A Novel Model of Unipolar Induction Phenomena Based on Direct Interaction Between Conductor Charges

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Abstract

Unipolar induction has been a heavily discussed phenomenon in the realm of electrodynamics, with research and experiments proposing and supporting different ways to explain the observed effects. This paper presents a novel model to predict induced electromotive forces in a Faraday generator, based on direct interaction between conductor charges. It is compared with predictions that are usually obtained through considerations of Lorentz force, flux linking or flux cutting rules. A standard apparatus provides additional experimental measurements that show good agreement with the theory.

1 Introduction

Since Faraday's discovery [1], unipolar induction has been a controversial phenomenon in electrodynamic theory that is described as paradoxical with competing theories and explanations suggested. To this day, no clear consensus has been reached [2–9] and it remains an interesting topic for debate [4–8]. A general set-up of a so-called Faraday-generator is shown in Figure 1, consisting of a cylindrical bar magnet and a conducting disk, across which the induced voltage is measured. Many explanations have been proposed to account for the electromotive force (EMF) that is observed when the disk is rotating in a magnetic field. These include: the hypothesis that the field remains stationary upon the magnet's rotation and the applicability of Faraday's law [10–13], the field rotating with the magnet and the view point that Faraday's law is not applicable to the problem [4, 5, 14–16], discussions about the relevance of Special Relativity [17–21] and/or General Relativity [22] to the problem, quantum mechanical explanations [23–26], as well as suggestions that distant galaxies warping space-time cause the induced voltage to appear [27].

From field theory, the paradox arises when movement of a magnet and its field lines are considered. When the magnet moves in a rectilinear fashion and a voltage is induced, it is deemed that the field lines must move with the magnet, but when the magnet rotates around its cylindrical axis, while the disk is kept stationary (Fig. 1), no voltage can be detected and the question as to whether the field lines move with the magnet or not becomes apparent. These two seemingly contradictory observations are the origins of what has been called "Faraday's paradox" in relation to unipolar induction. Whilst several theories have been proposed, they can largely be divided into two factions: The Moving Field Hypothesis (MFH), which maintains that the field lines rotate with the magnet, and the Stationary Field Hypothesis (SFH), which considers that the field lines remain stationary when the magnet is rotating.

There have been a number of experimental studies to investigate the behaviour of the field and which of the two theories is applicable [1, 4–8, 10–12, 14, 19, 20, 28–37]. However, from available

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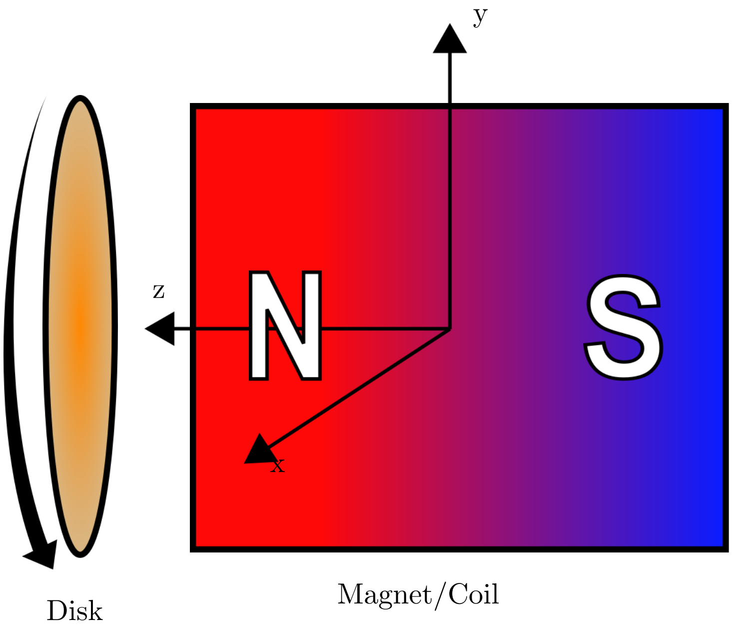


Figure 1: General layout of a Faraday-generator with a magnet or coil and a spinning conducting disk across which the induced voltage is observed.

experimental data, the problem remains unresolved as to which theory is the correct one since there are supporting results for both MFH and SFH. It has also been discussed where the seat of induction lies, if it is in the disk itself or in the closing wire of the measurement circuit. An argument has been made that for any unipolar machine the whole circuit including the closing wire needs to be considered [38].

Amongst the established theories, there is an approach by Montgomery [39, 40] to explain the phenomenon but not solely by consideration of the field entity alone. Instead, the motion of the conduction electrons and the conservation of energy are considered to help resolve the problem. In this regard, the field is not necessarily considered as a real physical entity, but rather as a mathematical tool indicating a vectorial map of possible interactions. It has been argued in the literature that field lines or 'lines of force' do not have to be continuous, individual or closed curves, only the knowledge of a local vector field with direction and magnitude is relevant to describe observable phenomena [41].

In the work of Zengel [9], four ways to derive the correct expression for the induced voltage $EMF = \omega R^2 B/2$ are given, which leaves some ambiguity as to which of the possible explanations can sufficiently satisfy the available experimental evidence. i.e., there exist multiple mathematical means by which one can correctly predict the observed voltage. In this work we have developed a new modelling approach to predict the induced EMF from a homopolar generator configuration based solely on the forces between charges in relative motion in the spinning disk in the presence of a stationary electromagnet. The model is based on the physical interaction between charges known to be present in this configuration.

Based on these considerations, this current work presents a novel mathematical model to predict the induced voltage in a Faraday-generator setup focussing on the movement of the interacting charges. It is compared to a standard field-model calculation and supported by experimental measurements.

2 Mathematical Models

In this section, expressions are derived for the induced voltage for a spinning disk and a stationary coil utilising two different approaches. The predictions of the two models will then be compared

with experimental data in the Results & Discussion section 4.
We will first develop our new model based on the relative movement of the available charges by applying Weber's force law [42] to calculate the induced EMF. Secondly, we will utilise the Lorentz force law to predict the induced voltage based on standard electromagnetic theory.

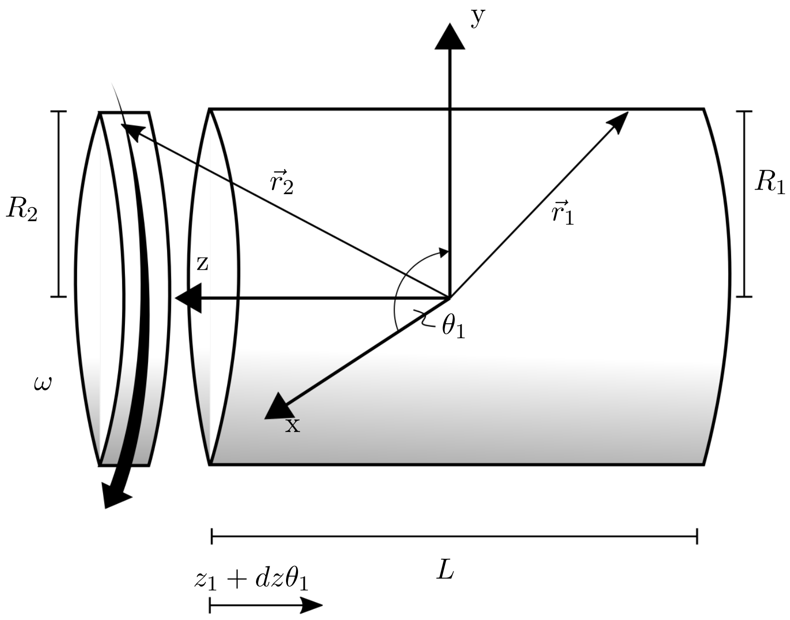


Figure 2: Diagram of coil geometry with radius R_1 and length L centred on a Cartesian coordinate system. A disk with radius R_2 is spinning with angular velocity ω and positioned on the z-axis at z_2. Vectors $\vec{r}_1$ and $\vec{r}_2$ describe the position of charges in the coil or disk with the help of polar angles θ_1 and θ_2, where θ_2 is the polar angle in the plane of the disk.

2.1 Direct Charge Interaction Model based on Weber's Force

Our derivation starts with any arbitrary coil which is centred on the origin of a Cartesian coordinate system within the laboratory reference frame (see Fig. 2). The coil has radius R_1, length L and number of windings N and it is supplied with a constant DC current I that confines the current flow into a helical motion through the windings. A current element travelling through the coil consists of a negative charge q_{1-} and a positive charge q_{1+}, where at any position the positive charge is stationary in the lattice and the negative charge is a conduction electron that is free to move. They can be described with the positional vector $\vec{r}_1$ as a helical parametrisation with the help of polar angle θ_1 in the coordinate system:

$$\vec{r}_1 = \begin{pmatrix} R_1 \cos(\theta_1) \\ R_1 \sin(\theta_1) \\ z_1 - dz\theta_1 \end{pmatrix} . \tag{1}$$

For a disk with radius R_2 positioned next to the coil, we can similarly write the positional vector $\vec{r}_2$ for its positive and negative charges which are equally distributed in the disk:

$$\vec{r}_2 = \begin{pmatrix} R_2 \cos(\theta_2) \\ R_2 \sin(\theta_2) \\ z_2 \end{pmatrix} , \tag{2}$$

which leaves us with a distance between the two:

$$\begin{aligned}\vec{r}_{12} =& \vec{r}_1 - \vec{r}_2 = \begin{pmatrix} R_1\cos(\theta_1) - R_2\cos(\theta_2) \\ R_1\sin(\theta_1) - R_2\sin(\theta_2) \\ z_1 - z_2 - dz\theta_1 \end{pmatrix}, \\ r_{12} =& |\vec{r}_1 - \vec{r}_2|. \end{aligned} \tag{3}$$

If we now consider the charges in the disk, q_{2+}, q_{2-}, moving with angular velocity ω due to the rotation of the disk; and the negative charge of a current element q_{1-} travelling through the coil with velocity $\vec{v}_1$ we get:

$$\begin{aligned}\vec{v}_1 =& \begin{pmatrix} -v_1 sin(\theta_1) \\ v_1 cos(\theta_1) \\ -v_1 dz/R_1 \end{pmatrix}, \qquad \vec{v}_2 = \begin{pmatrix} -\omega R_2 sin(\theta_2) \\ \omega R_2 cos(\theta_2) \\ 0 \end{pmatrix}, \\ \vec{v}_{12} =& \begin{pmatrix} -v_1 sin(\theta_1) + \omega R_2 sin(\theta_2) \\ v_1 cos(\theta_1) - \omega R_2 cos(\theta_2) \\ -v_1 dz/R_1 \end{pmatrix}. \end{aligned} \tag{4}$$

For any travelling current element in the coil, there is a positive lattice charge q_{1+} at the same position $\vec{r}_1$, only that it remains stationary, hence its velocity is zero.
Since we are dealing with circular motion the charges are subject to a certain acceleration $\vec{a}_1$ and $\vec{a}_2$, but these terms either cancel out in the model according to the equations below as is the case of $\vec{a}_2$ or are negligibly small since $R_1\vec{a}_1 \ll \omega R_2$ as is the case of the conduction electrons in the coil.
With these definitions for the four representative interacting charges q_{1+}, q_{1-}, q_{2+} and q_{2-}, we can now apply the Weber force law to predict the induced voltage. In a more general fashion, Wesley has previously considered unipolar induction with Weber's force but without applying it to any specific case [43]. We consider that such considerations are necessary since the distribution of charges is paramount to the Weber force.
In modern vector notation the Weber force can be written as:

$$\begin{aligned}\vec{F}_{21} =& \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\vec{r}_{12}}{r_{12}^3}\left(1 - \frac{3}{2c^2}\left[\frac{\vec{r}_{12}\vec{v}_{12}}{r_{12}}\right]^2\right. \\ &\left. + \frac{1}{c^2}\left(\vec{v}_{12}\vec{v}_{12} + \vec{r}_{12}\vec{a}_{12}\right)\right), \end{aligned} \tag{5}$$

where $\vec{r}_{12}$ is the relative position, $\vec{v}_{12}$ the relative velocity and $\vec{a}_{12}$ the relative acceleration between the charges q_1 and q_2, with the permittivity of free space ε_0 and the speed of light c. We can now find the four interaction forces between the positive and negative charge elements of the disk and coil, $\vec{F}_{2-1-}, \vec{F}_{2+1-}, \vec{F}_{2-1+}, \vec{F}_{2+1+}$. Since only the electrons are free to move in the disk and the positive charges are bound to the lattice, the forces $\vec{F}_{2+1-}, \vec{F}_{2+1+}$ acting on the positive charges in the disk will be countered by the lattice itself. Thus, only $\vec{F}_{2-1-}, \vec{F}_{2-1+}$ need to be considered to find the total force responsible for the EMF:

$$\vec{F}_{2-1-} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{r}_{12}}{r_{12}^3} \left\{ 1 - \frac{3}{2c^2} \frac{1}{r_{12}^2} \left[\begin{pmatrix} R_1\cos(\theta_1) - R_2\cos(\theta_2) \\ R_1\sin(\theta_1) - R_2\sin(\theta_2) \\ z_1 - z_2 - dz\theta_1 \end{pmatrix} \begin{pmatrix} -v_1 sin(\theta_1) + \omega R_2 sin(\theta_2) \\ v_1 cos(\theta_1) - \omega R_2 cos(\theta_2) \\ -v_1 dz/R_1 \end{pmatrix} \right]^2 \right.$$
$$+ \frac{1}{c^2} \left(\begin{pmatrix} -v_1 sin(\theta_1) + \omega R_2 sin(\theta_2) \\ v_1 cos(\theta_1) - \omega R_2 cos(\theta_2) \\ -v_1 dz/R_1 \end{pmatrix} \begin{pmatrix} -v_1 sin(\theta_1) + \omega R_2 sin(\theta_2) \\ v_1 cos(\theta_1) - \omega R_2 cos(\theta_2) \\ -v_1 dz/R_1 \end{pmatrix} \right.$$
$$\left.\left. + \begin{pmatrix} R_1\cos(\theta_1) - R_2\cos(\theta_2) \\ R_1\sin(\theta_1) - R_2\sin(\theta_2) \\ z_1 - z_2 - dz\theta_1 \end{pmatrix} \begin{pmatrix} a_{1x} - a_{2x} \\ a_{1y} - a_{2y} \\ a_{1z} - a_{2z} \end{pmatrix} \right) \right\}, \tag{6}$$

$$\vec{F}_{2-1+} = -\frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{r}_{12}}{r_{12}^3} \left\{ 1 - \frac{3}{2c^2} \frac{1}{r_{12}^2} \left[\begin{pmatrix} R_1\cos(\theta_1) - R_2\cos(\theta_2) \\ R_1\sin(\theta_1) - R_2\sin(\theta_2) \\ z_1 - z_2 - dz\theta_1 \end{pmatrix} \begin{pmatrix} \omega R_2 sin(\theta_2) \\ -\omega R_2 cos(\theta_2) \\ 0 \end{pmatrix} \right]^2 \right.$$
$$+ \frac{1}{c^2} \left(\begin{pmatrix} \omega R_2 sin(\theta_2) \\ -\omega R_2 cos(\theta_2) \\ 0 \end{pmatrix} \begin{pmatrix} \omega R_2 sin(\theta_2) \\ -\omega R_2 cos(\theta_2) \\ 0 \end{pmatrix} \right. \tag{7}$$
$$\left.\left. + \begin{pmatrix} R_1\cos(\theta_1) - R_2\cos(\theta_2) \\ R_1\sin(\theta_1) - R_2\sin(\theta_2) \\ z_1 - z_2 - dz\theta_1 \end{pmatrix} \begin{pmatrix} -a_{2x} \\ -a_{2y} \\ -a_{2z} \end{pmatrix} \right) \right\}.$$

We can then write the sum:

$$F_{sum} = \vec{F}_{2-1-} + \vec{F}_{2-1+}. \tag{8}$$

Further utilising the assumption $\omega R_2 \gg v_1$ and transitioning from discrete charges $q_1 v_1$ to continuous current elements $I R_1 d\theta_1$ leaves us with

$$\vec{F}_w = \frac{I q_2 R_1}{4\pi\varepsilon_0 c^2} \frac{\vec{r}_{12}}{r_{12}^3} \left\{ -\frac{3}{2} \frac{1}{r_{12}^2} \left[-2\omega R_2 sin(\theta_1) sin(\theta_2)(R_1 cos(\theta_1) - R_2 cos(\theta_2))^2 \right.\right.$$
$$+(2\omega R_2 cos(\theta_1) sin(\theta_2) + 2\omega R_2 sin(\theta_1) cos(\theta_2))(R_1 cos(\theta_1) - R_2 cos(\theta_2)(R_1 sin(\theta_1) - R_2 sin(\theta_2)$$
$$\left. -2\,\omega R_2 cos(\theta_1) cos(\theta_2)(R_1 sin(\theta_1) - R_2 sin(\theta_2))^2 \right]$$
$$\left. + \left[-2\omega R_2 sin(\theta_1) sin(\theta_2) - 2\omega R_2 cos(\theta_1) cos(\theta_2) \right] \right\} d\theta_1. \tag{9}$$

In order to estimate the corresponding EMF, (9) needs to be divided by q_2 and integrated along the radius R_2 of the disk, as this is the path along which the voltage difference is measured, so that we find the EMF to be:

$$EMF = \int_0^{R_{disk}} \int_0^{N\cdot 2\pi} \frac{I\omega R_1 R_2}{4\pi\varepsilon_0 c^2} \frac{\vec{r}_{12}}{r_{12}^3} \left\{ -\frac{3}{2} \frac{1}{r_{12}^2} \left[-2 sin(\theta_1) sin(\theta_2)(R_1 cos(\theta_1) - R_2 cos(\theta_2))^2 \right.\right.$$
$$+(2 cos(\theta_1) sin(\theta_2) + 2 sin(\theta_1) cos(\theta_2))(R_1 cos(\theta_1) - R_2 cos(\theta_2)(R_1 sin(\theta_1) - R_2 sin(\theta_2) \tag{10}$$
$$\left. -2\, cos(\theta_1) cos(\theta_2)(R_1 sin(\theta_1) - R_2 sin(\theta_2))^2 \right]$$
$$\left. + \left[-2 sin(\theta_1) sin(\theta_2) - 2 cos(\theta_1) cos(\theta_2) \right] \right\} d\theta_1 dR_2.$$

Equation (10) predicts the induced voltage in a Faraday-generator across the rotating disk (Fig. 1) for any stationary arbitrary coil for any position of the disk $\vec{r}_2$. It is numerically integrated in *MATLAB* (release 2020a, Mathworks, MA, USA); first for the angle θ_1 from 0 to $N \cdot 2\pi$ and then along radius R_2 with the help of the in-built *integral2* function. The angle θ_2 is fixed to one specific value between 0 and 2π, and for convenience the value $\pi/4$ has been chosen. This means that the radius is regarded along the y-axis, although the problem is rotationally symmetric and any arbitrary value can be chosen. To obtain the measurable EMF along the radius, the square root of the sum of the squared force components in x and y directions of (10) is then calculated, according to

$$EMF_R = \sqrt{EMF_x^2 + EMF_y^2}, \tag{11}$$

which gives a prediction for the expected value of induced voltage across the disk.
Furthermore, it is possible to transition (10) from a coil to a permanent magnet by approximating the integral according to the following:

$$\int_0^{N2\pi} (\dots) \approx N \cdot \int_0^{2\pi} (\dots), \tag{12}$$

and using the definition of remanence B_r

$$B_r = \mu_0 I \frac{N}{L}, \tag{13}$$

so that (10) becomes

$$EMF \approx \frac{BL\omega R_1 R_2}{4\pi} \int_0^{R_{disk}} \int_0^{2\pi} \{\dots\} \, d\theta_1 dR_2. \tag{14}$$

However, this is an approximation and there might be further modifications necessary before the equation can be fully applied to any permanent magnet setup.

2.2 Field Model

To predict the induced voltage with a standard approach based on field theory, we will show one possible way to derive the expression. For a more complete analysis the work of Zengel [9] is recommended. Consider the Lorentz force acting on the conduction electrons in the disk. In equilibrium, an electrostatic field $E(r)$ is set up such that the total Lorentz force on the charge is zero. That is,

$$F = -qE(r) - B_z(r)\omega R_2 q = 0 \tag{15}$$

giving

$$E(r) = -B_z(r)\omega R_2. \tag{16}$$

To obtain the EMF, once again we have to integrate between the center and radial distance, R_2, giving

$$EMF(R_2) = \omega \int_0^{R_{disk}} [B_z(R_2, z_2) \cdot R_2] dR_2. \tag{17}$$

For the calculation of the magnetic field B we are using a state-of-the-art model by Derby & Olbert [44] that has been incorporated into a *MATLAB* tool by D. Cébron [45]. The success of this model has been demonstrated by various scientists and has been applied to many topics in solenoid research [46–52]. The axial field component B_z is calculated in this model according to:

$$\begin{aligned} B_z = &\frac{B_r}{\pi} \frac{R_1}{R_1 + \rho} \frac{1}{\gamma + 1} \left[\chi_\pm \left(\mathcal{K} \left(\sqrt{1 - \zeta_\pm^2} \right) \right.\right. \\ &\left.\left. + \ \gamma \Pi \left(1 - \gamma, \sqrt{1 - \zeta_\pm^2} \right) \right) \right]_-^+, \end{aligned} \tag{18}$$

for $\zeta_+ < 1$, and for $\zeta_\pm \geqslant 1$ as

$$B_z = \frac{B_r}{\pi} \frac{R_1}{R_1+\rho} \frac{1}{\gamma(\gamma+1)} \left[\frac{\chi_\pm}{\zeta_\pm} \left(\gamma \mathcal{K} \left(\sqrt{1 - \frac{1}{\zeta_\pm^2}} \right) + \Pi \left(1 - \frac{1}{\gamma^2}, \sqrt{1 - \frac{1}{\zeta_\pm^2}} \right) \right) \right]_-^+, \tag{19}$$

with $\mathcal{K}$ and Π being the complete elliptic integral of the first and third kind and ρ the radial coordinate. Additionally, we have

$$\gamma = (R_1 - \rho)/(R_1 + \rho), \tag{20}$$

$$\zeta_\pm = \sqrt{\frac{(R_1-\rho)^2 + \xi_\pm^2}{(R_1+\rho)^2 + \xi_\pm^2}}, \tag{21}$$

$$\chi_\pm = \frac{\xi_\pm}{\sqrt{(R_1+\rho)^2 + \xi_\pm^2}}. \tag{22}$$

$$\xi_\pm = z_1 \pm L/2 \tag{23}$$

With this the field can be calculated at each position of the disk at a certain distance z_2 from the centre of the coil and a number of points along the radius that can be numerically integrated using the trapezium rule. It is found that dividing the path into a hundred equally spaced points along the radius gives sufficient accuracy and no further increase in accuracy is found by using additional points. Only the axial component of the magnetic field is relevant for (17) as by the right-hand rule it is the only component that contributes to the radial EMF.
Thus, all integrations are carried out with the help of numerical tools in Matlab, both for the direct approach and the field model. The predictions obtained this way can be seen in the Results and Discussions section (see Results & Discussion section 4) where they are compared with the experimental measurements.

3 Experiments

A disk made of brass was cut with grooves along the radius to measure the induced EMF from the centre to radial distances R_2 (0.5 , 1 , 1.5 , …, 4 cm) in different regions of the field. The disk is rotated by a DC motor at 1100 RPM ($\omega \approx 115\,\mathrm{rad\,s^{-1}}$) and positioned at two different points on the axis. At $z_2 = 0$ the disk sits in the centre of the coil and at $z_2 = 1\,\mathrm{cm}$ the disk aligns with the end of the coil. A short coil with $N = 320$ windings, length $L = 2\,\mathrm{cm}$ and mean radius $R_1 = 68\,\mathrm{mm}$ is supplied with 3 A of current and the setup can be seen in Figure 3. Measurements of the induced voltage are obtained by pressing copper wire against the disk and reading from a digital multimeter Hameg HMC8012 with an accuracy of around 0.01 mV for the given measurement range. The torque produced by the motor is strong enough to not slow the rotation and each measurement of voltage is repeated 20 times to obtain a statistical average. The measurements show good reproducibility, the mean and standard deviation can be seen in the Results & Discussion section. Thermoelectric effects are found to be negligible for the chosen set-up, no noticeable effect was detected for the given accuracy of measurement and measurement time frame. It is also found that interchanging the position of the probes only leads to a change in sign of the induced voltage, as is expected. For all reported measurements the positive probe is placed in the centre and the negative probe in the respective groove. Furthermore, a control experiment without the coil present shows that there is no measurable voltage induced in the spinning disk up to the range of measurement provided by the voltmeter. In conclusion the Earth's magnetic field or any influence of stray fields present in the laboratory can be ruled out as a potential source of error.

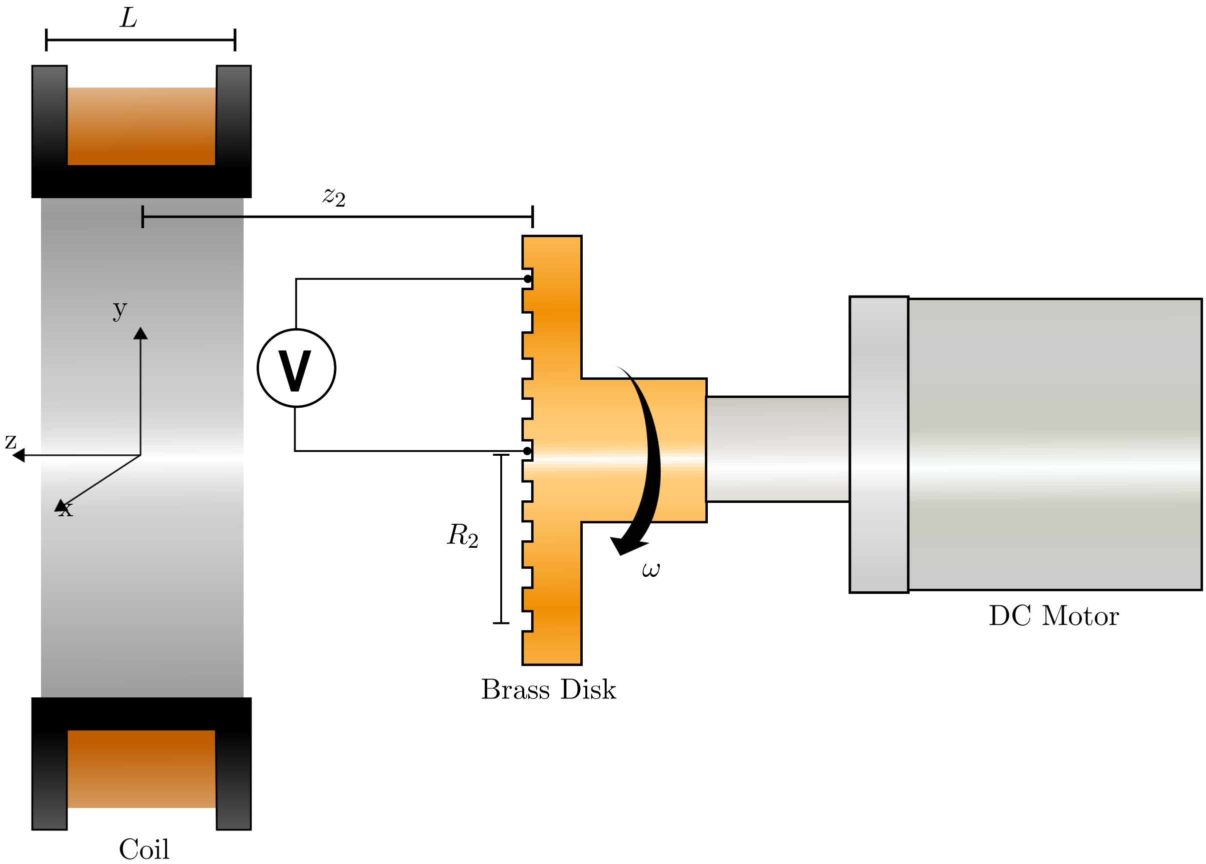


Figure 3: Experimental setup of a brass disk with grooves rotated by a DC motor and a short coil with 320 windings

4 Results & Discussion

Table 1 and Table 2 show the numerical values predicted with both the direct charge interaction model and the field model for the different radii alongside the measured values from the experiments. Table 1 shows results for the disk being positioned in the centre of the coil ($z_2 = 0$) and Table 2 shows values with the disk at the edge of the coil at $z_2 = 1\,\text{cm}$.

Table 1: Predicted and observed values of induced voltage in mV. Measurements were taken at the centre of the coil for different radii as indicated.

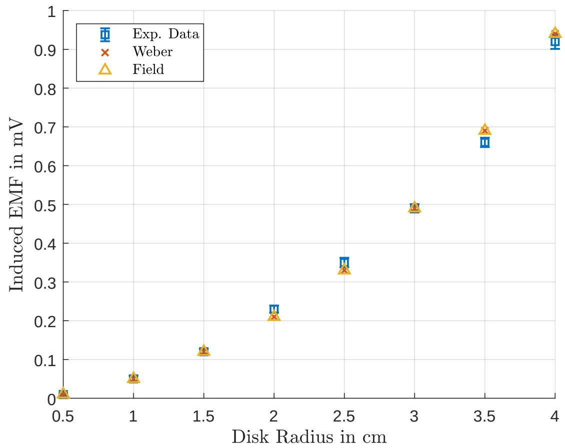
Radius in cm	Direct interaction model EMF in mV	Field model EMF in mV	Measured EMF in mV
0.5	0.01	0.01	0.001(± 0.002)
1	0.05	0.05	0.05(± 0.008)
1.5	0.12	0.12	0.12(± 0.008)
2	0.21	0.21	0.23(± 0.009)
2.5	0.33	0.33	0.35(± 0.012)
3	0.49	0.49	0.49(± 0.011)
3.5	0.69	0.69	0.66(± 0.012)
4	0.94	0.94	0.92(± 0.018)

As can be seen from the predicted and observed values, at any one position (R_2, z_2) both models predict the same induced voltage for the individual radius and position on the cylindrical axis. Further comparison of the predicted values with experimental measurements shows good

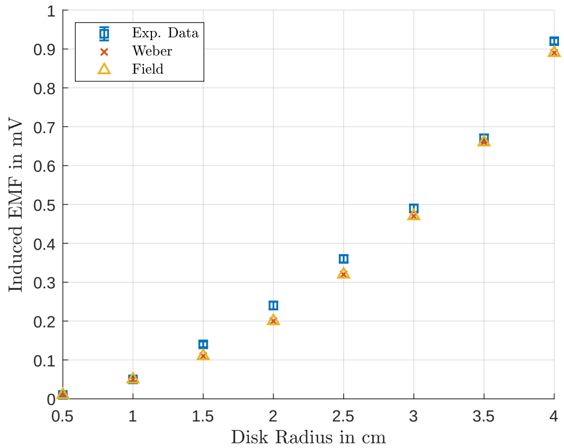
Table 2: Predicted and observed values of induced voltage in mV at the edge of the coil for different radii.

Radius in cm	Direct interaction model EMF in mV	Field model EMF in mV	Measured EMF in mV
0.5	0.01	0.01	0.01(± 0.003)
1	0.05	0.05	0.05(± 0.005)
1.5	0.11	0.11	0.14(± 0.007)
2	0.2	0.2	0.24(± 0.010)
2.5	0.32	0.32	0.36(± 0.009)
3	0.47	0.47	0.49(± 0.009)
3.5	0.66	0.66	0.67(± 0.010)
4	0.89	0.89	0.92(± 0.008)

agreement for both the centre of the coil (see Fig. 4a) and at the edge of the coil with $z_2 = 1\,\text{cm}$ (Fig. 4b). The general trend of the plotted values shows a clear R^2 dependence as expected.



(a)



(b)

Figure 4: Predicted and observed values of induced voltage shown as a function of Radius R_2. a) In the centre of the coil, b) at the edge of the coil

In some respect the performance of the direct interaction model as compared to field theory and the experimental measurements should not be surprising. Maxwell himself pointed out that it is possible to derive Faraday's law from Weber's theory [53]. We can see that the field model and the direct interaction model are both equivalent in predicting the induced EMF. The difference is, however, a conceptual one. An advantage of the Weber theory is, that it does not run into conceptual paradoxes concerning the possibility of a field spinning with a magnet/electromagnet, since the field entity is absent in Weber's formula. Any interaction responsible for the appearance of an EMF is a direct consequence of forces between electrical charges.

Weber's force law is beautifully simple yet profound. It is consistent with Ampere's law of current-element interaction, Newton's third law (action and reaction) and for the static case, it simply reduces to Coulomb's law. It has been demonstrated that it is consistent with Maxwell's equations [54–58]. Research into Weberian electrodynamics has received a significant contribution from Assis [38, 55, 59–61] and has been applied to several pure and applied

problems [43, 52, 57, 62–71].
Since we can obtain accurate predictions of the induced EMF using the direct interaction model, without involving the concept of fields, it raises the question whether the motion of the field lines and the magnetic field itself is important to the problem. Using a direct charge interaction approach, the magnetic field can be thought of as a useful metaphor [54]. If we only need to consider the relative motion of charges, then we can invoke the principal of Ockham's razor and choose that approach which makes the least assumptions and is of minimal complexity. Therefore, the Weber theory offers a possible resolution to the paradox that is synonymous with unipolar induction. Since only the interacting charges are considered and the field as a physical mediator is avoided, the question whether the field rotates or moves with the magnet never arises. The only consideration that needs to be made is the charge motion, which is unambiguous within a chosen frame of reference. The problem can then be analysed with the relative motions in the circuit of a Faraday generator setup [38] by considering the whole circuit including the disk and closing wire as part of the problem. According to Assis and Thober it is thus not the relative motion between disk and magnet, but the relative motion between disk and closing wire that determines if a voltage is induced or not. For the usually discussed three cases: i) spinning disk and stationary magnet; ii) stationary disk and spinning magnet; iii) spinning disk and spinning magnet; the direct-action approach predicts an induced EMF for cases i) and iii) but not for ii), in accordance with experimental observation. It is, however, an important consideration that Maxwell was heavily influenced and inspired by Faraday's experiments that led him to the idea of fields and their actions [72]. This has laid the conceptual foundation for modern physics with relativity theory, quantum field theory and aspects of particle physics which have had some excellent successes. Nonetheless, alternative perspectives can lead to new insight and understanding, especially if a problem is as heavily discussed as unipolar induction is. Interestingly recent experimental investigations [73] have shown unexpected behaviour when expanding the typical unipolar induction experiment to new cases. In such instances it appears that the observed results are difficult to predict and explain. A direct charge interaction model, based on the Weber force, may offer a possible way of explaining such results, although further research is needed.

5 Conclusion

A new model for unipolar induction has been developed. This has been compared with a Lorentz force approach based on field theory and verified against experimental data for a typical Faraday generator setup. Our approach utilises a direct-line-of-action force between the charges involved, where only the movement of the interacting charges is required to explain the observed phenomena. Such a method is beneficial in avoiding paradoxes arising from the consideration of fields. Although the two theories are conceptually different, it can be shown that they both provide the correct result for a homopolar generator. Among the different approaches to calculate induced voltage in the literature, we have shown that it is possible to obtain the voltage by only considering the charges directly involved in the interaction.
Weber's force, by solely considering direct particle interaction provides a foundation for electrodynamics consistent with electron theory, whereas the Maxwell-Lorentz theory is based on a continuum theory of matter and an all-pervading aether that has effectively been replaced by the electromagnetic field acting as a mediator for charge interaction. It is difficult, if not impossible, to distinguish the existence of such a mediator as the field itself cannot be measured directly. Nevertheless, a certain degree of corroboration exists between the two approaches and we consider that they are complementary. Future work will consider other cases where disk and magnet are spinning in unison, induction effects which involve conducting magnets and investigation of the seat of induction. Further experimental work should consider spinning magnets and fields in general, along with electron inertia effects consistent with the Weber model. New

experimental designs can help to narrow down the possible explanations.

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