**Study on thermal conductivity model of saline soil based on particle morphology**

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**Abstract:** Thermal conductivity of soils is significant on the thermal simulations in cold region engineering. Based on the generalized thermal conductivity for geotechnical materials, a thermal conductivity model for saline soil was proposed in this paper. At the micro level, the microstructure composition of sodium sulfate soils in the proposed model was redefined. Macroscopically, the variation of salt crystals and ice crystals with temperature and the relationship between the arrangements of components (i.e., liquid water, ice, soil particles) of saline soil were studied. Shape parameters *α, β, τ* were adopted to define the volume proportion of needle shape soil particles, spherical shape soil particles and disk shape soil particles, respectively. Besides, the proposed thermal conductivity model considered the heat loss due to the heat radiation in the measurement process. Furthermore, the accuracy of the proposed model was verified by the experimental data. The results showed that the soil particles can be defined as *SWCA* (i.e., soil particles, water, crystals mixture of salt and ice, air) four-layer concentric structure in the calculation of the thermal conductivity model of saline soil. Shape parameters have significant influence on simulation results. By comparing test data with calculated values, it can be found that at positive temperature, silty clay soil particles are mainly spherical (*β* = 0.5) and disc-shaped (*τ* = 0.5), while the sandy soil and soil-rock particles are more similar to the needle shape (*α* = 1.0). With the decrease of temperature, the particle parameters of silty clay soil change, *β* decreases to 0, *τ* increases to 0.75, and *α* increases to 0.25. However, at negative temperature, the soil particles of sandy soil and loess are still mainly needle-shaped (*α* = 1.0).

**Keywords:** porous media, soil particle shape, sodium sulfate soil, thermal conductivity

**1 Introduction**

Salinized soil is widely distributed around the world, such as the Qinghai-Tibet Plateau, the arid and cold regions in northwest China, as well as the Mediterranean Basin, California and Southeast Asia. Salinized soil will cause industrial and agricultural diseases, making salinized soil gradually become a research spot [1-5]. In particular, soil thermal conductivity, a key thermo-physical parameter in cold regions, has attracted great attention because it is often used in engineering numerical simulations [6-9]. With the decrease of soil temperature, the composition, content and internal structure of soils change dynamically, and finally causes the thermal conductivity of soil to change.

Many researchers have carried out on soil thermal conductivity based on the experiments and models. As early as 1945, Kersten [10] conducted thermal conductivity of 19 soils with different densities, temperatures and moisture content. Abu-Hamde and Reeder [11] used the single probe method to determine the thermal conductivity of saline soils, the results showed that the thermal conductivity of soils decreased with the increase of salts contents at given moisture content. The thermal conductivity of the ground at depths within the seasonally active zone at four sites across Canada was examined by Goodrich [12]. Midttømme and Elen Roaldset [13] measured the thermal conductivity of soil samples of different grain size fractions with a divided bar apparatus. In term of thermal conductivity model, Gori and Corasaniti [14] presented a model with quasi-spherical solid grain to evaluate the effective thermal conductivity of three-phase soils. Tsao [15] derived an equation to calculate the effective thermal conductivity of a two-phase heterogeneous material. Then, Cheng and Vachon [16] modified and extended the Tsao’s model [15] to predict the thermal conductivity of heterogeneous solid mixtures by the discontinuous phase volume fraction. Chulho et al. [17] analysed the effective thermal conductivity of unsaturated granular materials by the macroscopic pore structure network model with a randomly packed particle. Pei et al. [18] investigated soil parameters (i.e., porosity, dry density, water content and saturation degree) and established the multiple linear regression model to predict thermal conductivity of soil-rock media in cold regions. Based on phase transition and geometry approximation theory, a generalized thermal conductivity model considering the particle construction for geomaterials is proposed by Wang et al. [19]. Lu et al. [20] tested the thermal conductivity of aeolian sand sampled from the Tibetan Plateau and proposed a model as a function of the degree of saturation and the volume fraction of each phase to calculate the thermal conductivity of aeolian sand. Li et al. [21] selected 4 potential probability distributions models to evaluate the measured thermal conductivities of clay, from which the normal distribution performed best. Bi et al. [22] presents a generalized model for calculating the thermal conductivity of freezing soils by considering different connections between the soil pores and solid grain and between the unfrozen water and ice in the pores. Besides, Zhang et al. [23-24] measured the thermal conductivity of a silty clay by a QuickLine-30 Thermal Properties Analyser during a freezing-thawing process and proposed a matrix model to evaluate 16 other thermal conductivity models. However, the study of thermal conductivity models for saline soils considering the soil components and soil particle shape are quite scarce.

The solubility of sodium sulphate is very sensitive to the soil temperature, which causes the salt ice content in the sodium sulfate soil changed in cold regions, and leads to the thermal parameters (i.e., thermal conductivity) and other hydro-mechanicals parameters (i.e. permeability coefficient, elasticity modulus significantly fluctuate. Thus, this paper proposes a thermal conductivity model of sodium sulfate soil mixed with polymorphic soil particles. This model is based on the thermal conductivity theory of generalized geotechnical material and has included the characteristics variation of the sulphate salt ice content in saline soil. The morphological differences of soil particles in different soils and the variation characteristics of salt ice crystal content with temperature are considered in the model. Finally, the accuracy of the proposed model is verified by the experimental data.

**2 Description of the Model**

As a porous multiphase medium, the volume proportion of sodium sulfate soil change with temperature. As shown in Fig. 1, in the cooling process, when the soil temperature is lower than *Tc* (salt crystal precipitation temperature), the soil composition changes from Fig. 1(a) to Fig. 1(b), that is, salt crystals precipitate out. When the soil temperature decreased to *Tf* (freezing temperature of saline soil), the internal situation of soil changes from Fig. 1(b) to Fig. 1(c), that is, ice crystals precipitate out.



Fig. 1 Variation composition of sulphate soil with temperature decreased

From the above analysis, it can be seen that during the cooling process, the component and volume proportion of soil particle (i.e. liquid water, air, salt and ice crystal) would change as the soil temperature decreased. It would lead to the variation of the thermal conductivity of soils. If the salinized soil particles are assumed to be four-layer concentric ellipsoid, from which the innermost layer is the soil particles, and the outer layer being the *SWCA* structure of water, the mixture of salt crystals and ice crystals, and air (Fig. 2), the thermal conductivity of salinized soil can be evaluated by Equation 1 [24],

 （1）

in which,

 （2）

 （3）

 （4）

 （5）

where *λa*, *λs*, *λc*, *λl*, *λi* are the thermal conductivities of air, soil particles, salt crystals, salt solution and ice, respectively. *φs*, *φc*, *φl*, *φi* are the volume ratios of soil particles, salt crystals, salt solution and ice to the total volume of soil, respectively. *Nk(s)*, *Nk(c)*, *Nk(l)*, *Nk(i)*(*k=x, y, z*) refers to the depolarizing factors of soil particles, salt crystals, salt solution and ice in the *x, y,* and *z* directions, respectively.

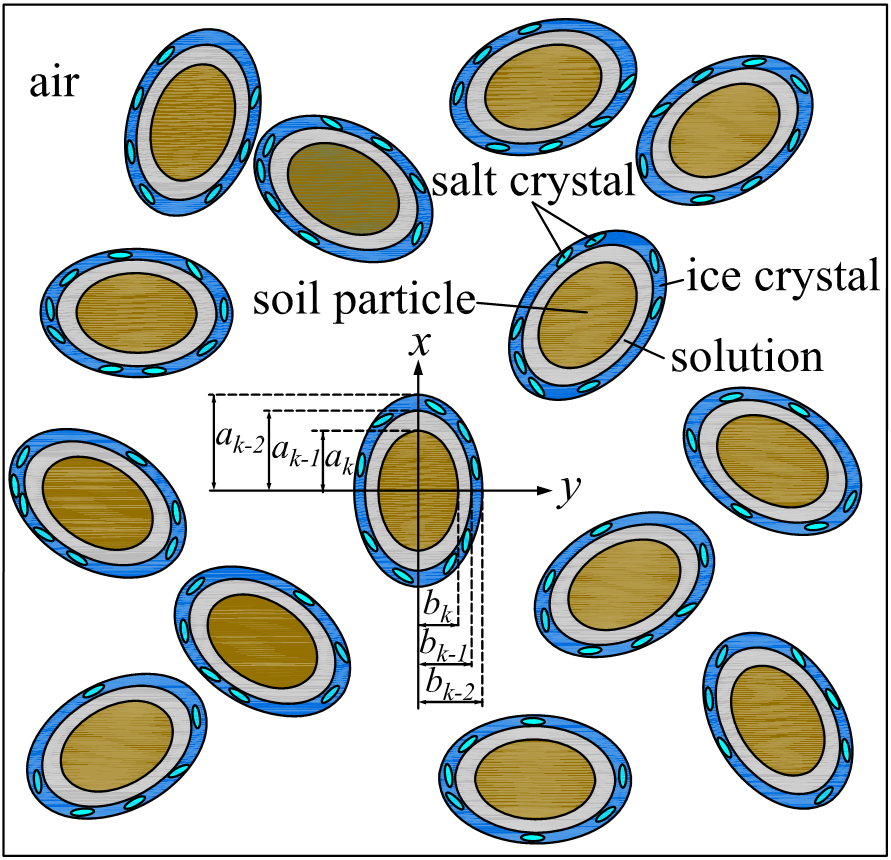


Fig. 2. Schematic diagram of *SWCA* four-layer confocal ellipsoid soil particles

The depolarizing factors *Nk(j)* (*j*=*s, l, c, i*) can be calculated by Sihvola and Kong [25],

 (6)

where *aK*,*bK* and *cK* are the half-axis size of the material in the *j*th layer of the reference ellipsoid (Fig. 2). *uj* is a scalar defining the *j*-layer material of the ellipsoid, and its value is a function of the volume of the ellipsoid containing the materials from *K-j* to *K* layers, the *K-j* to *K* refers to the outside materials (air) to inner material (soil particles and salt crystal mixture). The ellipsoid can be turned into any other shape by adjusting the axes of the ellipsoid model in the *x, y,* and *z* directions, and the geometric shapes of natural objects are usually represented as disk (*N*= 1,0,0), sphere (*N*= 1/3,1/3,1/3) or needle (*N*= 0,0.5,0.5).

In addition, during the heat transfer of soil samples, both particles and solutions dissipate heat through heat radiation (as shown in Fig. 3), resulting in a discrepancy between the calculated value *λc* and the experimented value *λe*. For cylindrical soil samples with diameter *d* and height *L*, the relationship between the calculated thermal conductivity of saline soil and its true value can be expressed as:

 (7)

where *δ* isstefan-boltzmann constant, 5.67×10-8Wm-2K-4. *ηs*, *ηl*, *ηi* are the emissivities of soil particles, water and ice crystals, respectively [26-28]. *Tm* is the soil temperature.



Fig.3 Schematic diagram of heat dissipation of cylindrical soil sample

By substituting Equation (1) into Equation (7), it can be obtained that,

 (8)

Previous studies have shown that when soil particles are needle shapes, the calculated results of frozen and unfrozen saline soil are close to the experimented values. While when soil particles are spherical or disk-shaped, the experimented values differ greatly from the calculated values [24]. However, the soil is a mixture of soil particles with multiple forms, and there is a certain difference between the actual situation and the assumption of needle-shaped soil particles.

To simplify the proposed model, it is assumed that the saline soil is a mixture of spherical, disk-shaped and needle-shaped soil particles. Thus, the thermal conductivity of the saline soil mixed with multi-morphological soil particles *λm* can be obtained as,

 （9）

Where *λtn*, *λts* and *λtd* represent the thermal conductivity of soils calculated by equation (4) when the soil particles are needle shaped (*N*=0,0.5,0.5), spherical (*N*=1/3,1/3) and disk-shaped (*N*=1,0,0), respectively. *α*, *β* and *τ* represent the volume proportion of spherical, disk-shaped and needle shaped particles, respectively. *α* + *β* +*τ* =1. When *α*, *β* and *τ*change in the range of [0, 1], the soil composed of different forms and proportions of soil particles can be simulated. To simplify the calculation, it is assumed that saline soil has 12 particle combinations as shown in Table 1.

Table 1 Soil particle morphology combination

|  |  |  |  |
| --- | --- | --- | --- |
| Code  number | Soil particle morphological parameters | | |
| *α-*needle shape | *β-*spherical shape | *τ-*disk shape |
| cb1 | 1.0 | 0 | 0 |
| cb2 | 0 | 1.0 | 0 |
| cb3 | 0 | 0 | 1.0 |
| cb4 | 0.75 | 0.25 | 0 |
| cb5 | 0.75 | 0 | 0.25 |
| cb6 | 0.25 | 0.75 | 0 |
| cb7 | 0.25 | 0 | 0.75 |
| cb8 | 0 | 0.75 | 0.25 |
| cb9 | 0 | 0.25 | 0.75 |
| cb10 | 0.5 | 0.5 | 0 |
| cb11 | 0.5 | 0 | 0.5 |
| cb12 | 0 | 0.5 | 0.5 |

Moreover, the solubility of sodium sulfate in soil is sensitive to temperature. As shown in Fig. 4, when the temperature is higher than 32.4°C, anhydrous sodium sulfate crystals will be precipitated from supersaturated sodium sulfate solution. However, when the temperature is lower than 32.4°C, the solubility of sodium sulfate decreases rapidly and mirabilite salt crystals are precipitated [29].



Fig. 4 Solubility phase diagram of sodium sulfate solution

When the solution temperature is within the range of 0~32.4°C, the formula for calculating the solubility of sodium sulfate is obtained by fitting the curve in Fig. 4,

*k*=0.3182*e*0.074*T*  (10)

According to the salt solution phase diagram and initial moisture content of sulphate saline soil, mirabilite's salt crystal volume content *θm* and salt solution volume content *θc* can be obtained through the equation (10) and the equation (12) respectively.

; *T*1> *T*2 (11)

; *T*1> *T*2 (12)

where *Mw* and *Mns* refers to molecular weight of water and *Na2SO4*, respectively. *T*1, *T*2 are the temperatures of sodium sulfate soil before and after cooling, respectively.*θ0* is the initial volumetric water content of soil and *θm* is the volume content of mirabilite. *ρc* is the density of mirabilite.

According to equation (12), the change diagram of the volume content of salt solution in saline soil with temperature is obtained (Fig. 5). From the Fig. 5, it can be seen that with the decrease of temperature, the volume of salt solution in saline soil decreases with different water content and dry density.



Fig. 5 Variation of volume content of salt solution in saline soil with temperature

It is assumed that the salt content in the salt solution of the saline soil is initially saturated. The relationship between the mirabilite salt content and temperature in the saline soil can be seen in Fig. 6. From the figure, it can be seen that with the decrease of temperature, mirabilite content in salinized soil increases continuously. While the mirabilite content increased with the initial salinity at the same temperature.



Fig. 6 Schematic diagram of volume content of mirabilite in saline soil with temperature (*θs* is initial volume salt content)

When the temperature of saline soil is below zero degree Celsius , the volume solution is dominated by the ice phase change, thus the change of ice crystal content must be considered. Thus, the unfrozen water content of the saline soil can be obtained from Equation (13) [24],

,  (13)

in which  (14)

 (15)

where *θr* is the residual volumetric unfrozen water content, which will not change with temperature, and *θn* is the volumetric water content of saline soil at zero degree Celsius; *g* is the acceleration of gravity; *Lwi* refers to the latent heat of freezing; *σiw* is the surface free energy between ice and water; *ri*refers to the radius of ice crystals. *ω*, *m* and *n* are Van Genuchten parameters, and different types of soil have different values [30], the suggested Van Genuchten parameters of soils are listed in Table 2.

Table 2 Van Genuchten parameters of soils

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Soil types | ω/mm-1 | | *n*/- | *m*/- | |
| Silty clay | 0.06 | 1.67 | | | 0.401 |
| Sand | 0.04 | 1.80 | | | 0.444 |
| Loess | 0.02 | 1.60 | | | 0.375 |

Therefore, the relationship between ice crystal content and temperature is,

,  (16)

**3 Research method**

**3.1 Evaluation of the proposed model**

In order to distinguish the applicability of the proposed thermal conductivity model, some statistical parameters, including coefficient of determination (*R*2), Root mean square error (RMSE) and mean absolute deviation (MD) are selected to evaluate the overall performance the model.

The coefficient of determination *R2* is a statistical parameter. *R2* is always between 0 and 1, *R2*=0 indicates that the model explains none of the variability of the response data around its mean, and *R2*=1 indicates that the model explains all the variability of the response data around its mean, respectively.

 (17)

where, *SSRegression* and *SSTotal* are the residual sum of squares and the total sum of squares, respectively. *n* is the number of data; *yi* is the measured thermal conductivity, each associated with a regression value *fi*.

Root mean square error (*RMSE*) is usually used to measure the difference between the value calculated by the model and the value observed. *RMSE* represents the sample standard deviation of the differences between the calculated values and the observed ones [33],

 (18)

Mean deviation is the absolute distance of the data set relative to its mean value, to measure the degree of data dispersion.

 (19)

Besides, to further evaluate the proposed model, the widely used average thermal conductivity models of soils are used. For porous media such as soils, when the thermal conductivity and components of each medium are known, the effective thermal conductivity can be calculated by the arithmetic mean model (Equation 14) and the geometric mean model (Equation 15) [24],

 (20)

 (21)

where, *φa* is the ratio of the volume of air to the total volume of soil.

**3.2 Experiments set up and conditions**

A database consisting of variety of soils (i.e., silty clay, sand, soil-rock, clay, sandstone, loess) was used in the study. Table 3 shows the physical properties of these soils. The thermal conductivity of saline soils with different salt content at different temperatures is measured. Measuring instrument includes TC-22 numerical heat conduction device, thermal constant analyzer (Mathis TCI, manufactured by Setaram Instrumentation Company, measurement range: 0.028 ~ 120 W·m−1·K−1, accuracy of measurement results is better than 1%) and Quickline-30 Thermal Parameter Analyzer (made in America, Manufactured by Anter Corporation, with an accuracy of ± 0.01 W·m−1·K−1). More details of this database could be found in references [17, 18, 20, 22, 24, 32].

**4 Results and analysis**

**4.1 Calculation of thermal conductivity of unfrozen soils**

The saline soil is considered to be a mixture of needle, disk and spherical soil particles, and the proportion of each soil particle in the soil sample is shown in Table 1. The thermal conductivity of clay, sand and loess was analysed by the proposed model. And the influence of the shape of soil particles on the thermal conductivity of soil with different sodium sulfate was studied.Detailed physical parameters, such as water content and salt content of soil samples, are shown in Table 3. The calculated results of thermal conductivity are shown in Fig. 7.

Table 3. Physical properties of unfrozen sodium sulfate soils

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Soil type | Volume of soil sample (length×width×height or diamiter×height,cm) | Temperature  /°C | Salt content  /% | Water content  /% | Dry density  /g·cm-3 | Test method |
| Silty clay [24] | 6.2×2 | 25 | 1.5,2,2.5,3 | 18.50 | 1.70 | plane heat source method |
| Silty clay [32] | 20×15×5 | 25 | 0,0.5,1,1.5,2 | 22.30 | 1.63 | hot-wire method |
| Silty clay [32] | 20×15×5 | 2,5,10,15,20,25 | 2 | 22.30 | 1.63 | hot-wire method |
| Silty clay [32] | 20×15×5 | 2.5.10.15.20.25 | 2 | 28.50 | 1.63 | hot-wire method |
| Silty clay [17] | 6.18×5 | 26 | 0 | 10.30 | 1.77 | hot-wire method |
| Sand [32] | 20×15×5 | 2,5,10,15,20,25 | 2 | 9.20 | 1.55 | hot-wire method |
| Sand [32] | 20×15×5 | 25 | 0,0.5,1 | 9.20 | 1.55 | hot-wire method |
| Sand [32] | 20×15×5 | 2.5,10,15,20,25 | 2 | 12.40 | 1.55 | hot-wire method |
| Soil-rock [17] | / | 10 | 0 | 11.56 | 2.03 | / |
| Soil-rock [17] | / | 10 | 0 | 17.89 | 1.61 | / |
| Soil-rock [17] | / | 10 | 0 | 15.77 | 1.75 | / |

For silty clay, sandy soil and the mixed medium of rock and soil particles in the positive temperature, the thermal conductivity is close to the experimented value when all the soil particles are assumed to be needle shape. And when all the soil particles are assumed to be disk-shaped, the calculated thermal conductivity value is much higher than the experimented value, while for the spherical shape, the calculated values are less than the experimented ones (Fig. 7). From Fig.7, it can also be seen that when the spherical and disk-shaped soil particles are equal as 50%, the calculated thermal conductivity of sodium sulfate soils is superior to the other combination.

The boxplot deviations of the calculated thermal conductivity values of particle shape combinations is shown in Fig.8. From the figure, it can be seen that when the soil particles combinations arecb1 (soil particles are all needle shape) or cb12(spherical shape and disk shape are all 0.5), the calculated thermal conductivity values are close to the experimented one (Fig. 8). The maximum positive deviation of silty clay is 0.047 (cb1 combination) and 0.098 (cb12 combination), respectively (Fig.8a). And the maximum negative deviation is -0.140 (cb1 combination) and -0.123 (cb12 combination), respectively (Fig.8a).

For mixed sand and rock soil particles, there are same variation trends as those of the silty clays (Fig.8a and Fig.8b). The thermal conductivity calculated by the spherical and disk-shaped soil particles are the lower and upper bound of the combined value, respectively. As the shapes of soil particles change, the thermal conductivity fluctuates between the upper and lower bounds (Fig.8b). When the soil particles combination are cb1 and cb12, the thermal conductivity deviation value is small (Fig.8b).

Further studies, as shown in Fig. 9, indicate that for unfrozen silty clay, the most accurate prediction is the arithmetic mean, followed by the combination of cb1 and cb12 in the new model (Fig. 9a). The R2 values under the three conditions are 0.934 (arithmetic mean model), 0.884 (cb1 combination) and 0.844 (cb12 combination), respectively. However, for the mixed soil samples of sand and rock particles, the accuracy of the arithmetic mean model is poor, and the R2 value is only 0.747 (Fig. 9b). The cb1 combination in the new model predicts best, with a R2 value of 0.856. This indicates that the shape of soil particles changes with the change of soil quality.



Fig. 7 Calculated values of thermal conductivity of sodium sulfate soils based on combination of different particles shapes in positive temperature



Fig. 8 Boxplot deviations of the calculated thermal conductivity values of twelve particle shape combinations from the experimented values

Fig. 9 The overall performances of the three models in positive temperature

**4.2 Calculation of thermal conductivity of frozen soils**

When the soil temperature is lower than the freezing temperature, salt and ice crystals will be separated out. Thus, the volume proportion of each component of the soil change with the soil temperature. The accuracy of the proposed model was evaluated by compared to the other two thermal conductivity models (i.e. arithmetic mean model and geometric mean model) in a negative temperature. Physical parameters, such as soil type, water content and salt content, are shown in Table 4.

Table 4. Physical properties of frozen sodium sulfate soils

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Soil type | Volume of soil sample (length×width×height or diamiter×height,cm) | Temperature  /°C | Salt content  /% | Water content  /% | Dry density  /g·cm-3 | Test method |
| Silty clay [23] | 6.3×7.2 | -19 | 0 | 12.98 | 1.50 | hot-wire method |
| Silty clay [23] | 6.3×7.2 | -13,-19 | 0 | 16.04 | 1.50 | hot-wire method |
| Silty clay [23] | 6.3×7.2 | -1.5,-2,-5,-8,-13,-19 | 0 | 18.13 | 1.50 | hot-wire method |
| Silty clay [23] | 6.3×7.2 | -1.5,-2,-5 | 0 | 19.89 | 1.50 | hot-wire method |
| Silty clay [23] | 6.3×7.2 | -2,-8,-13,-19 | 0 | 12.93 | 1.75 | hot-wire method |
| Silty clay [23] | 6.3×7.2 | -1.5,-2,-5,-8,-13,-19 | 0 | 16.06 | 1.75 | hot-wire method |
| Silty clay [23] | 6.3×7.2 | -1,-1.5,-2,-5,-8,-13 | 0 | 18.05 | 1.75 | hot-wire method |
| Silty clay [23] | 6.3×7.2 | -0.5,-1,-1.5,-2,-5,-8 | 0 | 19.91 | 1.75 | hot-wire method |
| Clay [21] | 6.18×5 | -1,-2,-5,-10 | 0 | 10.00 | 1.80 | hot-wire method |
| Sand [34] | 20×12×5 | -10 | 0,0.5,1,1.5,2,2.5,3,5 | 9.20 | 1.55 | hot-wire method |
| Sand [34] | 20×12×5 | -2,-5,-10,-15,-19 | 2 | 9.20 | 1.55 | hot-wire method |
| Sand [34] | 20×12×5 | -2,-5,-10,-15 | 2 | 9.20 | 1.55 | hot-wire method |
| Sandstone [19] | 6.18×5 | -5,-10,-20 | 0 | 6.04 | 2.27 | hot-wire method |
| Loess [34] | 20×12×5 | -10 | 0,0.5,1,1.5,2,2.5,3,5 | 16.5 | 1.65 | hot-wire method |





Fig. 10 Calculated values of conductivity of sodium sulfate soils based on combination of particles of different particles shapes in negative temperature

Fig. 10 shows the calculated thermal conductivity of soils from the three models. It can be seen that the soil particle morphology has a significant influence on the thermal conductivity (Fig. 10). When the particles of clay and silty clay are assumed to be needle shape or spherical shape, the calculated thermal conductivity values are much larger than the experimented values. However, when the soil particles are combined in different forms, the thermal conductivity of the soil sample increases with the proportion between the needle and disk-shaped soil particles in the soil. When the proportion between the needle soil particles and disk-shaped soil particles is 1:3, namely the cb7 combination of 0.25:0:0.75, the difference between the calculated and experimented thermal conductivity values is the smallest.

From Fig.11, it can also be seen that the thermal conductivity of soils varies from different soil types, this is from the fact that particle morphology of soil is different from different soil types. For the sand or sandstone, when the particles are all disk-shaped, the calculated thermal conductivity values is far greater than the experimented value. While when the particles are all spherical, the calculated thermal conductivity value is far less than the experimented value.

When soil particles are disk-shaped, the calculated thermal conductivity for the frozen clay is greater than the experimented values, the maximum deviation is of 0.269W·m-1K-1. When the needle and disk-shaped soil particles are mixed in 1:3 proportion (i.e., cb7 combination), the calculated thermal conductivity is the smallest, within the range of 0.103 W·m-1K-1 to -0.139 W·m-1K-1. In other combination cases, all calculated thermal conductivity are less than the experimented values. The maximum deviation is -1.505 W·m-1K-1 (Fig. 11a).

The calculated thermal conductivity of frozen sand has the maximum positive deviation of 1.373 W·m-1K-1 when the soil particles are disk-shaped. And the maximum negative deviation occurs when the soil particles are spherical, which is -1.319 W·m-1K-1 (Fig. 11b). In the other soil particle combinations, the thermal conductivity deviation values of saline soil are within the range of -1.319 W·m-1K-1 to 1.373 W·m-1K-1. In the case of spherical and disk-shaped soil particles and their related proportional combinations, the variation range of thermal conductivity deviation value is the smallest, which range from -0.100 W·m-1K-1 to 0.082 W·m-1K-1.





Fig. 11 Boxplot of deviations of the calculated thermal conductivity values of twelve particle shape combinations from the experimented data





Fig. 12 The overall performances of the three models in negative temperature

The evaluation coefficients *R2*, *RMSE* and *MAD* can well evaluate the accuracy of the model. When the soil particles are combined as cb7, the thermal conductivity *R2* of frozen clay calculated by the proposed model is 0.779, the values of *RMSE* and *MAD* are 0.071 and 0.059, respectively. From Fig. 12a, it can be seen that the values of *R2* for the proposed model is lower than those for the arithmetic average model and geometric average model, indicating the poor calculation (Fig. 12a). Fig. 12b and 12c show that under the cb1 combination of frozen sand, sandstone and loess, the calculation values of the proposed model are good.



Fig. 13 Relationship between soil thermal conductivity and volume

Additionally, the calculated thermal conductivity of soils in frozen and unfrozen states show that the thermal conductivity of soil samples has a certain relationship with their volume. As shown in Figure 13, it can be seen that the thermal conductivity of soil increases with the volume of the soil sample decreased, tends to as a whole. This is from the fact that in the process of heat conduction, the larger the volume of the soil sample, the greater the heat loss due to heat dissipation. As a result, the thermal conductivity of large-volume soil samples is smaller than the experimental value of small-volume soil samples.

The above study shows that the morphology of soil particles changes with soil types and environmental changes. In particular, the proportion of needle-shaped soil particles in frozen saline soil has increased significantly. This is due to the precipitation of needle-like salt crystals in the sodium sulfate soil as the temperature drops from positive to negative (Fig. 14). According to the SWCA four-layer confocal model of soil particles (Fig. 2), as the ambient temperature drops, the salt solution covered the soil particles undergoes a solid–liquid phase transformation and gradually forms needle-like mirabilite salt crystals, causing the entire soil particle model to change to needle-shaped. Therefore, the number of needle-shaped soil particles increases with decreasing temperature in saline soil.

However, soil particles in the natural state are diverse in shape, and they are only regarded as a mixture of needle, spherical and disc shapes, which is different from the actual situation. In addition, during the cooling process, the influence of the interaction of the salt and ice phase transformation on the morphology of soil particles is not considered in our study. Thus, how to establish a more comprehensive and reliable model of the thermal conductivity of saline soil will be worthy of further research.

**5 Conclusion**

Based on the generalized thermal conductivity model, a new thermal conductivity model of saline soil considered the heat dissipation due to heat radiation was proposed. And the accuracy of the new model is verified by the experimental data. The main conclusions are as follows:

1. In the positive temperature, the arithmetic mean model can predict the thermal conductivity of sulphate silty clay well. The particle morphology of salt-bearing sand and sandstone tends to be needle shaped, and the new model is the best model to calculate the thermal conductivity.
2. As the soil temperature decreases and enters the negative temperature state, the needle clay particles decrease while the disk clay soil particles increase, and when the volume ratio of two soil particles is 1:3, the thermal conductivity calculated by the proposed model is closer to the experimental value than other combination, arithmetic average model and geometric average model.
3. When sand and loess particles in negative temperature environment are considered as needles, the thermal conductivity calculated by the new model could be calculated with little difference from the experimented values. In particular, the *R2* of the prediction accuracy evaluation index of the new loess model was up to 0.959.
4. In the simulation calculation of sodium sulfate soil thermal conductivity, it is reliable to assume that the soil particles of loess, sand and soil-rock media are needle-shaped, and that clay and silty clay particles are a mixture of needle-shaped and disc-shaped soil particles.
5. The larger the volume of the soil sample, the greater the heat loss during the heat conduction process, resulting in a smaller measured value of its thermal conductivity.

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**Appendix A**

When soil temperature changes, the density of soil composition and its thermal conductivity also change. When the temperature dropped from 273K to 93K, pruppacher and Klett [35] obtained ice crystal *ρi* as，

 (A1)

The density of supercooled water can be obtained by fit the data of Hare and Sorensen [36]:

 (A2)

where the density of mirabilite crystal *ρc* is constant 1.46 [37].

The thermal conductivity of sodium sulfate solution can be calculated by the formula of thermal conductivity of electrolyte solution [38]:

 (A3)

where *μw* and *μet* are the mass fractions of water and electrolyte in solution respectively, *λw* and *λet* are the thermal conductivity of water and electrolyte, respectively. *λet* =0.284636，*κ* is a constant -7.8773×10-3 [38]. The thermal conductivity of water is related to temperature as below [39]:

(A4)

where *ν* =(*T*+273.15)/300 [39].

When the temperature changes within the range of 85 K~273 K, the formula for calculating the thermal conductivity of ice crystals can be obtained by fitting the data of Ahmad and Phillips [40]:

 (A5)

In addition, through fitting Montgomery's formula, it is known that within the range of 253K~313K, the calculation formula of air thermal conductivity is [41]:

 (A6)

Attached Table

See the attached table for the values or ranges of thermodynamic parameters in the text:

|  |  |  |  |
| --- | --- | --- | --- |
| Symbols | Name | Value or Range | Unit |
| *Tf* | freezing temperature of saline soils |  | K or °C |
| *Tc* | salt crystallization temperature |  | K or °C |
| *λ* | thermal conductivity of saline soils |  | W/(m·K) |
| *λa* | thermal conductivity of air |  | W/(m·K) |
| *λs* | thermal conductivity of soil particle |  | W/(m·K) |
| *λl* | thermal conductivity of salt solution |  | W/(m·K) |
| *λc* | thermal conductivity of salt crystal | 0.5 | W/(m·K) |
| *λi* | thermal conductivity of ice |  | W/(m·K) |
| *λw* | thermal conductivity of water |  | W/(m·K) |
| *λet* | thermal conductivity of electrolyte | 0.284636 [38] | W/(m·K) |
| *φs* | ratio of the volume of soil particles |  | dimensionless |
| *φl* | ratio of the volume of salt solution |  | dimensionless |
| *φc* | ratio of the volume of salt crystal |  | dimensionless |
| *φi* | ratio of the volume of ice |  | dimensionless |
| *φa* | ratio of the volume of air |  | dimensionless |
| *λt* | the true value of thermal conductivity |  | W/(m·K) |
| *δ* | stefan-boltzmann constant |  | W/(m-2·K-4) |
| *Ts* | soil temperature |  | K |
| *L* | height of cylindrical soil sample |  | m |
| *d* | section radius of cylindrical soil sample |  | m |
| *ηs* | emissivity of soil particle | 0.92~0.95 [27-29] | dimensionless |
| *ηl* | emissivity of water | 0.95~0.97 [27-28] | dimensionless |
| *ηi* | emissivity of ice | 097~0.98 [27-28] | dimensionless |
| *k* | sodium sulfate solubility |  | g/100g |
| *Tl* | sodium sulfate solution temperature |  | °C |
| *e* | natural constant | 2.718 | dimensionless |
| *Tl*1 | sodium sulfate solution temperature |  | °C |
| *Tl*2 | sodium sulfate solution temperature |  | °C |
| *θm* | volume content of mirabilite |  | dimensionless |
| *θ0* | volumetric water content of saline soil |  | dimensionless |
| *θl* | volumetric water content of saline soil |  | dimensionless |
| *Mns* | relative molecular mass of Na2SO4 | 142 | g/mol |
| *Mc* | relative molecular mass of mirabilite | 322 | g/mol |
| *Mw* | relative molecular mass of water | 18 | g/mol |
| *ρw* | density of water |  | g/cm3 |
| *ρc* | density of salt crystal | 1.46 [37] | g/cm3 |
| *θuw* | unfrozen water content of soil |  | dimensionless |
| *θr* | residual unfrozen water content |  | dimensionless |
| *θn* | unfrozen water content of soil zero degree Celsius |  | dimensionless |
| *ω* | Van Genuchten parameters |  | dimensionless |
| *m* | Van Genuchten parameters |  | dimensionless |
| *n* | Van Genuchten parameters |  | dimensionless |
| *ψ* | soil water potential |  | m |
| *T*\**f* | absolute temperature | 273.15 | K |
| *Tf* | freezing temperature of saline soils |  | K or °C |
| *ρi* | density of ice |  | g/cm3 |
| *ρw* | density of water |  | g/cm3 |
| *Lwi* | latent heat of water freezing | 333.7 | kJ/kg |
| *R* | gas constant | 8.314 | J/(K·mol) |
| *g* | acceleration of gravity | 9.8 | m/s2 |
| *T* | temperature |  | K or °C |
| *ri* | ice crystal curved surface radius |  | m |
| *aw* | water activity |  | dimensionless |
| *σiw* | surface free energy between ice and water |  | J/m2 |
| *λari* | thermal conductivity of saline soil |  | W/(m·K) |
| *λgeo* | thermal conductivity of saline soil |  | W/(m·K) |
| *μw* | the mass fractions of water in sulution |  | dimensionless |
| *μet* | the mass fractions of electrolyte in sulution |  | dimensionless |
| *κ* | the constant used to calculate *λl* |  | dimensionless |
| *Vs* | soil sample volume |  | cm3 |

**References**

[1] Wan X S, Gong F M, Qu M F, et al. Experimental study of the salt transfer in a cold sodium sulfate soil. KSCE Journal of Civil Engineering. 2019, 23(4):1573-1585.

[2] Hivon E G, Sego D C. Distribution of saline permafrost in the Northwest Territories, Canada. Canadian Geotechnical Journal. 1993, 30(3):506-514.

[3] Serrano R, Gaxiola R. Microbial models and salt stress tolerance in plants. Critical Reviews in Plant Sciences. 1994, 13(2):121-138.

[4] Shaterian J, Waterer D, Jong H D, et al. Differential stress responses to NaCl salt application in early-and late-maturing diploid potato clones. Environmental and Experimental Botany. 2005, 54(3):202-212.

[5] Wan X, Liu E L, Qiu E X. Study on ice nucleation temperature and water freezing in saline soils. Permafrost and Periglac Process. 2020, https://doi.org/10.1002/ppp.2081

[6] Ling F, Zhang T J, A numerical model for surface energy balance and thermal regime of the active layer and permafrost containing unfrozen water, Cold Regions Science and Technology. 2004, 38 (1) 1–15.

[7] Pei W S, Zhang M Y, Lai Y M, et al.. Evaluation of the ground heat control capacity of a novel air-L-shaped TPCT-ground (ALTG) cooling system in cold regions, Energy. 2019, 179, 655–668.

[8] He H L, Zhao Y, Dyck M F, et al,. A modified normalized model for predicting effective soil thermal conductivity, Acta Geotech. 2017, 12 (6) 1281–1300.

[9] Lai Y M, Pei W S, Zhang M Y, et al,. Study on theory model of hydro-thermal–mechanical interaction process in saturated freezing silty soil, Int. J. Heat Mass Transf. 2014, 78, 805–819.

[10] M.S. Kersten, Thermal properties of soils, Highway Research Board Special Re-port, 1949.

[11] Abu-Hamdeh N H, Reeder R C. Soil thermal conductivity: Effects of density, moisture, salt concentration, and organic matter[J]. Soil. soc. of Am.j, 2000, 64(4):1285-1290.

[12] Goodrich L E. Field measurements of soil thermal conductivity[J]. Canadian Geotechnical Journal, 1986, 23(1):51-59.

[13] Midttomme K, Roaldset E. The effect of grain size on thermal conductivity of quartz sands and silts[J]. Petroleum Geoence, 2015, 4(2):165-172.

[14] Tsao, Tsu-Ning G. Thermal Conductivity of Two-Phase Materials[J]. Industrial & Engineering Chemistry, 1961, 53(5):369-372.

[15] Cheng S C, Vachon R I. The prediction of the thermal conductivity of two and three phase solid heterogeneous mixtures[J]. International Journal of Heat & Mass Transfer, 1969, 12(3):249-264.

[16] Chulho L, Zhuang L, Dongseop L, et al. Evaluation of effective thermal conductivity of unsaturated granular materials using random network model[J]. Geothermics, 2017, 67: 76–85.

[17] Pei W S, Yu W B, Li S Y, et al. A new method to model the thermal conductivity of soil-rock media in cold regions: An example from permafrost regions tunnel[J]. Cold Regions Science and Technology, 2013, 95:11-18.

[18] Wang C, Lai Y M, Zhang M Y, et al. A generalized thermal conductivity model of geomaterials based on micro-structures[J]. Acta Geotechnica, 2018:1-14.

[19] Lu Y, Yu W, Hu D, et al. Experimental study on the thermal conductivity of aeolian sand from the Tibetan Plateau[J]. Cold Regions Science & Technology, 2017, 146:1-8.

[20] Li SY, Wang C, Shi L H, et al. Statistical characteristics of the thermal conductivity of frozen clay at different water contents[J]. Results in Physics, 2019, 13.

[21] Bi J, Zhang M Y, Lai Y M. A generalized model for calculating the thermal conductivity of freezing soils based on soil components and frost heave[J]. International Journal of Heat and Mass Transfer, 2020, 150.

[22] Zhang M Y, Lu J G, Lai Y M, et al. Variation of the thermal conductivity of a silty clay during a freezing-thawing process[J]. International Journal of Heat and Mass Transfer, 2018, 124:1059-1067.

[23] Zhang M Y, Bi J, Chen W W, et al. Evaluation of calculation models for the thermal conductivity of soils[J]. International Communications in Heat & Mass Transfer, 2018, 94:14-23.

[24] Wan X S, Zhong C M, Hazem SM, et al. Study on the thermal conductivity model of sodium sulfate soils[J]. Experimental Heat Transfer, 2020.

[25] Kong JA, Sihvola A H. Effective permittivity of dielectric mixtures[J]. IEEE Transactions on Geoscience & Remote Sensing, 1988. 26(4):420-429.

[26] Mikaél A. Bramson. Buchbesprechungen über: Infrared Radiation. A Handbook for Applications. (Ref. H. ELSÄSSER) [J]. Zeitschrift Fur Astrophysik, 1968, 69.

[27] Wolfe W L, Zissis G J. The Infrared Handbook. AnnArbor, Mich.: Environmental Research Institute of Michigan. 1978.

[28] Kern C D. Evaluation of infrared emission of clouds and ground as measured by weather satellites [J]. 1964.

[29] Derluyn H, Saidov T A, Espinosa-Marzal R M, et al. Sodium sulfate heptahydrate I: The growth of single crystals[J]. Journal of Crystal Growth, 2011, 329(1):44-51.

[30] Dall'Amico M, Endrizzi S, Gruber S, et al. A robust and energy-conserving model of freezing variably-saturated soil[J]. The Cryosphere, 2011.

[31] Hyndman R J, Koehler A B. Another look at measures of forecast accuracy[J]. International Journal of Forecasting, 2006, 22(4):679-688.

[32] Deng YS, He P, Zhou C L. An Experimental Research on the Thermal Conductivity Coefficient of Saline Soil[J]. Journal of Glaciology and Geocryology, 2004, 26(3):319-323.

[33] Pruppacher H R andKlett J D. Microphysics of Cloulds and Precipitation. New York, Bston, Dordrecht, London, Moscow: Kluwer Acadamic Pubishers, 1997.

[34] Hare D E, Sorensen C M. The density of supercooled water. II. Bulk samples cooled to the homogeneous nucleation limit[J]. Journal of Chemical Physics, 1987, 87(8):4840-4845.

[35] Kargel Jeffrey S. Brine volcanism and the interior structures of asteroids and icy satellites[J]. Icarus, 1991,94(2):368-390.

[36] Wang K Q, Sun X Z. A New Method for Calculating the Thermal Conductivity of Electrolyte Aqueous Solution[J]. Chemical Engineering, 2002, 30:60-62.

[37] Sharqawy M H. New correlations for seawater and pure water thermal conductivity at different temperatures and salinities[J]. Desalination, 2013, 313(11):97-104.

[38] Ahmad N, Phillips W A. Thermal conductivity of ice and ice clathrate[J]. Solid State Communications, 1987, 63(2):167-171.

[39] Montgomery R B. Viscosity and Thermal Conductivity of Air and Diffusivity of Water Vapor in Air[J]. Journal of the Atmospheric Sciences, 1947, 4(6):193-196.