Impact of Mobility on the Estimation of Primary Channel Activity Statistics

Shreyansh Shah*, Dhaval K. Patel*, Brijesh Soni*, Miguel López-Benítez†‡, Sagar Kavaiya*
*School of Engineering and Applied Science, Ahmedabad University, India
†Department of Electrical Engineering and Electronics, University of Liverpool, United Kingdom
‡ARIES Research Centre, Antonio de Nebrija University, Spain
Email: *{shreyansh.s, dhaval.patel, brijesh.soni, sagar.k}@ahduni.edu.in, †m.lopez-benitez@liverpool.ac.uk

***Abstract*—Dynamic Spectrum Access (DSA)/Cognitive Radio (CR) has emerged as an effective paradigm to solve problem of inefficient spectrum utilization. Spectrum sensing is the key in DSA/CR. Spectrum sensing decisions can be utilized to accurately estimate the primary channel activity statistics (PAS) like mean of idle/busy period, Duty cycle etc. Such estimated statistics can be used by CR to improve performance. However, when the secondary user (SU) is mobile, estimating these statistics becomes challenging. Taking this into account, this work provides a thorough review on the estimation of PAS for mobile SUs, considering vehicular scenario. The random way-point based mobility model is adopted for modelling SU mobility. Specifically, this work provides a set of closed form expressions for the estimated statistics under SU mobility as a function of the true PAS, SU velocity, initial distance between PU and SU, PU's protection range (R), SU's sensing range (S).**

***Index Terms*—Cognitive vehicular network, spectrum sensing, dynamic spectrum access, primary activity statistics, secondary user mobility.**

I. Introduction

Dynamic Spectrum Access / Cognitive Radio (DSA/CR) has been introduced as a revolutionary solution to the problem of inefficient spectrum utilization. DSA/CR aims at better utilization of spectrum by allocating frequency spectrum dynamically instead of static allocation of frequency spectrum. DSA/CR system allows unlicensed users to temporarily access the frequency channels of primary users (PUs) without creating any harmful interference to PUs [1].

Secondary users (SUs) in DSA/CR systems are required to know the primary channels occupancy patterns, so that they can use spectrum without causing any interference to the PUs. Spectrum sensing decisions can be utilized to estimate the statistical information about primary channel such as mean of past idle/busy periods, duty cycle of primary channel, underlying distribution which can be used for predicting future occupancy patterns of primary channel [2].

There have been many works in the literature that have estimated primary channel activity statistics (PAS) using spectrum sensing decisions. For instance, PAS like mean of idle/busy periods, and duty cycle was estimated using maximum likelihood estimation in [3]. While an in-depth analytical study for the estimation of PAS based on the underlying distributions of idle/busy period was carried out in [4]–[6]. Moreover, the use of PAS to further improve the sensing performance was reported in the recent work [7]. All the aforementioned studies have assumed the perfect spectrum sensing scenario. However, sensing errors may occur in a realistic scenarios, for instance when CR users are mobile, whereby perfect spectrum sensing may not be a valid assumption.

Simulation based detailed study under imperfect spectrum sensing for estimation of the PAS was carried out in [8]. While, in-depth analytical study was reported in a recent work in [9], and using deep learning aided LSTM autoencoder in [10]. All the above mentioned works have assumed the CR users to be stationary. However, for vehicular CR systems, the mobility of CR user is an important aspect that needs to be taken into consideration. Furthermore, the estimation of PAS can also be helpful and provide insights for improving the DSA/CR performance in vehicular CR systems. To the best of the authors' knowledge, the impact of mobility of CR users on the estimation of the PAS is yet to be reported in the literature.

In this paper, we have derived closed form expressions for estimating the number of false alarm and miss detection errors that DSA/CR system will have because of SU mobility and using that expressions for estimating PAS such as mean of estimated idle/busy period and duty cycle are derived as a function of true PAS, SU velocity, PU protection range, SU sensing range and sensing period. The major contributions of this paper are twofold and can be epitomized as follows:

- Firstly, we estimate the false alarm and miss detection probability for mobile SU. Utilizing the computed probabilities, the number of false alarm and miss detection errors occurring due to SU mobility are calculated.
- Secondly, with the aid of the calculated number of false alarm and miss detection errors, the performance analysis of the estimation of PAS like mean of idle/busy periods, duty cycle and opportunistic data rate under SU mobility is carried out. Furthermore, the derived analysis are verified by the Monte-Carlo simulations.

The remainder of this paper is organized as follows. In Section-II system model for SU mobility and sensing model is defined. In Section-III analysis for the estimation of PAS under SU mobility is carried out. Numerical results are discussed in Section-IV. Finally, concluding remarks are presented in Section-V.

II. System Model

A. System model for SU mobility

In this paper, we assume that there is one stationary PU with protection range R, and one mobile SU with sensing range S and velocity v. To avoid any harmful interference, the sensing range of SU is considered to be greater than or equal to protection range of PU ($S \geq R$). The initial distance between PU and SU is D_0 and after some time t, SU moves distance vt and at that time the distance between PU and SU is changed to D_1. For SU to detect PU and transmit

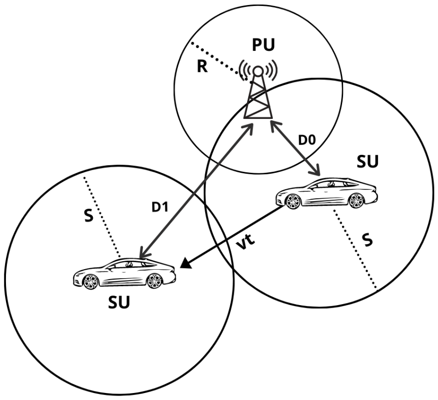


Fig. 1: System model

if channel is idle, it is necessary that PU be inside sensing range of SU, while, SU be outside the protection range of PU [11]. The time varying distance D between PU and SU at any time, determines whether the PU is inside or outside the sensing range of SU. Therefore, two events, Event "I" and Event "O" are defined as follows:

- EVENT "I": PU is inside SU's sensing range and SU is outside PU's protection range
- EVENT "O": PU is outside SU's sensing range

Important thing to note is that, if the SU is inside PU's protection range then also SU will be able to detect the PU. But, in this scenario, even if the channel is free, SU is not allowed to transmit on this channel at any cost [1]. That is why for event "I", we are only considering the area in which SU can detect PU and transmit if channel is free.

From the perspective of SU, the channel alternates between two states: idle (no activity) and busy (occupied). The PU channel activity can be modeled by two state birth-death process [12]. For Event "I", hypothesis can be defined as:

$$y_I(t) = \begin{cases} n(t), & H_0 \\ h(t)x(t) + n(t), & H_1 \end{cases} \tag{1}$$

For Event "O" SU only receives noise regardless of the state of PU. In this scenario the hypothesis turns into following:

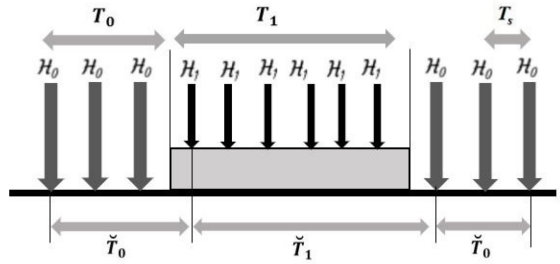
$$y_O(t) = n(t), \quad H_0, H_1 \tag{2}$$

where $y_I(t)$ is the signal that SU receives given Event "I", $x(t)$ denotes the transmitted PU signal, $n(t)$ is the additive white Gaussian noise (AWGN), $h(t)$* represents channel gain, $y_O(t)$ is the signal that SU receives given Event "O".

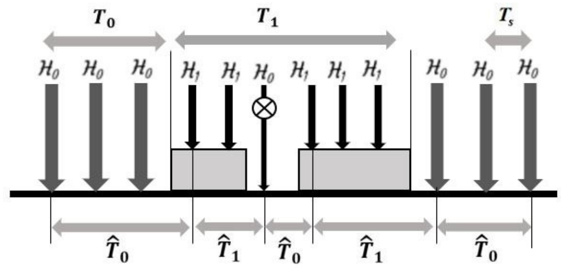
B. Sensing Model

This information can be estimated by DSA/CR system by periodically performing spectrum sensing with a sensing period of T_s. As a result of every sensing event, a binary decision (idle/busy) is made regarding the state of PU. Fig. 2(a) shows the ideal case of spectrum sensing. However due to SU mobility, which generates the false alarm and miss detection errors, the actual performance of DSA/CR system

*We assume the channel gain $h(t)$ to be constant during sensing period



(a) Ideal performance of DSA/CR system



(b) Actual performance of DSA/CR system under SU mobility

Fig. 2: Sensing Model

is shown in Fig. 2(b) where the impact of one miss detection error on estimation of busy period is shown. $\hat{T}_i$ represents the estimated period for the real period T_i (where i=0 refers for the idle period and i=1 refers for busy period). Furthermore, for the ease of analysis we assume interweave spectrum sharing approach.

III. ESTIMATION OF PU CHANNEL ACTIVITY STATISTICS UNDER SU MOBILITY

A. Impact of SU mobility

Event "I" and Event "O" probability can be derived from the cumulative distribution function of the distance D between stationary PU and mobile SU. According to [13], the cumulative distribution function of the distance between static PU and mobile SU can be assumed as log-normal distribution i.e.,

$$F_D(d) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{d - \mu_d}{\sigma_d\sqrt{2}}\right)\right] \tag{3}$$

where, erf denote the error function, μ_d and σ_d denotes the mean and standard deviation of distance between static PU and mobile SU. Similarly, cumulative distribution function for sensing range and protection range are defined. As discussed earlier, that SU will only be able to detect PU and access free band when the distance D is between R and S. $\Pr(I)$ can be defined as:

$$\Pr(I) = \Pr(R < D \leq S) \tag{4}$$

With the aid of [13], above equation can be simplified as:

$$\Pr(I) = \frac{1}{2}\left[\text{erf}\left(\frac{\frac{S-D_0}{v} - \mu_t}{\sigma_t\sqrt{2}}\right) - \text{erf}\left(\frac{\frac{R-D_0}{v} - \mu_t}{\sigma_t\sqrt{2}}\right)\right] \tag{5}$$

Similarly, $\Pr(O)$ can be defined and is calculated as:

$$\Pr(O) = \Pr(S \leq D) = 1 - \frac{1}{2}\left[1 + \text{erf}(\frac{\frac{S-D_0}{v} - \mu_t}{\sigma_t\sqrt{2}})\right] \tag{6}$$

where, μ_t and σ_t are the mean and standard deviation of time distribution, S is the SU's sensing range, R is the PU's protection range, v is the SU's velocity and erf is the error function.

B. Miss Detection and False Alarm Probabilities

The estimated miss detection probability for mobile SU, as a function of $\Pr(I)$ and $\Pr(O)$ can be defined with the aid of [14] as:

$$\begin{aligned} P_{m,v} =& \Pr(\lambda \leq Th|H_1, I)\Pr(I)\Pr(ON) \\ &+ \Pr(\lambda \leq Th|H_1, O)\Pr(O)\Pr(ON) \\ =& \Pr(ON)[\Pr(m|I)\Pr(I) + \Pr(m|O)\Pr(O)], \end{aligned} \tag{7}$$

where $P_{m,v}$ is the estimated miss detection probability for a given SU with velocity, $\Pr(m|I)$ and $\Pr(m|O)$ represents conditional miss detection probability of PU being inside and outside of sensing range of SU, λ represents the energy of received signal, Th represents decision threshold, $\Pr(ON)$ represent that PU is actually present (busy state).

Similarly, the estimated false alarm probability can be defined with the aid of [14] as:

$$\begin{aligned} P_{f,v} &= \Pr(\lambda > Th|H_0)\Pr(OFF) \\ &= \Pr(\lambda > Th|H_0, I)\Pr(I)\Pr(OFF) \\ &+ \Pr(\lambda > Th|H_0, O)\Pr(O)\Pr(OFF) \\ &= \Pr(f|I)\Pr(OFF), \end{aligned} \tag{8}$$

where $P_{f.v}$ is the estimated false alarm probability for a given SU velocity, $\Pr(f|I)$ denotes conditional false probability that PU is inside the sensing range of SU and $\Pr(OFF)$ denotes that PU is actually absent (idle state).

Furthermore, for a given SU, the conditional probability $\Pr(f|I)$ in terms of Q function can be given by:

$$\begin{aligned} \Pr(f|I) &= \Pr(\lambda > Th|H_0, I), \\ &= Q\left(\frac{Th - E(\lambda|H_0, I)}{\sqrt{\mathrm{Var}(\lambda|H_0, I)}}\right), \end{aligned} \tag{9}$$

where $E(\lambda|H_0, I) = n\sigma_n^2$ and $\mathrm{Var}(\lambda|H_0, I) = 2n\sigma_n^4$ with n=2 degrees of freedom in vehicular network. Similarly, the conditional probability $\Pr(m|I)$ in terms of Q function can be given by:

$$\begin{aligned} \Pr(m|I) &= \Pr(\lambda \leq Th|H_1, I) \\ &= 1 - Q\left(\frac{Th - E(\lambda|H_1, I)}{\sqrt{\mathrm{Var}(\lambda|H_1, I)}}\right) \end{aligned} \tag{10}$$

where $E(\lambda|H_1, I) = n(\sigma_n^2+\sigma_s^2)$ and $\mathrm{Var}(\lambda|H_1, I) = 2n(\sigma_n^2+\sigma_s^2)^2$

To summarize, till now we have derived an expression for estimating the probability of false alarm ($P_{f,v}$) and the probability of miss-detection($P_{m,v}$) under SU mobility. In the next section, using these probabilities, the total number of false alarm and miss detection errors are calculated and using the calculated total errors, PAS are estimated.

C. Estimation of the Mean Period

Mean of the idle/busy periods is a key statistical moment of the PAS. Assume we have a given set $\{\breve{T}_{i,n}\}_{n=1}^{N}$ of N periods estimated under perfect spectrum sensing. Then, conventionally their mean can be calculated by [9]:

$$E(\breve{T}_i) \approx \breve{m}_i = \frac{1}{N}\sum_{n=1}^{N}\breve{T}_{i,n} \tag{11}$$

However, this conventionally calculated mean will be extremely unreliable (far below the true mean). The reason for this is that any false alarm or miss detection error will divide the original period duration T_i into smaller sub duration. From Fig. 2(b), we can observe that, a single miss detection error can lead to corruption in the estimation of the busy period T_1 by furcating it into three sub duration. It divides T_1 into $\hat{T}_1$, $\hat{T}_0$ and $\hat{T}_1$ and the period duration of two $\hat{T}_1$ will depend on the position of the error within T_1 and the period duration of $\hat{T}_0$ which is equal to the sensing period (T_s).

Because of this phenomenon, the number of idle/busy periods observed by DSA/CR system under SU mobility N_v will be greater than the original number of periods N. So, we can say that $N_v \neq N$. As shown in Fig 2(b), one miss-detection error produces two short periods of $\hat{T}_1$ and one short period of $\hat{T}_0$, from the original T_1. Therefore, each miss-detection error would produce one additional estimated busy period $\hat{T}_1$ and one additional estimated idle period $\hat{T}_0$ and this is the reason why the number of idle/busy period observed by DSA/CR system with SU mobility (N_v) will be greater than the original number of periods N. Because of this phenomenon, it is not possible to estimate PAS using conventional method. Thus, for finding these statistics accurately, we first find the mean of busy period by taking the primary channel periods illustrated in Fig 2(b) and then later on generalize it for idle and busy periods. Estimated mean of busy periods can be given by :

$$\hat{m}_1 = \frac{1}{2}\sum_{n=1}^{2}\hat{T}_{1,n} = \frac{\hat{T}_{1,1} + \hat{T}_{1,2}}{2} = \frac{T_1 - \hat{T}_0}{2} = \frac{T_1 - T_s}{2} \tag{12}$$

The mean of the busy periods in fig 2(b) can be calculated by subtracting $\hat{T}_0$ from true busy period (T_1). The denominator part, represents the total number of busy periods observed by DSA/CR under SU mobility, which is equal to the number of true busy periods (N) plus one (for each miss-detection error). Extending this analysis to any number of miss-detection error within the whole set of busy period, the estimated mean of the busy period can be given by:

$$\hat{m}_1 = \frac{\sum_{n=1}^{N} T_{1,n} - N_{md}T_s}{N + N_{md}} \tag{13}$$

where N_{md} represents the total number of miss detection errors, which can be calculated as:

$$N_{md} = \frac{\sum_{n=1}^{N} T_{1,n}}{T_s} \cdot P_{m,v} \tag{14}$$

where $P_{m,v}$ is the estimated miss detection probability for a particular SU velocity.

The analysis done till now has assumed no false alarm errors, however false alarm errors will also produce additional busy periods in the observed set. Same logic can be applied for analysis of false alarm errors and (13) can be rewritten by taking false alarm errors into consideration as:

$$\hat{m}_1 = \frac{\sum_{n=1}^{N} T_{1,n} - N_{md}T_s + N_{fa}T_s}{N + N_{fa} + N_{md}} \tag{15}$$

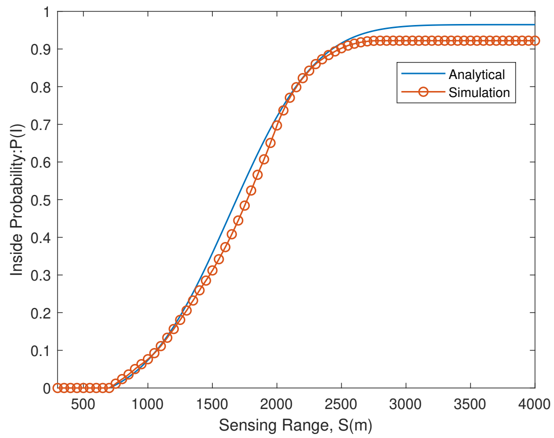


Fig. 3: Probability of PU being inside the SU's sensing range

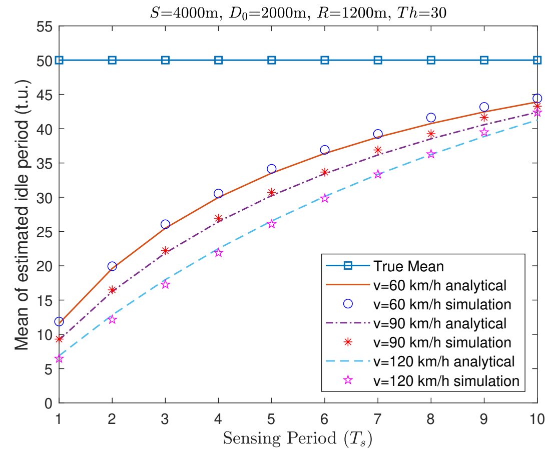


Fig. 4: Mean of estimated idle period ($\mathbb{E}(\hat{T}_0)$) for different values of velocity

where N_{fa} represents the total number of false alarm errors, which can be calculated as:

$$N_{fa} = \frac{\sum_{n=1}^{N} T_{0,n}}{T_s} \cdot P_{f,v} \tag{16}$$

where $P_{f,v}$ is the estimated false alarm probability for a particular SU velocity. Note that the term $\sum_{n=1}^{N} T_{i,n}$ can be rewritten as:

$$\sum_{n=1}^{N} T_{i,n} = N m_i \tag{17}$$

By substituting (14) and (16) into (15) and rewriting $\sum_{n=1}^{N_v} T_{i,n}$ using (17), we obtain:

$$\hat{m}_1 = \frac{m_1 (1 - P_{m,v}) + m_0 P_{f,v}}{1 + \frac{m_0}{T_s} P_{f,v} + \frac{m_1}{T_s} P_{m,v}} \tag{18}$$

There are some cases for which false alarm or miss detection error will not produce any additional errors. These cases are analysed in detail in [9], to make this work self contained, the final closed-form expression of the mean of estimated busy periods under SU mobility can be expressed as:

$$\mathbb{E}(\hat{T}_1) = \frac{\mathbb{E}(T_1)(1 - P_{m,v}) + \mathbb{E}(T_0) P_{f,v}}{1 + \left(\frac{\mathbb{E}(T_0)}{T_s} - 2\right) \grave{P}_{f,v} + \left(\frac{\mathbb{E}(T_1)}{T_s} - 2\right) \grave{P}_{m,v}} \tag{19}$$

where $\grave{P}_{f,v}$ and $\grave{P}_{m,v}$ are given by:

$$\grave{P}_{f,v} = P_{f,v} \left(\frac{1 - 2P_{f,v}}{1 - P_{f,v}} \right) \tag{20}$$

$$\grave{P}_{m,v} = P_{m,v} \left(\frac{1 - 2P_{m,v}}{1 - P_{m,v}} \right) \tag{21}$$

The closed form expression in (19) provides a mathematical relationship between mean of estimated busy period and estimated false alarm and miss detection probability, which indirectly provides a mathematical relationship between estimated mean and the velocity of the SU (as $P_{m,v}$ and $P_{f,v}$ is found using (7) and (8) which contains impact of SU velocity and several other parameters). Furthermore, the mean of estimated idle period can be found by similar analysis and it is given by:

$$\mathbb{E}(\hat{T}_0) = \frac{\mathbb{E}(T_0)(1 - P_{f,v}) + \mathbb{E}(T_1) P_{m,v}}{1 + \left(\frac{\mathbb{E}(T_0)}{T_s} - 2\right) \grave{P}_{f,v} + \left(\frac{\mathbb{E}(T_1)}{T_s} - 2\right) \grave{P}_{m,v}} \tag{22}$$

D. Estimation of the Duty Cycle

Duty Cycle (DC) can be defined as the ratio of total busy sensing decisions over the total number of sensing event [15]. In [4], another method has been proposed for estimating DC (Ψ) of primary channel based on mean of idle/busy periods as:

$$\Psi = \frac{\mathbb{E}(T_1)}{\mathbb{E}(T_1) + \mathbb{E}(T_0)}. \tag{23}$$

Expressions for mean of estimated busy and idle periods are given in (19) and (22). Accordingly, estimated duty cycle($\hat{\Psi}$) in terms of mean of estimated idle and busy periods can be expressed as:

$$\hat{\Psi} = \frac{\mathbb{E}(\hat{T}_1)}{\mathbb{E}(\hat{T}_1) + \mathbb{E}(\hat{T}_0)} \tag{24}$$

Opportunistic data rate R_b can be calculated using DC, channel bandwidth W and channel efficiency η as $R_b = (1 - \Psi)W\eta$. Estimated opportunistic data rate helps the DSA/CR system to assign the primary channel with highest opportunistic data rate that can be offered to SU. Using (24), opportunistic data rate can be estimated as:

$$\hat{R}_b = (1 - \hat{\Psi})W\eta \tag{25}$$

IV. Numerical and Simulation Results

In this section, simulation and analytical results are presented in order to validate the analysis carried out in this work. For modelling SU velocity, random way-point mobility model is used with square area of $5000 \times 5000m^2$, with PU being fixed at center. Approximately 10^6 random way-points are generated. These random way-points are treated as mobile SUs and for each way-point its distance from PU is calculated and consequently we calculate $\Pr(I)$. In Fig. 3 the impact of SU mobility on $\mathrm{P}(I)$ is illustrated. Results shown in Fig. 3 are calculated analytically using (5) as well as by means of simulation. We can observe that as sensing range increases the probability of PU being inside SU's sensing range increases.

Fig. 4 shows the mean of the estimated idle period compared to original mean of idle period. Mean of estimated idle period is calculated using (22). For the simulation similar approach given in [8] is used, except that here probability of miss detection ($P_{m,v}$) and Probability of false alarm ($P_{f,v}$) are

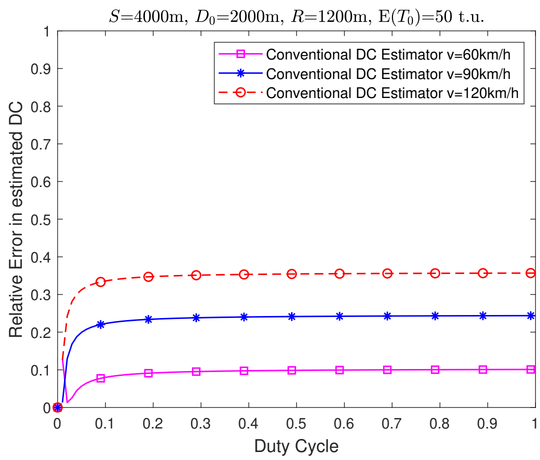


Fig. 5: Relative error in the estimated DC for different SU velocity (T_s = 5 t.u.)

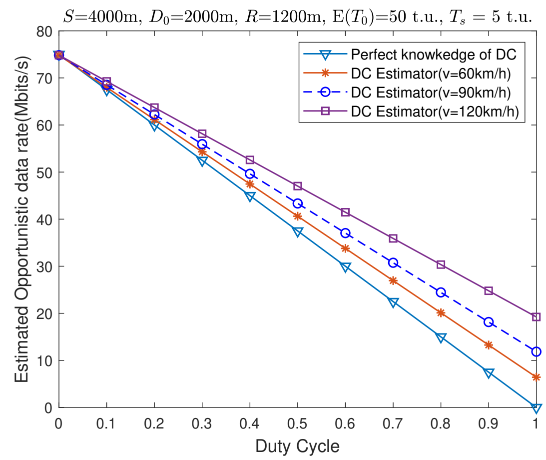


Fig. 6: Estimated Opportunistic data rate

calculated using (7) and (8) by using the appropriate value of velocity, sensing range, protection range and the initial distance between PU and SU. As expected, for low SU mobility we are more accurately able to estimate idle period compared to high SU mobility. Moreover as sensing period increases, the accuracy in the estimated idle mean period increases. This is because a longer sensing period reduces the number of sensing events per time unit and therefore how often sensing errors occur. In Table I, mean of estimated idle period and estimated DC (Ψ = 0.5 and T_s = 5 t.u.), for different SU velocity is shown.

TABLE I: Comparison of mean of estimated idle period and estimated DC for different velocity

Velocity(km/h)	70	80	90	100	110
$\mathbb{E}(\hat{T_0})$	33.25	32.15	30.23	28.31	27.03
DC	0.63	0.64	0.66	0.67	0.69

Fig. 5 the plot for relative error in estimated DC under SU mobility for whole range of possible DC is illustrated. The relative error in the estimated DC under mobile SU can be calculated with respect to its original value as $|\hat{\Psi} - \Psi|/\Psi$. We can observe that for low values of SU velocity, error in estimated DC is low, and after a certain point the relative error in estimated duty cycle gets constant. Fig. 6 shows the estimated data rate for mobile SU for $W = 25$ MHz and $\eta = 3$ bits/Hz as a function of DC. As observed, higher velocity leads to an overestimation of the available opportunistic data rate, which needs to be taken into account in system designs.

V. Conclusions

Performance of DSA/CR can be improved using PAS which can be estimated using spectrum sensing. However when SU is mobile, these estimated PAS can be inaccurate. In this regards, this paper provides thorough analysis of impact of SU mobility on estimation of PAS. In particular, we first estimate the false alarm and miss detection probability for mobile SU and compute the number of false alarm and miss detection errors occurring due to SU mobility are calculated. With the aid of the calculated number of false alarm and miss detection errors, the performance analysis of the estimation of PAS like mean of idle/busy periods, duty cycle and opportunistic data rate under SU mobility is carried out. Furthermore, the derived analysis are verified by the Monte-Carlo simulations. Numerical results suggest that as the velocity of SU increases, the error in estimated statistics increases. This work provides a realistic framework, offers a concrete study, and can be used while designing the vehicular CR systems.

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