 3500$ respectively:

$$P(\lambda|Y = j) = \lambda^{\mu_{post}-1} \cdot exp^{-\lambda\nu_{post}} \tag{15}$$

whereby $\mu_{post} = j + \mu$ and $\nu_{post} = t + \nu$. In the literature, μ_{post} was determined to be 23 while ν_{post} was determined to be 3500. From there, Zubair and Zhijian (2013) calculated the mean operational failure rate, λ_{mean}, to be 0.0067 per hour using the following equation to obtain the mean value from a Gamma distribution:

$$\lambda_{mean} = \frac{\mu_{post}}{\nu_{post}} \tag{16}$$

4. Results and Discussions

4.1. *Estimation of Demand Failure Probability, θ*

The TMCMC-Slice algorithm was implemented on the Posterior distribution, $P(\theta|X = k)$, alongside with the TMCMC algorithm. Sample sizes of $N = 500$, 1000, 5000, and 10000 were obtained from the Posterior distribution with 0 Burn-in length. The sampling process executed by both samplers took 2 iterations each. The resulting sample histograms for each value of N from the respective samplers are superimposed for comparison purposes and are presented in Figures 2, 3, 4, and 5. The demand failure probability, θ, obtained from literature and Electric de France (EDF) are presented in Table 2 while the numerical results obtained from TMCMC and TMCMC-Slice algorithms are presented in Tables 3 and 4.

From the numerical results presented in Tables 3 and 4, the Coefficient of Variation (CoV) of

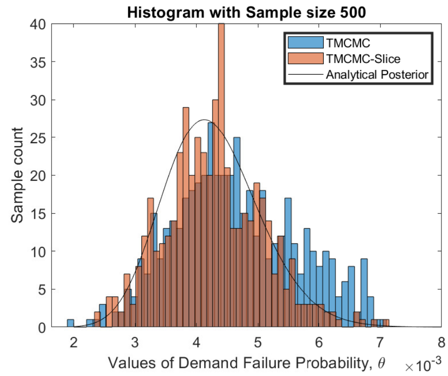

Fig. 2. Resulting histogram of θ samples obtained with sample size $N = 500$.

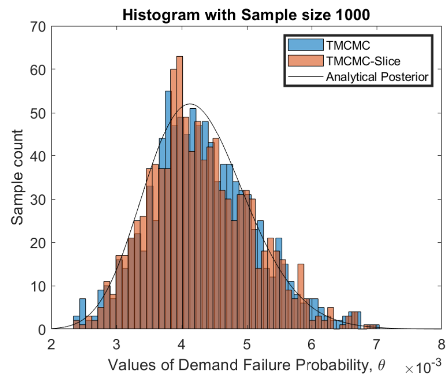

Fig. 3. Resulting histogram of θ samples obtained with sample size $N = 1000$.

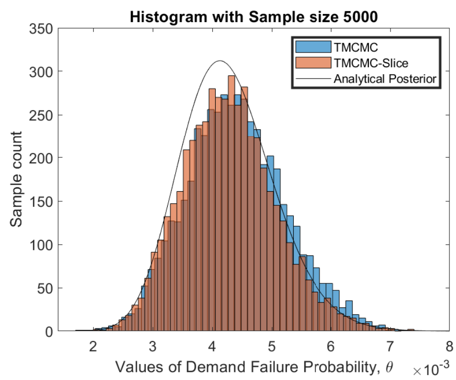

Fig. 4. Resulting histogram of θ samples obtained with sample size $N = 5000$.

Table 2. Demand failure probability, θ, values obtained from literature and EDF.

Name	Demand Failure Probability, θ
Literature	4.30×10^{-3}
EDF	3.40×10^{-3}

Source: Zubair and Zhijian (2013), and Electric de France (EDF).

Table 3. Numerical results from TMCMC sampler.

N	θ_{mean}	θ_{std}	CoV (%)	Time (sec)
500	4.65×10^{-3}	1.01×10^{-3}	21.70	1.70
1000	4.32×10^{-3}	8.16×10^{-4}	18.87	3.32
5000	4.41×10^{-3}	8.29×10^{-4}	18.81	16.68
10000	4.36×10^{-3}	8.39×10^{-4}	19.24	33.49

Table 4. Numerical results from TMCMC-Slice sampler.

N	θ_{mean}	θ_{std}	CoV (%)	Time (sec)
500	4.29×10^{-3}	8.01×10^{-4}	18.69	4.58
1000	4.29×10^{-3}	7.92×10^{-4}	18.48	9.16
5000	4.28×10^{-3}	7.82×10^{-4}	18.25	45.52
10000	4.27×10^{-3}	7.78×10^{-4}	18.21	94.15

the estimate of θ is consistently lower for the TMCMC-Slice than TMCMC algorithm. This indicates a relatively higher degree of precision of the estimate by the former and this observation is supported by the superimposed histograms from both samplers in Figures 2 to 4 where it can be observed that the histogram in red shows a smaller spread in general compared to the histogram in blue. The Two-sample KS test is then performed for each value of N at 5 % significance level and the resulting p-values for $N = 500$, 1000, 5000, and 10000 are 6.77×10^{-8} , 1.30×10^{-1}, 4.06×10^{-11}, and, 9.52×10^{-17} respectively. This indicates that the null hypothesis is rejected for all values of N , except for $N = 1000$, implying that the samples do not follow the same distribution despite having a similar histogram shape profiles as illustrated in Figures 2 to 5. There is insufficient evidence for $N = 1000$ to reject the null hypothesis.

One notable observation is that the computational time elapsed for the TMCMC sampler with Metropolis-Hastings algorithm is significantly shorter compared to that for the TMCMC sampler with Slice sampling algorithm. This comes despite the latter not requiring the need for a Proposal distribution as well as the need to compute the tuning parameter, $\mathbf{\Sigma}_j$, at each iteration j. One reason to account for this is due to the difference in the number of sampling sequence undertaken by the Slice sampling algorithm and the Metropolis-Hastings sampling algorithm. The slice sampling algorithm involves the need undergo two separate sampling sequences before obtaining a new sample, θ_i from the starting seed sample of the chain, θ_0. As mentioned earlier in Section 2, the Slice sampling algorithm first needs to sample for an auxiliary variable, y, uniformly between the values of 0 and the Posterior value at the starting seed sample, $P(\theta_0|D)$, and then sample across the regions of the sample space, θ, where $P(\theta_0|D) > y$. The Metropolis-Hastings algorithm, on the other hand, undertakes just one sequence of sampling from the Proposal distribution, centered about the seed sample θ_0, to obtain the new proposed sample, θ_i, and then passing θ_i through the acceptance criteria to determine the next sample of the chain.

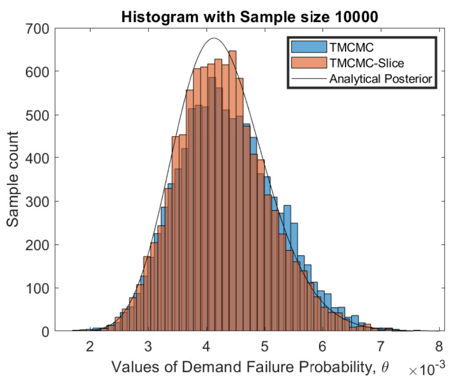

Fig. 5. Resulting histogram of θ samples obtained with sample size $N = 10000$.

4.2. *Estimation of Operational Failure Rate, $\boldsymbol{\lambda}$*

The TMCMC-Slice algorithm was implemented on the Posterior distribution, $P(\lambda|Y = j)$, alongside with the TMCMC algorithm. Sample sizes of $N = 500$, 1000, 5000, and 10000 were obtained from the Posterior distribution with 0 Burn-in length. The sampling process executed by both samplers took 1 iterations each. The resulting sample histograms for each value of N from the respective samplers are superimposed for comparison purposes and are presented in Figures 6, 7, 8, and 9. The operational failure rate, λ, obtained from literature and Electric de France (EDF) are presented in Table 5 while the numerical results obtained from TMCMC and TMCMC-Slice algorithms are presented in Tables 6 and 7.

From the numerical results presented in Tables

6 and 7, the Coefficient of Variation (CoV) of the estimate of λ is consistently lower for the TMCMC-Slice than TMCMC algorithm. This indicates a relatively higher degree of precision of the estimate by the former and this observation is supported by the superimposed histograms from both samplers in Figures 6 to 9 where it can be observed that the histogram in red shows a smaller spread in general compared to the histogram in blue. The Two-sample KS test is then performed for each value of N at 5 % significance level

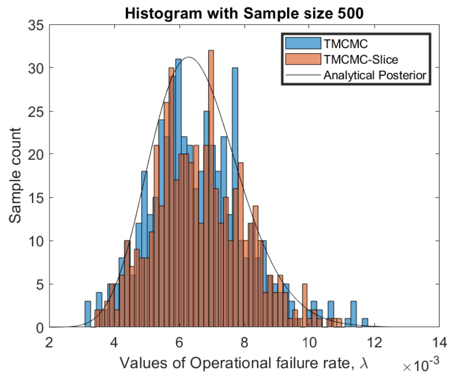

Fig. 6. Resulting histogram of λ samples obtained with sample size $N = 500$.

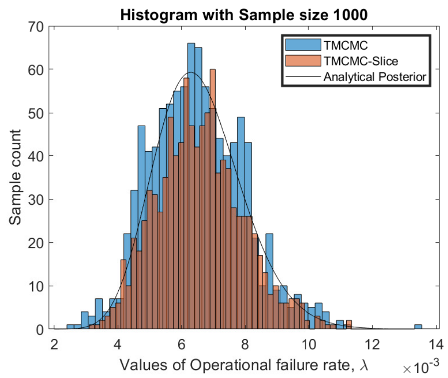

Fig. 7. Resulting histogram of λ samples obtained with sample size $N = 1000$.

Table 5. Operational Failure Rate, λ, values obtained from literature and EDF.

Name	Operational Failure Rate, λ (hr^{-1})
Literature	6.70×10^{-3}
EDF	7.70×10^{-3}

Source: Zubair and Zhijian (2013), and Electric de France (EDF).

Table 6. Numerical results from TMCMC sampler.

N	λ_{mean} (hr^{-1})	λ_{std} (hr^{-1})	CoV (%) (%)	Time (sec)
500	6.56×10^{-3}	1.49×10^{-3}	22.79	2.13
1000	6.51×10^{-3}	1.48×10^{-3}	22.77	4.23
5000	6.49×10^{-3}	1.43×10^{-3}	22.03	21.43
10000	6.55×10^{-3}	1.44×10^{-3}	22.01	42.77

Table 7. Numerical results from TMCMC-Slice sampler.

N	λ_{mean} (hr^{-1})	λ_{std} (hr^{-1})	CoV (%)	Time (sec)
500	6.61×10^{-3}	1.40×10^{-3}	21.18	3.55
1000	6.57×10^{-3}	1.34×10^{-3}	20.47	7.03
5000	6.57×10^{-3}	1.37×10^{-3}	20.93	35.23
10000	6.57×10^{-3}	1.36×10^{-3}	20.74	71.09

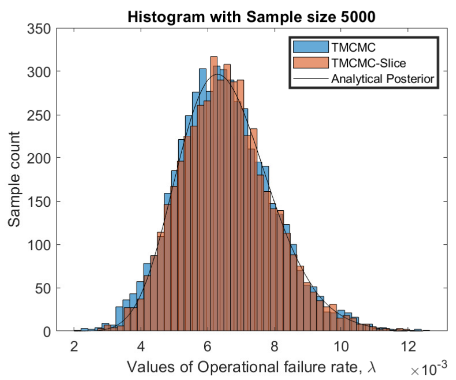

Fig. 8. Resulting histogram of λ samples obtained with sample size $N = 5000$.

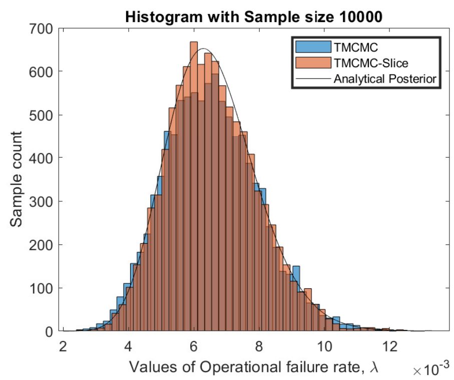

Fig. 9. Resulting histogram of λ samples obtained with sample size $N = 10000$.

and the resulting p-values for $N = 500$, 1000, 5000, and 10000 are 7.09×10^{-1} , 1.30×10^{-1}, 1.02×10^{-2}, and 2.50×10^{-3} respectively. This indicates that the null hypothesis is rejected for the case of $N = 5000$ and 10000 whilst there is insufficient evidence to reject the null hypothesis for the case of $N = 500$ and 1000. This implies that for $N = 5000$ and 10000, there is sufficient evidence to suggest that the samples do not follow the same distribution despite having a similar histogram shape profiles as illustrated in Figure 8.

Like in the previous study, the computational time elapsed for the TMCMC sampler with Metropolis-Hastings algorithm is significantly shorter compared to that for the TMCMC sampler with Slice sampling algorithm. The reason behind this is as explained in Section 4.1.

5. Conclusion

In this paper, the TMCMC-Slice sampler devised by Zhang and Yang (2014) is adopted applied on a Bayesian model updating problem presented by Zubair and Zhijian (2013) whereby the reliability parameters, demand failure probability and operational failure rate, of the EDG in Daya Bay NPP are to be updated. For each of the two reliability parameters, the TMCMC sampler algorithm devised by Ching and Chen (2007) is implemented alongside for comparison. The comparison was done on the basis of the accuracy of the estimated results with respect to the literature values, the Coefficient of Variation (CoV) of the estimation, the computational time elapsed , as well as the closeness of the distributions of the samples obtained from both samplers.

In both studies, it can be seen that both TMCMC samplers yield results which are close to the determined values by Zubair and Zhijian (2013). However, for both reliability parameters, the degree of the precision associated with the results obtain from the TMCMC-Slice sampler is relatively higher compared to the TMCMC sampler which indicates an advantage the former has over the latter for this case study. In addition, it was also observed that the total computation time elapsed for the TMCMC-Slice sampler was found to be longer compared to the TMCMC algorithm in both studies. This is attributed to the fact that the Slice sampling algorithm has to also automatically adjust the step-size of the random-walk procedure whereas for the Metropolis-Hastings algorithm, the step-size is already manually selected by the user. This presents a key limitation of the TMCMC-Slice sampler. Furthermore, the Two-sample KS Test indicated that there is sufficient evidence to reject the null-hypothesis that the samples from both algorithms follow the same distribution.

Further works which can be done from this point on would include looking into ways to improve the computational efficiency of the TMCMC-Slice algorithm as well as accounting for the reason as to why the Two-sample KS Test provided sufficient evidence to reject the null hypothesis.

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