Bayesian Model Updating in Time Domain with Metamodel-based Reliability Method

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Abstract: In this study, a two-step approximate Bayesian computation (ABC) updating framework using dynamic response data is developed. In this framework, the Euclidian and Bhattacharyya distances are utilized as uncertainty quantification (UQ) metrics to define approximate likelihood functions in the first and second steps, respectively. A new Bayesian inference algorithm combining Bayesian updating with structural reliability methods (BUS) with the adaptive Kriging model is then proposed to effectively execute the ABC updating framework. The performance of the proposed procedure is demonstrated with a seismic-isolated bridge model updating application using simulated seismic response data. This application denotes that the Bhattacharyya distance is a powerful UQ metric with the capability to recreate wholly the distribution of target observations and the proposed procedure can provide satisfactory results with much-reduced computational demand compared with other well-known methods, such as transitional Markov chain Monte Carlo (TMCMC).

Introduction

Bayesian model updating using observed dynamic response data has a broad range of applications in a number of engineering fields (Beck and Ktafygiotis 1998; Ktafygiotis and Beck 1998; Cheung and Beck 2009; Jensen et al. 2013; Rocchetta et al. 2018). In Bayesian model updating, uncertainties in both the simulation and observation procedures should be appropriately considered; hence, uncertainty quantification (UQ) metrics are significant in order to comprehensively and quantitatively measure the stochastic discrepancy between model predictions and observations.

In the context of UQ, parameters are categorized according to the involvement of aleatory or/and epistemic uncertainties as (Kennedy and O'Hagan 2001; Crespo et al. 2014):

I) Parameters without any uncertainties, appearing as explicit constants;
II) Parameters with only aleatory uncertainty, appearing as random variables with fully determined probability characteristics such as density functions and distribution coefficients;
III) Parameters with only epistemic uncertainty, appearing as unknown-but-fixed constants bounded by given intervals;
IV) Parameters with both aleatory and epistemic uncertainties, appearing as imprecise random variables with only vaguely determined probability characteristics.

Both Categories III and IV parameters are considered in Bayesian model updating, whose target is not a single set of the crisp parameter values, but a reduced space of epistemic uncertainty such as reduced intervals of Category III parameters and reduced bounds of the cumulative probability function (CDF) of Category IV parameters.

The geometric discrepancy between model predictions and observations caused by Category III parameters can be quantified using the classical Euclidian distance as the UQ metric. On the other hand, quantifying the stochastic discrepancy caused by Category IV parameters requires a more comprehensive UQ metric capable of capturing a higher level of statistical information. The Bhattacharyya distance (Bhattacharyya 1946) has been recently investigated as such a potential UQ metric (Bi et al. 2017). The Bhattacharyya distance is a stochastic measure between two sets of random samples, i.e., model predictions and observations, and accounts for their probability distributions.

Bi et al. (2019) developed a Bayesian model updating framework, in which the Bhattacharyya distance was employed as the UQ metric to define an approximate but efficient likelihood function based on the approximate Bayesian computation (ABC) method (Turner and Van Zandt 2012; Safta et al. 2015). This framework was demonstrated upon a three degree of freedom (DOF) spring-mass system example and showed to have a potential to recreate wholly the target observations. While the target outputs in this example is scalar modal responses, the direct computation of the Bhattacharyya distance becomes infeasible for high-dimensional dynamic responses because of so-called curse of dimensionality. A dimension reduction procedure is thus proposed in this study to calculate the Bhattacharyya distance for such dynamic responses.

On the other hand, Markov chain Monte Carlo (MCMC) algorithms are generally accepted as the most attractive Bayesian inference tools (Beck and Au 2002; Cheung and Beck 2009). Of particular importance among these algorithms is transitional Markov chain Monte Carlo (TMCMC) (Ching and Chen 2007; Betz et al. 2016) and Bi et al. (2019) also utilized TMCMC to perform the ABC updating framework. Although TMCMC is quite flexible and general, it requires a large number of model evaluations for calculating the likelihood function. In the ABC updating framework, the approximate likelihood function is calculated based on the Bhattacharyya distance, and the Bhattacharyya distance evaluation requires random samples of model predictions, which is generally generated by Monte Carlo (MC) sampling. Therefore, the number of model evaluations is extremely large compared with the general model updating and the computational cost becomes excessive in cases of time-consuming model evaluations, which are often involved in predicting dynamic responses.

Straub and Papaioannou (2015) recently provided a formulation called Bayesian updating with structural reliability methods (BUS). The key idea of this formulation is to transform the Bayesian updating problem into an equivalent reliability problem, allowing to obtain samples from posterior distributions as conditional samples located into the failure domain of this reliability problem. By employing Subset simulation techniques (Au and Beck 2001), BUS has shown great efficiency in estimating posterior distributions (Betz et al. 2018). Moreover, its efficiency depends on the choice of the so-called likelihood multiplier. Whereas the optimal multiplier ensuring the best acceptance rate is generally unknown, it can be defined a priori for the proposed approximate likelihood function. Hence, BUS has a potential to be efficiently integrated with the ABC updating framework.

At the same time, BUS can further improve its efficiency by applying metamodeling techniques (Giovanis et al. 2017). Among various types of the metamodels, the adaptive Kriging model has been shown to be one of the most accurate and efficient methods in solving reliability problems (Echard et al. 2011; Echard et al. 2013; Huang et al. 2016). However, the failure probability associated with the equivalent reliability problem in BUS is generally known to be extremely small. In such rare events, the adaptive Kriging model becomes significantly inefficient, since the number of candidate samples should be extremely large to ensure that enough samples are contained in the failure domain. On the other hand, Wei et al. (2019) recently proposed a new algorithm called AK-MCMC, in which the Kriging model is adaptively trained upon dynamically updated MCMC populations. This algorithm is in particular suitable for extremely rare events. The objective of this study is consequently to develop an efficient ABC updating framework using dynamic response data by combining BUS with the AK-MCMC algorithm.

The structure of this paper is as follows. In Section 2, we describes the dimension reduction procedure to evaluate the Bhattacharyya distance for high-dimensional dynamic response data, and the proposed ABC updating framework. Section 3 outlines the novel Bayesian inference algorithm combining BUS with the adaptive Kriging model based on the AK-MCMC algorithm. The principle and illustrative application is detailed in Section 4, using a model updating problem of a seismic-isolated bridge based on simulated seismic response data. The computational efficiency of the proposed scheme is also presented by comparing with the results using TMCMC. Finally, some conclusions are given in Section 5.

Approximation Bayesian Computation using Dynamic Response Data

Formulations of the Bhattacharyya Distance for Dynamic Response Data

In the context of Bayesian model updating, the investigating system can be expressed as:

$$\mathbf{y} = h(\mathbf{x}) \tag{1}$$

where $\mathbf{x} = [x_1, x_2, \cdots, x_n]$ is a column vector of n input parameters; $\mathbf{y} = [y_1, y_2, \cdots, y_m]$ is a column vector of the output features as m-dimensional dynamic responses; $h(\cdot)$ is the simulator (e.g. finite element model). The uncertainties of the system are first characterized by uncertain input parameters in various categories (refer Section 1) and then propagated through the simulator into the uncertain output features. In general, randomly sampled values of the parameters and features are used in Bayesian model updating. Suppose the required sample size is N_{sim}, the simulator h is executed N_{sim} times to generate the sample set of the simulated features $\mathbf{Y}_{sim} \in \mathbb{R}^{N_{sim} \times m}$:

$$\mathbf{Y}_{sim} = \left[\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_{N_{sim}}\right]^T, \text{with } \mathbf{y}_i = [y_{1i}, y_{2i}, \cdots, y_{mi}], \forall i = 1, 2, \cdots, N_{sim} \tag{2}$$

In addition to the simulated features, observed features are required as the target of model updating. Suppose the number of observations is N_{obs}, the sample set of the observed features has a similar structure as Eq. (2), where only the number of rows is changed: $\mathbf{Y}_{obs} \in \mathbb{R}^{N_{obs} \times m}$. The objective of Bayesian model updating can be expressed as to minimize the stochastic discrepancy between $\mathbf{Y}_{obs}$ and $\mathbf{Y}_{sim}$ by updating the uncertainty characteristics of the input parameters.

In the following, possible UQ metrics are defined to capture the discrepancy between $\mathbf{Y}_{obs}$ and $\mathbf{Y}_{sim}$. The very classical Euclidian distance metric is expressed as:

$$d_E(\mathbf{Y}_{obs}, \mathbf{Y}_{sim}) = \sqrt{(\overline{\mathbf{Y}}_{obs} - \overline{\mathbf{Y}}_{sim})(\overline{\mathbf{Y}}_{obs} - \overline{\mathbf{Y}}_{sim})^T} \tag{3}$$

where $\overline{\mathbf{Y}}_{\blacksquare}$ is a row vector of means of the features. The Euclidian distance is a point-to-point distance capable to capture the geometric discrepancy caused by Category III parameters. On the other hand, in the presence of Category IV parameters, it is desirable to employ a more comprehensive metric capable to consider a higher level of statistical information from the sample sets.

The Bhattacharyya distance is herein proposed as such a stochastic metric to robustly measure the degree of overlap between distributions of two sample sets. Its original definition is given as:

$$d_B(\mathbf{Y}_{obs}, \mathbf{Y}_{sim}) = -\log\left[\int_{\mathbb{y}} \sqrt{p_{obs}(y)p_{sim}(y)}\,\mathrm{d}y\right] \tag{4}$$

where $p_{\blacksquare}(y)$ is the probability density function (PDF) of each feature sample; $\mathbb{y}$ is the m-dimensional feature space; $\int_{\mathbb{y}} \blacksquare\,\mathrm{d}y$ is the integration performed over the whole feature space. Differently from the Euclidian distance, the Bhattacharyya distance considers not only the means but also the variances, covariances, and even the distribution shapes of the samples sets. Nevertheless, the direct evaluation of Eq. (4) is not feasible because precise estimation of the PDF is generally unavailable, especially for applications where experiments are difficult or expensive. Bi et al. (2019) hence proposed the so-called binning algorithm to evaluate the probability mass function (PMF) of a discrete distribution, such that the discrete Bhattacharyya distance is used instead. The PMF is a function which maps the possible values of a discrete random variable to the probabilities of their occurrence (Grimmett and Stirzaker 2001). The discrete Bhattacharyya distance is defined as (Patra et al. 2015):

$$d_B(\mathbf{Y}_{obs}, \mathbf{Y}_{sim}) = -\log\left\{\sum_{i_m=1}^{n_{bin}} \cdots \sum_{i_1=1}^{n_{bin}} \sqrt{p_{obs}\left(b_{i_1,i_2,\cdots,i_m}\right)p_{sim}\left(b_{i_1,i_2,\cdots,i_m}\right)}\right\} \tag{5}$$

where $p_{\blacksquare}(b_{i_1,i_2,\cdots,i_m})$ is the PMF value of the bin $b_{i_1,i_2,\cdots,i_m}$. The bin has m subscripts because it is generated under a m-dimensional joint PMF space. More detailed information of the binning algorithm can be referred to Bi et al. (2019).

In this study, the output features are assumed to be very high-dimensional dynamic responses. In such circumstances, the direct evaluation of Eq. (5) becomes infeasible since the total number of bins is exponentially increasing with the dimension m due to so-called curse of dimensionality. To overcome this obstacle, a dimension reduction procedure consisting of the following steps is herein proposed (Kitahara et al. 2020).

1) Define the window length L and divide $\mathbf{y}_i$, $\forall i = 1, 2, \cdots, N_{sim}$ into $\lceil m/L \rceil$ intervals, where $\lceil \blacksquare \rceil$ denotes the upper integer of the investigating values. This is also applied to $\mathbf{Y}_{obs}$;
2) Compute the root mean square (RMS) values of each interval $\mathbf{R} = [R_1, R_2, \cdots, R_{\lceil m/L \rceil}]$ and generate the sample set of the RMS values $\mathbf{R}_{\mathbf{Y}_{sim}} \in \mathbb{R}^{N_{sim} \times \lceil m/L \rceil}$:

$$\mathbf{R}_{\mathbf{Y}_{sim}} = \left[\mathbf{R}^1_{\mathbf{Y}_{sim}}, \mathbf{R}^2_{\mathbf{Y}_{sim}}, \cdots, \mathbf{R}^{\lceil m/L \rceil}_{\mathbf{Y}_{sim}}\right], \text{with } \mathbf{R}^j_{\mathbf{Y}_{sim}} = \left[R_{1j}, R_{2j}, \cdots, R_{N_{sim}j}\right]^T, \forall j = 1, 2, \cdots, \lceil m/L \rceil \tag{6}$$

and $\mathbf{R}_{\mathbf{Y}_{obs}} \in \mathbb{R}^{N_{obs} \times \lceil m/L \rceil}$. Note that, $\mathbf{R}_{\mathbf{Y}_{obs}}$ has a similar structure as Eq. (6), where only the number of rows is changed;

3) Evaluate the Bhattacharyya distance d_{Bj} between two sample sets of the RMS values $\mathbf{R}^j_{\mathbf{Y}_{obs}}$ and $\mathbf{R}^j_{\mathbf{Y}_{sim}}, \forall j = 1, 2, \cdots, \lceil m/L \rceil$;
4) Obtain the RMS value of the Bhattacharyya distances and employ it as a UQ metric.

The principle of the window length L is that a smaller L leads to employing more detailed information of the target dynamic response data, while it leads to a larger computational demand at the same time. It is found that $L = 0.025 \cdot m$ is a reasonable choice in this study. This corresponds to the case where each RMS contains 2.5 % of the target signals.

Approximate Bayesian Computation

The ABC updating framework with the distance-based UQ metrics is summarized here. Bayesian model updating is based on the Bayes' theorem (Beck and Ktafygiotis 1998):

$$P(\mathbf{x}|\mathbf{Y}_{obs}) = \frac{P_L(\mathbf{Y}_{obs}|\mathbf{x})P(\mathbf{x})}{P(\mathbf{Y}_{obs})} \tag{7}$$

where $P(\mathbf{x})$ is the prior distribution of $\mathbf{x}$, representing the initial knowledge about the parameters $\mathbf{x}$; $P(\mathbf{x}|\mathbf{Y}_{obs})$ is the posterior distribution of $\mathbf{x}$, representing the updated knowledge about the parameters $\mathbf{x}$ based on the observed data; $P(\mathbf{Y}_{obs})$ is the normalized factor ensuring that the posterior distribution integrates to one; $P_L(\mathbf{Y}_{obs}|\mathbf{x})$ is the likelihood function of $\mathbf{Y}_{obs}$ for an instance of the parameters $\mathbf{x}$.

The likelihood function is the key component in Bayesian model updating, since it quantifies the degree of relevance of a model with a given instance of the parameters, by representing the possibility of the observations. Under the assumption of independence between each observation, the likelihood function in Eq. (7) is theoretically defined as:

$$P_L(\mathbf{Y}_{obs}|\mathbf{x}) = \prod_{k=1}^{N_{obs}} P(\mathbf{Y}_k|\mathbf{x}) \tag{8}$$

where $P(\mathbf{Y}_k|\mathbf{x})$ is the PDF value of the kth observed data $\mathbf{Y}_k$ conditional to the corresponding instance of the parameters $\mathbf{x}$. Note that, the precise estimation of the PDF requires a large number of simulated features. Consequently, an analytical formula of the likelihood in Eq. (8) demands a huge number of model evaluations and it can be almost infeasible for complex simulators.

The ABC method (Turner and Van Zandt 2012; Safta et al. 2015) is utilized to overcome the above obstacle by replacing the full likelihood with an approximate but efficient function containing the information of the observations and the instance of the parameters $\mathbf{x}$. In the approximate likelihood, any types of statistics can be used to measure the stochastic discrepancy between model predictions and observations (Turner and Van Zandt 2012); hence, it is natural to define it employing the distance metrics. Various functional formulas have been investigated in the literature for the ABC method, such as the Gaussian (Turner and Van Zandt 2012), sharp (Rocchetta et al. 2018), and Epanechnikov (Safta et al. 2015) functions. Nevertheless, the basic principle of the approximate likelihood is that it should return a high value when the distance metric is small, while it penalizes the $\mathbf{x}$ instance when its corresponding distance metric is large. In this study, an approximate likelihood function based on the Gaussian function is proposed as:

$$P_L(\mathbf{Y}_{obs}|\mathbf{x}) \propto exp\left\{-\frac{d^2}{\varepsilon^2}\right\} \tag{9}$$

where d is the distance metric; ε is the so-called width factor, which is a pre-defined coefficient controlling the centralization degree of the posterior distribution. Based on a series of tests in various applications, ε is determined to lie in the interval $[10^{-3}, 10^{-1}]$ (Patelli et al. 2017). A smaller ε corresponds to a more peaked posterior distribution which is more likely to converge to the true value but requires more computational demand for convergence.

By employing the Bhattacharyya distance, the proposed approximate likelihood function is capable of capturing comprehensive uncertainty information from both model predictions and observations. However, the Bhattacharyya distance in Eq. (5) will be infinite if the initial $\mathbf{Y}_{sim}$ is too far from $\mathbf{Y}_{obs}$, i.e., there is no overlap between the two sample sets, and thus cannot be directly employed in the likelihood. Hence, Bi et al. (2019) proposed the two-step ABC updating framework, in which a preliminary step with the Euclidian distance-based likelihood is employed to force an overlap between $\mathbf{Y}_{obs}$ and $\mathbf{Y}_{sim}$. The comprehensive uncertainty characteristics of the parameters are then further updated in the main step with the Bhattacharyya distance-based likelihood. This two-step framework is also utilized in this study and its detailed information can be referred to Bi et al. (2019).

Bayesian Updating with Adaptive Kriging Model

Bayesian Updating with Structural Reliability Methods (BUS)

In this section, the BUS formulation (Straub and Papaioannou 2015; DiazDelaO et al. 2017) is briefly reviewed. The BUS formulation is based on the conventional rejection principle. Let c denotes the so-called likelihood multiplier such that the following inequality holds for all the parameters $\mathbf{x}$:

$$cP_L(\mathbf{Y}_{obs}|\mathbf{x}) \leq 1 \tag{10}$$

In the above context, a sample distributed as the posterior distribution $P(\mathbf{x}|\mathbf{Y}_{obs}) \propto P_L(\mathbf{Y}_{obs}|\mathbf{x})P(\mathbf{x})$ in Eq. (7) can be generated by the following rejection principle:

1) Generate u uniformly distributed on $[0, 1]$ and $\mathbf{x}$ distributed as the prior distribution $P(\mathbf{x})$;
2) If $u < cP_L(\mathbf{Y}_{obs}|\mathbf{x})$, return $\mathbf{x}$ as a posterior sample. Otherwise, go back to Step 1).

Although the rejection sampling is theoretically viable, it becomes inefficient with increasing the number of observations due to the large rejection rate. Hence, BUS transforms the Bayesian updating problem into an equivalent reliability problem to maintain the advantage of the rejection principle but have much higher efficiency. Consider a reliability problem with uncertain parameters $(\mathbf{x}, u)$ according to the joint PDF $P(\mathbf{x})I(0 \leq u \leq 1)$. Here, $I(\cdot)$ denotes the indicator function, equal to one if its argument is true and zero otherwise. The limit state function and failure domain of this reliability problem can be defined as:

$$G = u - cP_L(\mathbf{Y}_{obs}|\mathbf{x}) \tag{11}$$

$$F = \{G < 0\} \tag{12}$$

The PDF of the failure sample $(\mathbf{x}', u')$ can be then obtained as:

$$p_{\mathbf{x}',u'}(\mathbf{x}, u) = p_F^{-1}P(\mathbf{x})I(0 \leq u \leq 1)I\big(u < cP_L(\mathbf{Y}_{obs}|\mathbf{x})\big) \tag{13}$$

where

$$p_F = \iint P(\mathbf{x}) I(0 \leq u \leq 1) I\big(u < cP_L(\mathbf{Y}_{obs}|\mathbf{x})\big) du d\mathbf{x}$$

is the failure probability of the reliability problem. In this formulation, the PDF of the failure sample $p_{\mathbf{x}',u'}(\mathbf{x}, u)$ and the failure probability p_F correspond to the posterior distribution $P(\mathbf{x}|\mathbf{Y}_{obs})$ and normalized factor $P(\mathbf{Y}_{obs})$ in Eq. (7), respectively. As a consequence, the samples for deriving the posterior distribution can be generated as the conditional samples falling into the failure domain by existing reliability analysis methods including Subset simulation (Au and Beck 2001).

A key component in BUS is the likelihood multiplier, since the acceptance rate in BUS is directly proportional to it. Hence, it should be selected as large as possible along with satisfying the inequality in Eq. (10) for all the parameters $\mathbf{x}$ and its optimal choice is defined as $c = [\max P_L(\mathbf{Y}_{obs}|\mathbf{x})]^{-1}$. While the optimal multiplier is generally unknown in advance, it can be defined as $c = 1$ for the proposed approximate likelihood function, because the approximate likelihood function is maximized when the distance metric is minimized to be zero. Therefore, BUS can be efficiently utilized as the Bayesian inference tool in the two-step ABC updating framework.

Adaptive Kriging-based BUS Algorithm

BUS has shown great efficiency in estimating the posterior distribution by employing Subset simulation techniques (Betz et al. 2018). However, the failure probability of the equivalent reliability problem in BUS becomes significantly small and can reach 10^{-6} or even smaller with increasing the number of observations. In such rare events, a large number of limit state function evaluations is required to estimate the failure probability even for Subset simulation. Moreover, in the main step of the ABC updating framework, the limit state function involves the Bhattacharyya distance evaluated based on random samples of model predictions. Consequently, BUS with Subset simulation demands a huge number of model evaluations and it can be almost infeasible for complex simulators.

BUS can further improve its efficiency by applying metamodeling techniques (Giovanis et al. 2017). Among various types of the metamodels, the adaptive Kriging model has been paid significant attention as one of the most accurate and efficient methods in solving reliability problems. It can be interpreted as the classification method for the failure domain by the Kriging model, also known as the Gaussian process model. In this model, the estimated responses follow a Gaussian distribution with the Kriging means and Kriging variances. The basic rationales of the kriging model can be found in Echard et al. (2011).

The key idea of the adaptive Kriging model is to adaptively identify samples close to the limit state function from the candidate MC samples based on the Kriging means and Kriging variances. The Kriging model trained by those samples enables to provide a precise classification for the failure domain and thus the failure probability can be efficiently estimated using this model. Nevertheless, the failure probability of the equivalent reliability problem in BUS is significantly small. In such rare events, the adaptive Kriging model becomes very inefficient, since the candidate sample pool should be enlarged to ensure that enough samples are contained in the failure domain.

Meanwhile, Wei et al. (2019) proposed a new algorithm called AK-MCMC. In this algorithm, the classification for a series of intermediate failure domains $F_i = \{G < b_i\}$ is provided. Here, b_i is the intermediate failure thresholds ($b_1 > b_2 > \cdots > b_m = 0$). An illustration of a two-dimensional case following the AK-MCMC algorithm is provided in Fig. 1. Fig. 1(a) illustrates its initial step as the classification for the initial intermediate failure domain $F_1 = \{G < b_1\}$ upon MC samples represented by the plots. The grey and black plots denote the arbitrary selected initial training samples and the additional training samples adaptively selected based on the Kriging means and Kriging variances, respectively. In addition, the dashed and solid lines show the initial intermediate failure surface and the Kriging model trained by the above samples, respectively. On the other hand, Fig. 1(b) illustrates the classification for the failure domain $F_m = \{G < b_m(= 0)\}$ upon MCMC samples represented as the squared points. Note that, this figure corresponds to the case where $m = 2$. As same as Fig. 1(a), the black plots denote the adaptively selected training samples and the dashed and solid lines show the failure surface and the Kriging model trained by all of the training samples. As shown in these figures, this algorithm provides the classifications for a series of intermediate failure domains, which will finally converge to the classification for the true failure domain, and is much more efficient than the direct classification for the failure domain. As a consequence, this algorithm enables to efficiently utilized for extremely rare events and thus it is expected to be suitable for BUS.

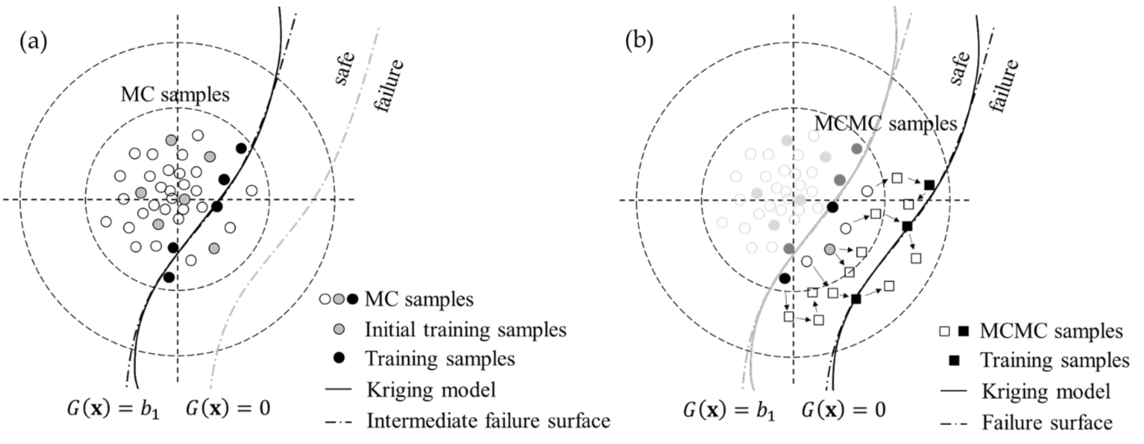

Fig. 1. Illustration of the AK-MCMC algorithm: (a) Classification for the initial intermediate failure domain; (b) Classification for the failure domain.

In this study, a new Bayesian inference algorithm is thus proposed herein by combining BUS with the adaptive Kriging model using the AK-MCMC algorithm. The flowchart of this algorithm is summarized in Fig. 2 and the procedure is described in detail as below:

1) Let $i = 1$. Generate an N MC samples population $\mathbf{W}_1$ of the parameters $(\mathbf{x}, u)$ according to the joint PDF $P(\mathbf{x})I(0 \leq u \leq 1)$;
2) Randomly select N_0 samples from $\mathbf{W}_1$ and evaluate the limit state function in Eq. (11) on these samples. Attribute these N_0 samples to the training samples population $\mathbf{W}_t$;
3) Train or update the Kriging model $\hat{G}_i(\mathbf{x}, u)$ with $\mathbf{W}_t$;
4) Predict the limit state function value for each non-training sample contained in $\mathbf{W}_i$ by the Kriging model $\hat{G}_i(\mathbf{x}, u)$. Obtain or update the intermediate failure threshold b_i based on the principle that $\lfloor p_0 N \rfloor$ samples in $\mathbf{W}_i$ is conditional on the intermediate failure domain F_i. Here, p_0 is the pre-defined target probability and $\lfloor \blacksquare \rfloor$ is the lower integer of the investigating values;
5) Compute the following learning function as:

$$U(\mathbf{x}, u) = |\mu_G(\mathbf{x}, u) - b_i| / \sigma_G(\mathbf{x}, u) \tag{14}$$

   where $\mu_G(\mathbf{x}, u)$ is the Kriging mean and $\sigma_G(\mathbf{x}, u)$ is the Kriging standard deviation. If the stopping criterion as $\min\big(U(\mathbf{x}, u)\big) \geq 2$ is satisfied for all the N samples, go to the next step. Otherwise, find the non-training sample in $\mathbf{W}_i$ with the minimum value of the learning function in Eq. (14) and evaluate the true limit state function. Attribute the sample to $\mathbf{W}_t$ and return to Step 3);
6) If $b_i \leq 0$, let $m = i$, save the Kriging model $\hat{G}_m(\mathbf{x}, u)$. Identify samples in $\mathbf{W}_m$ located into the failure domain F. Keep these samples as the seeds $\mathbf{W}_s$ and go to the next step. Otherwise, generate an N MCMC samples population $\mathbf{W}_{i+1}$ of the parameters $(\mathbf{x}, u)$ conditional on the intermediate failure domain F_i by calling the Kriging model $\hat{G}_i(\mathbf{x})$ based on the modified Metropolis-Hastings algorithm (Au and Beck 2001). Let $i = i + 1$ and $\hat{G}_i(\mathbf{x}) = \hat{G}_{i-1}(\mathbf{x})$, and return to Step 4).
7) Drawn N_p posterior samples in F with the seeds $\mathbf{W}_s$ by calling the Kriging model $\hat{G}_m(\mathbf{x}, u)$ based on the modified Metropolis-Hastings algorithm.

The learning function in Eq. (14) was proposed by Echard et al. (2011). Because the Kriging predictor follows a Gaussian distribution, $\Phi\big(U(\mathbf{x}, u)\big)$ denotes the probability of making a wrong classification on the sign of $\hat{G}(\mathbf{x}, u) - b_i$, where Φ is the standard normal cumulative distribution function. Thus, the stopping criterion ($\min(U(\mathbf{x})) \geq 2$) corresponds to the case that the probability of making a wrong classification on the sign of $\hat{G}(\mathbf{x}) - b_i$ is less than $\Phi(-2) = 0.023$.

The advantage of the proposed procedure is that it only needs a small number of evaluations to the computationally demanding limit state function in estimating posterior distributions. In addition, no prior information about the failure probability p_F is required for implementing this procedure, since the population size N depends on the target probability p_0 which is defined by the analyst in advance. Nevertheless, in the main step of the ABC updating framework, the stochastic property of the Bhattacharyya distance may cause inaccuracy in the classification of the failure surface by the Kriging model. Hence, the use of common random numbers (CRN) (Kleinman et al. 1999) is also employed in this step. CRN attempts to induce a positive correlation between the stochastic outputs (i.e. Bhattacharyya distances) for different inputs and thereby reduces the variance in the difference between the outputs; thus, it works to avoid the inaccuracy in the adaptive Kriging model.

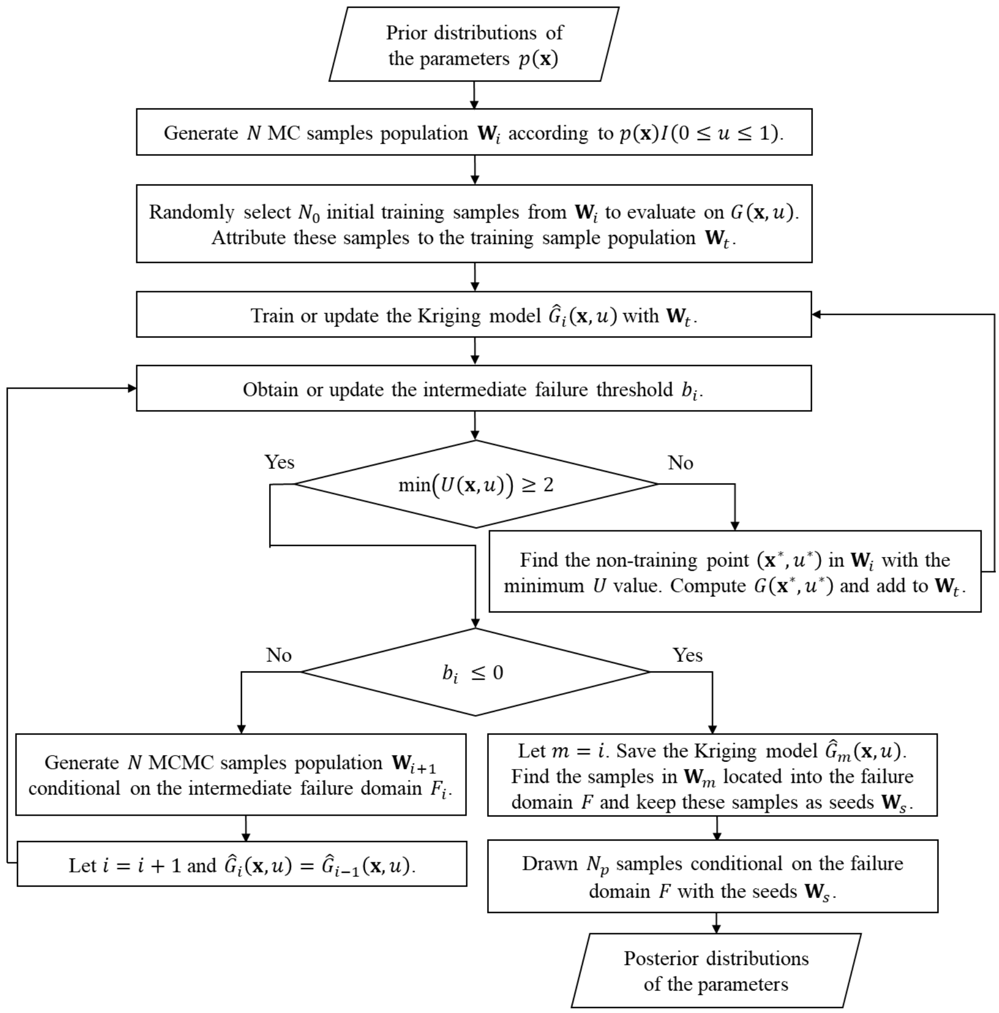

Fig. 2. Flowchart of the proposed Bayesian updating algorithm.

Numerical Example

Description of the Bayesian Updating Problem

The two-step ABC updating framework using dynamic response data is demonstrated on a model updating problem of a seismic-isolated bridge based on simulated seismic response data. The target bridge is a seismic-isolated bridge with lead rubber bearings designed based on Japan Road Association (JRA) (2016). Descriptions of the target bridge are listed in Table 1. The reinforced concrete (RC) pier with the rubber bearings is modeled as a 2-DOF lumped mass system shown in Fig. 3(a), in which the superstructure and RC pier are represented as lumped masses and the rubber bearings and RC pier are described as nonlinear horizontal springs. The boundary condition at the surface is assumed to be fixed. The rubber bearings are idealized by a bi-linear model with the ratio of the yield stiffness K_{B1} to the post-yield stiffness K_{B2} as 6.5:1 based on JRA (2004). On the other hand, the hysteresis and skeleton curves of the RC pier are idealized by a bi-linear model with the elasto-plastic characteristic and the stiffness degradation model (so-called Takeda model) (Takeda et al. 1970), respectively. Rayleigh damping is assumed in which damping ratios of the rubber bearings and RC pier are given as 0% and 2%, respectively.

Table 1. Descriptions of the target bridge.

Model parameter		Nominal value
Superstructure	Mass M_s (ton)	604.0
Rubber bearings	Yield strength (kN)	1118
	Yield stiffness K_{B1} (kN/m)	40000
	Post-yield stiffness K_{B2} (kN/m)	6000
RC pier	Mass M_P (ton)	346.2
	Yield strength (kN)	3374
	Yield stiffness K_P (kN/m)	110100
	Yield displacement (m)	0.0306
	Ultimate displacement (m)	0.251

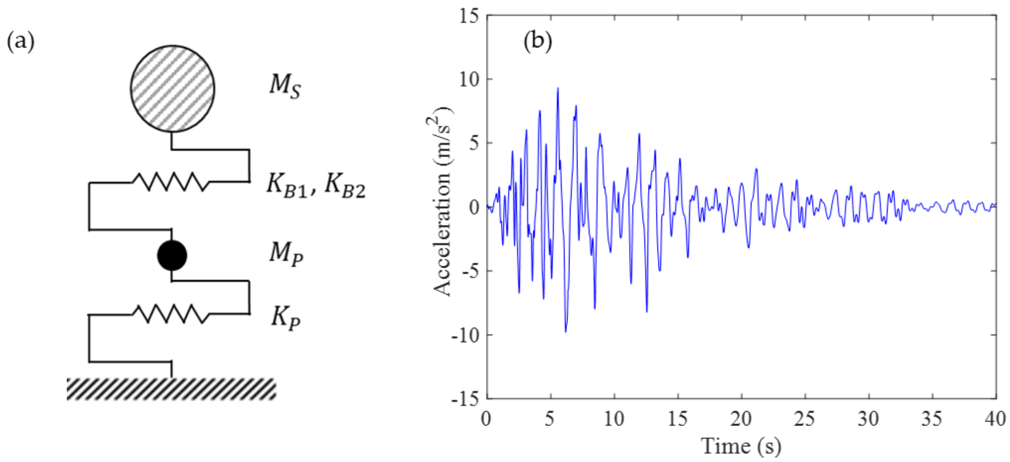

Fig. 3. (a) 2-DOF lumped mass system; (b) Time-history of the acceleration response at the superstructure.

The objective of the model updating problem is to capture the uncertainties in the post-yield stiffness of the rubber bearings K_{B2}, which characterize the nonlinear behavior of the target bridge under strong earthquakes, as well as in the other stiffness parameters K_P and K_{B1} by using simulated seismic response data. The remaining parameters are assumed to be fixed constants with the nominal values, as shown in Table 1. The time-history of the acceleration response at the superstructure subjected to the level-2 type-II-II-2 earthquake, shown in JRA (2016), is taken as the investigating output features whose uncertainties are driven by the uncertain parameters K_P, K_{B1}, and K_{B2}. Dynamic response analysis of the 2-DOF system is conducted by Newmark β method ($\gamma = 1/2$ and $\beta = 1/4$) with a time step $\Delta t = 0.001$ s. Fig. 3(b) illustrates a time-history of the acceleration response at the superstructure for the case where all parameters are considered as the nominal values in Table 1. The duration time of the time-history is 40 s with the time step $\Delta t = 0.001$ s; hence the output features are in the 40,000 dimensional-space. Both aleatory and epistemic uncertainties are involved in this system and are included by modeling K_P, K_{B1}, and K_{B2} as independent Gaussian random variables, where the means and standard deviations are not fixed but unknown lying within given intervals. According to the parameter categories in Section 1, K_P, K_{B1}, and K_{B2} are Category IV parameters, while the remaining parameters are Category I parameters. The intervals of the means μ and standard deviations σ associated to K_P, K_{B1}, and K_{B2} are detailed in Table 2.

Table 2. Uncertain characteristics and target epistemic inputs of the 2-DOF system.

Parameter	Uncertainty characteristic	Target value of epistemic input
K_P	Gaussian, $\mu_1 \in [0.5, 1.5]$, $\sigma_1 \in [0, 0.15]$	$\mu_1 = 1.0$, $\sigma_1 = 0.07$
K_{B1}	Gaussian, $\mu_2 \in [0.5, 1.5]$, $\sigma_2 \in [0, 0.15]$	$\mu_2 = 1.0$, $\sigma_2 = 0.07$
K_{B2}	Gaussian, $\mu_3 \in [0.5, 1.5]$, $\sigma_3 \in [0, 0.15]$	$\mu_3 = 1.0$, $\sigma_3 = 0.07$

The target of the updating procedure $\mathbf{Y}_{obs}$ is a set of the output features obtained by assigned target values of epistemic inputs μ_1, μ_2, and μ_3 and σ_1, σ_2, and σ_3 as shown in Table 2. Those target values are given based on Adachi (2002). The sample size of the observed features is set to be $N_{obs} = 100$, generated by evaluating the model 100 times with the model parameters sampled from their assigned Gaussian distributions with the target epistemic inputs.

In addition to the target values in Table 2, a set of initial values of the epistemic inputs is arbitrary selected within the pre-defined intervals but different from the target values. The sample size of the initial simulated features is set to be $N_{obs} = 500$, generated by evaluating the model 500 times with the model parameters sampled from their assigned Gaussian distributions with the initial epistemic inputs. Fig. 4 illustrates the relative positions of the target observed features and initial simulated features. RMS values of both the observed and simulated features for each interval divided based on the window length $L(= 0.025 \times 40000 = 1000)$ are computed and five arbitrary selected RMS values $\mathrm{RMS}_{\mathrm{ACC}}^{j}$ are shown in this figure. The diagonal subfigures compare histograms of the observed and initial simulated features. Due to the initial values of the epistemic inputs are intended assigned to be different from their target values, the scatters and histograms of the initial simulated features are clearly apart from those of the target observed features. Note that, Bayesian updating is not really started from those initial values, but from the prior distributions of the epistemic inputs, as shown in the second column of Table 2.

As shown in Fig. 4, the objective of the model updating herein is no longer a single updated point with maximum fidelity to a single observation point, but the updated means and variances of the parameter distributions which can represent simulated features as similar as the observed ones. To achieve this objective, both the Euclidian and Bhattacharyya distances are employed as metrics in the ABC updating framework. Moreover, this framework is executed using the proposed algorithm combining BUS with the adaptive Kriging model to efficiently estimate the posterior distributions.

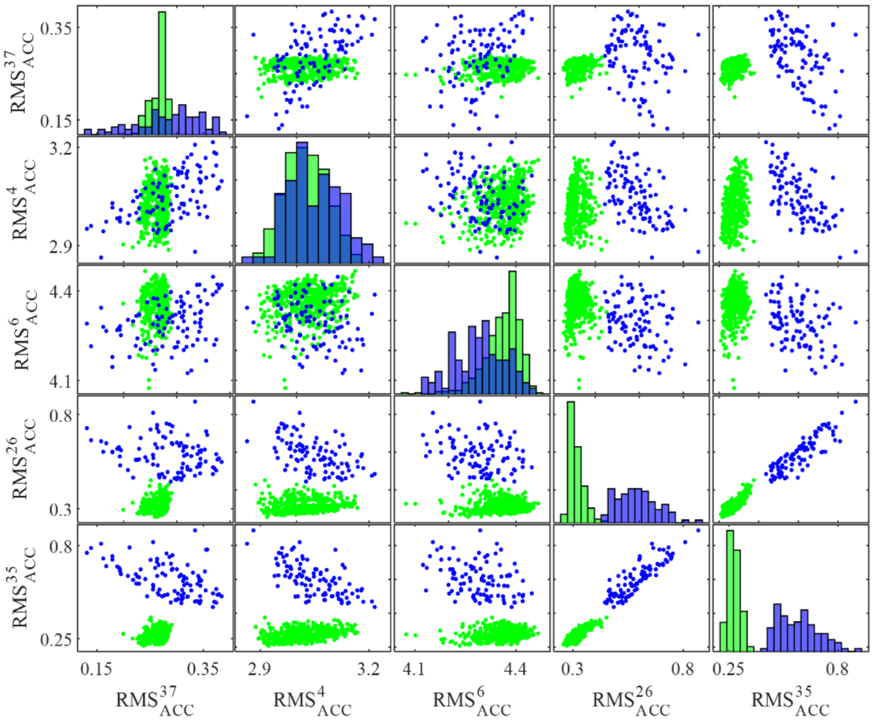

Fig. 4. Target observed scatters (in blue) and initial simulated scatters (in green); unit: $\mathrm{m/s^2}$.

Updating Results with the Euclidian Distance

In the first step where the Euclidian distance is taken as the metric, the geometric distance between the centre of mass of the two sample sets is measured, while the dispersion and distribution information of the sample sets cannot be considered. Thus, only the parameter means are considered as the uncertain parameters, whose prior distributions are set to be uniform based on the intervals in Table 1, and the model parameters are represented as those values, so that only the parameter means are updated in this step.

The parameters of the proposed algorithm are set to be $N = 3000$, $N_0 = 12$, $p_0 = 0.01$, and $N_p = 500$. The width factor in the approximate likelihood is set as $\varepsilon = 0.1$. Totally four intermediate failure surfaces are produced to finally provide the classification for the true failure domain. It implies that the failure probability of the equivalent reliability problem herein is reach around 10^{-8}. Even for such a challenging problem, the number of the total training samples is 229, selected by evaluating the limit state function associated with the Euclidian distance metric 229 times. The computation of the Euclidian distance needs a single model evaluation with the parameter means. Hence, only 229 model evaluations are required throughout this step.

As illustrated in Fig. 5, the posterior distributions of the parameter means well converge to their target values presented as the red lines. The horizontal axes of the figure are set to be as same as their prior intervals listed in Table 2. Table 3 presents the updated values of the parameter means which are obtained by estimating means of the posterior distributions. Percentage errors compared with the target values are also provided in the parentheses after the updated values.

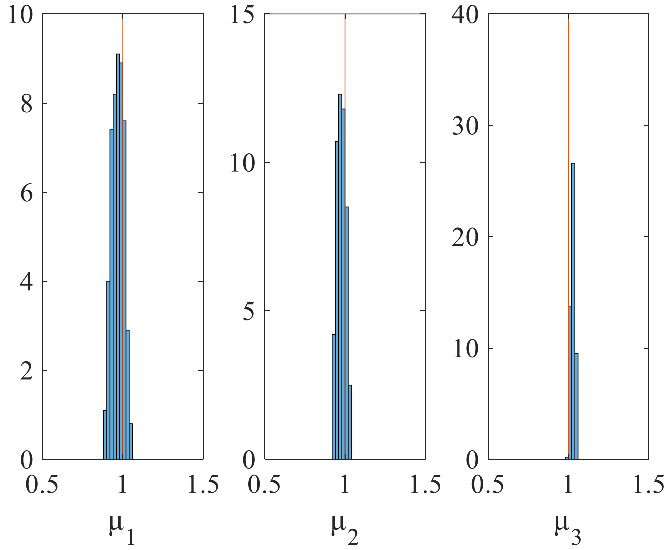

Fig. 5. Posterior distributions of parameter means after updating with the Euclidian distance.

Table 3. Updated epistemic inputs with both the Euclidian and Bhattacharyya distances.

Input	Target value	Updated value	
		With Euclidian distance	With Bhattacharyya distance
μ_1	1.0	0.9682 (3.18 %)	1.0125 (1.25 %)
μ_2	1.0	0.9772 (2.28 %)	1.0159 (1.59 %)
μ_3	1.0	1.0276 (2.76 %)	1.0024 (0.24 %)
σ_1	0.07	–	0.0604 (13.7 %)
σ_2	0.07	–	0.0572 (18.3 %)
σ_3	0.07	–	0.0813 (16.1 %)

Furthermore, Fig. 6 illustrates the relative positions of the target observed features and updated simulated features. The updated simulated features are obtained by evaluating the model 500 times with the model parameters sampled from their assigned Gaussian distributions with the updated means shown in Table 3 and variances arbitrary selected from their prior intervals. It can be seen that the simulated features are progressively sifted toward the observed features as a result of minimizing the Euclidian distance metric, corresponding to the maximization of the likelihood. Nevertheless, there are still some discrepancies between the observed and simulated features. These discrepancies are addressed in the next step using the Bhattacharyya distance as the metric.

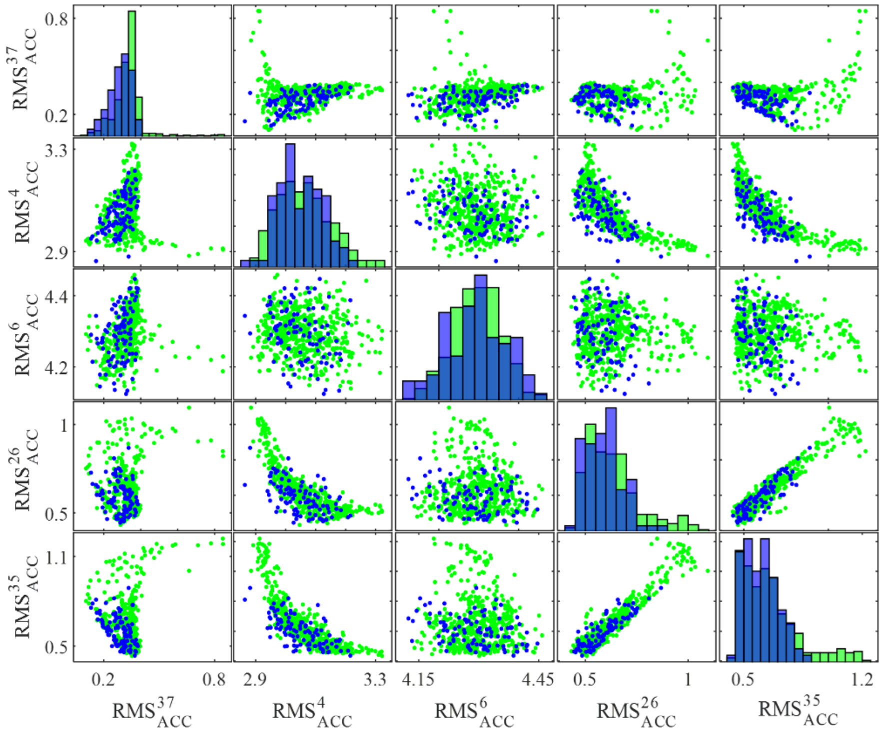

Fig. 6. Target observed scatters (in blue) and simulated scatters after updating with the Euclidian distance (in green); unit: $\mathrm{m/s^2}$.

Updating Results with the Bhattacharyya Distance

This section presents the second step where the Bhattacharyya distance is employed as the metric. The posterior distributions obtained in the first step are taken as the prior distributions of the parameter means in this step. At the same time, the prior distributions of the parameter variances are set to be uniform based on the intervals in Table 1. The model parameters are given as the assigned Gaussian distributions with the sampled means and variances, such that both the parameter means and variances are updated. In each computation of the Bhattacharyya distance, 100 random samples of the model parameters are generated and similarly 100 simulated features is obtained.

The parameters of the proposed algorithm are set to be same as those in the preliminary step. The width factor in the approximate likelihood is set to be $\varepsilon = 0.01$. After four intermediate failure surfaces are produced, the final Kriging model providing the classification for the true failure domain is obtained. It indicates that the failure probability of the equivalent reliability problem herein is also reach around 10^{-8}. The number of the total training samples is 521, selected by evaluating the limit state function associated with the Bhattacharyya distance metric 521 times. Differently from the first step, the Bhattacharyya distance evaluation requires 100 model evaluations. Hence, totally 52100 model evaluations are executed throughout this step.

Fig. 7 illustrates the finally updated posterior distributions of the epistemic inputs. The posterior distributions of the means are further updated to be more centralized to their target values compared with those in Fig. 5. This is caused by introducing the posterior samples in the first step as the prior samples in this step. More attention is paid to the posterior distributions of the standard deviations, which almost well converge to their target values presented as the red lines. The estimated means of these posterior distributions are listed in the last column of Table 3 as their updated values. The parameter means have quite high updating precisions with predicted errors less than 2%, while the parameter standard deviations show relatively large predicted errors more than 13%. This fulfils the general experience that dispersion information of parameters is much more difficult to be precisely updated than the means. Nevertheless, the finally updated simulated features obtained by evaluating the model 500 times with the model parameters sampled from their assigned Gaussian distributions with the updated epistemic inputs well coincide with the target observed features, as shown in Fig. 8. This demonstrates that the Bhattacharyya distance is a powerful UQ metric with the capability to recreate wholly the distribution of the target observations.

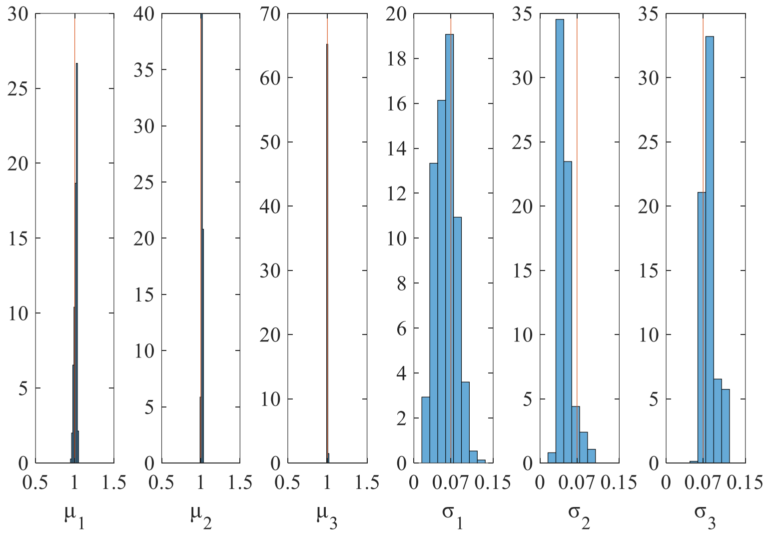

Fig. 7. Posterior distributions of epistemic inputs after updating with the Bhattacharyya distance.

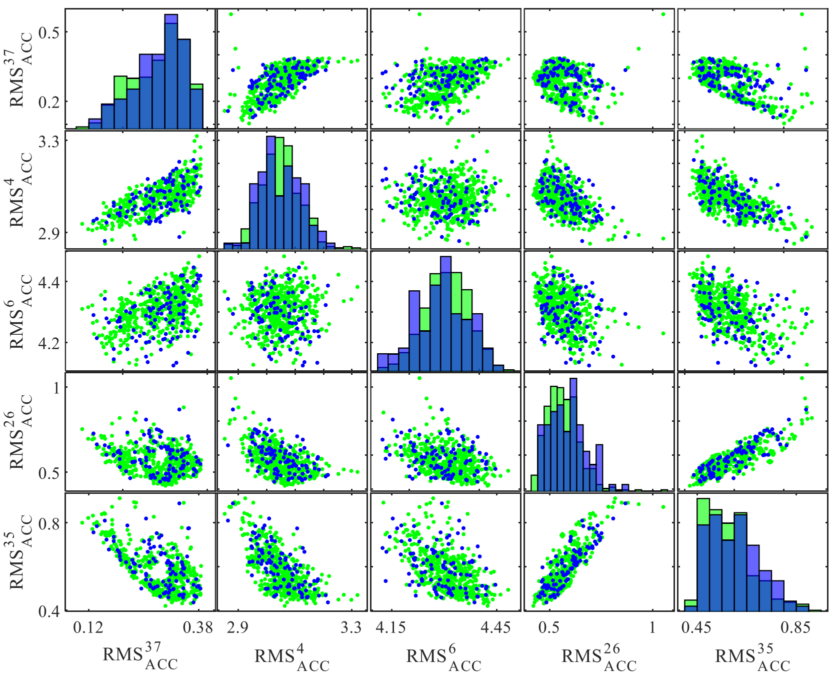

Fig. 8. Target observed scatters (in blue) and simulated scatters after updating with the Bhattacharyya distance (in green); unit: m/s^2.

Computational Efficiency

Finally, computational efficiency of the proposed procedure is demonstrated. For comparison, the two-step ABC updating framework is also executed using the TMCMC algorithm. The number of samples generated from the posterior distributions are set to be $N_p = 500$ as same as that in the proposed procedure. The width factors in the approximate likelihoods are set to be also same as those in the proposed procedure in order to keep the same computational demand for convergence as in the proposed procedure. All computations are processed using a local parallelization on a 12 cores machine installing an Intel core 2.10 GHz processor.

Table 4 summarizes the total computational time (in minutes) to reach convergence for both the first and second steps, in which the Euclidian and Bhattacharyya distances are used as metrics, respectively. In this context, computational efficiency is indicated as the ratio of the computational time using TMCMC and the proposed algorithm combining BUS with the adaptive Kriging model, and is provided in the parentheses after the computational time of the proposed algorithm. It can be seen that the second step with the Bhattacharyya distance needs much more computational demands than the first step with the Euclidian distance. The computational time in the second step is more than 300 times of that in the first step for TMCMC and is about 50 times of that in the first step for the proposed algorithm. It is obviously due to the necessity of MC sampling for each computation of the Bhattacharyya distance. Nevertheless, the difference in the computational time of those two steps is successfully reduced in the proposed procedure by confining the evaluation of the likelihood function only for the Kriging approximation.

Furthermore, it is noted that the proposed procedure reaches convergence with one-fifth of the computational time in the first step and with less than one-thirty of the one in the second step compared with TMCMC. This is mainly because the number of the model evaluations is significantly reduced by implementing the adaptive Kriging model in the proposed procedure. It should be noted that, the adaptive Kriging model enable to be also implemented in TMCMC. Nevertheless, several modifications are necessary to employ the adaptive Kriging model in TMCMC as the approximation of the likelihood function (Angelikopoulos et al. 2015; Jensen et al. 2017), because it was originally developed as the classification method in reliability problems. Meanwhile, the proposed algorithm transforms the Bayesian updating problem into the equivalent reliability problem; thus, the adaptive Kriging model is naturally implemented as the classification method. As a consequence, the proposed procedure combining BUS with the adaptive Kriging model enables to produce satisfied results with the much-reduced computational demand compared with TMCMC.

Table 4. Comparison of computational efficiency.

Method	Computational time (minutes)	
	With Euclidian distance	With Bhattacharyya distance
TMCMC	39.1	12437.5
BUS with the adaptive Kriging	7.9 (5.0)	394.4 (31.5)

Conclusions

In this study, the novel Bayesian inference algorithm combining BUS with the adaptive Kriging model is developed in order to effectively execute the two-step ABC updating framework using dynamic response data. The distance-based approximate likelihood function is capable to maximize the acceptance rate in BUS, since the optimal likelihood multiplier is straightforwardly applicable. Furthermore, to cope with the significant computational demand in the Bhattacharyya distance evaluation, the adaptive Kriging model based on the AK-MCMC algorithm is utilized to provide the classification for the limit state function associated with the Bhattacharyya distance. The AK-MCMC algorithm provides the classifications for a series of intermediate failure domains, which will finally converge to the classification for the true failure domain, and is much more efficient than the direct classification for the failure domain. The proposed procedure is demonstrated upon the seismic-isolated bridge model updating application using simulated seismic response data. This application denoted that the Bhattacharyya distance is a powerful UQ metric with the capability to recreate wholly the distribution of the target observations and that the proposed inference algorithm is enable to provide satisfactory results with much-reduced computational demand compared with TMCMC.

Data Availability Statement

All data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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