Minimizing Energy and Cost in Range-Limited Drone Deliveries with Speed Optimization

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Abstract

This paper introduces the Energy Minimizing and Range Constrained Drone Delivery Problem (ERDDP) in which drones are used to make deliveries to a number of customers and the drones themselves are transported by traditional vehicles that act as launch points. The ERDDP consists of (i) selecting the launch points from a potential set of sites from where drones will take off to serve a number of customers, (ii) assignments of customers to the launch points, and (iii) the speed at which drones are to travel between the customers and the launch points. The paper presents a nonlinear model for the ERDDP, which minimizes the total operational cost including an explicit calculation of the energy consumption of the drone as a function of the drone speed. The deliveries are limited by both a service time bound and the range of the drone. The model is reformulated using second order cone programming, and subsequently strengthened by the use of perspective cuts, that allows the use of off-the-shelf optimization software to solve the problem. Computational results are presented on a realistic data set that quantifies the effect of various parameters on location, assignment and speed decisions.

Keywords. drone delivery; energy consumption; freight transportation; second order cone programming; perspective cuts

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1 Introduction

Drones are a type of Unmanned Aerial Vehicle (UAV) that do not require a pilot on board. Initially used within military operations, recent technological developments allowed a wider usage of such vehicles for a variety of purposes, including deliveries from restaurants such as YO! Sushi (Willett 2013) and Domino's (Reid 2016), by transportation agencies (Overly 2017), construction companies (Balfour Beatty 2017) and airlines (Manalo 2015). Carriers have also started to make deliveries by using drones. In 2014, DHL prepared a report that discussed the potential of drone delivery in detail (DHL 2014) and successfully completed trials with their so-called parcelcopters in 2016 (DHL 2016). Amazon's new delivery system Prime Air, in which drones are used to provide faster service, was first trialled on 7 December 2016 (Amazon 2016). In a new delivery system introduced by UPS, drones take off from a truck that serves as a launch pad to deliver goods to customers (UPS 2017). One of the main benefits of drones is the potential reduction in the negative environmental impacts, such as CO_2 emissions, of the conventional means of transport (Goodchild & Toy 2018), although they can also provide economic benefits, particularly as drones can help to ease traffic congestion. It is therefore no surprise that Amazon and UPS are investing in their delivery network to utilize drones in a more efficient way (Desjardins 2018).

In such delivery systems, the drone needs to be prepared either at a depot or on a truck, depending on the network configuration of the carrier. The preparation entails battery installation and package placement, as well as coding the coordinates to which the delivery is destined. The drone takes off vertically from a given location, or a 'pad', and upon reaching a certain altitude, it cruises horizontally towards the direction of the customer. Upon arrival to the customer, it detects and lands on the drop-off point. After leaving the cargo at the drop-off point, it travels in a reverse manner to return to the initial point of departure, finalizing the delivery. Once back at initial position, the drone is prepared for a subsequent delivery. Due to safety issues and legislation, drones should be continuously monitored by the drone operator (or the driver, if launched from the truck) (Murray & Chu 2015).

Motivated by the developments described above, we propose, in this paper, a two-echelon delivery system that uses drones in combination of traditional means of transport. Trucks operate at the first echelon, acting as dispatch points for drones that travel at the second echelon to make the actual deliveries. Drones can either take off from a central depot or are transported by trucks to a parking area from where they are launched. In the latter case, each truck carries one drone to a designated parking area, from which the drone serves one customer and returns to the truck to be prepared for any subsequent delivery, as described above. The total distance that a drone traverses is limited by a given range that is a function of battery capacity and the drone speed. In this paper, we assume that, the battery on drone is used until it is empty and while the empty one is recharging, another battery is used in order to continue delivery operations without a break. Once all deliveries are made, the trucks return to the depot with their drones on board. The sequence of events is the same for drones operating from the depot directly, except that there is no need for transport to a parking location. The depot and the potential sites are collectively named as launch points. A visual representation of the delivery system just described is depicted in Figure 1, with one depot and five parking locations (indicated by 'P'), where the solid arrows represent truck movements, and dashed arrows represent drone deliveries between the launch points and the customers. A delivery structure of this type can be used in cases where the underlying transportation network has certain limitations. Examples include delivery in rural areas with insufficient roads or urban areas with heavy traffic congestion.

The Energy Minimizing and Range Constrained Drone Delivery Problem (ERDDP) is defined on the two-echelon delivery system described above, and consists of choosing launch points to use from a candidate set of sites, assigning customers to launch points, and determining the speed at which drones travel between customers and launch points, subject to all deliveries made within a given amount of time and the range of a drone. The problem minimizes the total variable cost that comprises the operational cost of trucks and the cost of energy consumption arising from the use of drones. The fixed costs of fleet acquisition including trucks and drones are not considered here as we assume the existence of a given fleet to start with, and instead present a parametric analysis by varying the number of vehicles used. Contrary to the literature on delivery problems with drones, the drone speed in the ERDDP is considered as a decision variable. The motivation is that the drone speed has a direct impact on the overall time spent, the range and the energy consumption of drones.

The paper makes four main contributions: (i) we introduce and formally define a delivery

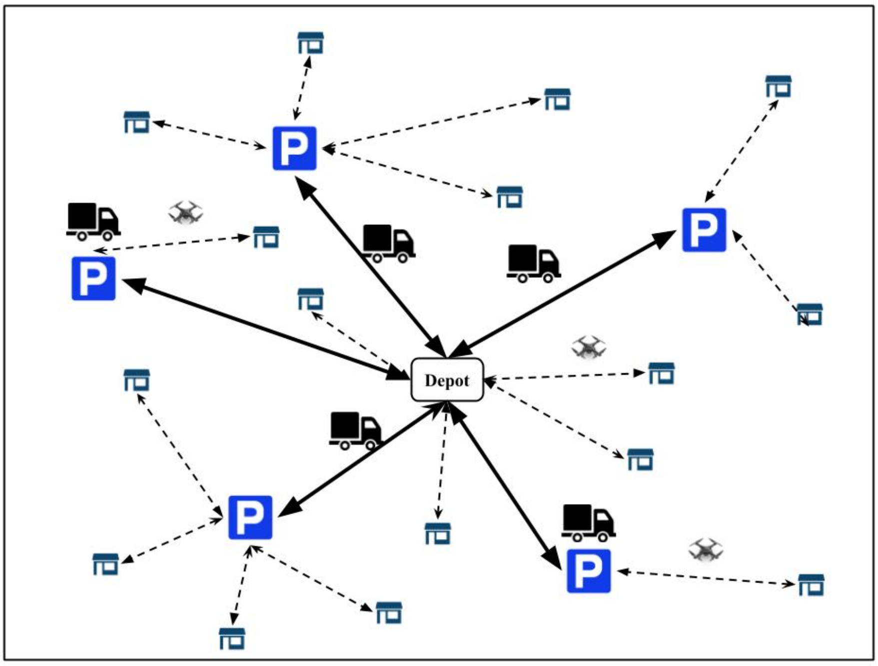

Figure 1: An illustration of the proposed drone delivery system

problem in which drones are used, where the speed of the drone is treated as a decision variable, (ii) we explicitly calculate the energy consumption of the drones by embedding an energy model within the delivery problem, (iii) we describe a nonlinear model for the problem and show that it can be reformulated as a second order cone programming formulation and strengthened by perspective cuts, and (iv) we present extensive computational experiments on the ERDDP on a data set that quantifies the impact of using drones in making deliveries. The remainder of the paper is structured as follows. Section 2 presents a review of the relevant literature concerning the use of drones within logistics network design. Section 3 presents a description of energy consumption and range calculation of a drone. Section 4 describes a nonlinear mathematical formulation given for the problem and presents the reformulation as second order cone programming problem. Section 5 presents results of computational experiments. The paper concludes in Section 6.

2 Related Literature

In line with the developments in drone deliveries, a body of literature has started to emerge that considers the use of drones within classical transportation problems. Murray & Chu (2015) introduced two drone assisted cargo delivery problems. The first is the flying sidekick traveling salesman problem (FSTSP). In the FSTSP, while a truck is serving a customer, a drone takes off from the truck, serves another customer, and returns to the truck either at the depot or another customer location on the route of the truck. The second problem concerns parallel drone scheduling traveling salesman problem (PDSTSP) where trucks and drones operate separately, i.e., while trucks are routed between customers, drones operate from the depot to perform dedicated deliveries. Murray & Chu (2015) point out that the flight endurance (i.e., range) of a drone is a function of its speed and present an analysis of the trade-offs between the drone speed and range. They conclude that the speed of a drone has a significant impact on drone delivery operations due to the impact on its range. Higher speeds are preferable even if they imply a lower drone range. In their analysis, they consider nine different speed and range combinations, as opposed to modeling drone range as a function of speed. The underlying setup of the FSTSP has led to other studies that focus on the coordination between trucks and drones (see, e.g., Wang et al. 2017, Poikonen et al. 2017, Carlsson & Song 2017, Agatz et al. 2018, Bouman et al. 2018, Yurek & Ozmutlu 2018), with some extending the problem to minimize the operational cost (see, e.g., Ha et al. 2018). For the PDSTSP, Mbiadou Saleu et al. (2018) developed an iterative two-step heuristic.

Dorling et al. (2017) described vehicle routing problems for drone delivery (VRPDD), where, in contrast to previous studies, customers are solely served by drones. The authors derive an energy consumption model for drones and analyzed the impact of payload and battery weight on energy consumption. The energy consumption used in Dorling et al. (2017) is only relevant to hovering and does not include that of the actual flight and the speed. Coelho et al. (2017) studied a multi-objective green UAV routing problem where charging stations are used to extend the limited range of drones. The proposed problem has seven different objective functions, one of which is the minimization of amount of energy required by the drone batteries. However, the energy required is estimated by using a simple function that depends on the drone speed instead of a consumption model. Boysen et al. (2018) studied drone scheduling for given truck routes, where they consider different variants of the problem based on the number of drones available and whether or not launch and landing points have to be the same. Kim et al. (2019) described a stochastic facility location model for drones, where uncertainty of the maximum flight distance is considered for a post-disaster humanitarian application. The uncertainty aspect of the problem is handled by a chance constraint. A delivery problem using drones has been introduced by Chauhan et al. (2019), who cast it as a capacitated facility location problem, where facilities act as launching sites, and where the aim is to maximize the demand served subject to battery range constraints. The model described by Chauhan et al. (2019) does not treat speed as a decision variable and consequently takes the form of an integer linear programming formulation solvable by an off-the-shelf solver. The authors also present several heuristics for the problem itself. Chowdhury (2018) studied a heterogeneous fixed fleet drone routing problem for inspection purposes in a post-disaster application. The problem includes different aspects of a drone trajectory such as hovering, turning, acceleration and deceleration, but the energy consumption model used is simple and the drone speed is chosen from as a set of predefined levels. Other studies include that of Yakıcı (2016), which is a location-and-routing problem for drones arising from a military application, and that by Guerriero et al. (2014) who studied a multi-objective unmanned aerial vehicle routing problem with soft time window constraints. For a comprehensive survey on optimization approaches for drones, we refer the reader to Otto et al. (2018).

Table 1 contrasts the studies just reviewed with our study based on the following factors: (i) the number of objective functions used (single or multiple), (ii) the nature of the solution methods (exact or heuristic), (iii, iv) the mode(s) of delivery, (v, vi) whether a factor or a microscopic energy consumption model is used or not, (vii, viii) whether a parameter or a model is used for the drone range or not, (ix) whether drone speed is considered as a decision or not, and (x) whether there is a time bound on deliveries or not.

The delivery system proposed in this study has a star-star topology. The problems defined in a star-star network which determine the number and location of concentrators (hubs) and also assignments of demand nodes to the concentrators are commonly known as the Concentrator Location Problem (CLP) (Fortz 2015). Although the proposed delivery system is very similar to the one in the CLP in terms of the network structure, the objective, constraints and problem dynamics are completely different. In particular, the objective function of the CLP only minimizes the fixed cost of establishing links between a central node and concentrators, and concentrators and demand nodes (Klincewicz 1998), whereas in our aim is to minimize the

Table 1: Features of transportation problem with drones

Reference	Objective	Method	Delivery Type		Energy	.	Range		Drone Speed	Time Bound
			Drones	Drones & Trucks	Factor	Micro	Parameter	Model		
Murray & Chu (2015)	Single	Exact & Heuristic		✓			✓			
Wang et al. (2017)	Single	-		✓						
Poikonen et al. (2017)	Single	-		✓			✓			
Carlsson & Song (2017)	Single	Heuristic		✓			✓			
Agatz et al. (2018)	Single	Exact & Heuristic		✓			✓			
Bouman et al. (2018)	Single	Exact		✓						
Yurek & Ozmutlu (2018)	Single	Exact & Heuristic		✓			✓			
Ha et al. (2018)	Single	Exact & Heuristic		✓			✓			
Mbiadou Saleu et al. (2018)	Single	Heuristic		✓			✓			
Dorling et al. (2017)	Single	Heuristic	✓			✓		✓		✓
Coelho et al. (2017)	Multi	Heuristic	✓		✓			✓	✓	
Boysen et al. (2018)	Single	Exact	✓							
Kim et al. (2019)	Single	Heuristic	✓				✓			
Chauhan et al. (2019)	Single	Exact & Heuristic	✓		✓		✓			
Chowdhury (2018)	Single	Exact & Heuristic	✓		✓			✓		✓
Yakıcı (2016)	Single	Exact & Heuristic	✓				✓			
Guerriero et al. (2014)	Multi	Exact & Heuristic	✓							✓
Our Study	Single	Exact	✓			✓		✓	✓	✓

variable cost of operations. Contrary to our study, the CLP satisfies the demand and capacity constraints, but ignores the time bound and range constraints. Another major difference is that the number of concentrators is not fixed and in some instances, the fixed cost of opening a concentrator included in the objective function. Finally, the main application area of the CLPs is telecommunication.

The CLP can be seen as a special case of the hub location problem (HLP) where no routing cost between two concentrators are considered (Contreras & Fernández 2012). Labbé & Yaman (2008) study the HLP in a star-star network arising in a telecommunication application. The objective is to minimize the total cost including the fixed cost of locating hubs and establishing links between hubs and demand nodes, and the variable cost of routing the flow between hubs and the central node. One of the main difference from our study is that the demand is satisfied between customers, but in our setting, a central entity satisfies the demand of customers. Another similar problem employing a star-star hub network topology is studied by Alumur et al. (2012) which presents a hierarchical multimodal hub location problem with time-definite deliveries. The proposed network consists of three layers, two of which are star-shaped and on which, two different transportation modes are utilized, namely ground (truck) and air (airplane). Finally, the proposed network in our study can also be considered as a one-to-many distribution network with terminals. A problem in such a network is solved by a continuous approximation model in Campbell (1993).

To conclude, the delivery system used in our problem can be also considered as the combination of the systems used in the FSTSP proposed by Murray & Chu (2015) and the VRPDD studied by Dorling et al. (2017). The system proposed here consists of two types of vehicles, namely trucks and drones, as in the FSTSP. However, in our setting, and similar to the VRPDD, only drones take an active role in delivery operations, as opposed to the FSTSP where both drones and trucks are used. Furthermore, and in contrast to the VRPDD, drones only visit one customer at a time, as is the case in the FSTSP.

3 Formulating Drone Energy Consumption and Range

This section describes a model for calculating the energy required during the flight of a drone over a given distance, and the range of a drone in terms of the maximum distance it can traverse under limited battery capacity. We do not explicitly model the energy consumed during takeoff and landing as this is a constant if there are a fixed number of deliveries to be made (as is the case in our setup). We consider the use of a commercial quadcopter, with four rotors to lift and propel the drone, that is designed to carry light packages. We make the assumption that the drone is able to travel at a constant speed between a pair of locations, and whilst windy conditions may impact the actual speed attained, there are ways to adjust the thrust during the journey to maintain stable speed.

Following the above assumptions, we only calculate the energy consumption during the hover and flight of the drone, for which we use the power consumption formulation given by Zeng et al. (2019). The required power $P(v)$ (in kg.m^2/s^3) to move the drone as a function of the drone speed v (in m/s) is given as follows:

$$P(v) = \frac{\delta}{8}\rho s A \Omega^3 r^3 \left(1 + \frac{3v^2}{U_{tip}^2}\right) + (1+k)\frac{W^{3/2}}{\sqrt{2\rho A}}\left(\sqrt{1+\frac{v^4}{4v_0^4}} - \frac{v^2}{2v_0^2}\right)^{1/2} + \frac{1}{2}d_0 \rho s A v^3, \quad (1)$$

where δ is the profile drag coefficient, ρ (in kg/m^3) is the air density, s is the rotor solidity, A (in m^2) is the rotor disc area, Ω (in radians/s) is the blade angular velocity, r (in m) is the rotor radius, U_{tip} (in m/s) is the speed of the rotor blade, k is the incremental correction factor to induced power, v_0 (in m/s) is the mean rotor induced velocity in hover, d_0 is the fuselage drag ratio and W (in kg) is the total weight carried. It includes the empty weight m_{air} of the aircraft (in kg) excluding the battery, the weight m_{batt} of the battery (in kg), and the weight m_{pack} of a package (in kg) carried only on the forward journey. Thus, $W = m_{air} + m_{pack} + m_{batt}$ on the forward journey to a customer and $W = m_{air} + m_{batt}$ on the return. Equation (1) represents the propulsion power consumption, and consists of three components, namely the blade profile, and induced and parasite power, which are used to overcome the profile drag of the blades, the induced drag of the blades and the fuselage drag, respectively. To simplify the above formula, let $P_0 = \frac{\delta}{8}\rho s A \Omega^3 r^3$ and $P_i = (1+k)\frac{W^{3/2}}{\sqrt{2\rho A}}$ be associated with the blade profile power and induced power during hovering. Then, Zeng et al. (2019) approximate equation (1) by applying the first-order Taylor approximation as follows:

$$P(v) \approx P_0\left(1 + \frac{3v^2}{U_{tip}^2}\right) + P_i\frac{v_0}{v} + \frac{1}{2}d_0\rho s A v^3. \tag{2}$$

Here, we convert the power consumption formula (1) to an energy consumption formula $E(v,d)$ (in kg.m^2/s^2 = 1 J) for a drone traveling a distance of d units (in m) at a constant speed v (in m/s), and calculated as $E(v,d) = P(v)t = P(v)\frac{d}{v}$:

$$E(v,d) \approx P_0\left(\frac{d}{v} + \frac{3dv}{U_{tip}^2}\right) + P_i\frac{v_0 d}{v^2} + \frac{1}{2}d_0\rho s A d v^2. \tag{3}$$

To simplify the above formula, let $\mu_1 = P_0$, $\mu_2 = P_0\frac{3}{U_{tip}^2}$, $\mu_3 = P_i v_0$, and $\mu_4 = \frac{1}{2}d_0\rho s A$. Then, the total amount of energy E (in J) can be given as follows:

$$E(v,d) = \mu_1\frac{d}{v} + \mu_2 d v + \mu_3\frac{d}{v^2} + \mu_4 d v^2. \tag{4}$$

Here, we remark that only parameter μ_3 is affected by the total weight which includes the package weight in the forward journey, but not in the return journey. Therefore, we define two new parameters μ_3^1 and μ_3^2, where the former includes the package weight and the latter does not.

Reference values of the parameters used for a rotary-wing drone energy consumption model introduced above are given in Table 2 (Zeng et al. 2019, Alibaba 2019).

By using the parameter values provided in Table 2 and equation (4), we show the relationship between drone speed and the resulting energy consumption in unit distance with and without the

Table 2: Reference values used for calculating energy consumption of a rotary-wing drone

Notation	Description	Reference Values
δ	Profile drag coefficient	0.012
ρ	Air density (kg/m^3)	1.225
s	Rotor solidity	0.05
A	Rotor disc area (m^2)	0.503
Ω	Blade angular velocity (radian/s)	300
r	Rotor radius (m)	0.4
U_{tip}	Tip speed of the rotor blade (m/s)	120
k	Incremental correction factor to induced power	0.1
m_{air}	Aircraft weight (kg)	2.04
m_{pack}	Package weight (kg)	1
m_{batt}	Battery weight (kg)	0.89
v_0	Mean rotor induced velocity in hover (m/s)	4.03
d_0	Fuselage drag ratio	0.6

package on board in Figure 2, which shows that the optimal drone speed (v_{opt}) that minimizes the energy consumption is approximately 74.65 km/h and 70.13 km/h, respectively.

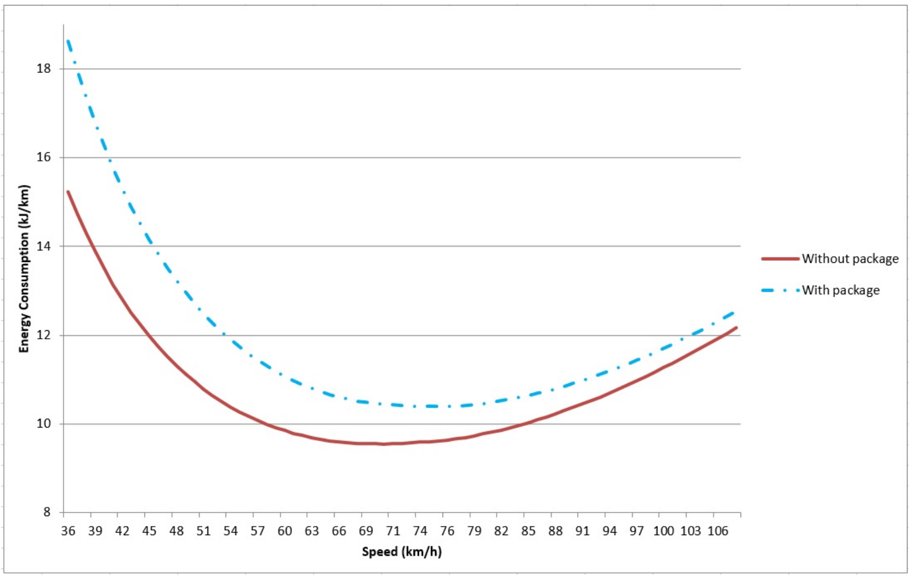

Figure 2: Energy consumption as a function of drone speed

We now present a way in which the range R (in m) of a drone that represents the maximum total distance that a drone can travel can be calculated. The range also depends on the energy consumption, for which reason we resort to the earlier stated parameters μ_1, μ_2, μ_3 and μ_4 in expressing the range of the drone by Stolaroff et al. (2018) as follows:

$$R(v) = \frac{m_{batt} s_{batt} \theta}{(\frac{\mu_1}{v} + \mu_2 v + \frac{\mu_3}{v^2} + \mu_4 v^2) f}, \tag{5}$$

where s_{batt} (in J) is the energy capacity of the battery per mass, θ is the depth of discharge, and f is the safety factor to reserve energy for unusual conditions. Using $\Theta = m_{batt} s_{batt} \theta / f$ to simplify the formulation, the range $R(v)$ (in m) of a drone traveling at a constant speed v (in m/s) can be given as follows:

$$R(v) = \frac{\Theta}{\frac{\mu_1}{v} + \mu_2 v + \frac{\mu_3}{v^2} + \mu_4 v^2}. \tag{6}$$

Reference values of the parameters introduced above for range calculations of a rotary-wing drone are given in Table 3 (Stolaroff et al. 2018). Based on these parameter values, we use the equation (6) to calculate the maximum distance of a drone that can travel without recharging battery. Given the parameter values, Θ equals 320,400 J. At the optimal speed (v^*) with the package (20.74 m/s = 74.65 km/h), the maximum distance ($R(v^*) = \frac{\Theta}{\frac{\mu_1}{v^*} + \mu_2 v^* + \frac{\mu_3}{(v^*)^2} + \mu_4 (v^*)^2}$) a drone can travel is approximately 30.83 km. Then, given the speed value, the maximum travel time $T(v)$ (in minutes) of a drone can be calculated by the following equation: $T(v) = R(v)/v$. At the optimal speed v^*, the maximum travel time ($T(v^*) = R(v)/v$) equals 24.78 minutes, approximately. Without package on board, at the optimal speed v^* (19.48 m/s = 70.13 km/h), the maximum distance and the maximum flight time equal 33.55 km and 28.70 minutes, respectively.

Table 3: Reference values used for calculating range of a rotary-wing drone

Notation	Description	Reference Values
s_{batt}	Energy capacity (density) per mass of the battery (J/kg)	540,000
θ	Depth of discharge	0.8
f	Safety factor to reserve energy	1.2

As it can be observed from equations (4) and (6), speed is a primary determinant of the energy consumption and the range of a drone. For this reason, we consider drone speed as a decision variable in the formulations that will be presented in the next section which will allow us to minimize energy consumption.

4 Mathematical Formulations for the ERDDP

The Energy Minimizing and Range Constrained Drone Delivery Problem (ERDDP) is defined on a complete directed graph $G = (N, A)$ where $N = \{0, 1, ..., n\}$ is the set of nodes and includes the depot shown by node 0, and $A = \{(i, j) : i, j \in N, i \neq j\}$ is the set of arcs. We also define a set $P = \{1, ..., m\}$ of potential parking locations for trucks, and a set $C = \{m + 1, ..., n\}$ of customers. The set $L = \{0\} \cup P$ of launch points consists of the depot and potential parking areas. All customers should be served within a predetermined time limit U. The preparation time of a drone for a delivery is shown by τ. The flying distance on arc $(i, j) \in A$ is denoted by d_{ij}, which is assumed to be symmetric as would be the case for air travel. The driving distance on arc $(i, j) \in A$ is denoted by c_{ij}. The maximum speed limits for drones is denoted by v_{max}. We assume that the (average) speed of the truck is fixed and denote it by $\overline{v}$. Parameters c_e and c_d represent the unit energy cost and the unit operational cost per distance, respectively, and p is the (fixed) number of launch points available for drones to take off.

In this section, we present a mathematical programming formulations for the ERDDP. The decision variables are defined as follows: If a candidate site $j \in L$ is used as a launch point, then a binary variable y_j is equal to 1, and equal to 0 otherwise. A binary variable x_{ij} equals 1 if a drone that takes off from a launch point $j \in L$ to serve customer $i \in C$, and equals 0 otherwise. The speed that the drone travels between customer $i \in C$ and launch point $j \in L$ is denoted by the continuous and nonnegative variable v_{ij}.

A formulation for the ERDDP is given as follows:

(ERDDP)

$$\text{Minimize} \quad c_d \sum_{j \in L} (c_{0j} + c_{j0}) y_j \tag{7}$$

$$+ \quad c_e \sum_{i \in C} \sum_{j \in L} \left[2\mu_1 \frac{d_{ij}}{v_{ij}} + 2\mu_2 d_{ij} v_{ij} + \frac{\mu_3^1 d_{ji} + \mu_3^2 d_{ij}}{(v_{ij})^2} + 2\mu_4 d_{ij} (v_{ij})^2 \right] x_{ij} \tag{8}$$

subject to

$$\sum_{j \in L} x_{ij} = 1 \qquad \forall i \in C \tag{9}$$

$$x_{ij} \leq y_j \qquad \forall i \in C, j \in L \tag{10}$$

$$\sum_{j \in L} y_j = p \tag{11}$$

$$\left[2\mu_1 \frac{d_{ij}}{v_{ij}} + 2\mu_2 d_{ij} v_{ij} + \frac{\mu_3^1 d_{ji} + \mu_3^2 d_{ij}}{(v_{ij})^2} + 2\mu_4 d_{ij} (v_{ij})^2\right] x_{ij} \leq \Theta \qquad \forall i \in C, j \in L \tag{12}$$

$$\frac{c_{0j}}{\overline{v}} y_j + \sum_{k \in C} \left(\frac{2d_{kj}}{v_{kj}} + \tau\right) x_{kj} - \frac{d_{ij}}{v_{ij}} x_{ij} - M_j(1 - x_{ij}) \leq U \qquad \forall i \in C, j \in L \tag{13}$$

$$v_{ij} \leq v_{max} \qquad \forall i \in C, j \in L \tag{14}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in C, j \in L \tag{15}$$

$$y_j \in \{0, 1\} \qquad \forall j \in L. \tag{16}$$

The first term (7) of the objective function minimizes the total operational cost associated with trucks and the second term (8) minimizes the total amount of energy caused by drones. Constraints (9) ensure that each customer will be assigned to a launch point and served by a drone takes off from that launch point. Constraints (10) guarantee that no drone will serve any customer from a launch point if it does not take off from that point. Constraint (11) fixes the number of launch points to be used to p. Constraints (12) limit the drone to stay within its range during travel. Constraints (13) ensure that each customer will be served within the predetermined time limit U, where M_j is a sufficiently large constant that can be set equal to $\max_{i \in C}\{d_{ij}/v_{opt}\}$. The first part of the left hand side of the inequality represents the journey time of the truck from the depot to a launch point. The second part represents the time of all drone journeys between the truck at a given launch point and all customers assigned to it. The return journey from customer i to the launch point j is subtracted in third part as this time does not need to be included in the time limit U. Constraint (13) is added for every pair of customers and launch points to ensure all customers are served within the time limit as we do not keep track of the sequence of customers served from a given launch point. The last part of the left hand side of that inequality is to ensure that the constraint is not active if customer i is not assigned to a launch point j, i.e., if the variable x_{ij} equals zero. Therefore, we subtract the last part (M_j) to be sure that each customer will be served within the time limit in the case where x_{ij} is zero. Constraints (14) ensure that the drone speed remains between the allowable limits. Constraints (15) and (16) define the domains of the binary variables.

4.1 Second order cone programming

In this section, we show how the formulation introduced above for the ERDDP can be reformulated as a second order cone program (SOCP), which in turn could be solved using off-the-shelf commercial optimizers. Different types of inequalities such as linear, quadratic, hyperbolic, etc. can be represented as a second order cone constraint. For applications of the SOCP problems, we refer the reader to Lobo et al. (1998), Alizadeh & Goldfarb (2003).

The formulation of the ERDDP is nonlinear due to the second term of the objective function (8) and constraints (12) and (13). We introduce three new auxiliary decision variables, namely w_{ij}, u_{ij} and z_{ij}, using which we reformulate the model as follows:

(ERDDP-SOCP)

$$\begin{aligned}\text{Minimize} \quad & c_d \sum_{j \in L} (c_{0j} + c_{j0}) y_j \\ + \quad & c_e \sum_{i \in C} \sum_{j \in L} \left[2\mu_1 d_{ij} w_{ij} + 2\mu_2 d_{ij} v_{ij} + (\mu_3^1 d_{ji} + \mu_3^2 d_{ij}) z_{ij} + 2\mu_4 d_{ij} u_{ij} \right] \end{aligned} \tag{17}$$

subject to (9)–(11), (15)–(16),

$$\left[2\mu_1 d_{ij} w_{ij} + 2\mu_2 d_{ij} v_{ij} + (\mu_3^1 d_{ji} + \mu_3^2 d_{ij}) z_{ij} + 2\mu_4 d_{ij} u_{ij} \right] \leq \Theta x_{ij} \qquad \forall i \in C, j \in L \tag{18}$$

$$\frac{c_{0j}}{\overline{v}} y_j + \sum_{k \in C} (2 d_{kj} w_{kj} + \tau x_{kj}) - d_{ij} w_{ij} - M_j (1 - x_{ij}) \leq U \qquad \forall i \in C, j \in L \tag{19}$$

$$v_{ij} \leq v_{max} x_{ij} \qquad \forall i \in C, j \in L \tag{20}$$

$$x_{ij} \leq w_{ij} v_{ij} \qquad \forall i \in C, j \in L \tag{21}$$

$$v_{ij}^2 \leq u_{ij} \qquad \forall i \in C, j \in L \tag{22}$$

$$w_{ij}^2 \leq z_{ij} \qquad \forall i \in C, j \in L. \tag{23}$$

The reformulation entails using the transformations $w_{ij} = x_{ij}/v_{ij}$, $u_{ij} = v_{ij}^2$ and $z_{ij} = w_{ij}^2$ in expression (8), which yields (17). To see why the formulations ERDDP and ERDDP-SOCP are equivalent, consider the following.

The first part of the expression (8) includes the term $\frac{x_{ij}}{v_{ij}}$, which is replaced by w_{ij}. In the second part of (8), we show that removing the variable x_{ij} from the expression $v_{ij}x_{ij}$ does not change the formulation since constraints (18) and (20) ensure that if x_{ij} equals 0, then v_{ij} equals 0. On the other hand, if x_{ij} equals 1, then both expressions are the same. Therefore, we can safely remove x_{ij} from second part of (8). In the third part of (8), which includes $\frac{x_{ij}}{v_{ij}^2}$, since x_{ij} is a binary variable, if x_{ij} equals 0, then x_{ij}^2 equals 0, and if x_{ij} equals 1, then x_{ij}^2 equals 1. Therefore, one can use $\frac{x_{ij}^2}{v_{ij}^2}$ instead of $\frac{x_{ij}}{v_{ij}^2}$. By definition, $\frac{x_{ij}^2}{v_{ij}^2}$ equals w_{ij}^2, which in turn equals z_{ij}. Finally, in the last part of expression (8), one can remove x_{ij} from the term $v_{ij}^2 x_{ij}$ without changing the formulation using the same reasoning for the second part of expression (8).

Constraints (22) and (23) are quadratic inequalities, which can be rewritten as a second order cone constraint in the form $\left\| \begin{bmatrix} 2v_{ij} \\ 1 - u_{ij} \end{bmatrix} \right\| \leq (1 + u_{ij})$ since $u_{ij} \geq 0$ and $v_{ij} \geq 0$, and in the form $\left\| \begin{bmatrix} 2w_{ij} \\ 1 - z_{ij} \end{bmatrix} \right\| \leq (1 + z_{ij})$ since $w_{ij} \geq 0$ and $z_{ij} \geq 0$, respectively. Constraint (21) is also a nonlinear constraint, which can be reformulated as:

$$x_{ij}^2 \leq w_{ij} v_{ij} \qquad \forall i \in C, j \in L. \tag{24}$$

Constraint (24) is a hyperbolic inequality and it can be represented as a second order cone constraint in the following form $\left\| \begin{bmatrix} 2x_{ij} \\ v_{ij} - w_{ij} \end{bmatrix} \right\| \leq (v_{ij} + w_{ij})$ since $x_{ij} \geq 0$, $v_{ij} \geq 0$ and $w_{ij} \geq 0$.

Furthermore, constraints (22) and (23) can be strengthened by using perspective cuts (Günlük & Linderoth 2012) in the following forms, respectively,

$$v_{ij}^2 \leq u_{ij} x_{ij} \qquad \forall i \in C, j \in L \tag{25}$$

$$w_{ij}^2 \leq z_{ij} x_{ij} \qquad \forall i \in C, j \in L. \tag{26}$$

Constraints (25) and (26) ensure that decision variables v_{ij} and w_{ij} will be zero if there is no drone that launches from launch point $j \in L$ and serves customer $i \in C$. Both (25) and (26) are hyperbolic inequalities and can be rewritten as second order cone constraints as $\left\| \begin{bmatrix} 2v_{ij} \\ u_{ij} - x_{ij} \end{bmatrix} \right\| \leq (u_{ij} + x_{ij})$ since $v_{ij} \geq 0$, $u_{ij} \geq 0$ and $x_{ij} \geq 0$, and as $\left\| \begin{bmatrix} 2w_{ij} \\ z_{ij} - x_{ij} \end{bmatrix} \right\| \leq (z_{ij} + x_{ij})$ since $w_{ij} \geq 0$, $z_{ij} \geq 0$ and $x_{ij} \geq 0$, respectively.

5 Computational Study

In this section, we present detailed analyses on the solutions of the ERDDP. First, we describe the data set. Second, we evaluate the performance of the perspective cuts on the ERDDP formulation. We then present analyses on the effect of changing the number p of available launch points and the time limit U, the effect of allowing the use of multiple drones, and, finally, the value of optimizing speed. All experiments were carried out on a Linux OS environment with Dual Intel Xeon E5-2690 v4 14 Core 2.6 GHz processors with 128 GB of RAM using IBM ILOG CPLEX Optimization Studio version 12.7.1 as the solver.

5.1 Data set

The data used for the computational analysis is drawn from a distribution problem arising in the Kartal district located in the east (Anatolian) side of Istanbul, Turkey. This district can be considered as a dense urban area with a total land area of 38.54 km^2 and a population of 500,000 people. In this data set, the average and maximum distances from launch points to customers vary between 2.59 and 3.98 km, and 4.90 and 6.64 km, respectively. At the optimal speed of 74.65 m/s, the average and maximum flying times vary between 2.08 and 3.2 minutes, and 3.94 and 5.34 minutes, respectively.

The data set is described in Kilci et al. (2018), and includes one depot (triangle), three schools and four hospitals that collectively correspond to seven potential parking locations (squares), and 37 centers of population each of which is a demand point (circles) (Figure 3). Pairwise distances for driving and for flying have been extracted using Google Maps and the visual basic codes in Analystcave (2014), respectively, where the latter takes into account the elliptic structure of the earth.

Based on the regulations of the General Directory of Highways of Turkey, the average truck speed $\overline{v}$ is set equal to 45 km/h. To ensure same day deliveries and competitiveness on service levels, the time bound U is set equal to 2, 1 and 0.5 hours. We set the maximum speed limit v_{max} as 30 m/s (108 km/h) (Zeng et al. 2019). As in Murray & Chu (2015), the setup time τ of a drone for a delivery is set equal to two minutes. The average monthly salary of a truck driver in Turkey is approximately 2,500 Turkish Lira (₺) and the workload per month is 120

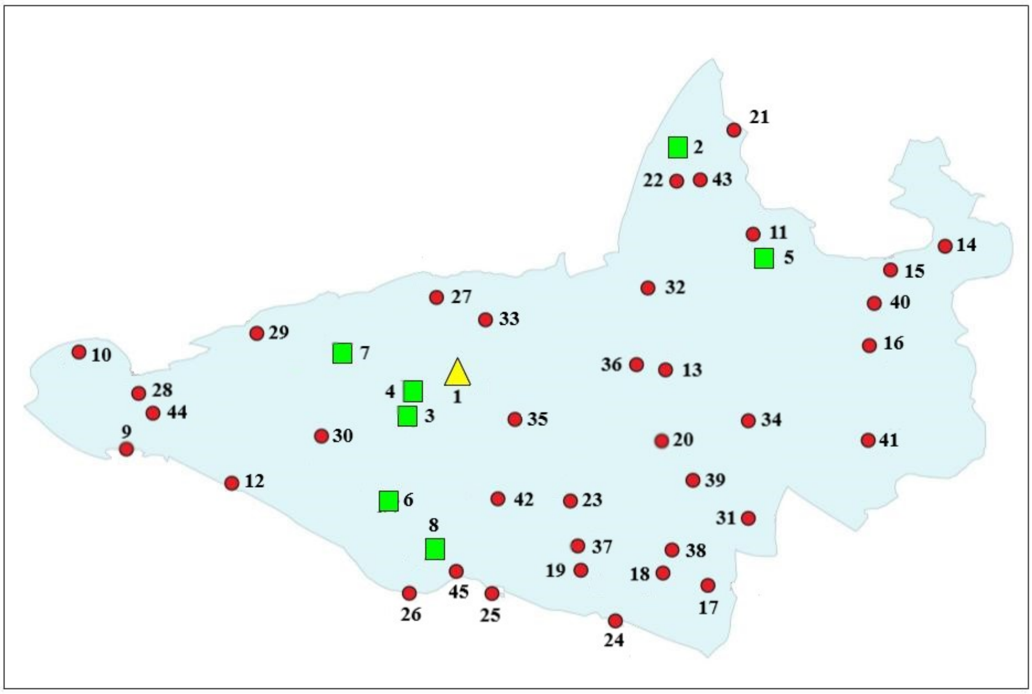

Figure 3: The locations of depot, parking areas and customers on Kartal map

hours, which, using the average truck speed, translates into 0.463 ₺/km as the driver cost per distance. Fuel cost per distance for a conventional truck in Turkey is approximately 0.6 ₺/km, which results in 1.063 ₺/km (0.18 \$/km) as the unit operational cost c_d per distance for a truck. Dorling et al. (2017) calculate the unit energy cost by dividing the cost of the battery by the energy density. The purchasing cost of the battery is approximately 377.136 ₺ (65 \$) (Alibaba 2019) and the energy density (capacity) of the battery is 540 kilojoule (kJ) and the cycle number is 500 (Tektronix 2020), resulting in a unit purchasing cost equal to 0.0013968 ₺/kJ and the unit electricity cost (for recharging battery) is approximately 10 cent/kWh, which equals to 0.000001634 ₺/kJ. The unit energy cost c_e equals to 0.0013984 ₺/kJ (0.00023773 \$/kJ). The description and typical values of parameters are summarized in Table 4. Other parameters are as shown in Tables 2 and 3.

5.2 Performance of Perspective Cuts

This section analyzes the impact of perspective cuts on the solution time of the ERDDP. In the experiments, we use three different values for the time bound U as 2, 1 and 0.5 (hours) and eight different values for the number of launch points as $p = 1, 2, 3, 4, 5, 6, 7, 8$ resulting in a total of

Table 4: Parameter Values

Notation	Description	Typical Values
v^t	Truck speed (km/h)	45
U	Time bound (hours)	0.5, 1, 2
v_{max}	Maximum drone speed (km/h)	108
τ	Setup time of a drone (s)	120
c_d	Unit operational cost per distance for a truck ($/km)	0.18
c_e	Unit energy consumption cost ($/kJ)	0.00023773

24 instance. Since in eight of these 24 instances, there is no feasible solution, we only report the remaining 16 instances. Table 5 presents the comparison results of the ERDDP formulation with and without perspective cuts. A common time limit of two hours has been imposed on the solution time of each instance.

Table 5: Performance of perspective cuts on instances with $|C| = 37$ and $|P| = 7$

		No Pers Cuts		Pers Cuts
U (hr)	p	Sol Time (s)	Gap (%)	Sol Time (s)
2	2	4.31		1.49
2	3	7200.00	6.69	1.12
2	4	7200.00	3.87	4.70
2	5	7200.00	2.76	3.91
2	6	7200.00	1.91	1.53
2	7	25.11		5.04
2	8	10.70		3.49
1	3	151.74		27.53
1	4	17.88		16.33
1	5	14.69		10.70
1	6	7200.00	2.09	10.55
1	7	5.31		3.80
1	8	12.20		5.28
0.5	6	7200.00	0.52	3471.61
0.5	7	7200.00	1.75	1609.30
0.5	8	25.60		30.87
Average		3166.72	2.80	325.45

While the formulation without perspective cuts finds optimal solutions in nine out of 16 instances within two-hour time limit, the formulation with perspective cuts optimally solves all instances within one hour, which we use in the rest of the experiments.

5.3 Analysis on the ERDDP solutions

This section presents the results of further analyses on the solutions of the ERDDP using five randomly generated sets of instances with different number of customers and potential parking areas, using the configurations ($|C|$, $|P|$) as (10, 2), (15, 3), (20, 4), (25, 5) and (30, 6). For each configuration, we have generated five instances. The number p of launch points ranges between one and $|P|$ + 1 in order to evaluate the solutions for each possible number of drone. The summary of the results are given in Table 6 which presents the average values over five instances for each configuration. The table also presents the results for the original data set where the configuration ($|C|$, $|P|$) is (37, 7). The time limit for the solver is set as two hours. Columns titled "T. Cost", "E. Con.", "T. Dis.", "D. Dis.", "M. Dis.", "A. Spe.", and "S. Time" stand for the optimal objective value, the total cost ($) calculated using the formulae (7) and (8), the total amount of energy consumption (in kJ) calculated using the formulae (8) but without multiplying it with c_e, the total distance traveled by trucks (in km) calculated using the formulae (7) but without multiplying it with c_d, the total distance traveled by drones (in km), the maximum distance traveled by a drone (in km), the average drone speed (in km/h) over each journey, and the solution time (in s), respectively.

The results presented in Table 6 suggest that an increase on the number of launch points increases the total cost due to the increase in the total distance traveled by trucks. On the contrary, increasing the number of launch points leads to a reduction in the amount of energy required by drones. This, in turn, is due to the reduction in the total distance traveled by drones and the customers being much closer to launch points, enabling the drones to attain speeds that are closer to the globally optimal speed (around 72.50 km/h) that minimizes the convex expression (4). For small p values, the average drone speed can increase up to 79.03 km/h as can be seen in Table 6. For all instances in which $p = 1$ and where the total distance traveled by truck is equal to zero, the drone is launched from the depot. The results also indicate that when several launch points are used, the depot is not necessarily used as a launch point. Moreover, using additional launch points for drones reduces the maximum distance traveled by a drone. Another interesting observation is the behavior of solution time, which seems to be affected by $|C|$, but not so much affected by the number p of launch points available.

Table 6: Summary of the results with a varying number of launch points

$\|C\|$	$\|P\|$	p	T. Cost (\$)	E. Con. (kJ)	T. Dis. (km)	D. Dis. (km)	M. Dis. (km)	A. Spe. (km/h)	S. Time (s)
10	2	1	0.13	563.49	0.00	56.42	4.28	72.53	0.27
10	2	2	1.01	522.26	4.88	52.29	4.28	72.50	0.28
10	2	3	3.15	420.12	16.88	42.06	3.47	72.50	0.22
15	3	1	0.19	805.15	0.00	80.63	4.25	72.54	0.54
15	3	2	0.84	766.27	3.63	76.73	4.25	72.53	0.48
15	3	3	1.69	744.70	8.37	74.56	4.25	72.53	0.46
15	3	4	3.63	640.42	19.22	64.12	3.71	72.50	0.45
20	4	1	0.26	1073.52	0.00	105.80	4.59	79.03	0.68
20	4	2	0.95	998.98	3.93	100.05	4.59	72.55	0.64
20	4	3	1.93	953.22	9.41	95.46	4.59	72.52	0.65
20	4	4	3.32	817.70	17.30	81.89	4.16	72.51	0.62
20	4	5	5.69	656.10	30.62	65.70	3.07	72.50	0.61
25	5	1	Infeasible						
25	5	2	0.95	1294.95	3.54	129.66	4.57	72.88	0.84
25	5	3	1.79	1258.20	8.28	126.00	4.57	72.52	0.96
25	5	4	2.83	1198.56	14.10	120.03	4.57	72.52	0.93
25	5	5	4.23	1045.81	22.04	104.74	4.21	72.52	0.96
25	5	6	6.53	836.69	35.05	83.79	3.28	72.51	0.88
30	6	1	Infeasible						
30	6	2	1.02	1583.70	3.54	158.59	4.79	72.89	0.94
30	6	3	1.73	1555.78	7.53	155.80	4.79	72.74	0.91
30	6	4	2.63	1514.48	12.58	151.67	4.79	72.74	1.47
30	6	5	3.86	1385.34	19.51	138.74	4.79	72.52	3.07
30	6	6	6.01	1016.79	31.90	101.83	3.18	72.51	2.75
30	6	7	8.52	981.39	45.84	98.28	3.18	72.51	2.20
37	7	1	Infeasible						
37	7	2	1.11	1992.05	3.54	199.51	4.90	72.67	1.49
37	7	3	1.82	1948.80	7.53	195.18	4.90	72.66	1.12
37	7	4	2.72	1899.05	12.58	190.19	4.90	72.66	4.70
37	7	5	3.90	1772.48	19.24	177.51	4.90	72.67	3.91
37	7	6	5.20	1653.38	26.58	165.59	4.90	72.67	1.53
37	7	7	7.32	1156.89	38.96	115.87	3.10	72.51	5.04
37	7	8	9.82	1099.43	52.90	110.10	3.10	72.51	3.49

5.4 Impact of the time limit

The aim of this section is to investigate the impact of varying the delivery time bound on the solution of the ERDDP. In the base case, the delivery time bound (U) is set equal to two hours, which we reduce first to one hour and then to 30 minutes. The results are presented in Tables 7 and 8 where we report the percent deviation of each of the metrics from those obtained by the two-hour time bound instances.

Table 7: Results with a one-hour time bound

$\|C\|$	$\|P\|$	p	T. Cost (%)	E. Con. (%)	T. Dis. (%)	D. Dis. (%)	A. Spe. (%)
10	2	1	3.08	3.08	0.00	0.00	14.27
10	2	2	0.02	0.12	0.00	0.11	0.02
10	2	3	0.00	0.00	0.00	0.00	0.00
15	3	1			Infeasible		
15	3	2	0.08	0.33	0.00	0.25	1.31
15	3	3	0.02	0.13	0.00	0.11	0.48
15	3	4	0.00	0.00	0.00	0.00	0.00
20	4	1			Infeasible		
20	4	2	0.85	3.37	0.00	2.02	7.83
20	4	3	0.17	1.32	0.00	1.28	1.01
20	4	4	0.05	0.55	0.00	0.54	0.27
20	4	5	0.00	0.00	0.00	0.00	0.00
25	5	1			Infeasible		
25	5	2	74.07	8.60	106.45	-9.42	43.18
25	5	3	0.39	2.33	0.00	2.26	1.33
25	5	4	0.19	1.81	0.00	1.69	1.59
25	5	5	0.07	0.98	0.00	0.90	0.58
25	5	6	0.00	0.00	0.00	0.00	0.00
30	6	≤ 2			Infeasible		
30	6	3	1.18	5.48	0.00	2.97	12.75
30	6	4	0.46	3.35	0.00	3.20	1.69
30	6	5	0.25	2.84	0.00	2.60	2.76
30	6	6	0.00	0.05	0.00	0.00	0.51
30	6	7	0.00	0.05	0.00	0.00	0.51
37	7	≤ 2			Infeasible		
37	7	3	35.87	10.74	44.43	-9.18	45.40
37	7	4	0.78	4.73	0.00	4.54	2.40
37	7	5	0.50	4.66	0.00	4.56	1.80
37	7	6	0.35	4.56	0.00	4.50	1.36
37	7	7	0.01	0.24	0.00	0.00	3.05
37	7	8	0.00	0.01	0.00	0.00	0.69

As can be seen in Table 7, reducing U from two hours to one hour makes the instances with

$(|C|, p) = (15, 1), (20, 1), (30, 1), (30, 2), (37, 1)$ and $(37, 2)$ infeasible. Also, setting U as one hour increases the total cost and the total energy consumption by 3.50% and 1.79% on average, respectively. For small p values, the increase in the total cost and the total energy consumption can be up to 163.55% and 23.44%, respectively. Tightening the time limit also affects the solutions in terms of the location of launch points and customer assignments as can be observed from the changes in the total distance traveled by trucks and drones, respectively. Especially, for the instances with $(|C|, p) = (25, 2)$ and $(37, 3)$, reducing the time limit to one hour increases the total distance traveled by trucks by 106.45% and 44.43% on average, respectively. For those instances, as the launch points become much closer to the customers, the total distance traveled by drones is decreased by 9.42% and 9.18%, respectively. For the remaining instance, while the location of launch points remains the same as the total distance traveled by trucks does not change and customers are assigned to the launch points further away as the total distance traveled by drones increases, except for large p values where the customer assignments remain the same. Finally, the speed of drones is increased by 4.40% on average, especially for small p values up to 45% increase in the drone speed is observed in order to make deliveries on time. Another interesting observation can be made for the instances with $(|C|, p) = (25, 2)$ and $(37, 3)$. Although the launch points become much closer to the customers as the total distance traveled by drones is decreased, the average drone speed has to be substantially increased (by around 45%) in order to make all deliveries within one hour. Analyzing the instance with $(|C|, p) = (37, 3)$ further indicate that three drones travel between launch points and customers at 105.65 km/h, on average including at the maximum speed limit (108 km/h) on several journeys.

Table 8 shows that reducing U further to 30 minutes yields more infeasible instances, namely $(|C|, p) = (10, 1), (15, 2), (20, 2), (20, 3), (25, 2), (25, 3), (30, 3), (30, 4), (37, 3), (37, 4)$ and $(37, 5)$. In this case, the total cost and the total energy consumption increase by 4.15% and 6.65% on average, respectively. For some instances with a larger number of customers such as 25, 30 and 37, the increase in the total cost and the total energy consumption are as significant as 50% and 30%, respectively. Similar to the cases with one hour-time limit, for the instances with $(|C|, p) = (25, 4), (30, 5)$ and $(37, 6)$, reducing the time limit to 30 minutes increases the total distance traveled by trucks and decreases the total distance traveled by drones, meaning that distances between the depot and launch points increase, while distances between customers

Table 8: Results with a 30-minute time bound

$\lvert C \rvert$	$\lvert P \rvert$	p	T. Cost (%)	E. Con. (%)	T. Dis. (%)	D. Dis. (%)	A. Spe. (%)
10	2	1			Infeasible		
10	2	2	0.84	5.20	0.00	1.54	14.38
10	2	3	0.03	0.64	0.00	0.52	0.51
15	3	≤ 2			Infeasible		
15	3	3	0.64	5.95	0.00	3.35	11.81
15	3	4	0.09	1.66	0.00	1.65	0.49
20	4	≤ 3			Infeasible		
20	4	4	0.58	8.92	0.00	7.25	7.72
20	4	5	0.11	3.63	0.00	2.48	4.68
25	5	≤ 3			Infeasible		
25	5	4	44.07	-6.08	49.78	-19.35	37.92
25	5	5	0.56	9.41	0.00	7.76	7.69
25	5	6	0.21	6.49	0.00	6.15	2.63
30	6	≤ 4			Infeasible		
30	6	5	16.55	13.37	16.97	-3.93	39.45
30	6	6	0.54	13.53	0.00	11.59	7.95
30	6	7	0.19	6.97	0.00	6.04	4.83
37	7	≤ 5			Infeasible		
37	7	6	18.28	9.75	18.98	-7.67	41.35
37	7	7	0.92	24.44	0.00	22.73	8.03
37	7	8	0.29	11.03	0.00	10.05	6.04

and launch points decrease. Another interesting result is that an increase in the total energy consumption is observed for all but the instances with ($|C|$, p) = (25, 4), for which the total energy consumption is reduced by 6.08% due to a 19.35% reduction in the total distance traveled by drones in spite of a 37.92% increases in the average drone speed. This reduction on the energy consumption, however, does not show its impact on the total cost, as it is increased by 44.07% due to a 49.78% increase in the total distance traveled by trucks. For the remaining instances with larger p values, while the total distance traveled by trucks remains the same, the total distance traveled by drones is increased, meaning that customers are assigned to the launch points further away, while the location of launch points does not change. Finally, the increase in the drone speed is 11.14% on average, but this could be up to 43.27% for instances with a larger number of customers especially for the instances with ($|C|$, p) = (25, 4), (30, 5) and (37, 6) where drones regularly travel more than 100 km/h, on average.

5.5 Allowing the use of multiple drones

In this section, we allow multiple drones to take off from a given launch point. To model this variant, we define a set $D = \{1, ..., d\}$ of drones and a new decision variable t_{ijd} that equals 1 if drone $d \in D$ takes off from a launch point $j \in L$ to serve customer $i \in C$, and 0 otherwise. In the formulation, we include the new constraints (27) below and replace constraints (13) with constraints (28) as below:

$$x_{ij} = \sum_{d \in D} t_{ijd} \qquad \forall i \in C, j \in L \tag{27}$$

$$\frac{c_{0j}}{\overline{v}} y_j + \sum_{k \in C} \left(\frac{2d_{kj}}{v_{kj}} + \tau\right) t_{kjd} - \frac{d_{ij}}{v_{ij}} t_{ijd} - M_j (1 - t_{ijd}) \leq U \qquad \forall i \in C, j \in L, d \in D. \tag{28}$$

Constraints (27) ensure that only one drone takes off from a launch point to serve a customer, and constraints (28) guarantee that each customer will be served within the predetermined time limit U.

We conduct computational experiments to analyze the impact of having several drones at each launch point. In order to clearly see the impact of using multiple drones, we impose a tighter time limit $U = 30$ minutes. Table 9 presents the comparisons between the case where only one drone can take off from each launch point and where two or three drones are used at each launch point. We report the percentage deviations of the latter cases from the former one for each of the metrics shown in Table 9. The computational experiments conducted in this section consist of five instances with ($|C|$, $|P|$) = (10, 2), (15, 3), (20, 4), (25, 5) and (30, 6) and the original instance (37, 7). The table reports the average results over five instances for each configuration and the original instance, and where p ranges between 1 and $|P|$ + 1.

Table 9: Results of the experiments with several drones per launch point

			1 drone					2 drones					3 drones				
$\|C\|$	$\|P\|$	p	T. Cost ($)	E. Con. (kJ)	T. Dis. (km)	D. Dis. (km)	A. Spe. (km/h)	T. Cost (%)	E. Con. (%)	T. Dis. (%)	D. Dis. (%)	A. Spe. (%)	T. Cost (%)	E. Con. (%)	T. Dis. (%)	D. Dis. (%)	A. Spe. (%)
10	2	1	Infeasible					Not applicable					Not applicable				
10	2	2	1.01	551.45	4.88	53.12	82.92	−0.83	−4.81	0.00	−1.50	−11.84	−0.83	−4.81	0.00	−1.50	−11.84
10	2	3	3.15	423.37	16.88	42.34	72.91	−0.03	−0.59	0.00	−0.52	−0.14	−0.03	−0.59	0.00	−0.52	−0.14
15	3	1	Infeasible					Infeasible					Not applicable				
15	3	2	Infeasible					Not applicable					Not applicable				
15	3	3	1.70	789.58	8.37	77.03	81.09	−0.63	−5.52	0.00	−3.15	−10.08	−0.64	−5.60	0.00	−3.23	−10.09
15	3	4	3.63	651.99	19.22	65.28	72.86	−0.09	−1.61	0.00	−1.60	−0.34	−0.09	−1.61	0.00	−1.60	−0.38
20	4	1	Infeasible					Infeasible					Not applicable				
20	4	2	Infeasible					Not applicable					Not applicable				
20	4	3	Infeasible					Not applicable					Not applicable				
20	4	4	3.34	892.21	17.30	87.72	78.10	−0.53	−7.73	0.00	−6.36	−6.29	−0.57	−8.12	0.00	−6.73	−6.70
20	4	5	5.70	680.07	30.62	67.33	75.90	−0.11	−3.41	0.00	−2.39	−4.22	−0.11	−3.39	0.00	−2.39	−4.23
25	5	1	Infeasible					Infeasible					Infeasible				
25	5	2	Infeasible					Not applicable					Not applicable				
25	5	3	Infeasible					Not applicable					Not applicable				
25	5	4	4.06	1109.70	21.03	95.38	100.02	−30.08	8.33	−32.77	25.73	−24.94	−30.18	6.67	−32.77	24.12	−26.34
25	5	5	4.26	1143.63	22.04	112.77	78.09	−0.51	−7.86	0.00	−6.63	−5.13	−0.56	−8.58	0.00	−7.19	−6.95
25	5	6	6.55	891.59	35.05	89.01	74.42	−0.21	−6.00	0.00	−5.74	−1.97	−0.21	−6.04	0.00	−5.74	−2.56
30	6	1	Infeasible					Infeasible					Infeasible				
30	6	2	Infeasible					Infeasible					Not applicable				
30	6	3	Infeasible					Not applicable					Not applicable				
30	6	4	Infeasible					Not applicable					Not applicable				
30	6	5	4.49	1561.55	22.81	132.62	101.12	−13.10	−8.01	−13.22	8.04	−27.43	−13.29	−10.28	−13.22	5.43	−28.03
30	6	6	6.04	1153.44	31.90	113.56	78.28	−0.53	−11.60	0.00	−10.16	−6.48	−0.53	−11.72	0.00	−10.30	−6.57
30	6	7	8.53	1049.24	45.84	104.18	76.01	−0.19	−6.37	0.00	−5.55	−4.24	−0.19	−6.48	0.00	−5.66	−4.51
37	7	≤ 2	Infeasible					Infeasible					Infeasible				
37	7	3	Infeasible					Not applicable					Not applicable				
37	7	4	Infeasible					Not applicable					Not applicable				
37	7	5	Infeasible					Not applicable					Not applicable				
37	7	6	6.15	1814.56	31.62	152.88	102.72	−16.51	0.91	−17.73	18.36	−25.72	−16.72	−2.32	−17.73	14.91	−27.38
37	7	7	7.38	1439.65	38.96	142.20	78.33	−0.82	−17.65	0.00	−16.65	−6.49	−0.91	−19.55	0.00	−18.52	−7.61
37	7	8	9.85	1220.68	52.90	121.16	76.90	−0.29	−9.67	0.00	−9.36	−3.60	−0.29	−9.98	0.00	−9.62	−4.18

The results indicate that using only one drone for each launch point leads to the infeasibility of 72 out of the 133 instances tested. Using two and three drones per launch point substantially reduces the number of infeasible solutions, resulting in only 29 and 16 infeasible instances, respectively. Further increase in the number of drones per launch point renders all instances feasible, and suggests that the primary cause of infeasibility is the time limit. Table 9 suggests that using two and three drones per launch point reduces the total cost by 3.12% and 3.15%

on average, respectively, which shows that while using a second drone has an impact, using a third drone has a relatively small effect on the solution values. This can also be seen from the total energy consumption and the average drone speed values. Launching a second and a third drone reduces the total energy consumption by 5.32% and 5.79%, and the drone speed by 8.27% and 8.71%, respectively. For instances with a larger number of customers, we observe a decrease around 30% in the drone speed meaning that with the help of additional drones, drones enable to travel at speeds that are closer to the optimal speed (72.50 km/h). Using more drones from each launch point also affects the solutions in terms of the location of launch points and customer assignments as the total distance traveled by drones is reduced by around 2% in both cases and the total distance traveled by trucks is decreased by around 3%. For the instances with ($|C|$, p) = (25, 4), (30, 5) and (37, 6), launching two drones decreases the total distance traveled by trucks by 32.77%, 13.22% and 17.73% on average, respectively, meaning that launch points are located closer to the depot. For the same instances, as the total distance traveled by drones is increased by 25.73%, 8.04% and 18.36%, respectively, launch points become further away from the customers. Similar to the interesting result found in Section 5.4, for instances with ($|C|$, p) = (25, 4), a 8.33% increase in the energy consumption is observed when two drones are allowed to take off from the same launch point due to the 25.73% increase in total distance traveled by drones in spite of a 24.94% decrease in the average drone speed. Although launching two drones increases drone energy consumption for this particular instance, a 32.77% decrease in the total distance traveled by trucks results in a 30.08% reduction on the total cost. For the remaining instance, while the location of launch points remains the same as the total distance traveled by trucks does not change, customers are assigned closer to the launch points as the total distance traveled by drones decreases. Similar to the results obtained in Section 5.4, we can say that increasing (decreasing) the time bound and increasing (decreasing) the number of drones per launch point have similar impacts on the solutions in terms of the location of launch points and customer assignments.

5.6 The value of using optimized vs. fixed speeds

To quantify the value of optimizing speed, we run the model where we set the speed to the fixed speed values from 50 km/h to 105 km/h in 5 km/h increments and the speed of 72.50 km/h as the optimal speed value (Table 6). The results of the tests, conducted on 16 instances each with 37 customers, are given in Table 10 in terms of the percentage change in the total cost, overall energy consumption, total distance traveled by trucks and drones and average speed. Additionally, the last column of the table reports the number of infeasible instances due to fixing speed.

Table 10: Results with varying fixed speed values

Fixed Speed (km/h)	T. Cost (%)	E. Con. (%)	T. Dis. (%)	D. Dis. (%)	A. Spe. (%)	Infeas.
50	2.16	19.94	0.35	1.24	-31.60	6
55	6.10	7.70	6.63	-1.61	-24.83	4
60	19.25	1.87	32.37	-2.66	-18.31	3
65	13.98	-1.38	23.70	-2.82	-11.50	3
70	4.77	-0.71	8.17	-0.76	-4.69	3
72.5	1.29	-0.15	1.97	-0.04	-1.29	3
75	1.27	-0.39	1.34	-0.35	1.66	2
80	0.13	1.43	0.00	0.30	8.44	2
85	4.12	0.63	5.42	-1.71	12.90	1
90	4.44	3.40	5.42	-1.93	19.54	1
95	3.15	7.50	3.09	-1.49	26.18	1
100	3.65	11.69	3.09	-1.73	32.82	1
105	4.08	15.30	3.03	-2.09	37.14	0

As Table 10 shows, while fixing the speed to the optimal value (72.50 km/h) yields a minimal change (a 0.15% decrease) in the total energy consumption, it results with a 1.29% increase in the total cost, which is higher than the increase (0.13%) when the speed is fixed as 80 km/h due to no change in the total distance traveled by trucks. Although reducing the fixed speed value leads to big increases in the total cost such as 65 km/h (13.98%) and 60 km/h (19.25%), smaller increases are observed when it is further decreased to 55 km/h (6.10%) and 50 km/h (2.16%) as this is an unexpected result, which can be explained by having additional infeasible instances. While for the speed values, which are closer to the optimal speed (72.50 km/h), a decrease in the energy consumption is observed, increasing or decreasing the fixed speed even further to extreme values results with large increases in the energy consumption up to 20%. One other disadvantage of using fixed speeds is the resulting infeasibility of some instances; especially for the small speed values such as 50 km/h, six out of 16 instances become infeasible. Even at the optimal speed value (72.50 km/h), three instances results with infeasibility. As the fixed speed values increases, the number of infeasible solution decreases even further and fixing the speed as 105 km/h for each journey results with no infeasible solutions with the burden of having 4.08% and 15.30% increases in the total cost and total energy consumption, respectively.

5.7 Testing on larger-size instances

Holland et al. (2017) state that real-life delivery problems involve 140–160 customers a day. In order to demonstrate the applicability of the problem on such instances therefore, we increase the number of customers in the Kartal data set to 150, and the time limit to either two and four hours to ensure feasibility. We keep the number of potential launch points to eight, and present the results in Table 11.

Table 11: Results of a larger instance with a varying number of launch points and time bound

U (hr)	p	T. Cost ($)	E. Con. (kJ)	T. Dis. (km)	D. Dis. (km)	M. Dis. (km)	A. Spe. (km/h)	S. Time (s)
4	≤ 2				Infeasible			
4	3	3.20	7750.54	7.53	682.08	5.53	98.43	1351.31
4	4	3.84	6591.49	12.58	659.80	5.53	73.32	101.17
4	5	4.95	6203.13	19.24	620.89	5.53	73.32	235.11
4	6	6.19	5855.67	26.58	586.09	5.53	73.32	291.22
4	7	8.00	4034.36	38.96	404.03	3.37	72.51	62.44
4	8	10.50	3940.69	52.90	394.64	3.37	72.51	58.75
2	≤ 4				Infeasible			
2	5	5.90	5856.33	24.97	490.34	4.11	103.70	1772.60
2	6	6.36	6533.50	26.58	611.64	5.53	90.38	1839.53
2	7	8.07	4332.75	38.96	430.76	4.30	76.39	364.26
2	8	10.54	4119.62	52.90	411.89	4.30	74.30	267.50

As Table 11 indicates, the instances with 150 customers are solved relatively quickly, with the maximum solution time for any instance being around 30 minutes. The results also suggest that the larger number of customers increases the total cost, energy consumption and drone distance as expected. Different than the results obtained from Table 6, decreasing p value does not always increase the energy consumption and the total distance traveled by drones as this can be seen in Table 11 when p is reduced to five with two-hour time bound. The reason for this unexpected result is related to the location of launch points used. When p equals six, all launch points are opened except launch points 2 and 5, which are located in the north of the district and far away from the other launch points (Figure 3). When p is reduced to five, then launch points 6 and 8 (relatively closer to other launch points in the center) are closed and launch point 5 is opened meaning that most customers located in north and east of the district are assigned to (closer) launch point 5 resulting in a reduction in the total distance traveled by drones and also in the energy consumption in the expense of an increase in the average drone speed that is 103.7 km/h.

6 Conclusions

In this paper, we introduced the Energy Minimizing and Range Constrained Drone Delivery Problem (ERDDP) where drones are used as the only mode of transportation to make deliveries to a number of customers. The proposed problem includes explicit calculations of drone energy consumption and the drone range, both of which depend on the drone speed, which itself is considered as a decision variable in the ERDDP. The problem is initially modeled as a nonlinear programming formulation, which is then reformulated using second order cone programming that can be solved using off-the-shelf optimizers. We also include the perspective cuts to strengthen the proposed formulation.

Extensive computational results have revealed a number of managerial implications, summarized as follows: First, for this operational problem, optimal solutions can be identified within reasonable computation times by solving the second order cone programming formulation. Second, although the use of the depot as a launch point incurs no operational truck cost, the depot may not always be the best launch point for a drone. Third, using a looser time limit or launching more than one drone from the same spot should always be considered by operators since it will help to reduce the drone speed resulting in a reduction in cost.

The original formulation described for the ERDDP has two components that naturally yield nonlinearities, namely in (i) the model used to compute energy consumption that is quadratic in speed, which is seen in the objective function and constraints, and (ii) the way in which travel time is computed in the constraints that limit the total time within which deliveries should be made. The presented formulation can still be used where it is possible to approximate the nonlinear energy consumption with a simpler expression, as may be the case when speeds are set as constant values. The latter may be more appropriate for applications involving conventional vehicles where a driver adjusts the vehicle speed. Our experience in this paper is that the former is a more accurate representation and one that allows for the calculation of the exact value of optimal speeds, which is particularly relevant for autonomous vehicles such as drones, where the desired (optimal) speed can be easily implemented through mechanisms such as drone navigation systems. On the other hand, time constraints will inevitably be nonlinear if speed is a decision variable, as has also been the case in other similar applications such as the speed optimisation problem (see, e.g., Hvattum et al. 2013). Nonlinear formulations come at the expense of requiring advanced tools for their solution. In this paper, we have shown that there are ways to reformulate the problem that help to ease the computational intractability. In other cases where such reformulations are not possible, approximate (simpler) expressions for energy consumption or time calculation can be used especially in cases where speed does not play such a crucial role.

One of the main assumptions made in this paper was that drones are able to maintain a chosen speed over the entire journey. However, uncertain external factors such as traffic, obstacles or wind may affect the speed and direction of the drone, which is a limitation of our study. The inclusion of such factors in drone delivery applications would make for an interesting extension that could be solved by stochastic optimization techniques, which we suggest for further research. One other assumption made in the paper was that trucks travel at predetermined (constant) speeds. A further research direction could treat truck speeds as decision variables, which would allow a more accurate representation of fuel consumption that could then be incorporated into the minimizing objective function. Finally, the models described here can be extended to humanitarian applications where the objective could instead be an equitable allocation of relief items.

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