27th International Congress
on Sound and Vibration
The annual congress of
the International Institute
of Acoustics and Vibration (IIAV)
11-16 July, 2021
ICSV27

Annual Congress of the International Institute of Acoustics and Vibration (IIAV)

SUPPLEMENTING THE MODAL ASSURANCE CRITERION (MAC) WITH THE PARTIAL MODAL VECTOR RATIO (PMVR) FOR THE VALIDATION OF FEM MODELS

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The Modal Assurance Criterion (MAC) is widely used for the validation of finite element models by assessing correlation between mode shapes from FEM and Experimental Modal Analysis (EMA). An important limitation of MAC is that it is sensitive to large values and insensitive to small values. In addition, MAC does not consider the relative response between the different parts of a FEM model. This paper introduces the Partial Modal Vector Ratio (PMVR) as a time efficient supplement to MAC for the validation of FEM models when there are correlation issues caused by the interaction of the different parts of a model. An example application is shown for a pile of three reinforced concrete beams stacked on top of each other without any bonding material.

Keywords: MAC, finite element, modal analysis

1. Introduction

The Modal Assurance Criterion (MAC) is widely used for the validation of finite element models by assessing correlation between mode shapes from FEM and Experimental Modal Analysis (EMA). An important limitation of MAC is that it is sensitive to large values and insensitive to small values [1]. Therefore, if one subset of the modal vector is significantly larger than the remaining subset of the modal vector, then the MAC value will be mainly determined by the former subset and any lack of correlation related to the latter will not be identified by MAC. The Partial Modal Assurance Criterion (PMAC) [2] can be used to give insight into individual subsets of the modal vector by applying MAC to each subset separately. However, MAC and PMAC only describe correlation between the mode shapes and do not consider the relative response between different parts of the model. In recent work this was found to be essential to assessing the interaction in junctions of concrete beams where the beams are stacked on top of each other without the existence of any bonding material [3]. This paper introduces the Partial Modal Vector Ratio (PMVR) as a time efficient supplement to MAC for the validation of FEM models when there are correlation issues caused by the interaction of the different parts of a model. A pile of three reinforced concrete beams is used to show the efficiency of PMVR by experimentally validating FEM models against EMA results in the frequency range up to 3.2 kHz.

2. Experimental work

2.1 Test specimens and setup

The pile consists of three concrete beams (2400 × 200 × 300 mm) that were reinforced with four or eight longitudinal steel bars of 16 mm diameter (see Fig. 1). The transverse reinforcement consists of 8 mm diameter stirrups placed at 200 mm centres along the beams. Beam 3 was designed to have one 100 mm wide concrete discontinuity in the middle of its length. To create the pile, beams 1, 2 and 3 were placed on top of each other without the presence of any bonding material. The lower surface of beam 1 rested upon a solid, square-section aluminium bar (25 × 25 mm).

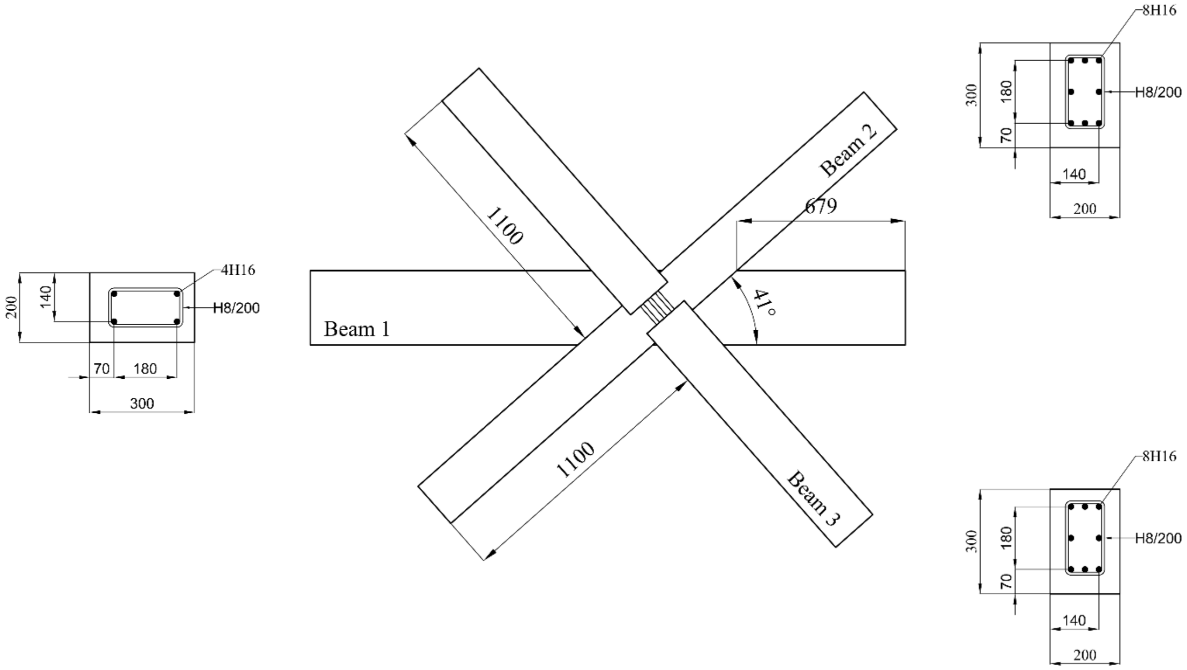

Figure 1: Test setup of the pile (units: millimetres).

2.2 Material properties

Table 1 shows the material properties of the beams. The density of the concrete for each beam was calculated by dividing the measured weight of the beams by the volume of concrete after extracting the weight of the steel reinforcement. The Young's modulus of the concrete was estimated through updating of the FEM model [3]. Material properties for steel and Poisson's ratio for concrete were taken from published data [4,5].

Table 1: Material properties of the concrete and steel used for beams 1, 2 and 3.

Material	Density, ρ (kg/m^3)		Young's modulus, E (N/m^2)	Poisson's ratio, ν (-)
Concrete (C25/30)	Beam 1	2329	36875E+06	0.2
	Beam 2	2245	32475E+06	
	Beam 3	2235	32475E+06	0.2
Steel (S500)	7800		200E+09	0.3

2.3 Modal analysis

EMA has been used to identify the modal characteristics of the pile using FFT analysis with 1 Hz frequency lines [3]. The beams were excited using an impact hammer (Brüel & Kjær Type 8200) and the out-of-plane response was measured using four accelerometers (Brüel & Kjær Type 4371).

3. FEM modelling

All finite element models used Abaqus software (Version 6.14) [6]. Eigenvalue extraction used the Lanczos solver in the frequency range from 1 Hz to 3.2 kHz and mode-based steady-state dynamic analysis was used to calculate vibration transmission between the beams in the pile. Only the experimentally validated modes were included into the mode-based analysis using direct damping determined from the modal damping identified in the experimental work. The nodes of the top surface of the beams were excited by sequentially applying a unit load at the intersections of a 100 mm square grid which approximately corresponded to the hammer positions used in EMA.

Solid element C3D20R (20 nodes) and the beam element B32 (3 nodes) were used to model the concrete and steel bars, respectively. The mesh density fulfils the requirement of at least six quadratic elements per bending wavelength. The linear spring element, SPRING1 was used to approximate the elastic support that the aluminium square bars provide to beam 1 and the stiffness of the springs was estimated to be 4.1E+05 N/m after model updating [3]. The contact between the beams was modelled using the surface-to-surface contact algorithm of Abaqus/Standard and was defined to have elastic normal behaviour. Two different FEM models were used for contact stiffness; FEM model No.1 and 2 with contact stiffness equal to 8.77E+07 N/m and 7.038E+08 N/m, respectively [3].

4. Validation criteria

4.1 Mode shape criteria

The Modal Assurance Criterion is used to assess the correlation between mode shapes from FEM and EMA using [7]

$$MAC(A,X) = \frac{\left|\{\varphi_X\}^{\mathrm{T}}\{\varphi_A\}^*\right|^2}{(\{\varphi_X\}^{\mathrm{T}}\{\varphi_X\}^*)(\{\varphi_A\}^{\mathrm{T}}\{\varphi_A\}^*)} \tag{1}$$

where X indicates the experiment, A indicates FEM, $\{\varphi_X\}$ and $\{\varphi_A\}$ are the column vectors of the degrees of freedom for the experimental and FEM mode shapes respectively, superscript T indicates the transpose and * is the complex conjugate.

The Partial Modal Vector Ratio is defined as the ratio in decibels of the squared modal vectors from EMA relative to FEM. For two subsets of the complete modal vector i and j, PMVR is given by

$$PMVR(A,X)_{i,j} = \left|10\log_{10}\left(\left(\frac{\langle|\{\varphi_X\}_i|^2\rangle}{\langle|\{\varphi_X\}_j|^2\rangle}\right)\Big/\left(\frac{\langle|\{\varphi_A\}_i|^2\rangle}{\langle|\{\varphi_A\}_j|^2\rangle}\right)\right)\right| \tag{2}$$

where $\{\varphi_A\}$ and $\{\varphi_X\}$ are subsets of the modal vectors from FEM and EMA respectively.

In this paper, a subset is defined as the vector containing the degrees of freedom of each of the beams that form the pile. Close agreement is defined as $PMVR \leq 5$ dB and reasonable agreement as 5 dB < $PMVR \leq 10$ dB [3].

4.2 Spatial – average transfer mobility ratio

Using data from EMA or FEM, the spatial-average transfer mobility ratio, $YR_{ji,i}$, for two beams i and j, with point force excitation on i is given by

$$YR_{ji,i} = 10\log_{10}\left(\frac{\left(\frac{1}{N}\sum_{k=1}^{N}\left|\frac{v_j}{F_i}\right|^2\right)}{\left(\frac{1}{N}\sum_{k=1}^{N}\left|\frac{v_i}{F_i}\right|^2\right)}\right) \tag{3}$$

where v is the velocity, F is the force, m is the mass, N represents the number of nodes in the FEM model or the number of accelerometer positions used in EMA.

5. Results

5.1 Eigenfrequencies

Figure 2 compares FEM models No.1 and 2 against EMA in terms of eigenfrequencies. Both FEM models show close agreement with differences less than 5% for most mode pairs.

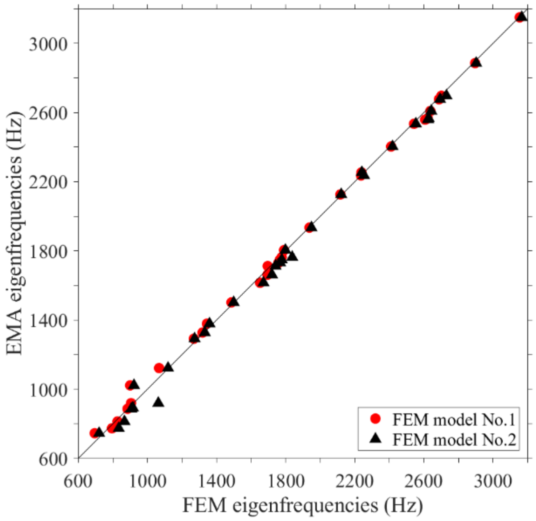

Figure 2: Comparison of FEM model No.1 and 2 against experimental eigenfrequencies.

5.2 Mode shapes

MAC results are shown in Figures 3a and 3b. For the first eight correlated mode pairs, MAC>0.8 for seven mode pairs with FEM model No.1 but only three mode pairs with No.2. Above the eighth mode, both FEM models showed equally close agreement with MAC > 0.8 for 17 of the 32 mode pairs.

In terms of PMVR, close agreement (≤ 5 dB) was achieved for 16% and 29% of the mode pairs from models No.1 and 2, respectively. It is seen that for model No.1, many PMVR values are >10 dB (see Figures 4a, b and c). Hence, whilst model No.1 had higher MAC values than No.2, PMVR indicates that model No.2 gives an improved representation of the interaction between the coupled beams.

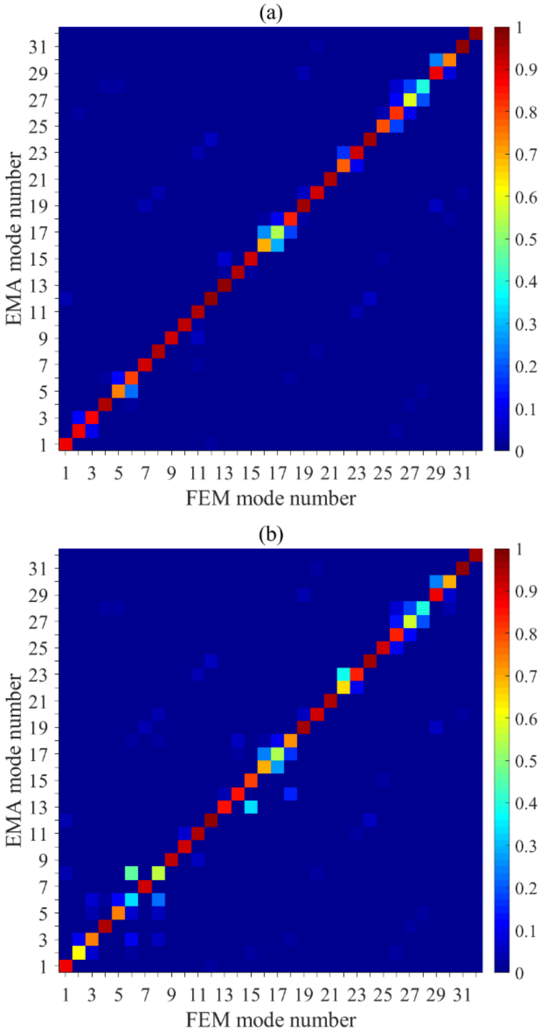

Figure 3: MAC values: (a) FEM model No.1 and (b) FEM model No.2.

5.3 Spatial-average transfer mobility ratio

Figure 5 compares FEM models No.1 and No.2 with EMA in terms of the spatial-average transfer mobility ratio. When the force is applied to beam 1 (Figures 5a and b), model No.2 shows closest agreement with EMA (differences less than 4 dB) whereas No.1 was offset with differences up to 12 dB on average. When the force is applied to beam 2 (Figure 5c) or beam 3 (Figure 5e), model No.2 also shows significantly closer agreement with EMA than No.1. However, when the force is applied to beam 2 (Figure 5d) or beam 3 (Figure 5f) and the velocity response is measured on beams 2 and 3, models 1 and 2 show equally reasonable agreement.

It is shown that FEM model No.2 is significantly better than FEM model No.1 for modelling vibration transmission between the beams in the pile. However, the FEM model that best represented the physical situation was identified by PMVR but not by MAC which led to a misleading validation by indicating that model No.1 was more accurate than No.2. MAC is very sensitive to large values, so any correlation problem caused by the interaction between the beams is not reflected in its value. For this reason, it is proposed here that PMVR is a computationally efficient supplement to MAC when validating FEM models where structural components are coupled by elastic connections of unknown stiffness.

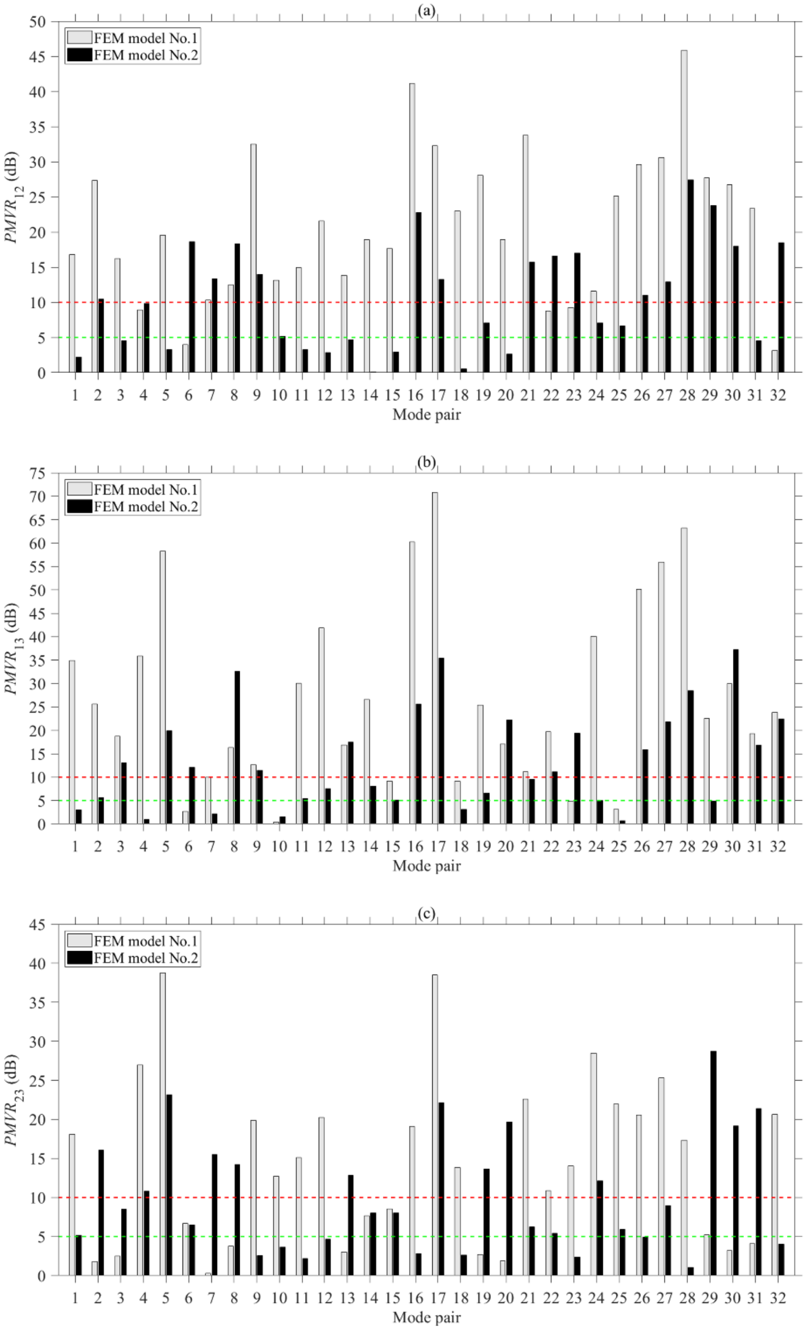

Figure 4: PMVR for FEM models No.1 and 2: (a) PMVR12, (b) PMVR13 and (c) PMVR23.

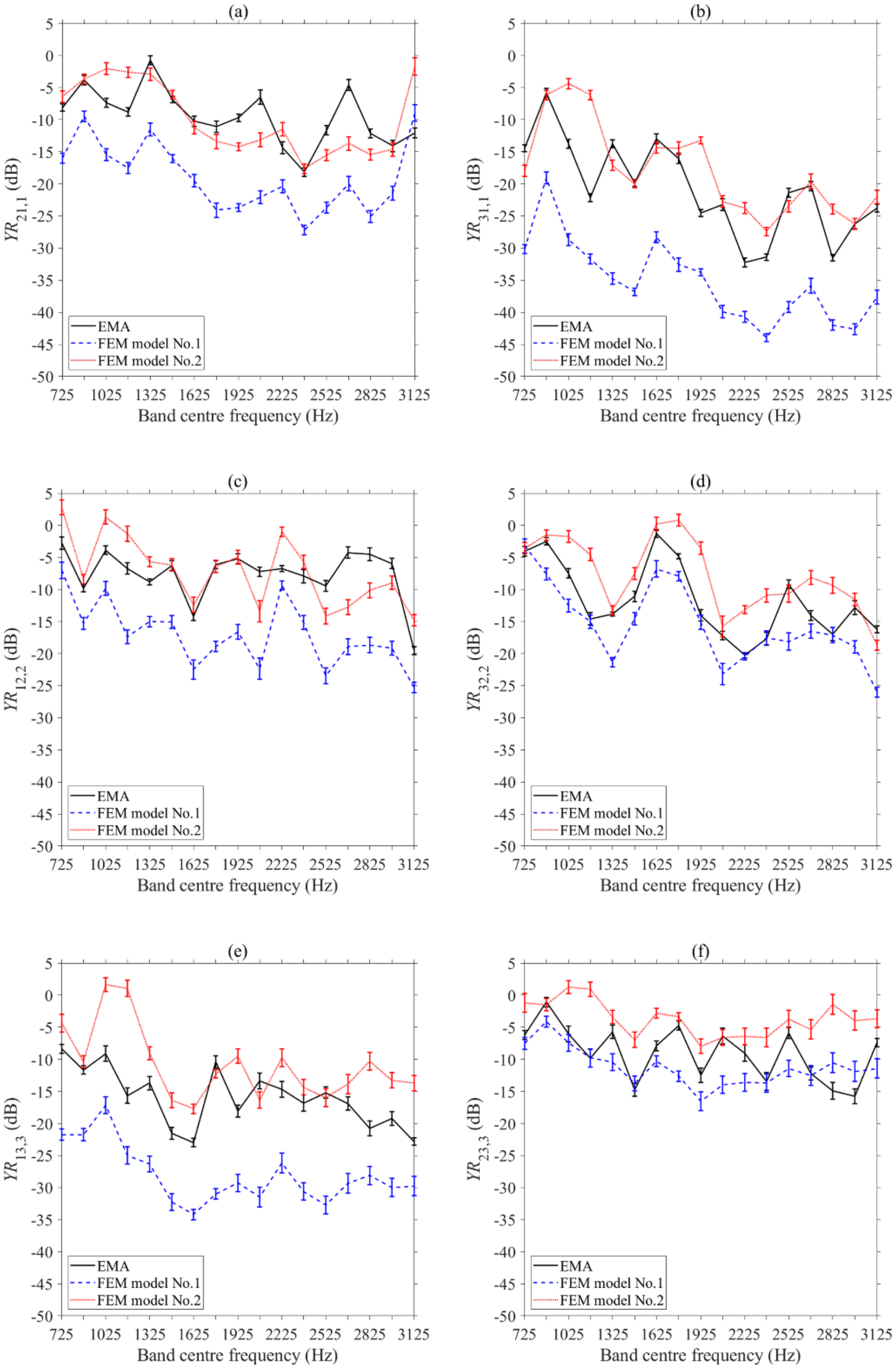

Figure 5: Spatial-average transfer mobility ratios: (a) $YR_{21,1}$, (b) $YR_{31,1}$, (c) $YR_{12,2}$, (d) $YR_{32,2}$, (e) $YR_{13,3}$ and (f) $YR_{23,3}$.

6. Conclusions

FEM models have been developed and validated with experimental modal analysis for a pile of three concrete beams that are stacked on top of each other without any rigid bonding material. These models were validated in terms of eigenfrequencies, mode shapes and spatial-average response. It was shown that MAC is not adequate to assess the validity of the FEM model as this led to misleading results whilst PMVR allowed identification of the FEM model that gave the most appropriate representation of the interaction between the coupled beams.

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