

Single equivalent approximation for multiple contact structure-borne sound sources in buildings

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The work reported in the paper addresses structure-borne sound transmission between multiple contact sources and non homogeneous plate receiver structures. This study concentrates on a practical method of predicting the installed structure-borne sound power from mechanical installations in lightweight buildings. The structure-borne sound power is a function of source activity, source mobility and receiver mobility, and all three quantities must be known to some degree. It is rarely practical to consider all transmission paths individually and in detail, and therefore, reduced data sets and less computationally demanding procedures are proposed. The paper examines how source data can be used to assemble single equivalent values, using spatial averages and magnitudes. Single equivalent values of receiver mobility also are proposed for lightweight, point-connected ribbed plate constructions. In case studies, the single equivalent values are used for predicting the structure-borne power in the installed condition.

1. Introduction

Lightweight building constructions, composed of e.g. composite or timber frame elements, offer economic and environmental advantages over heavyweight constructions. However, heavyweight constructions offer advantages with respect to airborne and structure-borne sound, which are more readily controlled, mainly by reason of high mass and stiffness. Careful design

of lightweight constructions is necessary to avoid excessive noise and vibration from installed machinery. However, practical methods of calculating structure-borne sound transmission are lacking.

In general, machines impart structure-borne power into connected and supporting structures through all contacts. The general expression of complex power for multi-point and multi component excitation, such as in [1], is given by:

$$\bar{W} = \bar{v}_{sf}^H [\bar{Y}_S + \bar{Y}_R]^{-H} [\bar{Y}_R] [\bar{Y}_S + \bar{Y}_R]^{-1} \bar{v}_{sf} \quad (1)$$

where \bar{v}_{sf} is the r.m.s. source free velocity vector, \bar{Y}_S and \bar{Y}_R is the complex mobility matrices of the source and the receiver, respectively. H is the Hermitian transpose, where $[]^H = []^{T*}$ and $[]^{-H} = [[]^{T*}]^{-1}$. The total power is the sum of the complex products of the forces and moments and their associated translational and rotational responses at the contacts of interest. Consideration of all transmission paths is rarely practical. Reduced data sets and less computationally demanding procedures are required, which give the total power to the accuracy appropriate for the particular engineering design requirement.

As indicated in equation (1), the dynamics of both the source and the receiving structure must be considered and they are seldom known in detail. However, reduced forms of source activity, source mobility and receiver mobility may not yield sufficiently accurate predictions because of large differences between contact conditions. Loss of phase information, resulting from using magnitudes of the source and receiver quantities, introduces uncertainties in the predicted structure-borne sound power [2]. Despite these potential penalties, single values are now considered, in the development of reduced measurement and prediction methods.

In the work described in this paper, full measurement data sets, of two sources and a receiver, are systematically reduced and comparisons made between the resultant approximate estimates of installed power and values obtained with full data. Single equivalent source and receiver values are proposed and estimates given of the resultant accuracy in the predicted power.

2. Single equivalent excitation

2.1 *Single equivalent source activity*

Source activity can be expressed as a free velocity vector, see equation (1), or as a blocked force vector. In seeking a single value of source activity, reference is made to indirect measurement methods, using reception plates [2]. If a source is attached to a thin high-mobility plate, it can be demonstrated that the source free velocity is obtained indirectly as the sum of

the squares of the free velocity over the contacts $\sum_{i=1}^N |v_i|^2$. This form of single value is used

throughout the following discussion. It is interesting to note that if the source is attached to a thick low-mobility plate, then the source blocked force is obtained indirectly as the sum of the

squares of the blocked force over the contacts $\sum_{i=1}^N |F_{bi}|^2$.

2.2 *Effective mobility*

Rather than base the discussion on the matrix representation of the installed power, given in equation (1), it is possible to preserve the simplicity of a single contact single component case by reference to the concept of the effective mobility [3, 4]. The effective mobility is based on the premise that the transmitted power can be obtained for each contact between the source and

receiving structure and each component of excitation, but where the influence of all other contacts and components is included.

For the case of mechanical and water installations in heavyweight buildings, forces perpendicular to the receiving structure are dominant [5, 6]. For ribbed and framed plate structures, associated with lightweight buildings, moments can assume importance at locations close to structural discontinuities [7, 8]. In Figure 1 is shown the moment induced powers for several locations of a fan unit on a timber joist floor. The fan unit and floor are described in detail later. The powers are normalised with respect to the power from the perpendicular forces. The moments M_x and M_y are about axes in the plane of the floor plate and parallel to the edges. The powers are calculated from measured data, obtained for the fan unit and floor separately.

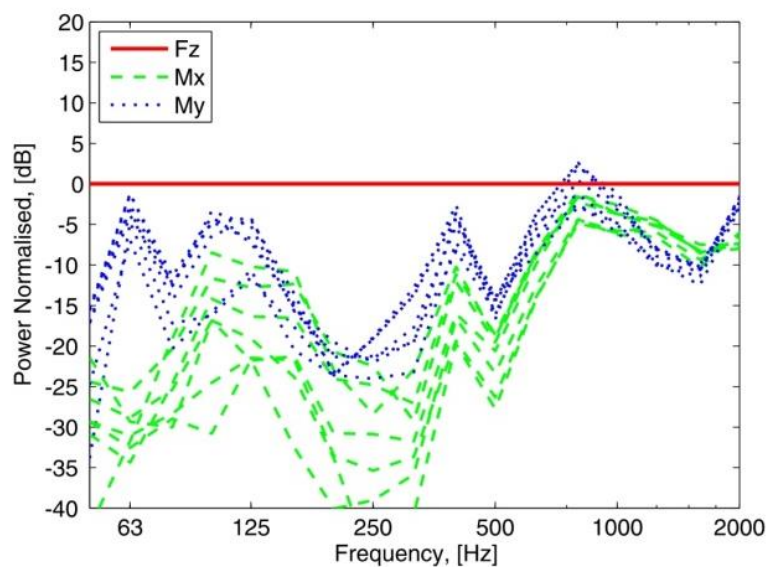


Figure 1: Power of the moment components, normalised with respect to the power of the perpendicular force components for ten locations of a fan unit on a timber joist floor, after [9].

The moment induced powers are significantly less than the force induced power, except in one third octave band (centred at 800 Hz), and can be neglected in general.

Therefore, assuming perpendicular forces only, the total power from a source S to a receiver R is the sum of the powers from N contacts:

$$P_{SR} = \sum_{i=1}^N |v_i|^2 \frac{\text{Re}(Y_{Ri}^{\Sigma})}{|Y_{Si}^{\Sigma} + Y_{Ri}^{\Sigma}|^2} \quad (2)$$

Point mobilities are replaced by effective point mobilities. Superscript Σ denotes *effective* in such a way that contributions from all other contacts are taken into account. The effective point mobility at the i^{th} contact is:

$$Y_i^{\Sigma} = Y_i + \sum_{\substack{j=1 \\ j \neq i}}^N Y_{ij} \frac{F_j}{F_i} \quad (3)$$

The first term on the right hand side of equation (3) is the point mobility at the i^{th} contact of interest. The second term is the sum of the transfer terms (i.e. the contributions to the velocity at the contact of interest from the forces at the other contacts). The effective mobility formulation of equation (2) will give the exact total power if the complex force ratios, see equation (3), are known.

However, if the effective mobilities are to be assembled prior to connection of the source to the receiver, then the force distribution F_j / F_i over the contacts cannot be known [10]. Even though it is the ratios of the forces, which are required, rather than the absolute values, this information still is not likely to be available. This is because the contact conditions (location

of the source, receiver plate geometry, edge conditions, etc.) will not be known in sufficient detail prior to installation.

In the absence of such information, simplifying assumptions are necessary. Although force ratios vary significantly, generally they do so about unity. Variations of an order of magnitude, above and below unity, are typical [10]. Therefore, it is assumed that the contact forces are of equal magnitude, to give a unit force ratio.

The phase difference between forces depends on the vibration behaviour of the source and also on the receiver properties. The receiving (plate) structure may be assumed to be of infinite extent, so that the receiver mobility varies relatively slowly and monotonically with position. The spatial variation in contact forces is then primarily due to the behaviour of the source. If a zero phase is assumed, i.e. the source is assumed to be rigid and moving in a bouncing mode, then equation (3) becomes the complex effective mobility:

$$Y_i^\Sigma = Y_i + \sum_{\substack{j=1 \\ j \neq i}}^N Y_{ij} \quad (4)$$

If the source is assumed to be rigid and moving in a rocking mode, then information about the centre of gravity and relative distances of the contact points is required. This condition was not considered, although it would be expected that the total power would be less, for a rocking mode, than for a bouncing mode.

At high frequencies, a resonant behaviour is likely for either or both of the source and receiver structures and a random phase difference between contact points can be assumed. This

assumption also applies for large distances between contacts, with respect to the governing wavelengths. In this case, the magnitude of the effective point mobility is approximated, according to [3]:

$$|Y_i^\Sigma| \approx \sqrt{|Y_i|^2 + \sum_{\substack{j=1 \\ j \neq i}}^N |Y_{ij}|^2} \quad (5)$$

In order to determine the sound power transmission via the i^{th} contact, the real part of the effective receiver mobility is required, which can be approximated by the real part of the point mobility [3]:

$$\text{Re}(Y_i^\Sigma) \approx \text{Re}(Y_i) \quad (6)$$

A further simplifying assumption, likely to occur at high frequencies, is that transfer terms (the second term of the right hand side of equation (5)) can be neglected. This is discussed in more detail in section 5.1.

2.3 Single equivalent mobility

The effective mobility allows the total installed power to be expressed as the sum of the individual contact powers (equation (2)). However, manufacturers view their products as single entities and seek an associated single value of source strength, along with single values of source and receiver mobility, required for prediction of the installed power. Consider the source quantities, free velocity and source mobility as two single equivalent values, also the receiver mobility as a single equivalent value. The total structure-borne sound power now is given as:

$$P_{SR} \approx \frac{\text{Re}\{Y_{Req}^\Sigma\}}{|Y_{Seq}^\Sigma + Y_{Req}^\Sigma|^2} \sum_{i=1}^N |v_i|^2 \quad (7)$$

The single equivalent free velocity is expressed as the sum of the squares of the magnitudes of free velocity over the source contacts $\sum_{i=1}^N |v_i|^2$. The single equivalent source mobilities are

expressed as the average of the effective point mobilities over the contacts, such that

$$Y_{eq}^\Sigma = \frac{1}{N} \sum_{i=1}^N Y_{ii}^\Sigma. \text{ This gives } Y_{Seq}^\Sigma \text{ for the sources and } Y_{Req}^\Sigma \text{ for the receiver.}$$

In proposing equation (7) for total installed power, it is recognised that the complex relationship between the source and receiver mobility terms is lost (see denominator in equation (2)). This is not a problem when the source and receiver mobilities are significantly different, but will yield an underestimate if, for example, the mobilities are complex conjugate. As a way forward, equation (7) for the approximate power was assumed to apply for all source-receiver mobility conditions and this assumption was examined by comparing the approximate and exact powers.

3. Single equivalent approximations of installed power

Single equivalent approximations of installed power were examined for a medium size fan unit and a whirlpool bath, and a timber joist floor. The approximate values of the total structure-borne power were compared with the powers using measured source free velocity at each contact, and measured point mobility at each contact and transfer mobility between contacts, for the source and receiver. For the fan unit, the free velocities at four mount points were recorded, along with the associated 4 x 4 complex mobility matrix. The fan base was of 3mm

steel plate, formed with two flanges of dimensions 350mm x 35mm, with two support points on each flange. The whirlpool bath, was on a 30mm hollow square section frame, supported on eight mount points [11, 12]

The receiver structure was a timber joist chipboard floor construction, without a ceiling plate. The floor dimensions were 4.55 m x 4.95m. The single layer of sheathing consisted of 21mm chipboard tongue and groove panels of dimensions 2.05m x 0.9m. The sheathing was supported by seven spruce joists with dimensions 192mm x 96mm, at a nominal spacing of 0.78m. The timber joist floor can be considered as a point-connected ribbed plate structure, with an expected large spatial variation in point mobility [13]. Therefore, spatial averaging, inherent in assembling single equivalent values, is expected to lead to increased uncertainty in the estimated power. Again, forces and associated velocities perpendicular to the floor were assumed dominant and that other components of excitation could be neglected. Simultaneously, single equivalent values of free velocity, source mobility and receiver mobility were assembled from the measurement data sets, according to equation (7).

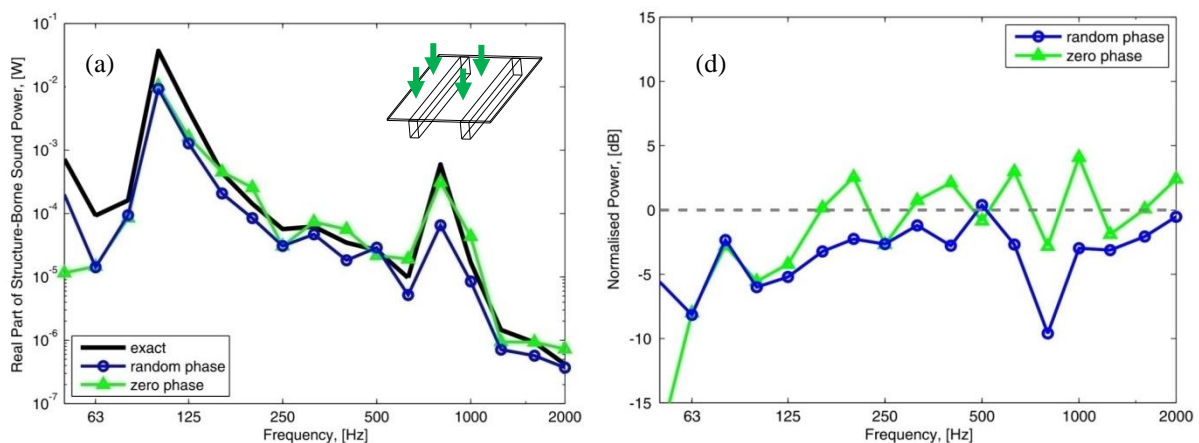
3.1 Approximate and exact powers

Single equivalent source and receiver mobilities were assembled, along with the sum of the squares of the magnitudes of free velocity according to equation (7). Both a zero phase difference assumption (i.e. between the contact forces, assumed to be of equal magnitude, equation (4)) and a random phase assumption (equation (5)) were considered. The *exact* total power from the source to the receiver structure was obtained from equation (2) as the sum of individual contact powers, calculated using complex effective source and receiver mobilities and the complex force ratios.

3.2 Fan unit on timber joist floor

Ten fan locations were considered, including where: two contacts are above a joist and two in a bay; four contacts are in the same bay; two each are in different bays; at the floor edges; in a corner.

In Figure 2 the approximate power, using single equivalent values and assuming random phase or zero phase difference, are shown with the exact power for three locations. Again, exact values were obtained using four complex free velocities at the contacts and the complex point and transfer mobilities, of the source and the receiver, for the same four contacts. Figures 2(a)-(c) show absolute values of power. Figures 2(d)-(f) show the approximate values, in dB (10 log), normalised with respect to the exact values. The maximum power is at 100 Hz with a second peak at 800 Hz. Above 800 Hz the power decreases with frequency to noise. In Figure 2(a), the fan is located with two contacts in one bay and two contacts in an adjacent bay, separated by a joist. Between 160 Hz and 2 kHz, the zero phase approximation gives fluctuations, of the order of ± 3 dB, about the exact value. The random phase assumption results in an underestimate of 3 dB over the same frequency range. Below 160 Hz, both assumptions give underestimates of the order of 5 dB.



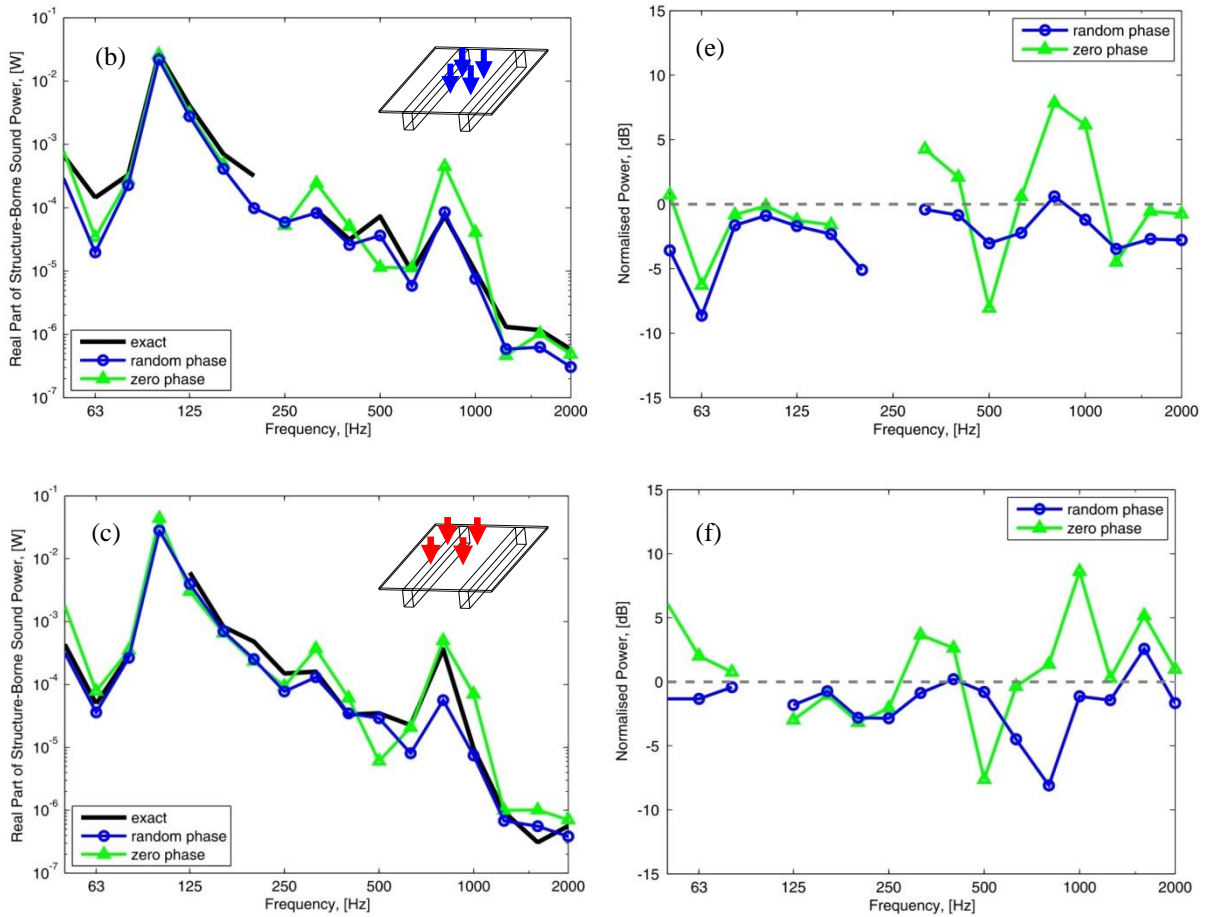


Figure 2: Structure-borne power from fan unit on a timber joist floor.

For four contacts in the same bay (Figure 2(b)), above 315 Hz, the zero phase approximation results in fluctuations about the exact power of the order of ± 8 dB. The ‘lost’ value at 250 Hz is the result of measurement error, yielding negative values of the real part of point mobility, which are not allowed. The random phase approximation again underestimates the exact power by 3 dB for frequencies above 63 Hz. Figure 2 (c) is for two contacts on a joist and two contacts in a bay. Below 250 Hz, the zero phase assumption provides a reasonable approximation. Above 250 Hz, there are fluctuations about the exact power of ± 8 dB. Except at 800 Hz, the random phase approximation gives an underestimate of 2 dB.

For the case considered, the random phase approximation leads to an underestimate and the zero phase approximation gives fluctuations about the exact value of installed power.

3.3 Whirlpool bath on timber joist floor

The whirlpool bath was considered for locations at a corner of the floor, at a long edge and in the centre, simulating installation conditions in buildings. The eight contacts consisted of four on an outer frame and four on an inner frame, subjected to a higher static load. A high mobility source condition again was assumed. Figure 3(a) is for the outer-frame contacts on joists and the inner frame contacts in bays. The whirlpool bath generates broadband excitation with a tonal component at 1 kHz. Above 1 kHz, the power decreases rapidly to noise. Above 160 Hz, the power is approximated by both phase assumptions, within 3 dB. Both give significant overestimates below 160 Hz.

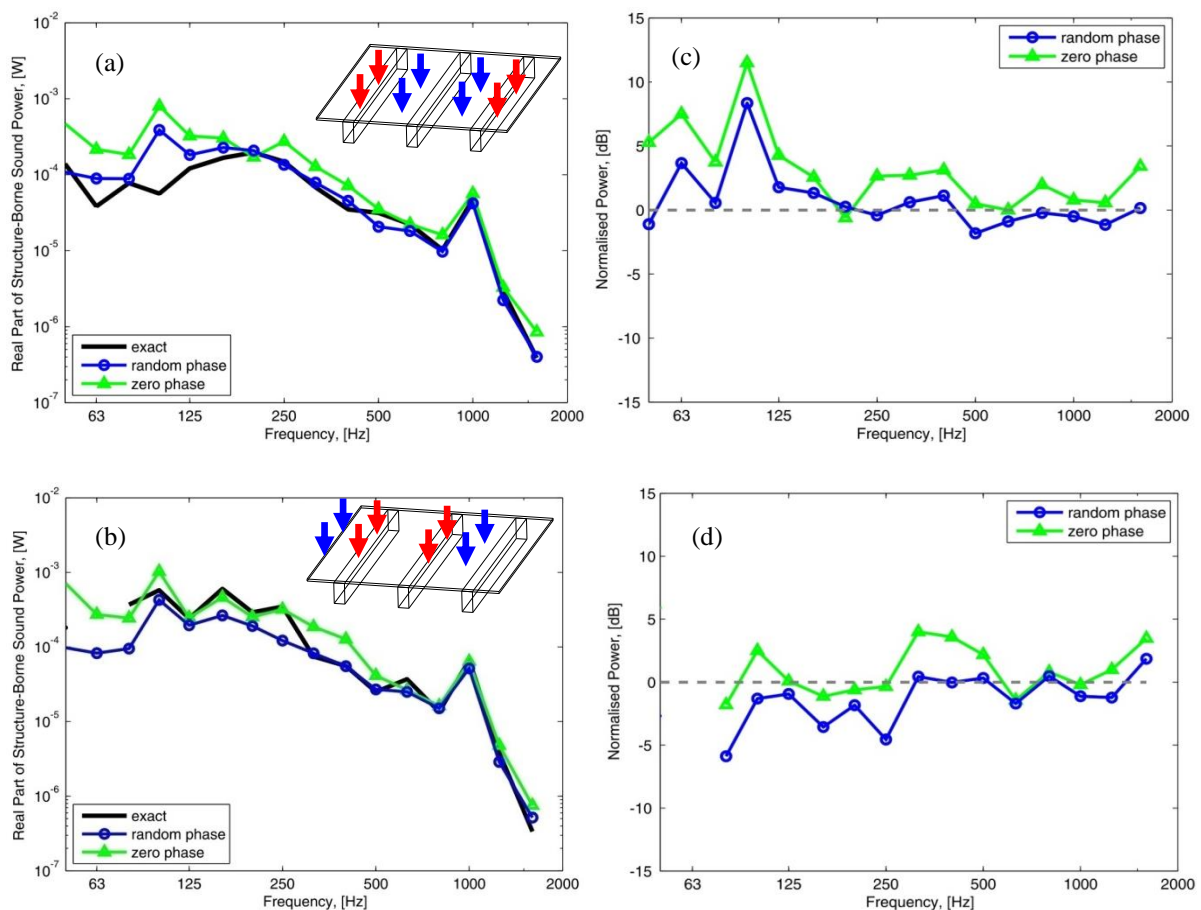


Figure 3: Structure-borne power from whirlpool bath on a timber joist floor.

Figure 3(b) is for the outer-frame contacts in bays and inner-frame contacts on joists. Overall, both approximations give agreements within 5 dB, above 80 Hz. Neither phase assumption approximates the exact value at low frequencies. At mid and high frequencies, the zero phase assumption gives approximations about the exact value whilst the random phase assumption tends to give an underestimate.

4. Spatial variation in installed power

When predicting the installed power from machines in lightweight buildings, the exact source location on rib-stiffened or framed plates is usually not known. The accuracy of single equivalent approximations therefore should be evaluated with respect to the likely spatial variation in installed power. Figure 4(a) shows the spatial variation in power for the fan at ten locations on the floor, corresponding to forty contact positions. Figure 4(b) is for three locations of the whirlpool bath, corresponding to twenty four contact positions. In both cases shown, again the powers are exact, in that complex values of the free velocity, and point and transfer mobility, were used.

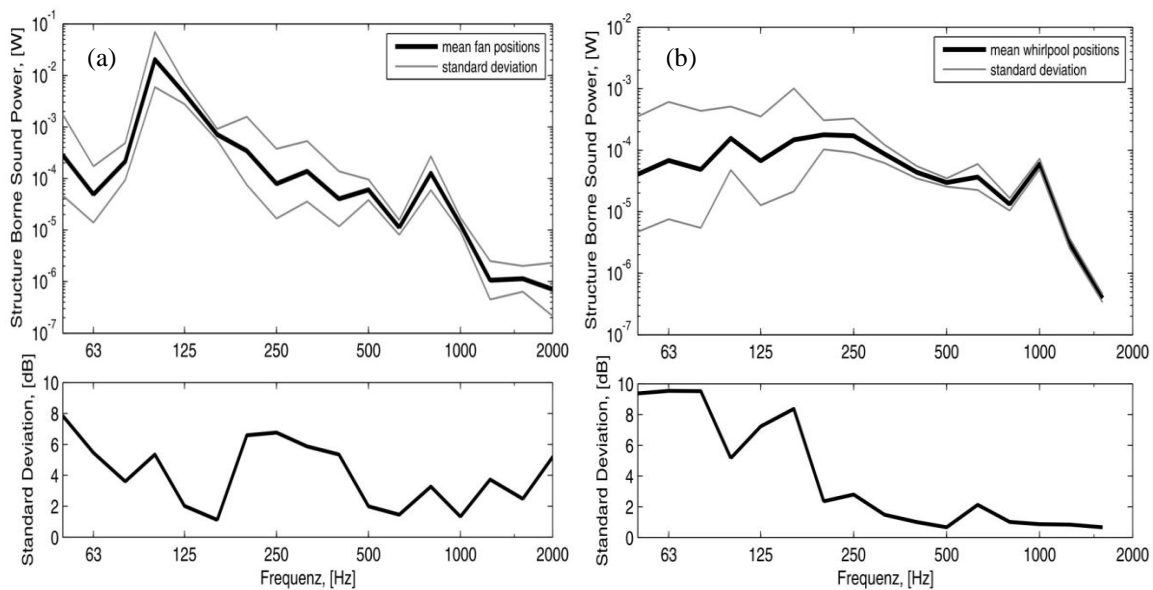


Figure 4: Mean and standard deviation of total exact power for (a) fan unit at ten locations, (b) whirlpool bath at three locations on a timber joist floor.

The numbers of locations are not large, due to the computational effort involved, and results are indicative, rather than statistically rigorous. However, a standard deviation of 10 dB is indicated at low frequencies, reducing to 3 dB at high frequencies and this is useful when discussing discrepancies between approximate and exact values.

Now consider the approximate estimates (equation (7)), normalised with respect to exact values, for the same sources and locations. The mean discrepancies, along with the associated variation are shown in Figure 5 for the zero phase approximation.

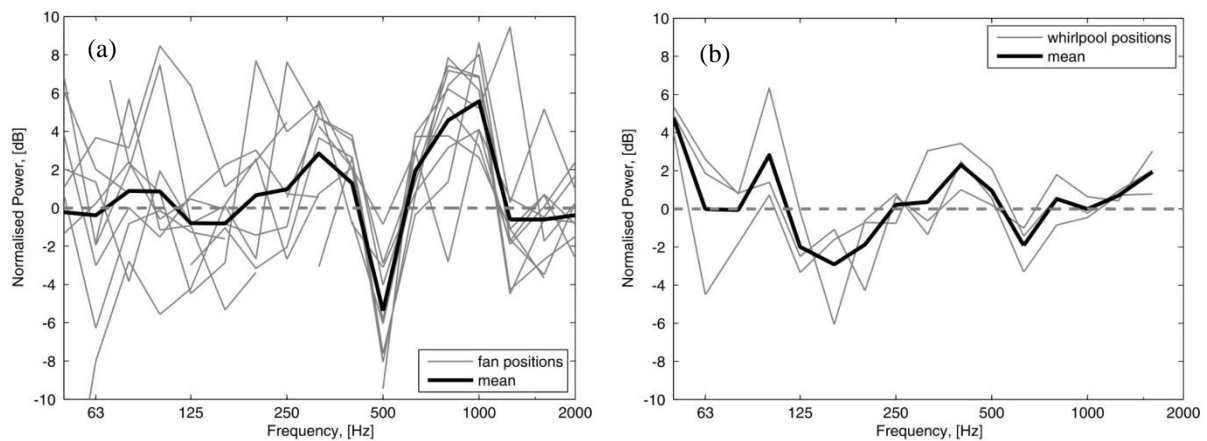


Figure 5: Mean and individual values of normalised power assuming zero phase; (a) fan unit at ten locations; (b) whirlpool bath at three locations.

For the fan unit, the average approximation is within 2 dB of the exact value below 400 Hz, with a range of 15 dB. There is a consistent underestimate around 500 Hz and an overestimate around 800 Hz, due to interference effects between the contacts. For the whirlpool bath, the average approximation is within 3 dB of the exact power, with the range decreasing from 12 dB at low frequencies to 4 dB at high frequencies.

Figure 6 shows results for the random phase approximation. For the fan unit, there is an average underestimate of 2 dB, with a range of 14 dB. For the whirlpool bath, the average underestimate is 4 dB at low frequencies and 1 dB at high frequencies. The range narrows from 8 dB at low frequencies to 1 dB at high frequencies.

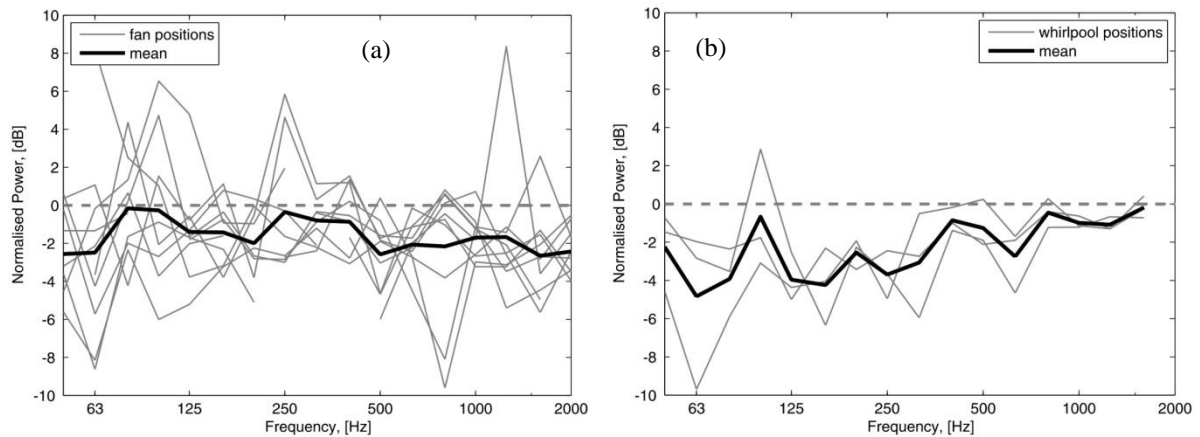


Figure 6: Mean and individual values of normalised power assuming random phase; (a) fan unit at ten locations; (b) whirlpool bath at three locations on a timber joist floor.

These results confirm the findings from previously considered single locations. The random phase approximation gives an underestimate of about 3 dB on average, while the zero phase approximation gives larger fluctuations about the exact power. Again, compare the average discrepancies in Figure 5 and Figure 6 with the expected spatial variation in *exact* power in Figure 4. If the location of such sources, particularly with respect to rib and frame elements, is not known, then the average approximate values are within the expected spatial standard deviation. However, individual approximate estimates can lie well outside the standard deviation. The range is likely to reduce in some situations, due to practical installation requirements. For example, when installing a boiler unit to a timber-frame wall, the contact (i.e. support) points will be on frames and the variation in contact condition will be smaller than for the cases considered.

5. Random phase estimate of source and receiver mobility

Since manufacturers and practitioners seek to measure and calculate required data as band-averaged (typically one third octave band) values, the random phase approximation, which does not require complex source data, was considered further. The observed underestimate of power could be due to either or both an underestimate of the real part of the single equivalent receiver mobility or to an overestimate of the magnitude of the single equivalent source and/or receiver mobility (see equation (7)). Figure 7 shows the average approximate values of the real part of the single equivalent receiver mobility, normalised with respect to exact values obtained from equation(3), for ten locations of the fan unit on the timber joist floor. Below 125 Hz, the underestimate is largely the result of neglecting the phase relationships between the transfer terms when calculating the real part of the effective mobility (see equation (6)). Above 125 Hz, the average agreement is within 2 dB of the exact value, with a range of 5 dB.

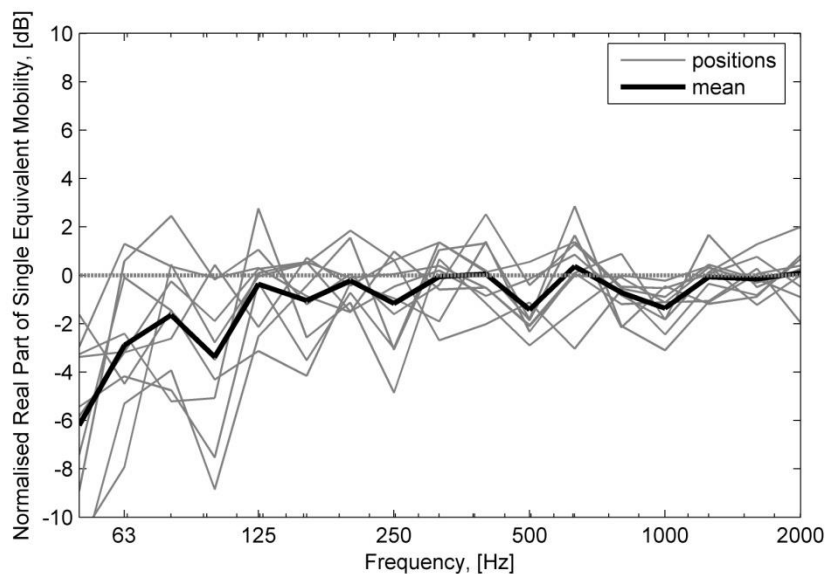


Figure 7: Mean and individual values of the approximate real part of single equivalent receiver mobility, normalised with respect to the exact value; fan unit at ten locations on timber floor.

In Figure 8, are shown the normalised random phase approximations of the single equivalent receiver mobility of the timber floor (8a) and of the magnitudes of single equivalent source mobility of the fan unit (8b). Note that the approximate single equivalent source mobility does not vary with location, but the approximate single equivalent receiver mobility does, as does the exact value. Therefore, the normalised values will vary with location. On average, the random phase approximation gives an overestimate of the magnitude of single equivalent mobility, by 2 dB for the receiver and 4 dB for the source. This mainly explains the underestimate of the total power.

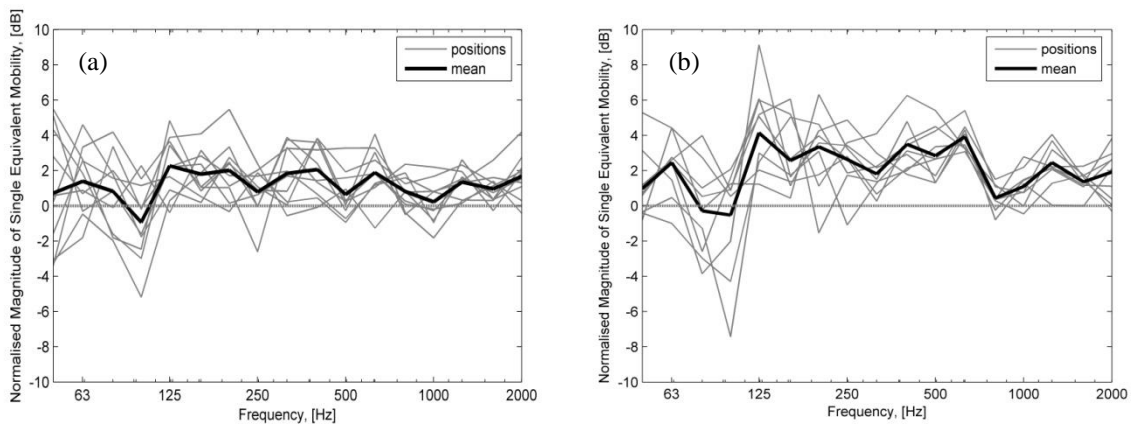


Figure 8: Mean and individual values of normalised magnitude of single equivalent mobility; (a) for floor; (b) for fan.

5.1 Neglect of transfer terms

A significant simplification, in the estimate of the installed power, would result by neglecting transfer terms, in the estimate of the real part and magnitude of the receiver mobility and the magnitude of the source mobility. The magnitude of the effective point mobility then is simply the magnitude of the point mobility. This, in effect considers the contacts as being independent of each other. Figure 9 shows the approximate power, normalised with respect to the exact power, for ten locations of the fan unit on the timber floor. Neglect of the transfer terms, i.e. the products of forces and transfer mobilities, gives an overestimate of the order of 3 dB, with

a range of 15 dB. As expected, the overestimate reduces with increase in frequency because of the reduced contribution of transfer terms.

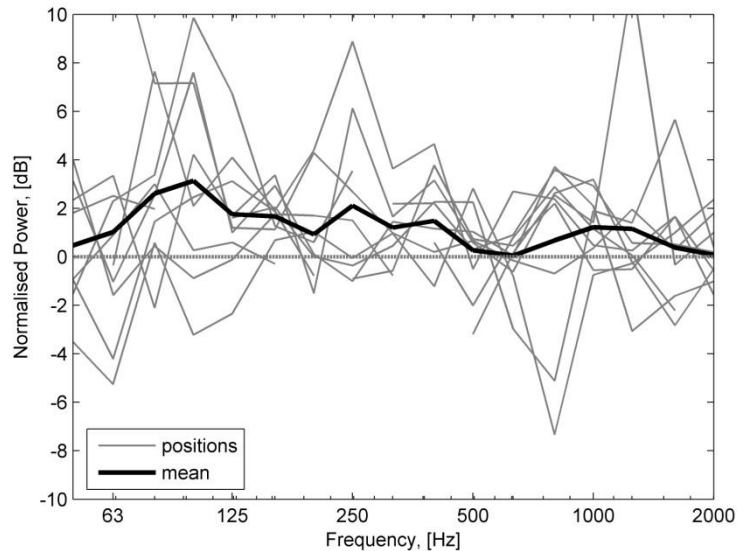


Figure 9: Mean and individual values of normalised power assuming independent contacts, for fan unit at ten locations.

Again, if reference is made to Figure 4, the average discrepancy lies within expected spatial standard deviation. Neglect of transfer mobility data significantly reduces the measurement and computational effort.

6. Concluding remarks

An approximate method is proposed for estimating the total structure-borne power from multi-contact sources in buildings. The concept of effective mobility is developed, to generate equivalent single values of source activity and source mobility, combined with the equivalent single receiver mobility. The complex interactions between contacts are represented by a unit contact force ratio and by either a zero phase difference or a random phase between forces. A

further simplification is explored where the contacts are assumed to be independent of each other.

As a receiver, a ribbed plate is considered, in the form of a timber joist floor. For the sources considered, a fan unit on four contacts and a whirlpool bath on eight contacts, the random phase approximation gives an underestimate of about 3 dB on average, whilst the zero phase approximation gives larger fluctuations about the exact power.

Using point mobilities only, the approximation gives an average overestimate of the order of 2 dB when compared to the exact power.

The accuracy of the approximate method has been related to the spatial variation in contact conditions, where a standard deviation of 10 dB is indicated at low frequencies, converging to 3 dB at high frequencies.

Of practical significance is the fact that the random phase assumption allows approximate values to be calculated as magnitudes and therefore as band-averages, such as one-third octave values.

Also of practical significance is the reduced measurement and computational effort, resulting from the approximations. For the cases considered, although the ranges are large, the average discrepancies between approximate and exact values of installed power are within the spatial variation of installed power.

The above comments are the result of a relatively small number of case studies, and any generalizations should be treated with caution. Only one timber floor construction has been considered and further case studies might be necessary for different construction types. An example would be prefabricated lightweight floor constructions, where the ribs are glued and screwed to the sheeting plates. The floor would behave as a line-connected ribbed plate, rather than the point-connected ribbed considered in this paper, and the effect on mobility of proximity to the ribs and thus on the spatial variation of mobility is likely to be greater.

However, the study indicates, that for mechanical installations on lightweight building structures, the approximate methods provide a feasible trade off between simplicity and accuracy.

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