**Stochastic model updating for assembled structures with bolted joints using a Bayesian method**

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**Abstract**: An efficient model-updating method based on Bayesian power spectrum sensitivity analysis is proposed to update the uncertain parameters of assembled structures. The dynamic equations of bolted assembled structures are derived using the substructure component model synthesis technique. The posterior probability density function of uncertain parameters of bolted joints is established by the Bayesian method, where the negative logarithmic likelihood function is taken as the objective function to be optimized. To improve the efficiency of the stochastic model-updating process, the pseudo-excitation method is introduced to derive analytically the expressions of the gradient vector and the Hessian matrix for the optimization. The proposed method is evaluated in the stochastic model updating of an assembled structure consisting of three beams. Then it is applied to model-updating of the simplified model of a rocket. The numerical results demonstrate that this approach can significantly reduce the computational cost and ensure computational accuracy.

**Keyword**：Stochastic model updating; Bayesian inference; Pseudo excitation method; Substructuring; Bolted assembly.

**1. Introduction**

Bolted structures, with the ease in assembly and disassembly, are one kind of typical joint structures. Their static analysis is now quite mature since complete theoretical models and numerical techniques have been developed. Compared with static analysis, dynamic analysis of bolted joint structures remains an open challenge since complex microscopic features exist at the interface of the connections. An efficient stochastic model updating and uncertainty quantification techniques are being developed to accurately predict the dynamic responses of bolted assembly structures (Brake 2018).

In recent years, there is widespread attention on stochastic model updating techniques (Rui, Ouyang, and Wang 2013; Abu Husain, Haddad Khodaparast, and Ouyang 2012; Huang and Chen 2019), especially those based on Bayesian methods, which have been applied to various fields such as modal parameter identification and damage assessment. Bayesian method for model updating can characterize the uncertainty associated with the underlying structure, quantify the uncertainty in parameter updating and provide a probabilistic model with robustness and rigour. Katafygiotis and Yuen proposed a Bayesian fast Fourier transform approach (Yuen and Katafygiotis 2003), a Bayesian power spectral approach (Katafygiotis and Yuen 2001), and a Bayesian time-domain approach (Yuen and Katafygiotis 2001). The above approaches present a general framework for Bayesian model updating methods. However, they have two main drawbacks that limit their applications in practice. On the one hand, the inverse of an ill-conditioned covariance matrix is involved, which makes the optimization process hard to converge. On the other hand, the scale of the resultant optimization problem explodes exponentially as the number of parameters increases. Focusing on the above computational issues, many improvements have been made. Au (2011) analyzed the mathematical characteristics of the objective function and proposed a frequency-domain fast Fourier transformation (FFT) Bayesian identification method, which led to the efficient computation, as the covariance matrix was computed analytically without resorting to a finite difference scheme. Yan and Katafygiotis (2015a) proposed a two-stage fast Bayesian spectral density method that extracts modal properties from separated and closely spaced modes. The method facilitates the convergence process by reducing the dimension of uncertain parameters. In the article (Yan and Katafygiotis 2015b) they further obtained a group of local mode shapes from full-scale operational modal tests with the different sensor setups, then the global mode shapes were formed by assembling above local mode shapes using a Bayesian statistical framework. The analytical gradient vector was derived by Yan and Katafygiotis (2015a, 2015b) to accelerate the convergence process.

In a Bayesian model updating, the uncertainty quantification of stochastic parameters is usually evaluated by the Laplace asymptotic approximation method or the Markov chain Monte Carlo (MCMC) sampling method (Beck and Katafygiotis 1998; Beck 2010; Solonen and Haario 2012). Beck and Katafygiotis (1998) pointed out that the posteriori probability density function (PDF) of the parameters can be approximated as a Gaussian distribution centred at most probable value (MPV) when sufficient data were available. Then, the covariance matrix could be approximated by the inverse of the Hessian matrix of the Bayesian objective function. However, when the parameters are of high dimensionality or a small amount of data is available, such an approximation is inappropriate. Along with the emergence of efficient random sampling methods such as MCMC, methods based on random sampling become popular. Based on the Metropolis-Hastings algorithm and a concept similar to simulated annealing, Beck and Au (2002) proposed an adaptive MCMC method, and Ching and Chen (2007) proposed the Transition MCMC method. Of course, there are some parametric reduced-order methods to deal with the sampling problem from high-dimensional parametric space. Bui-Thanh, Willcox, and Ghattas (2008) introduced greedy algorithm and proposed a model-constrained adaptive sampling methodology. This method solved the optimization problem with a gradient method , which is more efficient than an exhaustive search.

Most Bayesian methods are applied to the stochastic model updating problems of civil engineering structures (Zhu and Au 2018; Yan and Katafygiotis 2016). The model updating problem can be seen as an optimization problem where a finite element analysis (FEA) is required in each iteration. For assembled structures, the number of degrees-of-freedom (DOFs) of the finite element model is usually large, leading to high computational burden of FEA in each iteration. Additionally, the presence of numerous bolted joints results in a high-dimensional space of the optimization variables. The above two issues make the stochastic model updating process for assembled structures extremely time-consuming. To address the first issue, substructuring techniques can be used to improve the computational efficiency in FEA. For the second issue, the analytical gradient vector and Hessian matrix are derived to speed up the optimization process.

Substructuring is a general method for reducing the number of DOFs of complex structures in the FEA. As one of the typical methods, component model synthesis(CMS), which is based on modal parameters (Ingole and Chatterjee 2017) has been widely applied in various engineering structures (Battiato and Firrone 2020; Yuan *et al.* 2019). Although a CMS may produce modal truncation errors in the modal analysis of substructures, it can obtain more accurate FEA results when combined with model updating techniques. Papadimitriou and Papadioti (2013) combined the CMS and Bayesian inference in finite element model updating, Its effectiveness and efficiency were demonstrated in damage identification of a highway bridge. Fei *et al*. (2021) proposed a parametric modeling-based model updating strategy that considered uncorrelated and correlated modes as well as test data and applied it to update the model of an advanced turbofan engine stator casing.

In this article, along with the substructure technique, a Bayesian method based on the measured power spectrum density (PSD) information is also integrated to implement model updating of the assembly structure. The pseudo-excitation method (PEM) (Lin, Zhao, and Zhang 2001), which is a fast complete quadratic combination method, is used for stochastic dynamics analysis. It possesses high efficiency and accuracy than a traditional random vibration method in calculating the PSD response, which makes it an appealing method for analyzing complex structures (Zhang *et al*. 2009). Another notable advantage of the PEM is that the first and second order sensitivities of the PSD to the uncertain joint parameters can be analytically derived, making it possible to provide the gradient and the Hessian matrix for the resultant optimization problem. Such a sensitivity analysis can significantly improve the efficiency of the optimization process.

In conclusion, the uncertainty of the bolted joints is an intrinsic attribute, which mainly comes from the uncertain bolt preload, contact surface roughness, etc. There has been much research on the analysis of forward problems considering uncertain parameters of joints (*e.g.*, random matrices (Batou and Nabarrete 2018), random parameter models (Mignolet, Song, and Wang 2015)). However, there are few published works on the identification of stochastic parameters of the bolted joints. In this article, based on the Bayesian probabilistic framework, CMS and PEM, a Bayesian analytic sensitivity method is proposed, which overcomes the computational bottleneck problem in practical applications of Bayesian methods, and provides a solution for quantifying the uncertainty of the bolted joints.

The main sections of this article are organized as follows: in Section 2, a reduced-order dynamic equation of assembled structures is derived using the CMS. The PEM is used to solve the PSD response, the sensitivity of the pseudo displacement and PSD regarding uncertain joint parameters can be obtained. In Section 3, the posterior PDF of the uncertain joint parameters is established using the probabilistic characteristics of the PSD response and Bayesian inference. The negative log-likelihood function is taken as the objective function to determine the MPV. The gradient and the Hessian matrix of the objective function are derived using the sensitivity information calculated in Section 2. And the covariance matrix of uncertainty parameters is calculated by the inverse of the Hessian of the objective function at the MPV. Two numerical examples are given in Section 4 to validate the performance of the developed method. Finally, Section 5 concludes the article.

**2 Random vibration and sensitivity analysis of assembled structures based on PEM**

2.1 Reduced-order dynamic equation of the assembled structure using the CMS

As shown in Figure 1, The assembled structure is divided into 3 substructures, i.e., substructures A, B and bolted joints C. The research object of this article is a multi-bolt structure without macroscopic interface slip. Experimental studies (Bograd *et al*. 2011) reveal that the bolted joint exhibits approximately linear behaviour under small amplitude excitation in this type of assembled structure, which can be modelled with linear elements.



（a）coupled state；（b）uncoupled state

Figure 1. Substructures of assembled structure with bolted joints

The internal DOFs in substructures A and B are denoted as and respectively*,* which do not include DOFs in the interface. The interface DOFs coupled to the bolted joints are and .

 and are loads applied on substructure A and B. and are the number of DOFs with loads and acting on them, respectively, and . and are the internal forces between substructures A and B. The number of observation DOFs in the assembled structure is , of which there are and in substructures A and B respectively, and .

To simplify formulas and reduce unnecessary DOFs, three pairs of extraction matrices and, and,and, are introduced to extract the DOFs of the excitations, the DOFs of the sensors and DOFs of the bolted joints from substructures A and B, respectively. For example, extracts the DOFs of the from ,.

In this article, it is assumed that the mass of a substructure is much higher than that of bolted joints (Park and Chae 2008). Hence, the mass of the bolted joints can be neglected in the following derivations.

The equation of motion for substructures of the assembled structure is given by

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where ,, (\* represents A or B) are the mass matrix, damping matrix and stiffness matrix of substructure A or B, and **,** and are vectors of the acceleration, velocity and displacement responses of substructure A or B.

For the bolted structure shown in Figure 1, the linear coupling relationship between substructures A and B can be formulated as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

 is a general mathematical expression of the linear stiffness that depends on the method of modelling of the bolted joints. When the bolted joints are modeled as thin-layer elements, contains the elastic modulus, thickness of the thin layer, the contact area, etc. When they are modeled as spring elements with both longitudinal and torsional stiffness, the components of are the spring constants or correction factor of stiffness.

The equation of motion of the assembled structure in the coupled state is established by substituting Eq. (2) into Eq. (1) ：

|  |  |
| --- | --- |
|  | (3) |

According to the modal analysis method, the following transformation between modal and physical coordinates is often used

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

where and are modal coordinate vectors and and are normalized mode shape matrices of substructures A and B.

Substituting Eq. (4) into Eq. (3), the dynamic equation of the assembled structure under modal coordinate is established

(*t*) (5)

where：

(*t*)=

 and and and are diagonal matrices of the natural frequency and damping ratio of substructures A and B.

As for the DOFs corresponding to the sensors,

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

2.2 Random vibration analysis of assembled structures using PEM

The PEM transforms complex random loads into a deterministic generalized single-point simple harmonic excitation and the PSD of the assembled structure is obtained by the harmonic response analysis. The detailed procedure is as follows.

The excitation (*t*) in Eq. (5) is a stationary random excitation and its PSD is denoted by . Sinceis a Hermitian matrix, it can bedecomposed as

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

where superscript “H” represents the complex conjugate transpose; and are the eigen-pairs of the Hermitian matrix, which satisfy the following relationships

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

According to the PEM, the following pseudo excitations could be constructed

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

where “\*” represents the complex conjugate.

Substituting Eq. (9) into Eq. (5), one has

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

where and represent the pseudo excitations and the pseudo responses, respectively.

The pseudo displacement responses are obtained by solving Eq. (10)

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

Then, the PSD of the pseudo displacement response could be calculated by the following equation:

|  |  |  |
| --- | --- | --- |
|  |  | (12) |

The PSD of the DOFs of sensors is derived as

|  |  |  |
| --- | --- | --- |
|  |  | (13) |
|  |  | (14) |

2.3 Sensitivity analysis of the PSD of response

The first-order and second-order sensitivities of random responses can be calculated analytically and efficiently using the PEM. Many scholars have used this method and some have developed their own. Xu *et al*. (2009) derived the sensitivity equations of vehicle-road-bridge coupling systems for optimizing vehicle suspension systems. Zhang *et al.* (2013) proposed an optimization method for a riding comfort index, which also used the sensitivity equations of random responses to improve the optimization efficiency. In this article, the analytical gradient vector and Hessian matrix of negative logarithm of the likelihood function in terms of bolted joint parameters are derived by combining the Bayesian model updating method and the first-order and second-order sensitivities of the assembled structure.

To facilitate the optimization process in Section 3, the first and the second order sensitivity information is derived in this subsection.

Substituting Eq. (11) into Eq. (10), one gets

|  |  |  |
| --- | --- | --- |
|  |  | (15) |

Eq. (15) is rearranged as：

|  |  |  |
| --- | --- | --- |
|  |  | (16) |

where ,.

In this article, the focus is on the uncertainty of the bolted joints, and it could be seen from Eq. (16) that only contains the uncertain parameters. In the reanalysis of the stochastic model updating, only the bolted joints need to be reanalysed, while the analysis of substructures needs to be done only once for the modal analysis, and the substructures are independent and can be analysed in parallel computation. Let ,the first-order sensitivity of can be obtained by taking partial derivative to

|  |  |  |
| --- | --- | --- |
|  |  | (17) |

where is the frequency response function of the assembled structures.

And the second-order sensitivity is given by

|  |  |  |
| --- | --- | --- |
|  |  | (18) |

The first and the second order sensitivity of the PSD of the DOFs of sensors to the uncertain parameters can be solved using Eq. (17) and Eq. (18):

|  |  |
| --- | --- |
|  | (19) |
|  | (20) |
|  |

The Bayesian model updating in this article is based on the measured PSD data. Hence, the gradient vector and the Hessian matrix of the PSD with respect to the joint parameters are critical for the optimization problem. From Eq. (19) - Eq. (20), it can be seen that the gradient vector and Hessian matrix are related to , , and . and depend on . For example, if bolted joints are modelled as acting as linear springs and the coefficients of springs are selected as uncertain parameters , is a constant matrix without and is a zeros matrix. While and are intermediate matrices, which depend on , that must be computed when solving the dynamic response of the structure, the sensitivity analysis regarding the uncertain parameters does not increase the overall computational cost.

**3 Stochastic model updating of assembled structures based on Bayesian PSD method**

3.1 Basic theory of the Bayesian PSD method

The following derivation process is based on the dynamical model within a prescribed class of models. Considering segments of independent measured time histories denoted as . is the sampling time step and is the number of sampling points. is discrete response vector including observation DOFs, Then the measured time history data are transformed into the frequency domain by the FFT (Au 2016; Yuen 2010).

|  |  |
| --- | --- |
|   | (21) |

where ，, ‘Int’ denotes integer part of a real number. The superscript ‘’ indicates measured data.

The average PSD matrix is used as an estimator of the Bayesian inference，

|  |  |  |
| --- | --- | --- |
|  |   | (22) |

The relationship in the frequency domain between the measured data and the theoretical response can be expressed as follows

|  |  |  |
| --- | --- | --- |
|  |   | (23) |

where is the scaled FFT of the prediction error at frequency . is a zero-mean discrete (band-limited) white noise process with variance . The PSD of the prediction error is , in which is an identity matrix whose dimension is equal to the number of sensors.

The average PSD matrix estimator with sufficient measured data is asymptotically unbiased, which can be expressed as follows

|  |  |  |
| --- | --- | --- |
|  |  | (24) |

where denotes the mathematical expectation.

The research work (Yuen 2010; Au 2016) shows that the PDF of for given obeys the complex Wishart distribution of dimension with DOFs (only if ). The PDF can be given by:

(25)

where denotes the trace of the argument matrix.

Because and () are independent of each other in the selected frequency band, the PDF of is obtained as

(26)

According to the Bayesian inference framework, the posterior PDF of the bolted parameters is:

|  |  |  |
| --- | --- | --- |
|  |  | (27) |

where is a normalizing constant independent from . is the likelihood function; is a prior PDF of ,the posterior PDF is

|  |  |  |
| --- | --- | --- |
|  |  | (28) |

The objective function can be constructed by the negative logarithm of the likelihood function (NLLF):

|  |  |  |
| --- | --- | --- |
|  |  | (29) |

Quantifying the uncertainty of joint parameters is an important part of studying the dynamic model updating of assembled structures. This article does not only use the NLLF (Eq. (29)) to obtain the maximum likelihood estimation, but also use the Bayesian inference framework to obtain the posterior probability and quantify the uncertainty. An asymptotic approximation of the posterior PDF is used for quantifying the uncertainty.

3.2 Sensitivity analysis and optimization process of the Bayesian objective function

In recent years, intelligent stochastic search optimization methods, including ant colony algorithm (Villeneuve and Mavris 2012), genetic algorithm (Thorp and Pierson 2007; Fei *et al.* 2020) and simulated annealing method (Suman 2003) are popular. In addition, there is a class of optimization methods that use surrogate models (Cosenza and Block 2020; Lu *et al*. 2020) instead of the true model to increase the computational efficiency. Although these methods have been applied in some practical engineering problems, both intelligent optimization methods and surrogate models require high time costs. In order to save computing costs of optimization, the analytical expressions of the first-order and second-order sensitivities of the objective function are derived using the PEM and CMS. The sensitivity analysis of the objective function proceeds as follows.

The first-order partial derivative of to can be derived by Eq. (29):

|  |
| --- |
|  |
| (30) |

where is calculated by Eq. (19).

Further, the second-order partial derivative of can be given by

|  |
| --- |
|  |
|  |
| (31) |

Eq. (30) and Eq. (31) retain high accuracy since no simplification is made during the derivation. It is found that most of the above calculations are matrix operations of multiplication and addition and subtraction, and the most time-consuming calculations are to get the inverses of and . As stated previously, the calculation of is inevitable for the dynamic response. It is not an additional calculation cost due to the calculation of the sensitivity of the objective function, and the dimensions of have been reduced considerably by the CMS. The dimensions of are determined by the number of sensors and are not high. With the fact that is a symmetric matrix, the inversion process does not take much time. Figure 2 gives the flowchart of the proposed method.

****

Figure 2. The flowchart of the proposed method

**4 Numerical examples**

4.1 Example 1

As shown in Figure 3, an assembled structure involving three cantilever beams is considered. Beams ① and ② are taken as substructure 1 and beam ③ is taken as substructure 2. Beam ① and ② are both connected to beam ③ by two bolts. Each cantilever beam has a Young’s modulus , density , Poisson's ratio , and the geometric size and bolt positions are shown in Figure 3(b). Using the FEA, each cantilever beam is divided into 38 elements, and the overall structure has 684 DOFs. Each bolt is modelled using three extension springs and three rotation springs. Assuming that each finite element has one uncertain parameter, the uncertain parameters of the four bolts are . The stiffness matrix of the bolt is determined by the uncertain parameters and the spring stiffness and the relationship is given by

|  |  |
| --- | --- |
|  | (32) |

where and are coefficients of tension springs in corresponding local coordinates, and are coefficients of rotation springs in corresponding local coordinates. There are many ways to optimize sensor placement. The examples in this article are relatively simple. According to the main modes of the assembled structure, two sensors (observation DOFs) are placed and their positions are shown in Figure 3(a). In the Numerical examples, the prior PDF of is assumed to be multivariate uniform distributed in a non-zero finite interval.



(a) The positions of excitation and measured sensors



(b) The geometric size

Figure 3. The assembled structure involving three cantilever beams

4.1.1 Simulation of the measured time histories

When conducting numerical simulations, the measured time histories are generated by the theoretical PSD of the sensor nodes as the summation of triangular series, and then a zero-mean white noise process with PSD is added to the measured time histories (Shinozuka and Deodatis 1991). There is a subtle modelling error between the model for simulation data generation and the model for updating, which are reflected in the measurement uncertainty.

In this example, the actual value of is assumed to be and the PSD of the stationary stochastic excitation is . Then measured time histories of all channels are simulated that are divided into segments with a time duration 350s. Taking the FFT transform of each segment to obtain the measured signals in the frequency domain according to Eq. (21), the average measured PSD (as Figure 4(a)) is calculated from Eq. (22). Before updating, the parameter sensitivity analysis is performed, and it is found that the second peak of PSD is more sensitive to the joint parameters. Hence, example 1 takes the measured PSD with frequency band (as Figure a(b)) to implement the model updating.



 (a) Average measured PSD (b) Average measured PSD in

Figure 4. ‘Measured’ signal in frequency domain

4.1.2 Model updating of the bolted parameters

The numerical results are shown in Table 1, the standard deviations and coefficients of variation (COV) of the updated parameters are listed in the fourth and fifth columns, and a normalizing distance (Yuen 2010) which represents how many standard deviations the optimal estimate differs from the target value is reported in the last column. From Table 1, it can be seen that the target values are all closer to the actual target value , where the uncertain parameters and are closer to the target value compared with and . However, their standard deviations are larger, indicating greater dispersion, for example, the standard deviation of 0.0008 for and 0.0258 for . The difference in variance also shows to a certain extent that the interference of noise to the updated parameter statistics is not the same. The COV given in column 5 and the standard distance given in column 6 also reflect this conclusion.

Table 1. The results of stochastic model updating

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Parameter | Actual  | Optimal  | S.D. | COV |  |
|  | 1.000 | 0.9967 | 0.0008 | 0.0008 | 3.9133 |
|  | 1.000 | 1.0030 | 0.0009 | 0.0009 | 3.2512 |
|  | 1.000 | 1.0003 | 0.0258 | 0.0258 | 0.0112 |
|  | 1.000 | 0.9992 | 0.0258 | 0.0258 | 0.0302 |

4.1.3 Discussion on the accuracy of the CMS

According to the selection of the number of retained modes in substructures, different control groups (Case 1, Case 2 and Case 3) are set up to study the influence of the mode reduction method on the model updating results. The measured signals used here are simulated and generated by the results of the full-modal model, and the frequency band is . The results of the MPV, the updated power spectrum, the Coefficient of determination (it may also be known as the R-squared Value) calculated in the frequency band and the Coefficient of determination calculated in the frequency band are compared, respectively.

Table 2 shows a comparison of the model updating results of different cases. The first row sets up three cases. For example, in Case 1, the number of retained modes in substructures A and B are 400 and 210, respectively, and the total number of retained modes is 610. From rows 3 to 5 in Table 2, it can be seen that the optimal estimated values of the three cases are relatively accurate, and the model updating results of Case 1 and Case 2 are almost the same, and are closer to the actual values. Compared with Case 1 and Case 2, the results of Case 3 have certain errors from the actual values. This is mainly because Case 3 ignores a large amount of high-order modal information and the truncation error has a significant impact on the results.

Since the frequency band of the measured PSD for updating is , the results of the three cases do not deviate much in terms of . As for whose frequency band is , there are slight differences among the three cases, and the fitting accuracy of Case 3 is slightly poorer than those in Case 1 and Case 2. It can be considered that the model error caused by the substructure truncation in Case 3 has a limited impact on the model updating of the bolted joint parameters. The last row of Table 2 is a comparison of the calculation time of the three cases. The calculation time of Case 1 is 20 times more than that of Case 3.

Figure 5 gives the comparison of the average measured PSD and the updated PSD in the three cases. At the positions of the resonant frequencies under 125.7 rad/s, the updated PSD of all three cases matches the simulated measured PSD, indicating that the substructure truncation errors have less influences on the resonant peaks in this frequency band. In higher frequency bands such as the peak position within rad/s, the results of Case 3 match the measured PSD slightly poorly due to the higher-order modal truncation. However, usually the dynamical behaviour of the assembled structure is determined by the resonant properties of the first few orders, and a smaller truncation error is acceptable, considering the less computational time cost.

Table 2. The results of model updating of different cases

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Case 1** | **Case 2** | **Case 3** |
| Num. modal | SubA=400 | SubA=275 | SubA=150 |
| SubB=210 | SubB=140 | SubB=70 |
| Parameter | Actual  | Optimal  |
|  | 1.000 | 1.0004 | 0.9967 | 0.9725 |
|  | 1.000 | 0.9996 | 1.0030 | 1.0275 |
|  | 1.000 | 1.0024 | 1.0003 | 1.0979 |
|  | 1.000 | 0.9982 | 0.9992 | 0.9039 |
|  | 0.9998 | 0.9999 | 0.9997 |
|  | 0.9992 | 0.9971 | 0.9825 |
| Time(s) | 3057 | 1306 | 147 |



Figure 5. Comparison of average PSD, updating PSD in different cases（）

4.1.4 Discussion on the efficiency of proposed method

**（a）Reduce computational costs due to analytical gradient and Hessian：**The gradient vector and the Hessian matrix are calculated by the proposed method and the finite difference method, respectively. It can be seen from Table 3 that the final optimization results of the two methods are the same, but the calculation cost of the proposed method in this article is 20% of the cost of the finite difference method. For more complex structures with more parameters, the improvement of efficiency will be more obvious. Figure 6 compares the first-order optimality convergence of the two methods. It can be clearly seen from the figure that the proposed method in this article converges faster and needs fewer iteration steps, indicating high accuracy brought about by the use of the analytical gradient vector and Hessian matrix, which significantly speeds up the optimization process.

From the time-consuming analysis of the code, calculating the analytic gradient vector and Hessian matrix simultaneously, which are by-products of the NLLF equation, only add the calculation time by 4.7% compared with calculating the NLLF equation alone, while the finite difference method must to calculate the NLLF equation repeatedly.

**（b）Reduce computational costs due to** **Substructuring：**In this section, the reduced-order model of the control group comes from the mode decomposition of the assembled structure. PEM and Bayesian PSD with numerical gradient vector and Hessian matrix are used for stochastic dynamics analysis and model updating. For a fair comparison, the reduced-order models using the two methods have the same order, and the calculation results are shown in Table 3. As seen from the last four columns of Table 3, the computational efficiency of the control group is poorly low compared with the method of this article. The main reason is as follows. The method in the control group must reassemble and reanalyse the overall structure whenever the parameters change. However, the method in this article can reserve the reduced-order models of the uncoupled substructures, reanalyse the bolted joints only and then assemble the reduced-order models much smaller than the overall structure model, which has a great computational advantage.

Table 3 Results of model updating with analytical/ numerical gradient and Hessian

|  |  |  |
| --- | --- | --- |
| Method of the reduced-order model | CMS | the mode decomposition of the assembled structure |
| Sensitivity analysis | With analytical gradient and Hessian | With numerical gradient and Hessian | With numerical gradient and Hessian |
| Parameter | Optimal  | S.D. | Optimal  | S.D. | Optimal  | S.D. |
|  | 0.9725 | 0.0008 | 0.9725 | 0.0008 | 0.9852 | 0.0008 |
|  | 1.0275 | 0.0009 | 1.0275 | 0.0009 | 1.0186 | 0.0009 |
|  | 1.0979 | 0.0260 | 1.0979 | 0.0260 | 1.1189 | 0.0258 |
|  | 0.9039 | 0.0257 | 0.9039 | 0.0257 | 0.8658 | 0.0258 |
| Time(s) | 147 | 709 | 897 |



Figure 6. Frist-order optimality iterative history

4.2 Example 2

This example presents a study of an inter-stage structure of a simplified rocket, and updates the uncertain parameters of the explosive bolts between the rocket body stages. As shown in Figure 7, the structure of a two-stage rocket body can be simplified to a two-cylindrical shell structure, and the joints are eight explosive bolts distributed in a uniform ring around the centre, as shown in Figure 7(c). The geometric sizes of the structure are as follows: the height of each shell section is , the diameter is , the thickness is , the end frame width is , and the thickness . The material is a high-strength aluminium alloy with elastic Young’s modulus , density.

There are two cases with different excitation forms in the example (as shown in Table 4). Case 1 applies a single point random load at the middle of substructure B whose PSD is , as shown by the arrow in Figure 7(a). Case 2 is a multi-point random excitation that is a homogeneous and out-of-phase excitation applied at two different positions in the middle of substructure B. The time difference between the two excitations is *τ=*0.6 s, as shown by the arrows in Figure 7(a). Each case has two measuring points for collecting vibration data. Their positions are shown by the pentagram symbols in Figure 7(a).



 (a) simplified rocket structure and excitation position (b) the position of the explosive bolts

(c) schematic diagram illustrating the arrangement of explosive bolts

Figure 7. Schematic diagram of the simplified rocket

As shown in Figure 7(a), the two ends of the cabin are fixed. A commercial FEA software package is used to model the assembled structure (a total of 8688 elements) and construct the modal analysis. Modal results are read into the MATLAB program, and the measured signals are generated by simulation according to Section 4.1.

The results are reported in Table 5. It can be seen from columns 3 and 4 that there are deviations for in Case 1 and Case 2, especially for and . From columns 7 and 8, it can be seen that the distances between the best estimate and the actual value of Case 1 and Case 2 are within one standard deviation under certain noise intensity, and the results are both satisfactory. Figure 8 shows the iteration history of the first-order optimality in both cases. Case 1 takes fewer iterations than Case 2 to achieve convergence, which indicates that the multipoint excitation method possesses better convergent performance.

Table 4. The list of cases

|  |  |  |  |
| --- | --- | --- | --- |
|  | Excitation | Measured Channel | Signal-to-noise ratio |
| Case 1 |  | 1 | 25.3 |
| 2 | 20.7 |
| Case 2 |  | 1 | 25.6 |
|  | 2 | 23.8 |

Table 5. The results of model updating for exploding bolts

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Parameter | Actual  | Optimal  | S.D. |  |
| Case1 | Case2 | Case1 | Case2 | Case1 | Case2 |
|  | 1.0000 | 0.9737 | 0.9290 | 0.2161 | 0.1926 | 0.1218 | 0.3684 |
|  | 1.0000 | 0.9863 | 0.9615 | 0.1897 | 0.1384 | 0.0723 | 0.2778 |
|  | 1.0000 | 1.0743 | 1.0566 | 0.1991 | 0.1841 | 0.3733 | 0.3074 |
|  | 1.0000 | 0.9073 | 0.9399 | 0.1882 | 0.1337 | 0.4926 | 0.4491 |
|  | 1.0000 | 1.0134 | 1.0136 | 0.1657 | 0.1631 | 0.0810 | 0.0833 |
|  | 1.0000 | 1.2849 | 1.0886 | 0.2326 | 0.1551 | 1.2246 | 0.5714 |
|  | 1.0000 | 0.9433 | 0.9972 | 0.2257 | 0.2048 | 0.2513 | 0.0136 |
|  | 1.0000 | 0.8162 | 1.0250 | 0.2221 | 0.1455 | 0.8274 | 0.1717 |



Figure 8. Profiles of the first-order in Case1 and Case2

**5 Conclusions**

In this article, an efficient stochastic model updating method is proposed for the uncertain bolted joint parameters of assembled structures using the CMS, PEM and Bayesian PSD techniques. The main contributions of the article are threefold: (1) the CMS is introduced to obtain a reduced-order dynamic model of assembled structures; (2) the PEM is used for determining random vibrations of the assembled structure and deriving the sensitivity of the response PSD; (3) the analytical expressions of the gradient vector and Hessian matrix of the objective function are derived to speed up the optimization process.

The CMS greatly improves the computational efficiency, and the reason comes down to two aspects. On the one hand, if the traditional modal reduction techniques are used in the assembled structures, both the mode shapes of assembled structures and modal coordinates contain the uncertain bolted joint parameters. It is difficult to calculate the partial derivatives of mode shapes with respect to uncertain parameters. In this article, the CMS is used in substructures. The mode shapes of substructures are independent of the uncertain bolted joint parameters, and thus it is convenient and practically feasible to derive analytical expression of the gradient vector and Hessian matrix. On the other hand, the bolted joints containing uncertain parameters can be isolated and then analysed individually, avoiding repeated rean0alysis. However, it should be pointed out that ignoring high-order modes in the CMS results in relatively large truncation errors, making the results less accurate in high-frequency bands. When there is a large difference in natural frequencies between substructures, additional attention should be paid to the selection of number of retained modes; otherwise the response of assembled structures in relatively high-frequency bands can be inaccurate. In this situation, the accuracy can be improved by involving more modes or using high-accuracy CMS in relatively flexible substructures, one of the authors’ future research directions is to address this issue. In addition, using the data from experimental and field tests will make this research more convincing, which is also planned in the author’s future work.

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No potential conflict of interest was reported by the authors.

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**Data Availability Statement**

The data that support the findings of this study are available from the corresponding author, Y. Zhao, upon reasonable request.

**Reference**

Abu Husain,N., H. Haddad Khodaparast, andH. J.Ouyang. 2012. “Parameter Selection and Stochastic Model Updating Using Perturbation Methods with Parameter Weighting Matrix Assignment.” Mechanical Systems and Signal Processing 32: 135–152. doi:10.1016/j.ymssp.2012.04.001.

Au, S. K. 2011. “Fast Bayesian FFT Method for Ambient Modal Identification with Separated Modes.” Journal of Engineering Mechanics 137 (3): 214–226. doi:10.1061/(ASCE)EM.1943-7889.0000213.

Au, S. K. 2016. “Insights on the Bayesian SpectralDensityMethod forOperationalModal Analysis.” Mechanical Systems and Signal Processing 66-67: 1–12. doi:10.1016/j.ymssp.2015.04.023.

Batou, A., and A. Nabarrete. 2018. “Nonparametric Probabilistic Approach of Uncertainties with Correlated Mass and Stiffness Random Matrices.” Mechanical Systems and Signal Processing 111: 102–112. doi:10.1016/j.ymssp.2018.03.049.

Battiato, G., and C. Firrone. 2020. “AModal Based Reduction Technique forWide Loose Interfaces andApplication to a Turbine Stator.” Mechanical Systems and Signal Processing 139 (May): 106415. doi:10.1016/j.ymssp.2019.106415.

Beck, J. L. 2010. “Bayesian System Identification Based on Probability Logic.” Structural Control and HealthMonitoring 17 (7): 825–847. doi:10.1002/stc.424.

Beck, J. L., and S. K. Au. 2002. “Bayesian Updating of Structural Models and Reliability Using Markov Chain Monte Carlo Simulation.” Journal of Engineering Mechanics 128 (4): 380–391. doi:10.1061/(ASCE)0733-9399(2002)128:4(380).

Beck, J. L., and L. S. Katafygiotis. 1998. “UpdatingModels and Their Uncertainties. I: Bayesian Statistical Framework.” Journal of Engineering Mechanics 124 (4). doi:10.1061/(ASCE)0733-9399(1998)124:4(455).

Bograd, S., P. Reuss, A. Schmidt, L. Gaul, and M. Mayer. 2011. “Modeling the Dynamics of Mechanical Joints.” Mechanical Systems and Signal Processing 25 (8): 2801–2826. doi:10.1016/j.ymssp.2011.01.010.

Brake, Matthew R. W. 2018. The Mechanics of Jointed Structures: Recent Research and Open Challenges for Developing Predictive Models for Structural Dynamics. Springer. Cham, Switzerland.

Bui-Thanh, T., K. Willcox, and O. Ghattas. 2008. “Parametric Reduced-Order Models for Probabilistic Analysis of Unsteady Aerodynamic Applications.” AIAA Journal 46 (10): 2520–2529. doi:10.2514/1.35850.

Ching, J. Y., and Y. C. Chen. 2007. “Transitional Markov Chain Monte Carlo Method for Bayesian Model Updating, Model Class Selection, and Model Averaging.” Journal of Engineering Mechanics 133 (7): 816–832. doi:10.1061/(ASCE)0733-9399(2007)133:7(816).

Cosenza, Z., and D. E. Block. 2020. “A Generalizable Hybrid Search Framework for Optimizing Expensive Design Problems Using Surrogate Models.” Engineering Optimization, 1–14. doi:10.1080/0305215X.2020.1826466.

Fei, C. W., H. Li, H. T. Liu, C. Lu, L. Q. An, L. Han, and Y. J. Zhao. 2020. “Enhanced Network Learning Model with IntelligentOperator for theMotion Reliability Evaluation of FlexibleMechanism.” Aerospace Science and Technology 107: 106306. doi:10.1016/j.ast.2020.106342.

Fei, C.W., H. Liu, S. Li, H. Li, L. An, and C. Lu. 2021. “Dynamic ParametricModeling-BasedModel Updating Strategy of Aeroengine Casings.” Chinese Journal of Aeronautics. doi:10.1016/j.cja.2020.10.036.

Huang, B., and H. Chen. 2019. “A New Approach for Stochastic Model Updating Using the Hybrid Perturbation-Garlekin Method.” Mechanical Systems and Signal Processing 129: 1–19. doi:10.1016/j.ymssp.2019.04.012.

Ingole, S. B., and A. Chatterjee. 2017. “Joint Stiffness Identification: A Three-Parameter Joint Model of Cantilever Beam.” The International Journal of Acoustics and Vibration 22 (1): 3–13. doi:10.20855/ijav.2017.22.1445.

Katafygiotis, L. S., and K. V. Yuen. 2001. “Bayesian Spectral Density Approach for Modal Updating Using Ambient Data.” Earthquake Engineering & Structural Dynamics 30 (8): 1103–1123. doi:10.1002/eqe.53.

Lin, J. H., Y. Zhao, and Y. H. Zhang. 2001. “Accurate and Highly Efficient Algorithms for Structural Stationary/Non-stationary Random Responses.” Computer Methods in Applied Mechanics & Engineering 191 (1/2): 103–111. doi:10.1016/S0045-7825(01)00247-X.

Lu, C., C. W. Fei, H. T. Liu, H. Li, and L. Q. An. 2020. “Moving Extremum Surrogate Modeling Strategy for Dynamic Reliability Estimation of Turbine Blisk with Multi-physics Fields.” Aerospace Science and Technology 106: 106112. doi:10.1016/j.ast.2020.106112.

Mignolet,Marc P., Pengchao Song, and X.Q.Wang. 2015. “A Stochastic Iwan-TypeModel for Joint Behavior Variability Modeling.” Journal of Sound and Vibration 349: 289–298. doi:10.1016/j.jsv.2015.03.032.

Papadimitriou, C., and D. C. Papadioti. 2013. “Component Mode Synthesis Techniques for Finite Element Model Updating.” Computers & Structures 126: 15–28. doi:10.1016/j.compstruc.2012.10.018.

Park, S. S., and J. Chae. 2008. “Joint Identification of Modular Tools Using a Novel Receptance Coupling Method.” The International Journal of Advanced Manufacturing Technology 35 (11-12): 1251–1262. doi:10.1007/s00170-006-0826-6.

Rui,Q., H. J. Ouyang, andH. Y.Wang. 2013. “An Efficient Statistically Equivalent ReducedMethod on StochasticModelUpdating.” Applied Mathematical Modelling 37 (8): 6079–6096. doi:10.1016/j.apm.2012.11.026.

Shinozuka, M., and G. Deodatis. 1991. “Simulation of Stochastic Processes by Spectral Representation.” Applied Mechanics Reviews 44 (4): 191. doi:10.1115/1.3119501.

Solonen, A., and H. Haario. 2012. “Model-Based Process Optimization in the Presence of Parameter Uncertainty.” Engineering Optimization 44 (7): 875–894. doi:10.1080/0305215x.2011.617817.

Suman, B. 2003. “Simulated Annealing-BasedMultiobjective Algorithms and Their Application for System Reliability.” Engineering Optimization 35 (4): 391–416. doi:10.1080/03052150310001597765.

Thorp, N. A., and B. L. Pierson. 2007. “Cluster Analysis After a Partial Genetic Algorithm Search.” Engineering Optimization 31 (2): 225–246. doi:10.1080/03052159808941371.

Villeneuve, F. J., andD.N.Mavris. 2012. “Aircraft Technology PortfolioOptimizationUsing ant ColonyOptimization.” Engineering Optimization 44 (11): 1369–1387. doi:10.1080/0305215x.2011.649747.

Xu, W. T., J. H. Lin, Y. H. Zhang, D. Kennedy, and F. W. Williams. 2009. “Pseudo-Excitation-Method-Based Sensitivity Analysis and Optimization for Vehicle Ride Comfort.” Engineering Optimization 41 (7): 699–711. doi:10.1080/03052150902752066.

Yan, W. J., and L. S. Katafygiotis. 2015a. “A Two-Stage Fast Bayesian Spectral Density Approach for Ambient Modal Analysis. Part I: Posterior Most Probable Value and Uncertainty.” Mechanical Systems and Signal Processing 54-55: 139–155. doi:10.1016/j.ymssp.2014.07.027.

Yan, W. J., and L. S. Katafygiotis. 2015b. “A Two-Stage Fast Bayesian Spectral Density Approach for Ambient Modal Analysis. Part II:Mode ShapeAssembly andCase Studies.”Mechanical Systems and Signal Processing 54-55: 156–171. doi: 10.1016/j.ymssp.2014.08.016.

Yan, W. J., and L. S. Katafygiotis. 2016. “Application of Transmissibility Matrix and Random Matrix to Bayesian System Identification with Response Measurements Only.” Smart Materials & Structures 25 (10): 105017. doi:10.1088/0964-1726/25/10/105017.

Yuan, J. J., E.H. Fadi, S. Loic, andW. Chian. 2019. “Numerical Assessment of Reduced OrderModeling Techniques for Dynamic Analysis of Jointed Structures with Contact Nonlinearities ASME.” Journal of Engineering for Gas Turbines and Power 141 (3): 031027. doi:10.1115/1.4041147.

Yuen, K. V. 2010. Bayesian Methods for Structural Dynamics and Civil Engineering. JohnWiley & Sons. Singapore.

Yuen, Ka-Veng, and Lambros S. Katafygiotis. 2001. “Bayesian Time–Domain Approach for Modal Updating Using Ambient Data.” Probabilistic Engineering Mechanics 16 (3): 219–231. doi:10.1016/s0266-8920(01)00004-2.

Yuen, K. V., and L. S. Katafygiotis. 2003. “Bayesian Fast Fourier Transform Approach for Modal Updating Using Ambient Data.” Advances in Structural Engineering 6 (2): 81–95. doi:10.1260/136943303769013183.

Zhang, Y. H., Q. S. Li, J. H. Lin, and F. W. Williams. 2009. “Random Vibration Analysis of Long-Span Structures Subjected to Spatially Varying Ground Motions.” Soil Dynamics & Earthquake Engineering 29 (4): 620–629. doi:10.1016/j.soildyn.2008.06.007.

Zhang, Y. W., Y. Zhao, Y. H. Zhang, J. H. Lin, and X. W. He. 2013. “Riding Comfort Optimization of Railway Trains Based on Pseudo-ExcitationMethod and SymplecticMethod.” Journal of Sound and Vibration 332 (21): 5255–5270. doi:10.1016/j.jsv.2013.05.018.

Zhu, Y. C., and S. K. Au. 2018. “Bayesian Operational Modal Analysis with Asynchronous Data, Part II: Posterior Uncertainty.” Mechanical Systems and Signal Processing 98: 920–935. doi:10.1016/j.ymssp.2017.05.023.

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