Equivalent Circuits and Analysis of a Generalized Antenna System

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***Abstract*— It is well-known that the maximum power can be transferred to a load when its impedance is the complex conjugate of the source impedance in an electrical or electronic system although the power transfer efficiency is only 50% in this case. Is it true for a radio system as well? This has been a question for decades. In this paper, the equivalent circuits of antenna systems are re-examined. It is demonstrated that the classical equivalent circuit models, based on the Thévenin or Norton theorem, are not suitable for a receiving antenna system since the antenna cannot be simply approximated by a voltage or a current source. A constant power source is required but not readily available. Thus, two new general power sources are introduced and examined. It is found that such a general power source model may be suitable for a transmitting antenna system but not accurate enough for a receiving antenna when the antenna loss is taken into account. A new special constant power source is therefore proposed, it is shown that this new equivalent circuit can well simulate the behaviour of a receiving antenna. When the load impedance is the complex conjugate of the antenna impedance, the maximum power is obtained, and the power transfer efficiency is also maximized simultaneously (up to 100% if the antenna system is lossless).**

***Index Terms*—Antennas, antenna modelling, equivalent circuits, radio system modelling.**

I. Introduction

RAdio systems are now everywhere and have become an essential part of our daily life. The most well-known examples are radio, TV, smartphones and radar. An antenna, as a vital element of these systems, is employed to radiate and/or receive radiowaves, thus electric signals (voltage and current) are converted to radiowaves (electric and magnetic fields), and vice versa. From the circuit point of view, an antenna is equivalent to an impedance which is a complex number: the real part is formed by the radiation resistance and loss resistance while the imaginary part is about the energy storage of the antenna. When the imaginary part is zero, the antenna operates at a resonant frequency. Ideally, the antenna impedance should be the same as the characteristic impedance of the transmission line connected to it, typically 50 ohms. To understand the role of an antenna in a radio system, an equivalent circuit is used in almost all antenna books [1-5]. At the transmitter (Tx), the antenna is considered as the load to the Tx source as shown in Fig. 1 where, for simplicity, the antenna is directly connected to the source without considering the characteristic impedance of the transmission line. The Thévenin equivalent circuit is given in Fig. 1(a) where the source is a constant voltage source with an internal impedance Z_S in series with the load/antenna impedance Z_L, while the Norton equivalent circuit is presented in Fig. 1(b) where the source is a constant current source in parallel with its internal impedance Z_S and the antenna impedance Z_L [1, 2]. When to use Thévenin or Norton equivalent circuit, it depends on the feature of the source in practice. It can be proved that these two circuits are equivalent when both source impedances are the same (Z_S) as shown in Fig. 1. The Thévenin equivalent circuit can be easily converted to the Norton equivalent circuit, and vice versa. The source voltage V_S and the source current I_S satisfy the following equation:

$$V_S = I_S Z_S \quad (1)$$

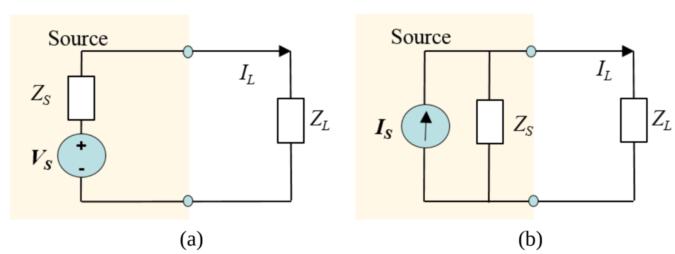

Fig. 1. The equivalent circuit of an antenna system: (a) The Thévenin equivalent circuit; (b) The Norton equivalent circuit.

Since the impedance

$$Z_S = R_S + jX_S;\ Z_L = R_L + jX_L \quad (2)$$

we can find that the average load power in both equivalent circuits is the same, which is:

$$P_L = \frac{|I_L|^2 R_L}{2} = \frac{1}{2}\frac{|V_S|^2 R_L}{(R_S+R_L)^2+(X_S+X_L)^2} = \frac{1}{2}\frac{|I_S|^2 R_L\left({R_S}^2+{X_S}^2\right)}{(R_S+R_L)^2+(X_S+X_L)^2} \quad (3)$$

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This power consists of both radiated and ohmic loss powers. It can be proved that the power is maximized when the derivatives of P_L to R_L and X_L are zero. In this case, the antenna impedance must be the complex conjugate of the source impedance:

$$Z_S = Z_L^* = R_S + jX_S = R_L - jX_L \tag{4}$$

Thus, when $R_S = R_L$ and $X_S = -X_L$, the maximum power is transferred to the load/antenna. Fig. 2 shows clearly how the power at the load varies with the magnitude of the load impedance Z_L. At the maximum, from (3), we have

$$\begin{aligned} P_L &= \frac{1}{8}\frac{|V_S|^2}{R_S} = \frac{1}{8}\frac{|V_S|^2}{R_L} \\ &= \frac{1}{8}\frac{|I_S|^2\left(R_S^2 + X_S^2\right)}{R_S} = \frac{1}{8}\frac{|I_S|^2\left(R_L^2 + X_L^2\right)}{R_L} \end{aligned} \tag{5}$$

This result is the well-known maximum power transfer theorem. But in this case, the maximum power transfer efficiency from the source to the load (defined as the ratio of the load power over the source power) is just 50%! That is, only half of the total source power is delivered to the antenna.

From Fig. 1(a), we can see that the power transfer efficiency can actually be increased by either increasing the antenna impedance or reducing the source impedance. Similarly, in Fig. 1(b), the power transfer efficiency can be increased by either reducing the antenna impedance or increasing the source impedance in the current source case. For both cases, the total load power would be reduced as a trade-off to increase the power transfer efficiency.

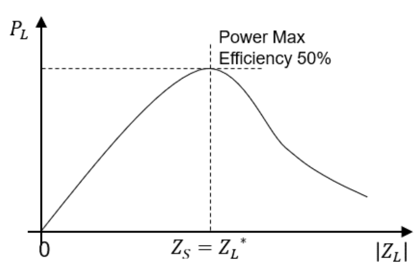

Fig. 2. The power at the load as a function of the magnitude of its impedance. At the peak, the power transfer efficiency is 50%

For a receiving antenna system, the power transfer efficiency is also just 50% in the perfectly matched case. That is, only half of the power received by the antenna is passed on to the receiver. A widely accepted explanation for the lower than expected power transfer efficiency is because of the re-radiation of the receiving antenna since a current is induced on the antenna which may re-radiate. This has been included in many popular antenna books [1-5]. But is this correct?

In this paper, we aim to deal with this question. The classical equivalent circuits of an antenna system will be re-examined. The remainder of the paper is organized as follows. In Section II, we are going to review other people's work on this interesting topic, and it is demonstrated that there is a problem with the classical model and concept. As a result, new equivalent circuits of two general constant power sources are introduced and discussed in Section III, but they are not accurate enough for a receiving antenna. In Section IV, a new equivalent circuit for a receiving antenna is introduced and discussed in detail. Conclusions are drawn in the final section.

II. Previous Studies

Friis' formula is widely used in RF engineering and communications. It links the Rx power to the Tx power through antenna gains, propagation distance and frequency. However, it has not taken the whole radio system into account: the source of the Tx and the load of the Rx are not included in this formula. The power transfer efficiency issue is not dealt with in this formula.

In practice, the power transfer efficiency of a radio system is a very important but complex issue since it has to take antennas and many other devices (such as filters and amplifiers) into account. Almost all university education and antenna books just introduce the simplified equivalent circuit using Thévenin's and Norton's Theorems as shown in Fig. 1. The limitation of the equivalent circuit of an antenna system was not discussed. However, this problem has drawn some researchers' attention [6-14]. Quite a few interesting papers have been published. The focus of these studies has been on the receiving antenna and its scattered and/or absorbed power. Love proposed an equivalent circuit for aperture antennas in 1987 [6]. It was probably the first time that a constant power source was used to replace a voltage or current source for an antenna equivalent circuit. But it did not draw much attention, possibly because his discussion was limited to aperture antennas, not general enough. A few papers on the equivalent circuit of receiving antennas were presented in this magazine in 2002 and 2003 [7-11]. Basically, Love considered Van Bladel's equivalent circuit in [7] not accurate for a receiving antenna [8] which stimulated Colin to conduct his own investigation [9]. Colin expressed his reservation on Love's equivalent circuit but agreed that there were limitations on using the Thévenin and Norton equivalent circuits for a receiving antenna. He gave an interesting discussion on the re-radiated and scattered power by a receiving antenna. Their discussion finished without an agreed solution [10, 11]. A tutorial article on the receiving and scattering properties of antennas was presented in [12]. They concluded that the Thévenin/Norton equivalent circuit model can be used to determine the received and re-radiated powers for the antenna, but they "cannot be used to provide an accurate estimate of the total scattered power for the general antenna, since they cannot predict residual or structural scattering." A more recent paper on this topic appeared in [13] where the complete equivalence of the Thévenin and the Norton circuits that describe the receiving properties of an N-port antenna was discussed using the Lorentz reciprocity theorem. Very recently, an equivalent model based on reciprocity theorem, consisting of a passive distributed transformer, was proposed for wire antennas (not for a general case) in both the transmit and receive modes [14]. It is apparent that the antenna equivalent circuit has been an interesting topic for many decades. But there does not seem to have a generally accepted solution, and all these papers did not discuss the dilemma of 50% power transfer efficiency. Thus, the accurate antenna equivalent circuit and the 50% power efficiency issue remain as an unsolved problem, and further work is required.

III. TWO GENERAL CONSTANT POWER SOURCES

Thévenin and Norton equivalent circuits were originally introduced for electrical and electronic systems where constant voltage and current sources are used. For example, a battery is a constant voltage source. These equivalent circuits may be applicable to a Tx antenna system, but they are definitely not suitable for an Rx antenna system since the receiving antenna is actually a **constant power source**. The power captured by an antenna is the product of the incoming power density and the effective area of the antenna [1-5]. For a given location and antenna, the received power is a constant and independent of the load impedance. Thus, a more accurate model for a receiving antenna should be a power source rather than a voltage or a current source.

What is the equivalent circuit of a constant power source? This is surprisingly not a well-studied subject. There is very little information available in the public domain. It is interesting to note that, when dealing with the susceptibility of a transmission line in electromagnetic compatibility, both a voltage source (induced by the incident electric field) and a current source (induced by the incident magnetic field) are used simultaneously to model the incident wave [15]. This combination could act as a power a source. With this in mind, let us first introduce two new equivalent constant power sources in Fig. 3 which are based on the definition: power = voltage × current. Fig. 3(a) shows a power source formed by a voltage source and a current source in series (similar to Love's model in [6]) while the power source in Fig. 3(b) is the combination of a voltage source and a current source in parallel. For simplicity, the power source is directly connected to a load with impedance Z_L. The power source impedance is divided into two, Z_1 and Z_2, which could be considered as the internal impedances of the voltage and current source impedance, respectively. We are going to study these two power sources separately, and evaluate if they are suitable for an antenna system.

3.1 For the series power source given in Fig. 3(a)

The total source impedance is: $Z_S = Z_1 + Z_2$. The total (average) power P_0 consists of the power dissipated at the source P_S and the power at the load P_L:

$$P_0 = P_S + P_L = (\frac{1}{2}|I_1|^2 R_1 + \frac{1}{2}|I_2|^2 R_2) + \frac{1}{2}|I_1|^2 R_L \tag{6}$$

Using the circuit theory to obtain:

$$I_1 = I_S + I_2 \tag{7}$$

$$V_S = Z_1 I_1 + Z_2 I_2 + Z_L I_1 \tag{8}$$

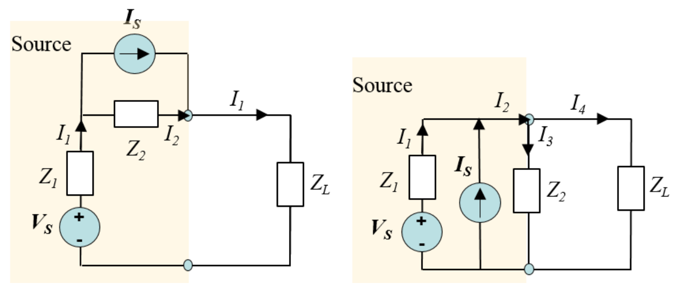

Fig. 3. Two constant power sources: (a) Series power source; (b) Parallel power source.

That means:

$$I_1 = \frac{V_S + Z_2 I_S}{Z_1 + Z_2 + Z_L}; I_2 = \frac{V_S - (Z_1 + Z_L) I_S}{Z_1 + Z_2 + Z_L} \tag{9}$$

We can find the total power:

$$P_0 = \frac{1}{2}\left|\frac{1}{Z_1 + Z_2 + Z_L}\right|^2 \{|V_S + Z_2 I_S|^2 (R_1 + R_L) + |V_S - (Z_1 + Z_L) I_S|^2 R_2\} \tag{10}$$

As a constant power, it is not a function of the load; thus, the following conditions should be satisfied:

$$\frac{\partial P_0}{\partial R_L} = \frac{\partial P_0}{\partial X_L} = 0 \tag{11}$$

After a lengthy process, in order to ensure that (11) is valid for all load impedance Z_L, we must have:

$$X_2 = 0, \qquad V_S = I_S R_2 \tag{12}$$

This means that the voltage source V_S and current source I_S are linked through R_2 only. The total power of the source is therefore proven to be:

$$P_0 = \frac{1}{2}|V_S I_S| = \frac{1}{2}|I_S|^2 R_2 \tag{13}$$

It is determined by $\boldsymbol{V_S, I_S, R_2}$, but not $\boldsymbol{Z_1}$ (i.e. it does not affect the total power in the circuit). The power delivered to the load is

$$P_L = \frac{1}{2}|I_1|^2 R_L = \frac{1}{2}\left|\frac{V_S + Z_2 I_S}{Z_1 + Z_2 + Z_L}\right|^2 R_L = \frac{2|V_S|^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2} \tag{14}$$

where

$$R_S = R_1 + R_2, \qquad X_S = X_1 \tag{15}$$

Compared with (3), this power is four times larger than that in (3) which is resulted from the change of the source model. The total voltage produced by the source is actually $2V_S$, not V_S.

Now let us examine three special cases.

1) *The perfectly matched case*: as we did for (3), it can be proved that the power delivered to the load is maximized when the load impedance is the complex conjugate of the source impedance, i.e. $R_L = R_S$ and $X_L = -X_S$. Using (14), the maximum power delivered to the load can therefore be obtained as:

$$P_{L_{max}} = \frac{|V_S|^2}{2R_S} = \frac{|V_S|^2}{2(R_1 + R_2)} \tag{16}$$

The power transfer efficiency in this case is:

$$\eta = \frac{P_L}{P_0} = \frac{R_2}{(R_1 + R_2)} \tag{17}$$

If $R_1 = 0$, the load power is increased to the maximum, the same as the source power and we can find that $I_2 = 0$: this means that there is no current passing through the resistance R_2 and no power dissipation at the source! The power transfer efficiency from the source to the load, as defined in (17), is now 100%, not 50%. Here we have demonstrated that the total source power can now be fully delivered to the load when the perfect matching condition in (4) is met. It is interesting to note that **the role of R_1 is about the loss of the source. If R_1 is zero in the perfectly matched case, there is no power dissipation at both R_1 and R_2.** However, if R_1 is not zero, there will be power dissipation, and the power transfer efficiency will be smaller than 100%.

2) *The open-circuit (OC) case*: the equivalent circuit in Fig. 3(a) can be simplified to Fig. 4(a) in this case. The total source power $P_0 = |I_S|^2 R_2/2$ is only dissipated at the internal impedance R_2 and no power is consumed by the load and R_1.

3) *The short circuit (SC) case*: the equivalent circuit in Fig. 3(a) becomes Fig. 4(b). The total power:

$$P_0 = P_S = \left(\frac{1}{2}|I_1|^2 R_1 + \frac{1}{2}|I_2|^2 R_2\right) \tag{18}$$

It is dissipated at both internal resistances R_1 and R_2 which is different from the open-circuit case. If $R_1 = 0$, I_2 is equal to I_S and all power is only consumed by R_2, which is the same as the OC case.

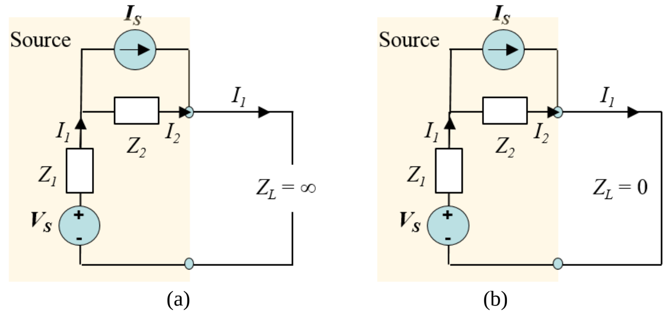

Fig. 4. The equivalent circuits of two special cases (a) an OC load (b) a SC load.

Up to now, we have examined the series power source case presented in Fig. 3(a), understood the role of each element and demonstrated that it can indeed provide a constant power given by (13) which can be 100% transferred to the load when the internal resistance $R_1 = 0$. A very important message is that, **when a power source is used, the maximum power and maximum power efficiency can be obtained at the same time** – this is very different from a voltage source and a current source.

3.2 For the parallel power source given in Fig. 3(b)

The two internal impedances are in parallel, thus the total source admittance is: $G_S = G_1 + G_2$, the impedance is in parallel with the current and voltage sources and can be expressed as:

$$Z_S = \frac{Z_1 Z_2}{Z_1 + Z_2} = R_S + jX_S \tag{19}$$

Similar to the series power source, the total (average) power P_0 consists of the power dissipated at the source P_S and the power at the load P_L:

$$P_0 = P_S + P_L = \left(\frac{1}{2}|I_1|^2 R_1 + \frac{1}{2}|I_3|^2 R_2\right) + \frac{1}{2}|I_4|^2 R_L \tag{20}$$

Using the circuit theory, we can find that the total power can be expressed as:

$$P_0 = \frac{1}{2}\left\{|I_1|^2 R_1 + \left|\frac{V_S - Z_1 I_1}{Z_2}\right|^2 R_2 + \left|\frac{V_S - Z_1 I_1}{Z_L}\right|^2 R_L\right\} \tag{21}$$

The expression for I_1 is given by:

$$I_1 = \frac{(Z_2 + Z_L)V_S - Z_2 Z_L I_S}{Z_1 Z_2 + Z_2 Z_L + Z_L Z_1} \tag{22}$$

Thus, the total power in (21) can be determined by the current and voltage sources and the impedances in Fig. 3(b). Since the total power should be a constant and not a function of the load, it should satisfy equation (10), which results in the following conditions:

$$X_1 = 0, \qquad V_S = I_S R_1 \tag{23}$$

This means that the voltage V_S and current I_S are linked through R_1 only. The total power of the source is, therefore:

$$P_0 = \frac{1}{2}|V_S I_S| = \frac{1}{2}|I_S|^2 R_1 \tag{24}$$

It is determined by $\boldsymbol{V_S, I_S, R_1}$, but not $\boldsymbol{Z_2}$. It is interesting to note that this power $\frac{1}{2}|V_S I_S|$ is the same as the power provided by the series power source given by (13) although the roles of internal impedances are changed.

Similar to the series power source, for the parallel power source:

1) *The perfectly matched case*: it can be proved that the power delivered to the load is maximized when the load impedance is the complex conjugate of the source impedance, i.e. $R_L = R_S$ and $X_L = -X_S$. The total source power can also be 100% (not 50%) delivered to the load in this case. It is interesting to note that **the role of $\boldsymbol{R_2}$ is about the loss of the source. If $\boldsymbol{R_2}$ is zero, there is no power dissipation at both $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$.** However, if R_2 is not zero, there will be power dissipation at both R_1 and R_2, and the power transfer efficiency will be smaller than 100%.

2) *The OC case*: This is equivalent to the SC case of Fig. 3(a).

3) *The SC case*: This is equivalent to the OC case of Fig. 3(a).

3.3 Discussions

In this section, we have introduced the equivalent circuits of two general constant power sources given in Fig. 3. Are they suitable for a Tx and Rx antenna system?

For a Tx antenna system, the source is normally complicated. It is possible to use the equivalent circuit of a voltage, or a current or a power source introduced here to approximate the source connected to the Tx antenna. Which one is the most appropriate source – depending on the specific system. For example, if an antenna were connected to a signal generator or a network analyzer, the source could be approximated by a constant power source as shown in Fig. 3, while the Tx antenna could be simply represented by a load impedance Z_L in an equivalent circuit. The power transfer efficiency could be up to 100%.

For an Rx antenna system, the Rx antenna acts as a constant power source – the power is determined by the incoming wave power density and the antenna effective area. Can we use a source in Fig. 3 to represent the receiving antenna? This is a critical question of this study.

Mathematically, the antenna impedance can be expressed as:

$$Z_A = R_A + jX_A = R_r + R_{loss} + jX_A \tag{25}$$

where R_r is the radiation resistance and reflects the ability of the antenna for the energy conversion, R_{loss} is the loss resistance and represents the ohmic or dielectric loss of the antenna. This loss is independent of the load connected to the antenna and should be included in the equivalent circuit of the power source. From the above discussion, we have seen that:

1) For Fig. 3(a), R_1 represents the loss resistance of the source, thus it should be equivalent to R_{loss} of the antenna. It seems to work for both the perfectly matched case and the SC case. However, for the open-circuit case as shown in Fig. 4(a), the loss resistance R_1 (i.e. R_{loss}) is not linked to the power, which is wrong for a receiving antenna. Because, as shown in Fig. 5, the received power into the antenna P_{in} is first partially consumed by the antenna due to its loss R_{loss} (it is also linked to the antenna radiation efficiency) and then absorbed by the load R_L. If it is not perfectly matched, some power will be reflected from the load back to the antenna and go through another round of power consumption by R_{loss} and finally re-radiated back to space via R_r (equivalent to R_2). Thus, the total power consumed in the circuit should include the loss. When it is perfectly matching, the power contains the antenna loss once. For any other cases, it consists of the antenna loss twice plus the re-radiation. Thus, Fig. 3(a) is not accurate enough to take the antenna loss into account.

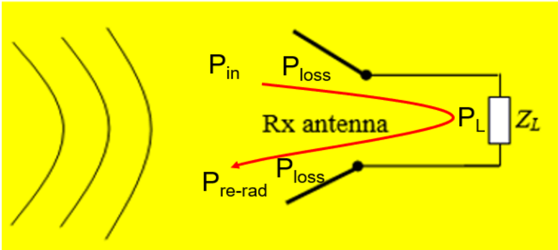

Fig. 5. The power going through a receiving antenna.

2) For Fig. 3(b), similarly, it does not properly take the antenna loss into account for the SC case where the power is only dissipated at the internal impedance R_1 and no power is consumed by R_2 which is linked to the antenna loss resistance in this case. Thus, Fig. 3(b) is also not accurate enough to represent a receiving antenna system.

Although the two general constant power sources introduced in Fig. 3 could be used for a Tx antenna system, for an Rx antenna, they have difficulties in taking the antenna loss fully into account, thus, a more accurate circuit model is required for an Rx antenna.

IV. The Proposed Receiving Antenna Model

As discussed in Fig. 5, the power delivered to the load is the received power taking away the loss power of the antenna through the ohmic loss and re-radiation. Thus, the antenna loss resistance is present for any load impedance. A new equivalent circuit model for a receiving antenna is proposed and shown in Fig. 6 where R_r, R_{loss}, R_L, X_A, and X_L represent the antenna radiation resistance, the antenna loss resistance, the load resistance, the antenna reactance and the load reactance, respectively. The model is similar to the parallel power source model in Fig. 3(b), the only change is the parallel loss resistance R_2 (which is R_{loss} for the antenna) is now placed in series with the current source to ensure that the antenna loss is included for all cases as we will see later.

As shown in Fig. 5, the antenna loss power appeared twice which is also reflected in Fig. 6 by the loss resistance R_{loss}.

It is worth noting that the voltage source V_0 and the current source I_0 are linked by the relationship presented in (1) ($V_0 = I_0 Z_A$ and $Z_A = R_r + R_{loss} + jX_A$ is the antenna impedance for the case in Fig. 6).

To obtain the load power (P_L) and re-radiated power (P_{rrad}), we can apply the superposition method [16]. First, we replace the current source I_0 with an OC to find the load current (I_{L1}) and the antenna current (I_{A1}) as shown in Fig. 7(a). Then, we replace the voltage source V_0 with an SC to find the load current (I_{L2}) and the antenna current (I_{A2}) as shown in Fig. 7(b). The overall load current is $I_L = I_{L1} + I_{L2}$ and the overall antenna current is $I_A = I_{A1} - I_{A2}$ (due to the opposite direction).

From Fig. 7(a), we have

$$I_{L1} = I_{A1} = \frac{V_0}{(R_r + R_{loss} + R_L) + j(X_A + X_L)} \quad (26)$$

From Fig. 7(b)

$$I_{L2} = \frac{I_0(R_r + R_{loss} + jX_A)}{(R_r + R_{loss} + R_L) + j(X_A + X_L)} \quad (27)$$

$$I_{A2} = \frac{I_0(R_L + jX_L)}{(R_r + R_{loss} + R_L) + j(X_A + X_L)} \quad (28)$$

Note that I_{A2} flows counter-clockwise (CCW) while I_{L1}, I_{A1} and I_{L2} flow clockwise (CW).

Since $I_L = I_{L1} + I_{L2}$, $I_A = I_{A1} - I_{A2}$, the load power P_L and the re-radiated power P_{rrad} can be expressed as:

$$P_L = \frac{1}{2}|I_L|^2 R_L = \frac{2|V_0^2|R_L}{(R_r + R_{loss} + R_L)^2 + (X_A + X_L)^2} \quad (29)$$

$$P_{rrad} = \frac{1}{2}|I_A|^2 R_r = \frac{\frac{1}{2}|V_0 - I_o Z_L|^2 R_r}{(R_r + R_{loss} + R_L)^2 + (X_A + X_L)^2} \quad (30)$$

The load power in (29) and the load powers obtained from Thévenin and Norton equivalent circuits shown in Fig. 1 are plotted against load impedance (Z_L) and presented in Fig. 8. The powers are normalized to the maximum input power in each case while the load impedance is normalized to the antenna impedance (Z_A). Assuming lossless antennas ($R_{loss} = 0$), it is clear that in Thévenin and Norton equivalent circuits, the load powers for both cases are the same as shown in Fig. 8 (they are overlapping) and reach up to 50% of the input power at the perfectly matching condition ($|Z_A| = |Z_L|$), but it reaches 100% for the proposed model which agrees with the behaviour of a lossless antenna in the receiving mode.

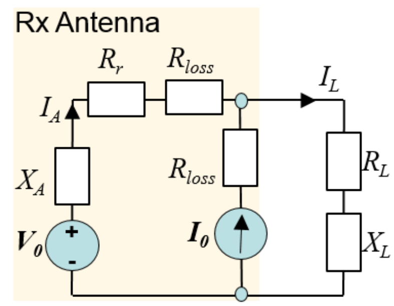

Fig. 6. The proposed receiving antenna model.

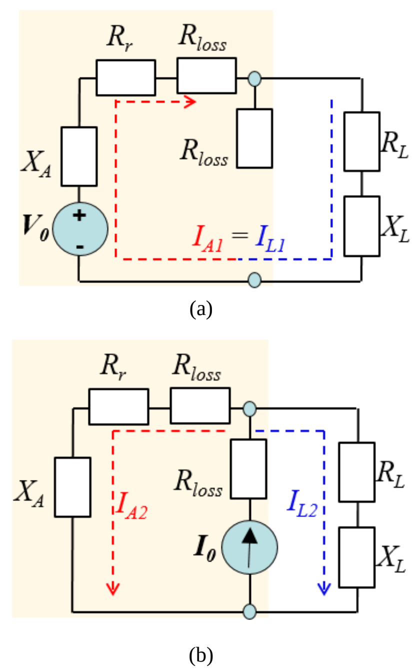

Fig. 7. Applying the superposition method to the proposed model by (a) replacing I_0 with an OC (b) replacing V_0 with an SC

Regarding the loss power, there are two loss resistances in our proposed model which is a bit more complex than normal. A load-independent loss power (which is the first P_{Loss} in Fig. 5) is due to the loss resistance (R_{loss}) in series with I_0 as shown in Fig. 6. It can be expressed as $\frac{1}{2}|I_0|^2 R_{loss}$. Furthermore, an additional loss power (which is the second P_{Loss} in Fig. 5) exists due to the loss resistance in series with V_0. This loss power equals to $\frac{1}{2}|I_A|^2 R_{loss}$ – this is load-dependent as we will see later: $I_A = 0$ when it is perfectly matched. The antenna total loss power (P_{Loss}) is the sum of these two losses (load-independent and load-dependent) and can be expressed as:

$$P_{Loss} = \frac{1}{2}|I_0|^2 R_{loss} + \frac{1}{2}|I_A|^2 R_{loss} \quad (31)$$

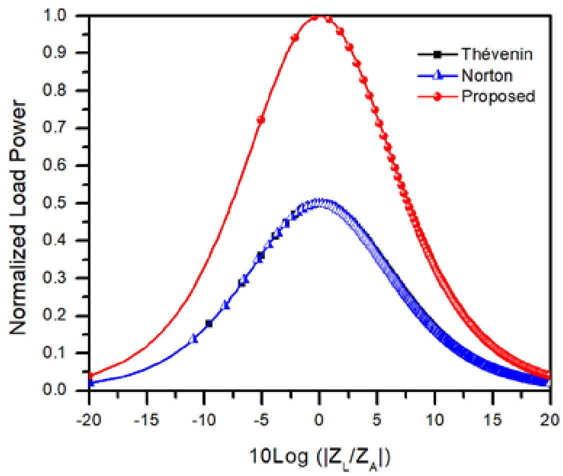

Fig. 8. The normalized load power as a function of the impedance ratio of the load and antenna on a logarithmic scale.

This can be explained using Fig. 5: in the receiving antenna system, when the received signal travels to the load, the load-independent loss occurs due to the finite conductivity of the antenna. Once the signal is delivered to the load, a portion of it may be reflected in correspondence to the mismatch between the antenna impedance and the load impedance. This portion is reflected back to the antenna to be re-radiated. Hence, the load-dependent loss occurs.

For the case $Z_A \neq Z_L$, the antenna current I_A exists and hence P_{rrad} exists. The larger the difference between Z_A and Z_L (i.e. more mismatching), the larger the value of I_A due to the dominance of one of its components (I_{A1} or I_{A2}) over the other. Hence, more power is re-radiated, and less power is consumed by the load. This typically implies the behaviour of an antenna in the receiving mode.

Fig. 9 shows various powers for an antenna with 80% antenna efficiency which are obtained using (29), (30) and (31). All the plotted powers are normalized to the input power. It is clear that both the input power and the load-independent loss power are constant (black and green horizontal lines respectively) with the variation of Z_L for a fixed Z_A. This demonstrates their independency of the antenna load impedance. The other powers are dependent on the load. The load power reaches its maximum (80% of the input power) in the perfect matching case while it decreases when deviating from the matching point until it vanishes at the OC and SC cases. The load-dependent loss power is zero at the perfect matching case because of the absence of the reflected power from the load and it increases until it reaches its maximum value at the OC and SC loads (to be the same as the load-independent loss power) because of the total reflection at the load. The re-radiated power is also zero at the perfect matching point and then increases when the impedance moves towards the edges. The maximum re-radiation occurs when the antenna is terminated with an OC or SC load. Note that in this case, the re-radiated power is 60% of the input power due to the doubled loss power (20% load-independent loss + 20% load-dependent loss). In general, at any load condition, the sum of the load power, load-independent loss power, load-dependent loss power and re-radiated power should equal to the constant input power.

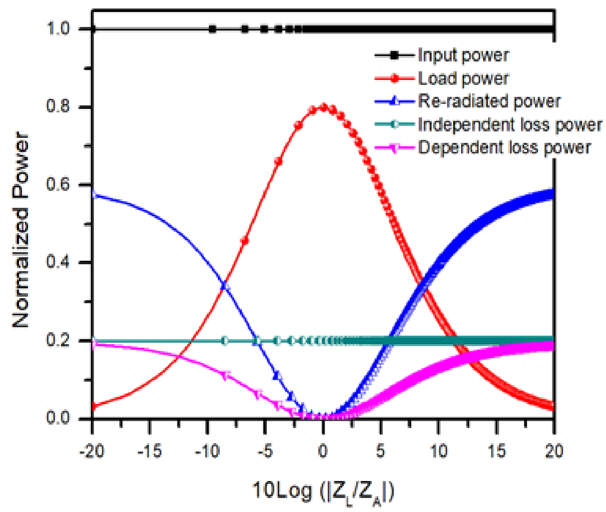

Fig. 9. The normalized various powers as a function of the impedance ratio of the load and antenna on a logarithmic scale for a lossy case.

Now let us consider the three special cases again.

1) *The perfectly matched case*: $R_r + R_{loss} = R_L$ and $X_A = -X_L$, the antenna current $I_A = 0$ as its two components I_{A1} and I_{A2} are equal in magnitude and opposite in direction. Thus, P_{rrad} is zero (i.e. no re-radiation). P_L in (29) is maximized and equals to:

$$P_{L_{max}} = \frac{V_0^2}{2R_L} \quad (32)$$

2) *The OC case*: $R_L = X_L = \infty$. Using equations (26), (27) and (28), $I_{A1} = I_{L1} = I_{L2} = 0$. Hence P_L is zero as shown in (29) and in Fig. 10(a) while $I_{A2} = I_0$. So, $I_A = I_0$ and flows CCW. The re-radiated power and the loss power are maximized and can be re-written using equations (30) and (31) as:

$$P_{rrad} = \frac{1}{2}|I_0|^2 R_r \quad (33)$$

$$P_{Loss} = |I_0|^2 R_{loss} \quad (34)$$

In this case, both the load-independent and load-dependent loss powers are identical and equal to $\frac{1}{2}|I_0|^2 R_{loss}$.

3) *The SC case*: $R_L = X_L = 0$, as in Fig. 10(b), $I_{L1} = I_{A1} = V_0/Z_A = I_0$, $I_{L2} = I_0$ and $I_{A2} = 0$. Although $I_L = 2I_0$, the load power P_L in (29) is zero due to the fact that $R_L = X_L = 0$. I_A, in this case, equals to I_0 and flows CW. Again, the re-radiated power and the

loss power are maximized and can be expressed by (33) and (34) respectively.

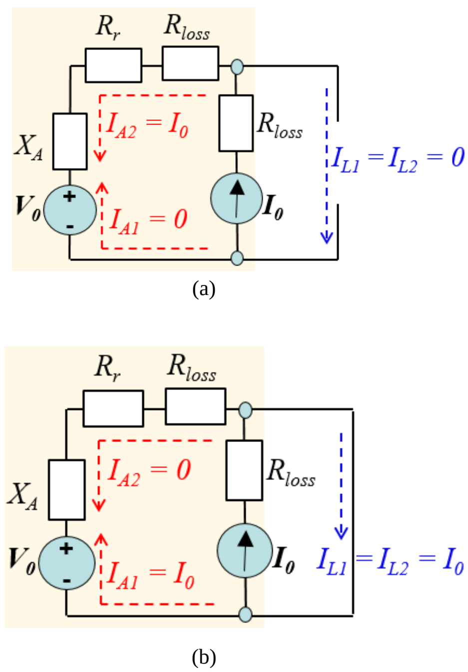

Fig. 10. The proposed receiving antenna model terminated by (a) an OC (b) an SC.

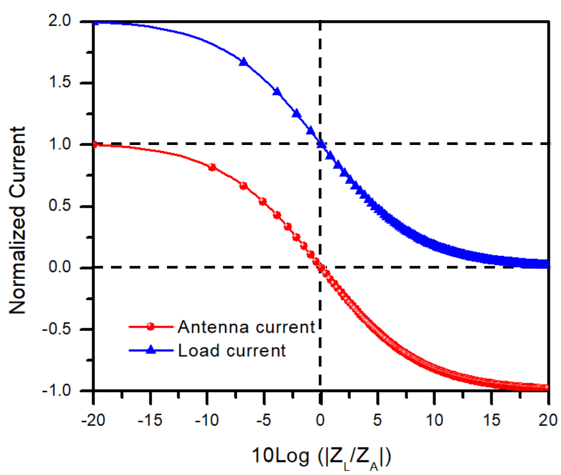

Fig. 11. The antenna current and the load current as a function of the impedance ratio of the load and antenna on a logarithmic scale

It is worth noting that although I_A has the same magnitude in both OC and SC cases, it has opposite directions (CCW and CW respectively). This can be linked to the fact that an OC has a reflection coefficient $\Gamma = 1$ while a SC results in a reflection coefficient $\Gamma = -1$. This can be better clarified by plotting the responses of I_A and I_L to the variation of Z_L using equations (26), (27) and (28) as shown in Fig. 11, where the currents are normalized to the value of the current source I_0. It is clear that I_A changes its sign when the load impedance changes from the SC (I_A = 1) to the OC (I_A = -1). It becomes zero at the perfect matching point (it means no re-radiation). This indicates the 180° phase change between the reflection coefficients at the two extreme cases (SC and OC loads). Moreover, load current I_L equals to twice of the source current when the load is the SC while it reaches zero when the load is the OC. These currents are not affected by the antenna impedance and they are the same for loss-free and lossy cases.

In case there is a doubt on the antenna input impedance (Z_{in}) in the proposed model, Thévenin theorem is applied by replacing I_0 with an OC and V_0 with an SC then looking at the antenna through its load terminals [16], as shown in Fig. 12. It is clear that $Z_{in} = R_r + R_{loss} + jX_A = Z_A$. This is a standard input impedance of an antenna and also proves the validity of equation (1).

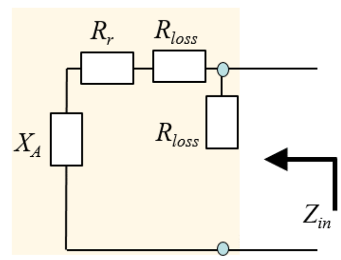

Fig. 12. The input impedance of the proposed antenna model using Thévenin theorem

Another very important issue is how to link the equivalent circuit source to the incoming radiowave (i.e. the fields E and H). If the the polarization of the wave is well matched with the antenna, the antenna received power should be the same as the equivalent source power, that is

$$P_A = \frac{1}{2}(E \times H^*)A_{eff} = \frac{1}{2}(V_0 \times I_0{}^*) \tag{35}$$

where A_{eff} is the effective aperture of the antenna. We can find that

$$V_0 = E\sqrt{\frac{Z_A}{\eta}A_{eff}} \quad , \quad I_0 = H\sqrt{\frac{\eta}{Z_A}A_{eff}} \tag{36}$$

where η is the medium intrinsic impedance (377 ohm in free space) and equals to the ratio of E and H. This agrees with source equation $V_0 = I_0 Z_A$, we can use (36) to link the incoming fields and the equivalent circuit source. It should be pointed out that here we have not taken the antenna scattered wave/power into account, since the wave scattering of the receiving antenna is dependent on the antenna configuration and structure [11, 12]. The power in (35) is the received power, excluding the scattered power. How to take the the scattered power into account could be a direction for future study.

V. Veridation and Conclusions

As pointed out earlier, the major difference between the power source models in Fig. 3 and the Rx antenna model in Fig. 6 is the loss. To better understand this point, we have plotted the normalized loss powers of the sources in Fig. 3(a) (the series power source) and in Fig. 6 (the proposed Rx antenna source) for the lossless and a lossy case (with an efficiency of 80%).

The load impedance changes from the SC to OC. The results are shown in Fig. 13. As expected, the loss powers for both sources are zero for the lossless case. However, they are different for the lossy case except when it is perfectly-matched (the normalized/relative loss power is 20% for both cases). As discussed in Fig. 6, we understand why the loss power is 40% for both OS and SC cases. For the source in Fig. 3(a), the loss power reaches its maximin $4\eta(1\text{-}\eta)$ = 64% at the SC and the minimum 0% at the OC (For Fig. 3(b), the results are just the opposite). This further confirms the validity of Fig. 6 and deficiency of Fig. 3 as the Rx antenna model.

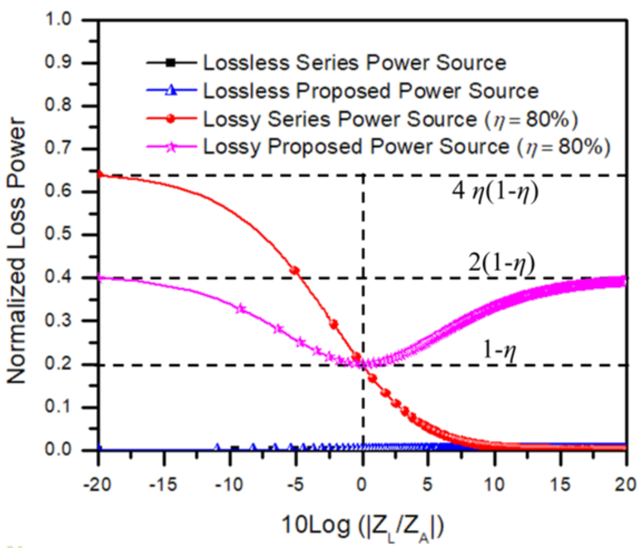

Fig. 13. Normalized loss power in two different power sources for the lossless and a lossy case.

To validate the proposed new antenna equivalent circuit, both simulation and measurements have been carried out. For example, two microstrip patch antennas resonating at 3 GHz with a realized gain of 6.3 dBi were used in an anechoic chamber as shown Fig. 14.

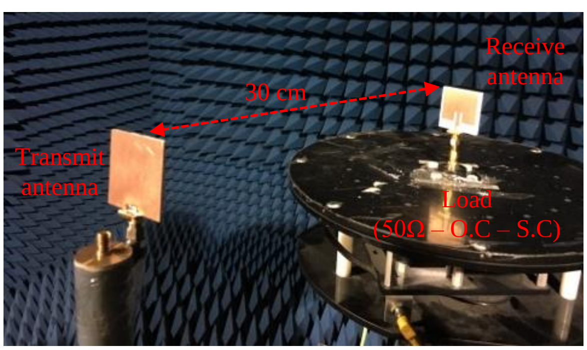

Fig. 14. Two patch antennas were used to measure the S11 with different loads.

The reflection coefficient (S_{11}) of the transmit antenna was measured for the standalone (no receive antenna), a well-matched receive antenna, an OC receive antenna and an SC receive antenna cases, respectively. The results are shown in Fig. 15 where we can see that

1) there was almost no difference in S_{11} between the standalone and matched load cases. This indicates that there was almost no re-radiation from the well-matched antenna (small chang may be due to scattered power from the antenna).

2) S11 was increased (at the resonant frequency) for the OC and SC cases, which was resulted from the re-radiations of the receive antenna.

All these results are in very good agreement with our equivalent circuit prediction and software simulation results.

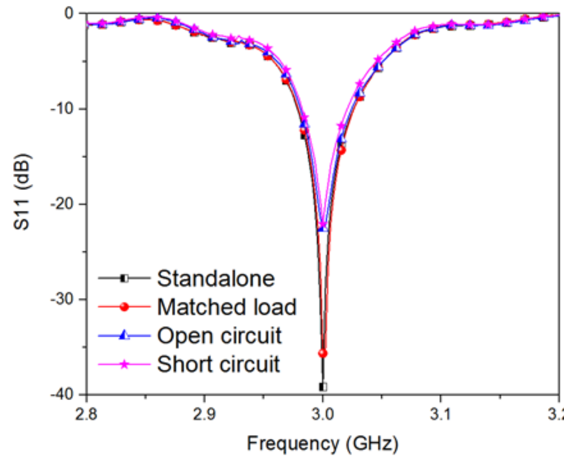

Fig. 15. Measured S11 of the Tx antenna when the Rx was connected to different loads.

There might be a question on if we could use the proposed receiving antenna model for a transmitting antenna. As discussed earlier, if the transmitting antenna system, is not a power source, thus the equivalent circuit in Fig. 6 (and Fig. 3) is not suitable. However, if it is a power source, we could use equivalent circuits in Fig. 3 and Fig. 6 – depending on the feature of the source in reality. When $Z_2 = \infty$ and $R_{loss} = 0$, Fig. 3(b) and Fig. 6 become the same.

Another question is linked to the use of the equivalent circuit for the proof of the reciprocity theorem. Some books (such as [1, 2]) used the voltage source which is not correct as we know now. It should be replaced by our proposed power source although the results are the same. A main impact of the proposed power source model is for us to better understand how an antenna works from the circuit point of view and obtain the power and power transfer efficiency accurately. One example is to use the new circuit model for rectenna design which is a hot topic for wireless energy harvesting and power transfer.

In this paper, we have studied the equivalent circuits of an antenna system. When it is used as a Tx antenna, the antenna acts as a complex load impedance to the source. The equivalent circuit of a Tx antenna system could have a constant voltage, current or power source as shown in Figs. 1, 3 or 6, depending on which source is actually used in reality. When it is used as an Rx antenna, the antenna acts as a constant power source. It has shown that the two new general power source models in Fig. 3 is not suitable for an Rx antenna system since they cannot take the antenna loss fully into account, thus another new special constant power source has been introduced and presented in Fig. 6 where the antenna loss power is formed by a load-independent part and a load-dependent part which have shown clear physical meanings. It has also shown that for an antenna system with a constant power source, the maximum power and maximum power transfer efficiency can be achieved at the same time when the load impedance is the complex conjugate impedance of the source/antenna. The power transfer efficiency could be 100% for a loss-free case.

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