Reward Shaping for Reinforcement Learning with Omega-Regular Objectives

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Abstract. Recently, successful approaches have been made to exploit good-for-MDPs automata—Büchi automata with a restricted form of nondeterminism for model free reinforcement learning, a class of automata that subsumes good for games automata and the most widespread class of limit deterministic automata [3]. The foundation of using these Büchi automata is that the Büchi condition can, for good-for-MDP automata, be translated to reachability [2]. The drawback of this translation is that the rewards are, on average, reaped very late, which requires long episodes during the learning process. We devise a new reward shaping approach that overcomes this issue. We show that the resulting a model is equivalent to a discounted payoff objective with a biased discount that simplifies and improves on [1].

1 Preliminaries

A nondeterministic Büchi automaton is a tuple $\mathcal{A} = \langle \Sigma, Q, q_0, \Delta, \Gamma \rangle$, where Σ is a finite alphabet, Q is a finite set of states, $q_0 \in Q$ is the initial state, $\Delta \subseteq Q \times \Sigma \times Q$ are transitions, and $\Gamma \subseteq Q \times \Sigma \times Q$ is the transition-based acceptance condition.

A run r of \mathcal{A} on $w \in \Sigma^{\omega}$ is an ω -word $r_0, w_0, r_1, w_1, \dots$ in $(Q \times \Sigma)^{\omega}$ such that $r_0 = q_0$ and, for i > 0, it is $(r_{i-1}, w_{i-1}, r_i) \in \Delta$. We write $\inf(r)$ for the set of transitions that appear infinitely often in the run r. A run r of \mathcal{A} is accepting if $\inf(r) \cap \Gamma \neq \emptyset$.

The *language*, L_A , of \mathcal{A} (or, *recognized* by \mathcal{A}) is the subset of words in Σ^{ω} that have accepting runs in \mathcal{A} . A language is ω -*regular* if it is accepted by a Büchi automaton. An automaton $\mathcal{A} = \langle \Sigma, Q, Q_0, \Delta, \Gamma \rangle$ is *deterministic* if $(q, \sigma, q'), (q, \sigma, q'') \in \Delta$ implies q' = q''. \mathcal{A} is *complete* if, for all $\sigma \in \Sigma$ and all $q \in Q$, there is a transition $(q, \sigma, q') \in \Delta$. A word in Σ^{ω} has exactly one run in a deterministic, complete automaton.

A Markov decision process (MDP) \mathcal{M} is a tuple (S, A, T, Σ, L) where S is a finite set of states, A is a finite set of actions, $T : S \times A \to \mathcal{D}(S)$, where $\mathcal{D}(S)$ is the set of probability distributions over S, is the probabilistic transition function, Σ is an alphabet, and $L : S \times A \times S \to \Sigma$ is the labelling function of the set of transitions. For a state $s \in S$, A(s) denotes the set of actions available in s. For states $s, s' \in S$ and $a \in A(s)$, we have that T(s, a)(s') equals $\Pr(s'|s, a)$.

A run of \mathcal{M} is an ω -word $s_0, a_1, \ldots \in S \times (A \times S)^{\omega}$ such that $\Pr(s_{i+1}|s_i, a_{i+1}) > 0$ for all $i \geq 0$. A finite run is a finite such sequence. For a run $r = s_0, a_1, s_1, \ldots$

we define the corresponding labelled run as $L(r) = L(s_0, a_1, s_1), L(s_1, a_2, s_2), \ldots \in \Sigma^{\omega}$. We write $\Omega(\mathcal{M})$ (Paths (\mathcal{M})) for the set of runs (finite runs) of \mathcal{M} and $\Omega_s(\mathcal{M})$ (Paths $_s(\mathcal{M})$) for the set of runs (finite runs) of \mathcal{M} starting from state s. When the MDP is clear from the context we drop the argument \mathcal{M} .

A strategy in \mathcal{M} is a function μ : Paths $\rightarrow \mathcal{D}(A)$ that for all finite runs r we have $\operatorname{supp}(\mu(r)) \subseteq A(\operatorname{last}(r))$, where $\operatorname{supp}(d)$ is the support of d and $\operatorname{last}(r)$ is the last state of r. Let $\Omega_s^{\mu}(\mathcal{M})$ denote the subset of runs $\Omega_s(\mathcal{M})$ that correspond to strategy μ and initial state s. Let $\Sigma_{\mathcal{M}}$ be the set of all strategies. We say that a strategy μ is *pure* if $\mu(r)$ is a point distribution for all runs $r \in$ Paths and we say that μ is *positional* if $\operatorname{last}(r) = \operatorname{last}(r')$ implies $\mu(r) = \mu(r')$ for all runs $r, r' \in$ Paths.

The behaviour of an MDP \mathcal{M} under a strategy μ with starting state s is defined on a probability space $(\Omega_s^{\mu}, \mathcal{F}_s^{\mu}, \Pr_s^{\mu})$ over the set of infinite runs of μ from s. Given a random variable over the set of infinite runs $f : \Omega \to \mathbb{R}$, we write $\mathbb{E}_s^{\mu} \{f\}$ for the expectation of f over the runs of \mathcal{M} from state s that follow strategy μ .

Given an MDP \mathcal{M} and an automaton $\mathcal{A} = \langle \Sigma, Q, q_0, \Delta, \Gamma \rangle$, we want to compute an optimal strategy satisfying the objective that the run of \mathcal{M} is in the language of \mathcal{A} . We define the semantic satisfaction probability for \mathcal{A} and a strategy μ from state s as:

$$\mathsf{PSem}^{\mathcal{M}}_{\mathcal{A}}(s,\mu) = \Pr{}^{\mu}_{s} \{ r \in \Omega^{\mu}_{s} : L(r) \in L_{\mathcal{A}} \} \text{ and } \mathsf{PSem}^{\mathcal{M}}_{\mathcal{A}}(s) = \sup_{\mu} \left(\mathsf{PSem}^{\mathcal{M}}_{\mathcal{A}}(s,\mu) \right)$$

When using automata for the analysis of MDPs, we need a syntactic variant of the acceptance condition. Given an MDP $\mathcal{M} = (S, A, T, \Sigma, L)$ with initial state $s_0 \in S$ and an automaton $\mathcal{A} = \langle \Sigma, Q, q_0, \Delta, \Gamma \rangle$, the *product* $\mathcal{M} \times \mathcal{A} = (S \times Q, (s_0, q_0), A \times Q, T^{\times}, \Gamma^{\times})$ is an MDP augmented with an initial state (s_0, q_0) and accepting transitions Γ^{\times} . The function $T^{\times} : (S \times Q) \times (A \times Q) \rightarrow \mathcal{D}(S \times Q)$ is defined by

$$T^{\times}((s,q),(a,q'))((s',q')) = \begin{cases} T(s,a)(s') & \text{if } (q,L(s,a,s'),q') \in \Delta \\ 0 & \text{otherwise.} \end{cases}$$

Finally, $\Gamma^{\times} \subseteq (S \times Q) \times (A \times Q) \times (S \times Q)$ is defined by $((s, q), (a, q'), (s', q')) \in \Gamma^{\times}$ if, and only if, $(q, L(s, a, s'), q') \in \Gamma$ and T(s, a)(s') > 0. A strategy μ on the MDP defines a strategy μ^{\times} on the product, and vice versa. We define the syntactic satisfaction probabilities as

$$\begin{split} \mathsf{PSyn}^{\mathcal{M}}_{\mathcal{A}}((s,q),\mu^{\times}) &= \mathrm{Pr}^{\,\mu}_{s}\{r \in \Omega^{\mu^{\times}}_{(s,q)}(\mathcal{M} \times \mathcal{A}) : \mathrm{inf}(r) \cap \Gamma^{\times} \neq \emptyset\} \hspace{0.1cm}, \hspace{0.1cm} \text{and} \\ \mathsf{PSyn}^{\mathcal{M}}_{\mathcal{A}}(s) &= \sup_{\mu^{\times}} \left(\hspace{0.1cm} \mathsf{PSyn}^{\mathcal{M}}_{\mathcal{A}}((s,q_{0}),\mu^{\times}) \right) \hspace{0.1cm}. \end{split}$$

Note that $\mathsf{PSyn}_{\mathcal{A}}^{\mathcal{M}}(s) = \mathsf{PSem}_{\mathcal{A}}^{\mathcal{M}}(s)$ holds for a deterministic \mathcal{A} . In general, $\mathsf{PSyn}_{\mathcal{A}}^{\mathcal{M}}(s) \leq \mathsf{PSem}_{\mathcal{A}}^{\mathcal{M}}(s)$ holds, but equality is not guaranteed because the optimal resolution of nondeterministic choices may require access to future events.

Definition 1 (**GFM automata [3]**). An automaton \mathcal{A} is good for MDPs if, for all MDPs \mathcal{M} , $\mathsf{PSyn}_{\mathcal{A}}^{\mathcal{M}}(s_0) = \mathsf{PSem}_{\mathcal{A}}^{\mathcal{M}}(s_0)$ holds, where s_0 is the initial state of \mathcal{M} .

For an automaton to match $\mathsf{PSem}^{\mathcal{M}}_{\mathcal{A}}(s_0)$, its nondeterminism is restricted not to rely heavily on the future; rather, it must be possible to resolve the nondeterminism on-the-fly.

2 Undiscounted Reward Shaping

We build on the reduction from [2,3] that reduces maximising the chance to realise an ω -regular objective given by a good-for-MDPs Büchi automaton \mathcal{A} for an MDP \mathcal{M} to maximising the chance to meet the reachability objective in the augmented MDP \mathcal{M}^{ζ} (for $\zeta \in]0, 1[$) obtained from $\mathcal{M} \times \mathcal{A}$ by

- adding a new target state t (either as a sink with a self-loop or as a point where the computation stops; we choose here the latter view) and
- by making the target t a destination of each accepting transition τ of $\mathcal{M} \times \mathcal{A}$ with probability 1ζ and

multiplying the original probabilities of all other destinations of an accepting transition τ by ζ .

Let

$$\begin{split} \mathsf{PSyn}_t^{\mathcal{M}^{\varsigma}}((s,q),\mu) &= \mathrm{Pr}_s^{\mu} \{ r \in \Omega^{\mu}_{(s,q)}(\mathcal{M}^{\zeta}) : r \text{ reaches } t \} \ , \quad \text{ and } \\ \mathsf{PSyn}_t^{\mathcal{M}^{\varsigma}}(s) &= \sup_{\mu} \left(\mathsf{PSyn}_t^{\mathcal{M}^{\varsigma}}((s,q_0),\mu) \right) \ . \end{split}$$

Theorem 1 ([2,3]). *The following holds:*

- 1. \mathcal{M}^{ζ} (for $\zeta \in]0,1[$) and $\mathcal{M} \times \mathcal{A}$ have the same set of strategies.
- 2. For a strategy μ , the chance of reaching the target t in $\mathcal{M}_{\mu}^{\zeta}$ is 1 if, and only if, the chance of satisfying the Büchi objective in $(\mathcal{M} \times \mathcal{A})_{\mu}$ is 1:
- PSyn^{\mathcal{M}^{ζ}} $((s_0, q_0), \mu) = 1 \Leftrightarrow \mathsf{PSyn}^{\mathcal{M}}_{\mathcal{A}}(s_0, q_0), \mu) = 1$ 3. There is a $\zeta_0 \in]0, 1[$ such that, for all $\zeta \in [\zeta_0, 1[$, an optimal reachability strategy μ for \mathcal{M}^{ζ} is an optimal strategy for satisfying the Büchi objective in $\mathcal{M} \times \mathcal{A}$:

$$\mathsf{PSyn}_t^{\mathcal{M}^\varsigma}((s_0, q_0), \mu) = \mathsf{PSyn}_t^{\mathcal{M}^\varsigma}(s_0) \ \Rightarrow \ \mathsf{PSyn}_{\mathcal{A}}^{\mathcal{M}}(s_0, q_0), \mu) = \mathsf{PSyn}_{\mathcal{A}}^{\mathcal{M}}(s_0)).$$

This allows for analysing the much simpler reachability objective in $\mathcal{M}_{\mu}^{\zeta}$ instead of the Büchi objective in $\mathcal{M} \times \mathcal{A}$, and is open to implementation in model free reinforcement learning.

However, it has the drawback that rewards occur late when ζ is close to 1. We amend that by the following observation:

We build, for a good-for-MDPs Büchi automaton \mathcal{A} and an MDP \mathcal{M} , the augmented MDP $\overline{\mathcal{M}}^{\zeta}$ (for $\zeta \in]0,1[$) obtained from $\mathcal{M} \times \mathcal{A}$ in the same way as \mathcal{M}^{ζ} , i.e. by

- adding a new sink state t (as a sink where the computation stops) and
- by making the sink t a destination of each accepting transition τ of $\mathcal{M} \times \mathcal{A}$ with probability 1ζ and

multiplying the original probabilities of all other destinations of an accepting transition τ by ζ .

Different to \mathcal{M}^{ζ} , $\overline{\mathcal{M}}^{\zeta}$ has an undiscounted reward objective, where taking an accepting (in $\mathcal{M} \times \mathcal{A}$) transition τ provides a reward of 1, regardless of whether it leads to the sink *t* or stays in the state-space of $\mathcal{M} \times \mathcal{A}$.

Let, for a run r of \mathcal{M}^{ζ} that contains $n \in \mathbb{N}_0 \cup \{\infty\}$ accepting transitions, the total reward be $\mathsf{Total}(r) = n$, and let

$$\begin{split} \mathsf{ETotal}^{\overline{\mathcal{M}}^{\zeta}}((s,q),\mu) &= \mathbb{E}_{s}^{\mu}\{\mathsf{Total}(r): r \in \varOmega_{(s,q)}^{\mu}(\overline{\mathcal{M}}^{\zeta})\} \ , \quad \text{ and} \\ \mathsf{ETotal}^{\overline{\mathcal{M}}^{\zeta}}(s) &= \sup_{\mu} \left(\mathsf{ETotal}^{\overline{\mathcal{M}}^{\zeta}}((s,q_{0}),\mu)\right) \ . \end{split}$$

Note that the set of runs with $\text{Total}(r) = \infty$ has probability 0 in $\Omega^{\mu}_{(s,q)}(\overline{\mathcal{M}}^{\zeta})$: they are the runs that infinitely often do not move to t on an accepting transition, where the chance that this happens at least n times is $(1 - \zeta)^n$ for all $n \in \mathbb{N}_0$.

Theorem 2. The following holds:

- 1. $\overline{\mathcal{M}}^{\zeta}$ (for $\zeta \in]0, 1[$), \mathcal{M}^{ζ} (for $\zeta \in]0, 1[$), and $\mathcal{M} \times \mathcal{A}$ have the same set of strategies.
- 2. For a strategy μ , the expected reward for $\overline{\mathcal{M}}_{\mu}^{\zeta}$ is r if, and only if, the chance of reaching the target t in $\mathcal{M}_{\mu}^{\zeta}$ is $\frac{r}{1-\zeta}$:

$$\mathsf{PSyn}_t^{\mathcal{M}^{\zeta}}((s_0, q_0), \mu) = (1 - \zeta)\mathsf{ETotal}^{\overline{\mathcal{M}}^{\zeta}}((s_0, q_0), \mu).$$

- 3. The expected reward for $\overline{\mathcal{M}}_{\mu}^{\varsigma}$ is in $[0, \frac{1}{1-\zeta}]$.
- 4. The chance of satisfying the Büchi objective in $(\mathcal{M} \times \mathcal{A})_{\mu}$ is 1 if, and only if, the expected reward for $\overline{\mathcal{M}}_{\mu}^{\zeta}$ is $\frac{1}{1-\zeta}$.
- 5. There is a $\zeta_0 \in]0, 1[$ such that, for all $\zeta \in [\zeta_0, 1[$, a strategy μ that maximises the reward for $\overline{\mathcal{M}}^{\zeta}$ is an optimal strategy for satisfying the Büchi objective in $\mathcal{M} \times \mathcal{A}$.

Proof. (1) Obvious, because all the states and their actions are the same apart from the sink state t for which the strategy can be left undefined.

(2) The sink state t can only be visited once along any run, so the expected number of times a run starting at (s_0, q_0) is going to visit t while using strategy μ is the same as its probability of visiting t, i.e., $\mathsf{PSyn}_t^{\mathcal{M}^{\zeta}}((s_0, q_0), \mu)$. The only way t can be reached is by traversing an accepting transition and this always happens with the same probability $(1 - \zeta)$. So the expected number of visits to t is the expected number of times an accepting transition is used, i.e., $\mathsf{ETotal}^{\overline{\mathcal{M}^{\zeta}}}((s_0, q_0), \mu)$, multiplied by $(1 - \zeta)$.

- (3) follows from (2), because $\mathsf{PSyn}_t^{\mathcal{M}^{\zeta}}((s_0, q_0), \mu)$ cannot be greater than 1.
- (4) follows from (2) and Theorem 1 (2).
- (5) follows from (2) and Theorem 1 (3).

3 Discounted Reward Shaping

The expected undiscounted reward for $\overline{\mathcal{M}}_{\mu}^{\zeta}$ can be viewed as a discounted reward for $(\mathcal{M} \times \mathcal{A})_{\mu}$, by giving a reward ζ^i to when passing through an accepting transition when *i accepting* transitions have been passed before. We call this reward ζ -*biased*.

Let, for a run r of $\mathcal{M} \times \mathcal{A}$ that contains $n \in \mathbb{N}_0 \cup \{\infty\}$ accepting transitions, the ζ -biased discounted reward be $\mathsf{Disct}_{\zeta}(r) = \sum_{i=0}^{n-1} \zeta^i$, and let

$$\begin{split} \mathsf{EDisct}^{\mathcal{M}\times\mathcal{A}}_{\zeta}((s,q),\mu) &= \mathbb{E}^{\mu}_{s}\{r\in \varOmega^{\mu}_{(s,q)}(\mathcal{M}\times\mathcal{A}):\mathsf{Disct}_{\zeta}(r)\} \hspace{0.1cm}, \hspace{0.1cm} \text{and} \\ \mathsf{EDisct}^{\mathcal{M}\times\mathcal{A}}_{\zeta}(s) &= \sup_{\mu} \left(\mathsf{EDisct}^{\mathcal{M}\times\mathcal{A}}_{\zeta}((s,q_{0}),\mu)\right) \hspace{0.1cm}. \end{split}$$

Theorem 3. For every strategy μ , the expected reward for $\overline{\mathcal{M}}^{\zeta}_{\mu}$ is equal to the expected ζ -biased reward for $(\mathcal{M} \times \mathcal{A})_{\mu}$: $\mathsf{EDisct}^{\mathcal{M} \times \mathcal{A}}_{\zeta}((s,q),\mu) = \mathsf{ETotal}^{\overline{\mathcal{M}}^{\zeta}}((s,q),\mu).$

This is simply because the discounted reward for each transition is equal to the chance of not having reached t before (and thus still seeing this transition) in $\overline{\mathcal{M}}_{\mu}^{\zeta}$.

This improves over [1] because it only uses one discount parameter, ζ , instead of two (called γ and γ_B in [1]) parameters (that are not independent). It is also simpler and more intuitive: discount whenever you have earned a reward.

References

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