The Empirical Relevance of the Shadow Rate and the Zero Lower Bound

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Abstract

This paper investigates the evolution of unconventional monetary policies under a binding ZLB constraint for the US economy. In doing so, this study provides a comprehensive empirical assessment on the economic and statistical implications of allowing conventional and unconventional monetary policies to work in mutually exclusive union using shadow rates. Shadow rate Taylor rules and policy counterfactuals implied by time-varying coefficient structural VAR models show: i) one can reconcile plausible economic results using shadow rates when short-term interest rates approach the ZLB; and ii) unconventional monetary policies are a viable response to recession and facilitate stability during economic recovery.

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1 Introduction

A binding zero lower bound (ZLB) constraint results in the monetary policy response to economic downturns being large scale asset purchases, more commonly known as Quantitative Easing (QE), and other lending facilities. These unconventional monetary policies are a direct substitute for changes in the short-term interest rate and aim to provide stimulus to the economy working through an array of channels; such as lowering the long-term end of the yield curve, and to promote trading on financial markets. The implications of the ZLB and the overall impact of unconventional monetary policies, are prominent issues in economic research which are highly relevant for central banks. Both policymakers and economists advocate, from an empirical perspective, that traditional and unconventional monetary policy work in mutually exclusive harmony (see e.g. Yellen (2016), Reifschneider (2016), and Wu and Xia (2016)).

There is a growing literature that examines the economic impact and effectiveness of unconventional monetary policies in isolation (see, among others Gambacorta et al. (2014) and Weale and Wieladek (2016)). Wu and Xia (2016) propose a shadow rate so that economists are able to examine conventional and unconventional monetary policies using data preceding the 2008 recession. The shadow rate, stemming from a term-structure model, co-moves strongly with the Federal funds rate when there is no binding ZLB constraint. When the Federal funds rate approaches its ZLB, thereby conveying no information regarding monetary policy stance, the shadow rate is negative. In particular, their study reveals that the shadow rate behaves similar to the Federal funds rate in models of monetary policy.

Theoretically, Wu and Zhang (2019b) show that a negative shadow rate has identical effects on economic quantities to unconventional monetary policies within a micro-founded New Keynesian model and demonstrate the data-consistent result that a negative supply shock is always contractionary[1]. Sims and Wu (2020) examine how substitutable QE and conventional monetary policy are in a New Keynesian model. Their results show that the observed increase of the Federal Reserve's balance sheet over the ZLB period provides stimulus equivalent to cutting the policy rate to -2%; in line with the empirical decline in the federal funds–shadow rate time series. Sims and Wu (2021) develop a DSGE framework permitting the analysis of QE, forward guidance, and negative interest rate policies. They show such policies can all stimulate output as much as conventional monetary policy. This burgeoning literature provides further substance that shadow rates are a feasible time-series to use in econometric models to overcome issues surrounding the ZLB.

The main contribution of this paper is to investigate the evolution of unconventional monetary policies under a binding ZLB constraint. In doing so, this study provides a comprehensive empirical assessment on the economic and statistical implications of allowing conventional and unconventional monetary policies to work in mutually exclusive union using shadow rates in structural VAR models. Focusing on the US economy, this study seeks to answer two main questions. First, can one reconcile economically plausible results regarding monetary policy

[1] From an international perspective, Wu and Zhang (2019a) propose an open-economy New Keynesian model that accommodates unconventional monetary policy. In particular, they resolve the lower bound anomaly in an open economy New Keynesian model that causes output and terms-of-trade to move in opposite directions following a supply shock.

stance under a binding ZLB constraint? Second, are unconventional monetary policies viable in facilitating economic stability following a deep recession?

Models using the Federal funds rate over the ZLB period yield a high degree of posterior uncertainty. Under a binding ZLB constraint, this indicates the Federal funds rate conveys no information regarding monetary policy stance. Consistent with Wu and Xia (2016), models that use shadow rates throughout the ZLB period, deliver economically plausible results for the monetary transmission mechanism. This paper quantifies the economic importance of unconventional monetary policy shocks from models using shadow rates. Specifically, structural variance decompositions uncover economically meaningful differences over the ZLB period in the percent of variance attributable to monetary policy shocks when using shadow rates.

This study extends on Wu and Xia (2016) from a methodological perspective. Using time-varying coefficient VAR models, this paper backs out model implied shadow rate Taylor rules that allow one to quantify how unconventional monetary policy relates to economic activity and inflation. Adopting this methodology permits one to examine how monetary policy reacts to economic activity and inflation in a dynamic manner. The results show a clear emphasis on economic activity in shadow rate Taylor rules over the ZLB period. Shadow rate Taylor rules also reveal the Federal Reserve's intention of supporting the recovery of the US economy; something that disappears under the conventional Taylor rule. More generally, these findings help us understand how unconventional monetary policies can act as an expansionary monetary policy tool available to central banks under a binding ZLB constraint; as well as promoting recovery from deep recessions.

In mapping to the structural model it also enables one to construct policy counterfactuals. Overall, counterfactual simulations support the findings from shadow rate Taylor rules with two main findings. First, the contraction in GDP growth would have been 1.14% larger in absolute terms followed by sustained volatility. Secondly there would have been a higher risk of entering a deflationary period in 2010-2012 (i.e. during QE2 and Operation Twist) had the Federal reserve placed no emphasis on economic activity over the ZLB period. These results conform with Kapetanios et al. (2012), Baumeister and Benati (2013) and Eberly et al. (2020) who show that without unconventional monetary policies, the respective recoveries of the UK and US economy would have been far more sluggish than realised. Overall, these exercises imply that unconventional monetary policies are a viable way to hinder contractions in GDP growth and also to facilitate economic stability in terms of lower GDP and inflation volatility.

This study also relates with an emerging area that examines economic performance at the ZLB (Wu and Zhang, 2019b; Garín et al., 2019). Debortoli et al. (2019) test the null hypothesis that there are no practical effects on economic performance under a binding ZLB constraint. Their findings suggest little difference in the response of macroeconomic fundamentals to shocks and that macroeconomic volatility was largely unaffected by the ZLB period. Their paper examines the transmission mechanism through impulse response analysis. The approach taken here tests both for differences in the transmission mechanism and economic importance of shocks. Another innovation is the specification here explicitly accounts for unconventional monetary policies using shadow rates; the former use long-term interest rates. Overall this study presents evidence consistent with Debortoli et al. (2019) and Garín et al. (2019). However the main novelty in this paper examines shocks from a fully identified multivariate structural model with a particular focus on monetary policy shocks themselves. Overall the results suggest that, accounting for unconventional monetary policies using the shadow rate, the sensitivity of macroeconomic fundamentals to monetary policy shocks has remained similar over the last 17 years.

All of the main findings are robust to three alternatives; the first uses alternative proxies for economy activity and inflation by using an output gap measure (Wu and Zhang, 2019b) and an alternative proxy for inflation. The second alternative extends the information set by constructing a macro-financial factor using a subset of the FRED-MD database (McCracken and Ng, 2016). Meanwhile the third, hinged upon a transaction model of money demand in the spirit of Benati (2019), incorporates a narrow measure of money into the baseline specification.

This work contributes to three main streams of literature. First, it contributes to the widespread work on monetary policy dynamics, which among many others, includes: Sims and Zha (2006); Primiceri (2005); Benati and Mumtaz (2007); Benati (2008); and Belongia and Ireland (2016). The main innovations of this paper are that the sources of time-variation are tested through a model selection experiment, and that the focal point is the conduct of monetary policy prior to, during, and following the 2008 recession.

Second, this work is pertinent to the growing literature on unconventional monetary policy analysis (see e.g. Kapetanios et al. (2012); Hamilton and Wu (2012); Baumeister and Benati (2013); D'Amico and King (2013); and Swanson and Williams (2014)). In general, this paper supports the consensus view that QE policies were a necessity in harbouring the recovery of the US economy (and others). Papers within this strand of literature typically examine unconventional monetary policies using interest rate spreads (Kapetanios et al., 2012; Baumeister and Benati, 2013) , or in isolation (Gambacorta et al., 2014; Weale and Wieladek, 2016). This paper relates well with Wu and Xia (2016) where conventional and unconventional monetary policies are captured by the shadow rate. The innovation here is the adoption of a methodology that allows monetary policy stance to change over time. This is a natural approach since the objective is to examine the evolution of unconventional policy throughout the ZLB period.

Third, this paper aligns closely with work arguing that the ZLB imposes little constraint on monetary policy. Notably, while central banks are unable to influence short-term policy rates when approaching the ZLB, they are able to influence economic outcomes through other avenues (see e.g. Bundick (2015) and Chen et al. (2012)). The empirical approach by Wu and Xia (2016) provides evidence in favour the ZLB having little constraint on monetary policy. The subsequent theoretical results in Wu and Zhang (2019b) and Wu and Zhang (2019a) show how economic performance is unaffected by the ZLB if unconventional monetary policies in New Keynesian models are tracked through shadow rates. Garín et al. (2019) test predictions of a New Keynesian model for supply shocks at the ZLB. Their analysis reveals that a binding ZLB constraint has no effect on the influence supply shocks have for expected inflation. Overall results presented here are consistent with the aforementioned. The novelty of these results show that one can reconcile plausible economic results using shadow rates over the ZLB period thereby emphasising the use of shadow rates in structural VAR models at tracking monetary policy stance under a binding zero lower bound constraint.

The remainder of this paper is structured as follows. Sections 2 outlines competing time-varying coefficient VAR models. Section 4 discusses economic data and reports the main empirical analysis. Section 5 tests the robustness of these results, and Section 6 concludes.

2 Competing Time-varying Coefficient VAR Models

Analysis begins with the class of time-varying coefficient VAR models with stochastic volatility. The general model with p=2 lags, and N=3 variables is

$$\mathbf{Y}_t = \beta_{0,t} + \beta_{1,t}\mathbf{Y}_{t-1} + \beta_{2,t}\mathbf{Y}_{t-2} + \mathbf{e}_t \equiv \mathbf{X}_t^{\prime}\mathbf{B}_t + \mathbf{e}_t \tag{1}$$

where Y_t is defined as $Y_t \equiv [y_t,\ \pi_t,\ \mathrm{r}_t]^{\prime}$, with y_t being annual growth rate of real GDP, π_t is the annual rate of consumer price inflation, respectively. $\mathbf{X}_t^{\prime}$ contains lagged values of $\mathbf{Y}_t$ and a constant. The VAR's time-varying coefficients are collected in the vector $\mathbf{B}_t$, and conditional on the roots of the VAR polynomial lying outside the unit circle evolve as a driftless random walk.

$$\mathbf{B}_t = \mathbf{B}_{t-1} + u_t, \quad u_t \backsim N(0, \mathbf{Q}_t) \tag{2}$$

Two different types of time-variation are considered regarding the covariance of the parameter innovations. In the first case, $\mathbf{Q}_t = \mathbf{Q}$ where $\mathbf{Q}$ is a full matrix allowing parameters across equations to be correlated. Note that when $\mathbf{Q}$=0, the model reduced to a constant parameter VAR with a stochastic volatility structure. In the second case, $\mathbf{Q}_t$ is a diagonal matrix, whose elements evolve as driftless geometric random walks:

$$\ln \mathbf{q}_{i,t} = \ln \mathbf{q}_{i,t-1} + \nu_t, \quad \nu_t \backsim N(0, \mathbf{Z}_{\mathbf{q}}) \tag{3}$$

This structure on the parameter innovations was first introduced in Baumeister and Benati (2013) under the premise that the conventional model of Primiceri (2005) tended to over-fit during periods of economic tranquillity, and under-fit during periods of economic distress. The innovations of the measurement equation, $\mathbf{e}_t$ are Normal with zero mean and time-varying covariance matrix $\mathbf{\Omega}_t$ which is factored as

$$\mathbf{\Omega}_t = \mathbf{A}_t^{-1}\mathbf{H}_t(\mathbf{A}_t^{-1})^{\prime} \tag{4}$$

where $\mathbf{A}_t$ is a lower triangular matrix with unit diagonal containing the contemporaneous relations between variables in the model, and $\mathbf{H}_t$ is a diagonal matrix containing the reduced-form stochastic volatility innovations. Collecting the non-unit non-zero elements of $\mathbf{A}_t$ and the diagonal elements of $\mathbf{H}_t$ in the vectors, $\mathbf{h}_t \equiv [\mathbf{h}_{1,t},\ \mathbf{h}_{2,t}, \mathbf{h}_{3,t}]^{\prime}$ and $\mathbf{a}_t \equiv [\mathbf{a}_{21,t},\ \mathbf{a}_{31,t}, \ldots, \mathbf{a}_{33,t}]^{\prime}$, they evolve as a geometric random walk and random walk respectively

$$\ln \mathbf{h}_{i,t} = \ln \mathbf{h}_{i,t-1} + \eta_t, \quad \eta_t \backsim N(0, \mathbf{Z}_{\mathbf{h}}) \tag{5}$$

$$\mathbf{a}_t = \mathbf{a}_{t-1} + \zeta_t, \quad \zeta_t \backsim N(0, \mathbf{S}) \tag{6}$$

The innovations in the model, collected in the diagonal matrix $\mathbf{V}$, are jointly Normal and the structural shocks, $\mathbf{v}_t$ are such that, $\mathbf{e}_t \equiv \mathbf{A}_t^{-1}\mathbf{H}_t^{\frac{1}{2}}\mathbf{v}_t$. $\mathbf{S}$ is a block diagonal matrix, which implies that the non–zero and non–unit elements of $\mathbf{A}_t$ that belong to different rows evolve independently. This is a simplifying assumption that permits estimation of $\mathbf{A}_t$ equation by equation. The models are estimated using Bayesian methods where the initial conditions are calibrated using the posterior mean of the coefficients and covariance matrix from a Bayesian VAR model estimated using the first 20 years of data. The Appendix provides specific information with regards to our choice of priors and an outline of the Markov-Chain Monte Carlo (MCMC) posterior simulation algorithm.

2.1 Structural Identification

The structural models are identified using a variant of Algorithm 1 in Arias et al. (2018) that stems from the Rubio-Ramirez et al. (2010) algorithm. Specifically, following Arias et al. (2018) and Rubio-Ramirez et al. (2010), the time-varying structural impact matrix, $\mathbf{A}_{0,t}$ is calculated in the following manner. Given the current state of the economy, the eigenvalue-eigenvector decomposition of the VAR's time-varying covariance matrix at time t is, $\mathbf{\Omega}_t = \mathbf{P}_t\mathbf{D}_t\mathbf{P}'_t$. Then an N × N matrix K is drawn from the $N(0,1)$ distribution and its $\mathbf{QR}$ decomposition is computed. Normalising the elements of the diagonal matrix $\mathbf{R}$ to be positive; the matrix $\mathbf{Q}$ is a matrix whose columns are orthogonal to one another. The time-varying structural impact matrix is computed as $\mathbf{A}_{0,t} = \mathbf{P}_t\mathbf{D}_t^{\frac{1}{2}}\mathbf{Q}'$. More detail is in the Supplementary Appendix.

The economy is subject to three shocks: a supply shock, $\mathbf{v}_t^{\mathbf{s}}$; a demand non policy shock, $\mathbf{v}_t^{\mathbf{d}}$; and a monetary policy shock, $\mathbf{v}_t^{\mathbf{mp}}$ by imposing contemporaneous sign restrictions for each time period following those in Belongia and Ireland (2016). Table 1 reports the contemporaneous response of our variables to each identified shock. All structural inference is carried out in a generalised framework, following Koop et al. (1996), thereby accounting for all sources of model uncertainty; further details are in the Appendix.

Table 1: **Contemporaneous Sign Restrictions**
Notes: This table reports the contemporaneous response of real GDP growth, y_t; inflation, π_t; and the interest rate, i_t with respect to a supply shock, $\mathbf{v}_t^{\mathbf{s}}$; a demand non policy shock, $\mathbf{v}_t^{\mathbf{d}}$; and a monetary policy shock, $\mathbf{v}_t^{\mathbf{mp}}$ respectively. "x" denotes no restriction imposed.

	$\mathbf{v}_t^{\mathbf{s}}$	$\mathbf{v}_t^{\mathbf{d}}$	$\mathbf{v}_t^{\mathbf{mp}}$
y_t	$\geq$	$\geq$	$\leq$
π_t	$\leq$	$\geq$	$\leq$
r_t	x	$\geq$	$\geq$

2.2 Structural Monetary Policy Rules

Structural monetary policy rules are obtained from the reduced-form estimates by factoring the reduced-form covariance matrices as

$$\mathbf{\Omega}_t = \bar{\mathbf{P}}_t^{-1}\bar{\mathbf{D}}_t\bar{\mathbf{D}}_t'(\bar{\mathbf{P}}_t^{-1})' \tag{7}$$

where $\bar{\mathbf{P}}_t$ and $\bar{\mathbf{D}}_t$ are 3×3 matrices of the form

$$\bar{\mathbf{P}}_t = \begin{bmatrix} 1 & -\mathbf{p}_t^{y,\pi} & -\mathbf{p}_t^{y,\mathrm{r}} \\ -\mathbf{p}_t^{\pi,y} & 1 & -\mathbf{p}_t^{\pi,\mathrm{r}} \\ -\mathbf{p}_t^{\mathrm{r},y} & -\mathbf{p}_t^{\mathrm{r},\pi} & 1 \end{bmatrix}, \quad \bar{\mathbf{D}}_t = \begin{bmatrix} \mathbf{d}_{y,t} & 0 & 0 \\ 0 & \mathbf{d}_{\pi,t} & 0 \\ 0 & 0 & \mathbf{d}_{\mathrm{r},t} \end{bmatrix} \tag{8}$$

where $\bar{\mathbf{P}}_t$ is a matrix with unit diagonal elements and the structural impact coefficients within the equations of the model. The matrix $\bar{\mathbf{D}}_t$ is diagonal and contains the volatility of the structural innovations. Therefore the structural representation of our models may be written as

$$\bar{\mathbf{P}}_t \mathbf{Y}_t = \mathbf{G}_{0,t} + \mathbf{G}_{1,t}\mathbf{Y}_{t-1} + \mathbf{G}_{2,t}\mathbf{Y}_{t-2} + \bar{\mathbf{D}}_t \mathbf{v}_t \tag{9}$$

where $\mathbf{G}_{0,t} = \bar{\mathbf{P}}_t \beta_{0,t}$ and $\mathbf{G}_{j,t} = \bar{\mathbf{P}}_t \beta_{j,t}$ for $j = 1, 2$. and $\mathbf{v}_t$ is the 3×1 vector of structural innovations where $\mathbf{v}_t \backsim N(0, I_3)$. The third row of (9) delivers the structural monetary policy rule. The third row of $\bar{\mathbf{P}}_t$ contains the structural impact coefficients within the monetary policy rule associated to GDP growth, $\mathbf{p}_t^{\mathrm{r},y}$; inflation, $\mathbf{p}_t^{\mathrm{r},\pi}$. These coefficients represent the contemporaneous response of the interest rate to movements in GDP growth and inflation. Ignoring the structural shock and its volatility, the third row of (9) delivers the structural monetary policy rule of the model.

$$\mathrm{r}_t = \mathbf{g}_0^{\mathrm{r},t} + \mathbf{p}_t^{i,y} y_t + \mathbf{g}_{1,t}^{\mathrm{r},y} y_{t-1} + \mathbf{g}_{2,t}^{\mathrm{r},y} y_{t-2} + \mathbf{p}_t^{\mathrm{r},\pi} \pi_t + \mathbf{g}_{1,t}^{\mathrm{r},\pi} \pi_{t-1} + \mathbf{g}_{2,t}^{\mathrm{r},\pi} \pi_{t-2} + \mathbf{g}_{1,t}^{\mathrm{r},\mathrm{r}} \mathrm{r}_{t-1} + \mathbf{g}_{2,t}^{\mathrm{r},\mathrm{r}} \mathrm{r}_{t-2} \tag{10}$$

As can be seen, this takes a similar form to the monetary policy rule proposed in Taylor (1993). More specifically, it allows the interest rate to respond to movements in economic activity and inflation, whilst also capturing the interest rate smoothing effect documented in Belongia and Ireland (2016). Since the coefficients are allowed to vary throughout time, this specification permits an investigation into the weight the Federal Reserve places on its objectives for inflation versus economic activity; as well as how stringent they were in adhering to their (model implied) systematic behaviour.

3 Results

3.1 Economic Data

The proxies for economic activity used in this study are US data on real GDP and the Consumer Price Index (CPI). Following the recommendation in Wu and Xia (2016), and similar to Wu and Zhang (2019b), the interest rate is proxied by either the Federal funds rate or a shadow rate, which is a spliced series of the Federal funds rate with the estimated shadow rate of Wu and Xia (2016). GDP and the CPI are converted into annual growth rates, the interest rate is untransformed; data plots can be found in the Supplementary Appendix.

It is noteworthy to mention that, among others, Gertler and Karadi (2015) advise using Treasury yields with longer maturities, such as the 1-year or 2-year Treasury yields to proxy monetary policy under a binding ZLB constraint. Over the ZLB period (2008Q4–2015Q4), the contemporaneous correlation of the shadow rate and 2-year Treasury yield and the shadow rate and 1-year Treasury yield are 0.69 and 0.81 respectively; thereby capturing similar information. The advantage of the shadow rate over these Treasury yields is that during QE2, the 1-year and 2-year Treasury yields are essentially zero and thus also suffer from a binding ZLB constraint. A second advantage, noted in Wu and Zhang (2019b), is the strong resemblance between the shadow rate and Federal Reserve's balance sheet[2].

3.2 Model Evaluation

To evaluate the fit of our competing models, the Bayesian deviance information criterion (DIC) proposed in Spiegelhalter et al. (2002) is used. The DIC consists of two terms, one evaluating the fit of the model, and a penalty term for model complexity. Specifically, the DIC is given by

$$\mathrm{DIC} = \bar{\mathrm{D}} + \mathrm{pD} \tag{11}$$

where $\bar{\mathrm{D}} = -2\mathbb{E}(\ln \mathrm{L}(\boldsymbol{\Lambda}_i))$, the measure of fit, is equal to minus two multiplied by the expected value of the log likelihood evaluated over the draws of the MCMC, and $\mathrm{pD} = \bar{\mathrm{D}} + 2\ln \mathrm{L}(\mathbb{E}(\boldsymbol{\Lambda}_i))$, is the measure of model complexity; with $\ln \mathrm{L}(\mathbb{E}(\boldsymbol{\Lambda}_i))$ being the log likelihood evaluated at the posterior mean of parameter draws. The lower the DIC, the better the model fit. For time-varying coefficient VARs with stochastic volatility, the DIC is estimated using a particle filter that evaluates the likelihood function to deal with the non-linear interaction of the stochastic volatilities (Mumtaz and Sunder-Plassmann, 2013). Restricted variants of time-varying coefficient VAR models, as well as a two-regime Markov Switching VAR (MSVAR) are examined to identify whether the data suggests the need for such flexibility, and indeed the source(s) of time-variation. Restricted variants of the time-varying coefficient models include: a conventional Bayesian VAR; a time-invariant coefficient VAR with stochastic volatility and constant contemporaneous relations; a time-invariant coefficient VAR with stochastic volatility and time-varying contemporaneous relations (i.e. a time-varying covariance matrix); and a time-varying coefficient VAR with constant covariance matrix[3].

Table 2 reports the estimated DIC statistics, along with the measure of model complexity, pD, and the expected value of the log likelihood evaluated over posterior draws of the parameters, $\mathbb{E}(\ln \mathrm{L}(\boldsymbol{\Lambda}_i))$. Panel A contains statistics generated models using the Federal funds rate as the short-term rate of interest. Panel B shows analogous statistics for models using the shadow rate.

As can be seen in Table 2, models that fit the data best, according to DIC statistics, are the time-varying coefficient VAR model with a full, time-invariant, covariance matrix of parameter innovations. It is also evident that these models possess the largest expected log likelihood relative to restricted variants of the TVP VAR models. However, note that models using

[2]Available on request are results using the 2-year Treasury rate in place of the Federal Funds rate; as expected conclusions in the form of a high degree of posterior uncertainty around impulse response functions during QE2 and Operation Twist are present due to the 2-year Treasury rate approaching the ZLB during these periods.

[3]These models were all estimated with standard priors within the literature. In particular, BVARs were estimated with a Minnesota prior on the coefficients, models with constant covariance matrices were assumed to have inverse-Wishart priors (see e.g. Koop and Korobilis (2010)), and those with time-varying parameters or stochastic volatility were estimated using analogous priors to the time-varying coefficient VAR models as outlined in the Appendix.

Table 2: **DIC statistics for Competing Models**

Notes: This table reports model evaluation statistics for competing time-varying coefficient VAR models, and restricted variants. In Panel A, each model is estimated using the Federal funds rate as the short-term interest rate. Panel B shows analogous estimates where the Federal funds rate is replaced with the shadow rate as in Wu and Xia (2016). DIC is the estimated Bayesian DIC statistic proposed in Spiegelhalter et al. (2002); pD is the measure of model complexity which Spiegelhalter et al. (2002) notes as the effective number of parameters within the model; and $\mathbb{E}(\ln \mathrm{L}(\mathbf{\Lambda}_i))$ is the expected value of the log likelihood function evaluated at the posterior draws of the model parameters. Row 1 of Panels A and B refer to time-varying coefficient VAR model of Primiceri (2005) where the covariance matrix of parameter innovations is time-invariant, but parameters across equations are allowed to be correlated. Row 2 refers to TVP VAR models where the covariance matrix of the parameter innovations is diagonal, with each element along the main diagonal evolving as a geometric random walk. Row 3 refers to a two-regime Markov-Switching VAR, and Rows 4-7 of Panels A and B refer to restricted variants of the time-varying coefficient models.

A: Models using Fed Funds Rate, r = i_t	DIC	pD	$\mathbb{E}(\ln \mathrm{L}(\mathbf{\Lambda}_i))$
TVP VAR $\mathbf{Q}_t = \mathbf{Q}$, with stochastic volatility	88.32	45.50	-21.41
TVP VAR $\mathbf{Q}_t = \mathbf{Q}_t$, with stochastic volatility	99.76	38.44	-30.66
Two-regime MS-VAR	190.19	35.67	-77.26
TVP VAR constant covariance matrix	1034.41	63.99	-485.21
BVAR with time-varying covariance matrix	161.92	43.43	-59.25
BVAR with stochastic volatility	170.72	49.58	-60.57
constant coefficient BVAR	1758.59	44.59	-857.00
B: Models using Shadow Rate, r = i_t^s	**DIC**	**pD**	**$\mathbb{E}(\ln \mathrm{L}(\mathbf{\Lambda}_i))$**
TVP VAR $\mathbf{Q}_t = \mathbf{Q}$,with stochastic volatility	116.13	20.32	-47.91
TVP VAR $\mathbf{Q}_t = \mathbf{Q}_t$, with stochastic volatility	148.66	43.47	-52.59
Two-regime MS-VAR	216.20	38.34	-89.3251
TVP VAR constant covariance matrix	1091.2	52.68	-519.26
BVAR with time-varying covariance matrix	171.99	35.47	-68.26
BVAR with stochastic volatility	181.21	41.74	-69.73
constant coefficient BVAR	1762.55	44.40	-859.07

the Federal funds rate, when benchmarked against their analogous model using the shadow rate, have lower estimated DIC statistics. Regarding model complexity, it is clear they are similar across all restricted variants of time-varying coefficient models, with the largest model complexity being associated to TVP VARs with a constant variance matrix. This, corresponding with Koop et al. (2009), highlights the importance of modelling time-variation in the volatility of macroeconomic variables. Based on the results provided in Table 2, the conventional time-varying coefficient VAR models of Primiceri (2005) are estimated using the Federal funds rate and shadow rate respectively.

3.3 Monetary Policy Dynamics

Figure 1 plots the posterior median and 80% point-wise equal-tailed probability bands of the impulse response functions of real GDP growth and inflation with respect to a monetary policy shock at selected dates[4]. The dates cover 2009Q2–2017Q4 which corresponds to periods both during, and following, a binding zero lower bound constraint in the US. The first four dates are the mid points of QE periods implemented by the Fed (i.e. 2009Q2, 2011Q1, 2012Q2, 2013Q2), with 2012Q2 being the mid point of Operation Twist. The final date chosen is 2017Q4 which represents the post zero lower bound period.

It is clear that the response of GDP growth and inflation with respect to a monetary policy shock are qualitatively similar in 2009Q2 and 2017Q4. This is to be expected in 2017Q4 as the ZLB constraint no longer binds. However, there is a less clear explanation for the similarities we observe in 2009Q2, since the US economy exhibits a binding ZLB constraint from 2008Q4. A possible explanation for this is to note that: i) the model consists of two lags; and ii) the MCMC Gibbs sampling algorithm mixes future and past realisations (Benati and Mumtaz, 2007). Therefore around the period the US approaches the ZLB one expects to see these similarities in the posterior distribution of impulse response functions. Note that as time progresses, one expects to see these similarities disappear; we also see this in the slightly higher degree of posterior uncertainty stemming from the model using the Federal funds rate.

The second, third, and fourth row confirm the expectation that similarities between impulse response functions disappear over the ZLB period. This phenomenon is found in Wu and Xia (2016). We can see that there is a substantially higher degree of posterior uncertainty in the response of GDP and inflation when the ZLB is binding. The differences in posterior uncertainty from the model using the shadow rate and the model using the Federal funds rate is highest during Operation Twist and QE3. Again this is expected since the Federal funds rate remains constant at effectively zero, thus providing no information about monetary policy stance[5].

To assess the statistical significance between differences in the transmission of monetary policy shocks stemming from each respective model, the probability that the four quarter accu-

[4]The impulse response functions of the short-term interest rates are not reported as they are uninteresting; however, they are available on request. The only noteworthy point regarding these is that response of the Federal funds rate with respect to a monetary policy shock is substantially more persistent, relative to the response of the shadow rate to a monetary policy shock over selected dates. Statistical evidence of this is reported in Table 3 of the paper.

[5]Available on request are estimates of impulse response functions using 1-year and 2-year Treasury yields. These convey the same message as in Figure 1. This is completely expected since during QE2, these yields are approaching the ZLB.

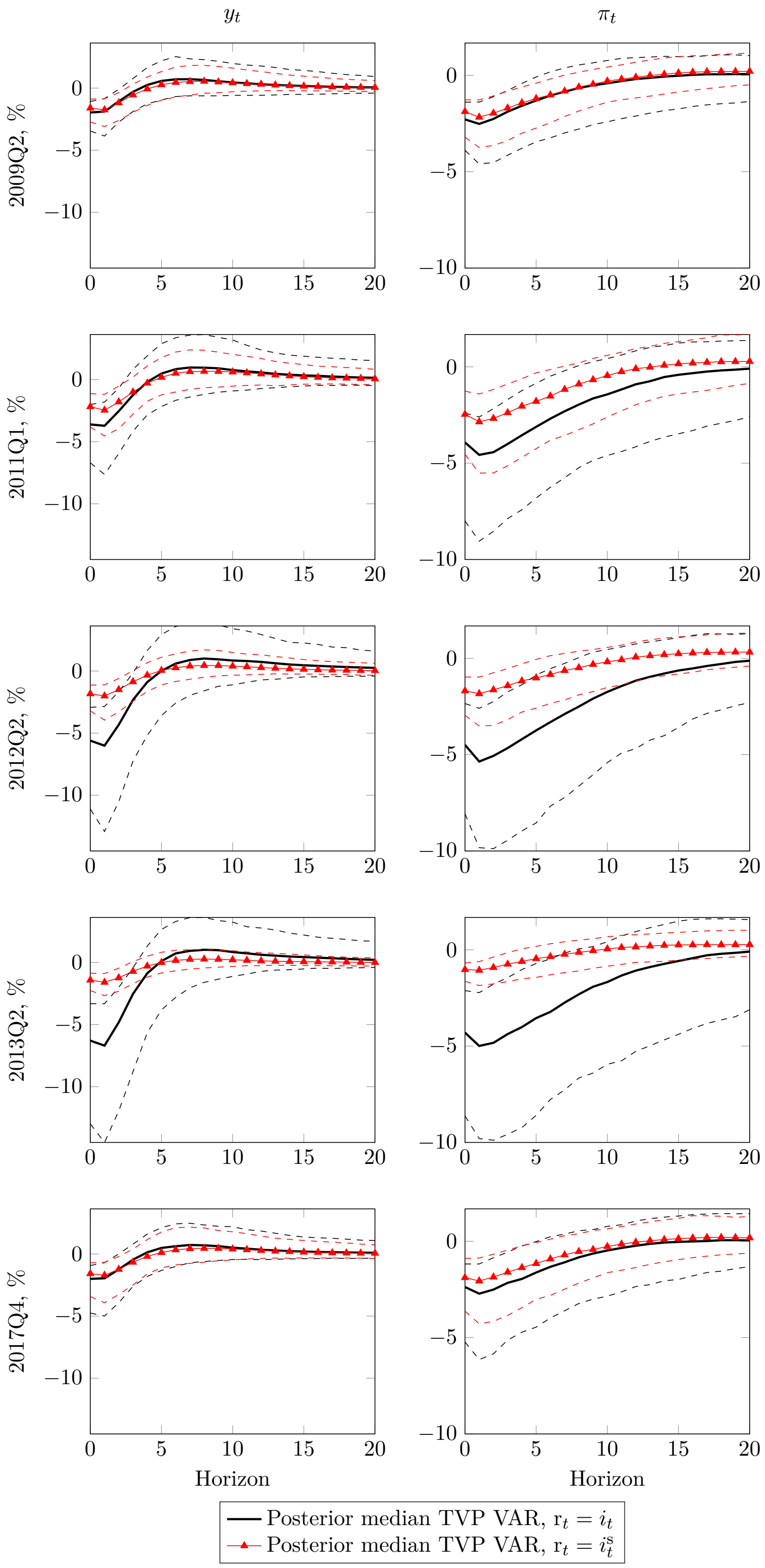

Figure 1: **Impulse Response Functions of Monetary Policy Shocks for GDP growth and Inflation during and after the Zero Lower Bound Period**

Notes: This figure plots the posterior median and 80% point-wise equal-tailed probability bands of the impulse response functions of real GDP growth, y_t; and Consumer price inflation, π_t. In each model, the short-term interest rate is proxied by either the Federal funds rate, $r_t = i_t$, or the shadow rate, $r_t = i_t^s$ across selected dates. The consecutive rows in the figure correspond to 2009Q2, 2011Q1, 2012Q2, 2013Q2, and 2017Q4. Note that the first four rows, all correspond to the mid-points of Quantitative Easing periods, with 2012Q2 representing the mid-point of Operation Twist. 2017Q4 represents the post zero lower bound period. Impulse responses are plotted over a 20 quarter horizon (x-axis) and expressed in % (y-axis).

mulated impulse response functions for variable $x = \{y_t,\ \pi_t,\ \mathrm{r}_t = \{i_t,\ i_t^{\mathrm{s}}\}\}$ from the TVP VAR model using the Federal funds rate during period T is greater than the analogous 4 quarter accumulated impulse response function implied by the model using the shadow rate of Wu and Xia (2016) are tested. Following Cogley et al. (2010), a statistical difference is observed if the probability is lower (greater) than 0.1 (0.9). A value lower (greater) than 0.1 (0.9) indicates that the four quarter accumulated response is smaller (greater) when using the Federal funds rate. Given that a monetary policy shock causes contractions to GDP and inflation, probabilities lower than 0.1 indicate that the contraction to the variable is larger.

Three factors emerge from Table 3. First, the response of the Federal funds rate, relative to the response of the shadow rate, is more persistent both during and following the ZLB period. Second, models using the shadow rate suggest that inflation is statistically less sensitive to monetary policy shocks from 2009 to 2017. Thirdly, during QE2, Operation Twist, and QE3, the contraction in GDP with respect to monetary policy shocks is relatively more subdued when using the shadow rate[6].

Table 3: **Assessing Statistical Differences in the Transmission of Monetary Policy Shocks throughout and after the Zero Lower Bound Period. The Federal Funds Rate vs. the Shadow Rate**
This table reports the probability that the four quarter accumulated response of variable $x = \{y_t, \pi_t, \mathrm{r}_t = \{i_t, i_t^{\mathrm{s}}\}\}$ with respect to a monetary policy shock, from the TVP VAR model where the short-term interest rate is the Federal funds rate, $\mathrm{IRF}_{x,T}^{\mathrm{MP},4,\mathrm{r}=i_t}$, is greater than the four quarter accumulated response of variable x, with respect to a monetary policy shock, from the TVP VAR model using the shadow rate of Wu and Xia (2016) as a proxy for the short-term interest rate for a given time period T, $\mathrm{IRF}_{x,T}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{s}}}$. Therefore: $\Pr\left(\mathrm{IRF}_{x,T}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,T}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{s}}}\right)$. A statistical difference is observed when the probability is less (greater) than 0.1 (0.9). A value lower (greater) than 0.1 (0.9) implies that the IRF implied by the TVP VAR using the Federal funds rate (shadow rate) in time period T is smaller (larger) than that implied by the TVP VAR using the shadow rate (Federal funds rate).

$x=$	y_t	π_t	r_t
$\Pr\left(\mathrm{IRF}_{x,2009Q2}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2009Q2}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{s}}}\right)$	0.88	0.99	1.00
$\Pr\left(\mathrm{IRF}_{x,2011Q1}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2011Q1}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{s}}}\right)$	0.97	0.99	0.95
$\Pr\left(\mathrm{IRF}_{x,2012Q2}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2012Q2}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{s}}}\right)$	0.99	1.00	0.83
$\Pr\left(\mathrm{IRF}_{x,2013Q2}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2013Q2}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{s}}}\right)$	1.00	1.00	0.80
$\Pr\left(\mathrm{IRF}_{x,2017Q4}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2017Q4}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{s}}}\right)$	0.85	0.96	0.97

To explore the economic importance of monetary policy shocks, frequency domain structural variance decompositions of US macroeconomic variables are computed; details of the procedure are provided in the Appendix. Figure 2 plots the posterior median and 80% highest posterior density intervals of the proportion of variance of US macroeconomic variables attributable to monetary policy shocks. From density intervals, there are negligible statistical differences between models using the Federal funds rate and shadow rate[7].

[6]Note that these results hold when examining 8, 12, and 20 quarter accumulated responses.

[7]It is worth noting here that a feature of these class of models are large error bands. However, our variance decompositions are consistent with Benati and Mumtaz (2007) and Benati (2008), in that monetary policy shocks play a negligible role in the Great Moderation.

However, there are economically significant differences from posterior median estimates over the ZLB period. Specifically, during QE2 the proportion of variance in GDP growth explained by monetary policy shocks using the shadow rate is 24% which is some 6% higher than that implied by the model using the Federal funds rate. During the very same period, expectedly, the percent of variance explained by monetary policy shocks of the shadow rate is 24.5%; 10% higher than the percent of variance explained by monetary policy shocks of the Federal Funds rate. Furthermore, in 2015Q4 as the Federal Reserve announces rises in the short-term interest rate, the proportion of inflation variance explained by monetary policy shocks from the model using the shadow rate is double that of the model using the Federal funds rate.

On the whole, these results indicate that models using short-term interest rates that are subject to a binding ZLB constraint overstates the transmission of monetary policy shocks when the constraint binds. The analysis presented in Figure 1 and Table 3 imply, from both a statistical and economic perspective, significant differences when replacing the Federal funds rate with its shadow rate alternative. Furthermore, the variance decompositions reveal economically meaningful differences in the importance of monetary policy shocks. The increased proportion of variance explained by monetary policy shocks from the TVP VAR using the shadow rate highlights that unconventional monetary policies can be effective tools in response to deep recession. Taken together these results provide further substance to those in Wu and Xia (2016) whilst also demonstrating empirical validation of the theoretical findings in Wu and Zhang (2019b).

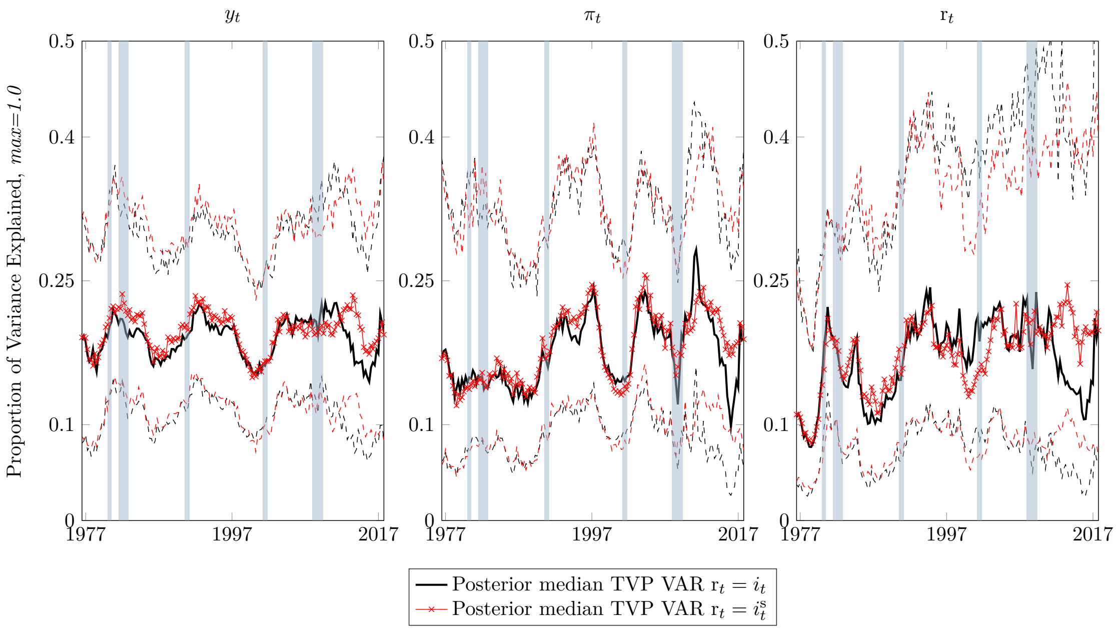

Figure 2: **Frequency Domain Structural Variance Decomposition: The Importance of Monetary Policy Shocks from 1976Q3–2017Q4**

Notes: This figure plots the posterior median and 80% point-wise equal-tailed probability bands of the percent of variance explained by monetary policy shocks for real GDP growth, y_t; Consumer price inflation, π_t; and the short term interest rate, $\mathrm{r}_t = \{i_t, i_t^{\mathrm{s}}\}$, implied by TVP VARs using the Federal funds rate, $\mathrm{r}_t = i_t$; and the shadow rate, $\mathrm{r}_t = i_t^{\mathrm{s}}$, from 1976Q3–2017Q4. Grey bars indicate NBER recession dates.

3.4 Dynamic Taylor Rules

Wu and Zhang (2019b) demonstrate theoretically that the impact of unconventional monetary policies for the economy is identical to a negative shadow rate. From an empirical perspective, they show the Taylor rule is a good description of shadow rate dynamics[8]. In the context of this study, the time-varying Taylor rule and shadow rate Taylor rule are computed by mapping from the reduced-form model to the structural model using algorithm 1 in Arias et al. (2018) and can be thought of as an extension on the empirical illustration in Wu and Zhang (2019b). In particular this exercise allows parameters, and volatilities to change over time. In doing so, this analysis assesses the dynamic monetary policy response to economic activity and inflation during the ZLB period whilst accounting for unconventional monetary policies in a structural multivariate model.

Figures 3 and 4 focus on the impact and long-run coefficients implied by the Taylor rule from each respective model. Figure 3 shows the impact coefficients, $\mathbf{p}_t^{r,y}$, $\mathbf{p}_t^{r,\pi}$ thereby tracking the contemporaneous responses of the Federal funds rate and shadow rate to movements in GDP and inflation. Figure 4 plots the long-run coefficients, computed in the same manner as Belongia and Ireland (2016), that capture the change in the Federal funds/shadow rate following a permanent one percentage point increase in GDP growth or inflation.

It is clear there are substantial differences in the impact and long-run coefficients associated to GDP growth implied by the Taylor rule and shadow rate Taylor rule during, and following, the ZLB period. The contemporaneous response of the shadow rate to movements in GDP throughout 2009–2015, from posterior median estimates, fluctuates around 0.4. Comparing this with the (posterior median) contemporaneous response of the Federal funds rate over the same period at around 0.1. From posterior median estimates, there is also a clear divergence in the long-run coefficients associated to GDP from the Taylor rule and shadow rate Taylor rule from 2009–2017[9]. During 2014, the impact of a permanent one percentage point increase in GDP growth yields an increase in the shadow rate of 4%; double that of the increase in the Federal funds rate. These differences in structural impact and long-run coefficients associated to GDP growth are attributable to the fact that the Federal funds rate cannot summarise expansionary monetary policy under a binding ZLB constraint.

It is also evident that there are negligible differences in the impact and long-run coefficients for inflation implied by the Taylor rule and shadow rate Taylor rule. Consistent with Belongia and Ireland (2016), the impact and long-run coefficients on inflation from both models exhibit a gradual downward trend, which becomes particularly prominent when the Fed funds rate hits the ZLB. In both cases, there is a surge in the impact and long-run coefficients, in 2015Q4, when the Federal Reserve started to move away from the ZLB. The evolution of the impact and long-run coefficients associated to inflation reflect a combination of two factors. First is relatively low inflation rates throughout the ZLB period which suggests the Federal Reserve had no need to alter policy stance through (unconventional) monetary policies as a result of

[8]Note that Wu and Zhang (2019b) estimate the shadow rate Taylor rule using the output gap and inflation. The subsequent section reports estimates using the output gap as a robustness exercise.

[9]The differences begin to appear following the burst of the dot-com bubble in 2001. This could arise due to the fact that estimates are conditional on the full sample, therefore the Gibbs sampler in this case could be mixing the future and past around this period such that the break is nuanced.

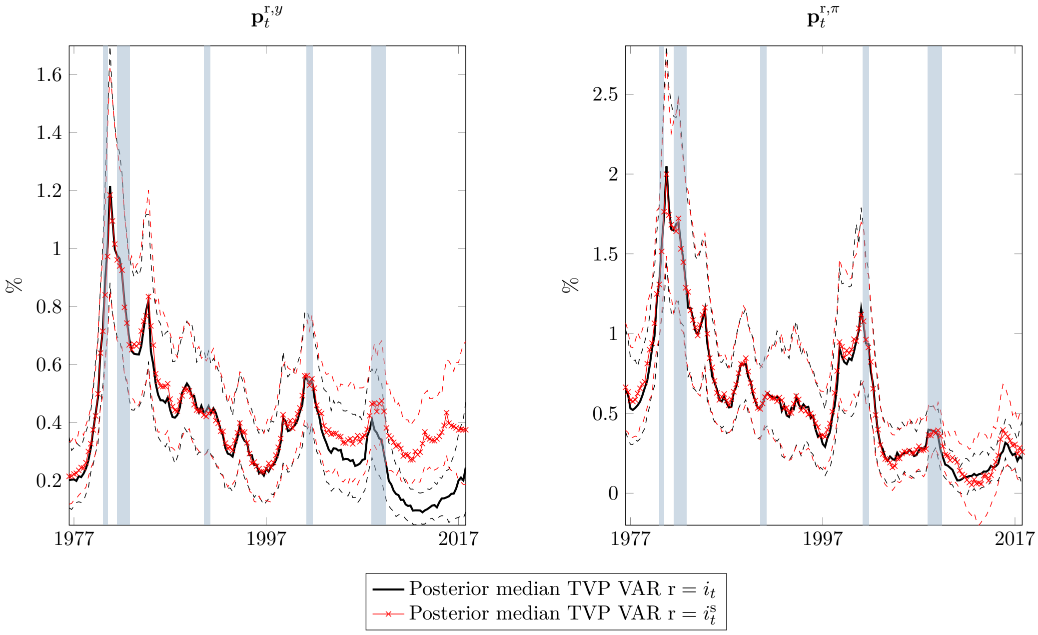

Figure 3: **Structural Impact Coefficients Implied by Structural Monetary Policy Rules from 1976Q3–2017Q4**

Notes: This figure plots the posterior median and 80% point-wise equal-tailed probability bands of the structural impact coefficients associated to real GDP growth, $\mathbf{p}_t^{r,y}$ and Consumer price inflation, $\mathbf{p}_t^{r,\pi}$. In each model, the short-term interest rate is proxied by either the Federal funds rate, $r_t = i_t$, or the shadow rate, $r_t = i_t^s$. Grey bars indicate NBER recession dates.

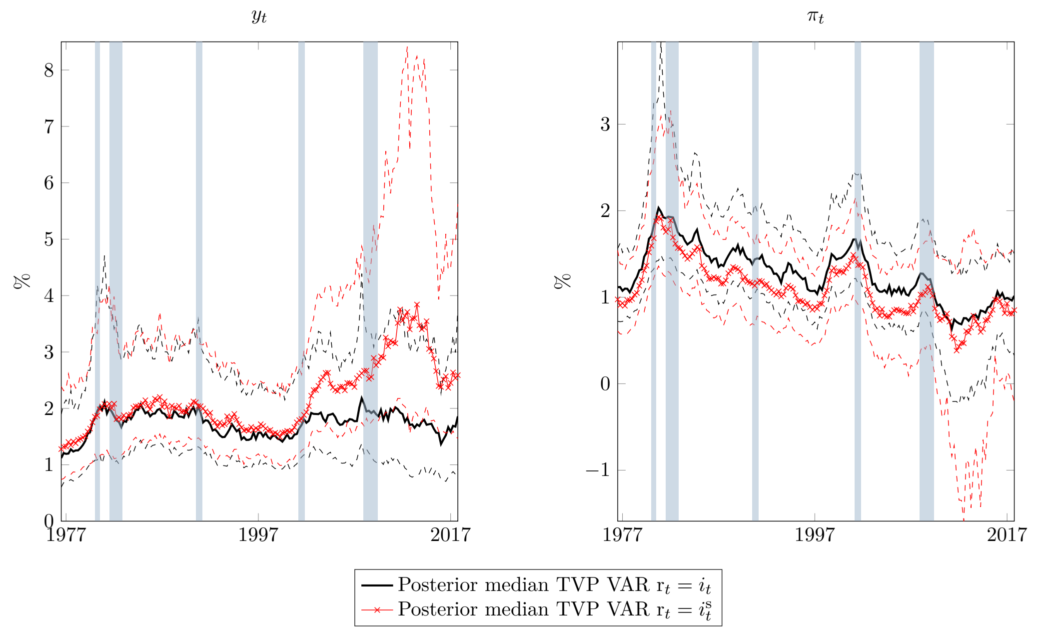

Figure 4: **Long-run Coefficients Implied by Structural Monetary Policy Rules from 1976Q3–2017Q4**

Notes: This figure plots the posterior median and 80% point-wise equal-tailed probability bands of the long-run coefficients associated to real GDP growth, y_t and Consumer price inflation, π_t as implied by the structural monetary policy rule. In each model, the short-term interest rate is proxied by either the Federal funds rate, $r_t = i_t$, or the shadow rate, $r_t = i_t^s$. Grey bars indicate NBER recession dates.

imminent inflationary pressures. Second, the Federal Reserve is prioritising the facilitation of, or hindering further declines to, GDP growth in response to the 2008 recession.

Table 4: **Assessing Statistical Differences in Taylor Rules throughout and Following the Zero Lower Bound Period. The Federal Funds Rate vs. the Shadow Rate**
Panel A of this table reports the probability that the structural impact coefficient associated to variable $x = \{y_t,\ \pi_t,\ \mathrm{r}_t = \{i_t,\ i_t^{\mathrm{S}}\}\}$, within the structural monetary policy rule from the TVP VAR model where the short-term interest rate is the Federal funds rate at period T, $\mathbf{p}_T^{\mathrm{r}x,\mathrm{r}=i_t}$, is greater than the structural impact coefficient associated to variable x, within the structural monetary policy rule from the TVP VAR model using the shadow rate of Wu and Xia (2016) as a proxy for the short-term interest rate at period T, $\mathbf{p}_T^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}$. Therefore: $\Pr\left(\mathbf{p}_T^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_T^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$. Panel B of this table reports analogous statistics, but for the long-run coefficients associated to variable x, $\mathrm{LRC}_T^{\mathrm{r}x}$, implied by the structural monetary policy rules from TVP VARs using the Federal funds rate and shadow rate respectively. Therefore: $\Pr\left(\mathrm{LRC}_T^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_T^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$. A statistical difference is observed when the probability is less (greater) than 0.1 (0.9). A value lower (greater) than 0.1 (0.9) implies that the structural impact/long-run coefficient implied by the TVP VAR using the Federal funds rate (shadow rate) in time period T is less than that implied by the TVP VAR using the shadow rate (Federal funds rate).

A: Structural Impact Coefficients		
$x=$	y_t	π_t
$\Pr\left(\mathbf{p}_{2009Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2009Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.25	0.37
$\Pr\left(\mathbf{p}_{2011Q1}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2011Q1}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.12	0.26
$\Pr\left(\mathbf{p}_{2012Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2012Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.04	0.60
$\Pr\left(\mathbf{p}_{2013Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2013Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.02	0.58
$\Pr\left(\mathbf{p}_{2017Q4}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2017Q4}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.33	0.42
B: Long Run Coefficients		
$x=$	y_t	π_t
$\Pr\left(\mathrm{LRC}_{2009Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2009Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.29	0.54
$\Pr\left(\mathrm{LRC}_{2011Q1}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2011Q1}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.28	0.50
$\Pr\left(\mathrm{LRC}_{2012Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2012Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.23	0.56
$\Pr\left(\mathrm{LRC}_{2013Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2013Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.23	0.54
$\Pr\left(\mathrm{LRC}_{2017Q4}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2017Q4}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.35	0.56

Having established the economic significance of the differences between the Taylor rule and shadow rate Taylor rule, Table 4 assesses the statistical credibility of differences between monetary policy rules from both models during and following the ZLB period. Panel A reports the probability that the impact coefficients associated to GDP growth and inflation from the conventional Taylor rule during period T=\{2009Q2, 2011Q1, 2012Q2, 2013Q2, 2017Q4\} is greater than the analogous impact coefficient implied by the shadow rate Taylor rule. Therefore, the probability that the structural impact coefficient of variable $x = \{y_t,\ \pi_t\}$ during time T implied by the Taylor rule is greater than the comparable impact coefficient implied by the shadow rate Taylor rule is denoted as $\Pr\left(\mathbf{p}_T^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_T^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$. Panel B reports the same probabilities, but for the long-run coefficients, LRC, denoted as $\Pr\left(\mathrm{LRC}_T^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_T^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$. Statistical differences emerge in the impact coefficients associated to GDP growth throughout Operation Twist and QE3. Specifically during Operation Twist $\Pr\left(\mathbf{p}^{\text{r}y,\text{r}=i_t}_{2012Q2} > \mathbf{p}^{\text{r}y,\text{r}=i^{\text{S}}_t}_{2012Q2}\right)$=0.04, and during QE3, $\Pr\left(\mathbf{p}^{\text{r}y,\text{r}=i_t}_{2013Q2} > \mathbf{p}^{\text{r}y,\text{r}=i^{\text{S}}_t}_{2013Q2}\right)$=0.02[10].

In general, the conventional Taylor rules and shadow rate Taylor rules provide a clear depiction of the fact that the Federal funds rates harbours no information regarding (expansionary) monetary policy stance under a binding ZLB constraint. The shadow rate Taylor rule however, clearly captures expansionary unconventional monetary policies implemented in response to the Great Recession whilst retaining the characteristics of the conventional Taylor rule prior to the ZLB era. From both an economic and statistical perspective, the structural impact coefficients and long-run coefficients throughout the ZLB period, suggest that the Federal Reserve placed a larger emphasis on economic activity relative to inflation; something that is absent when looking at the conventional Taylor rule.

To provide an idea of how unconventional monetary policies, and monetary policy stance, affect realised the performance of the economy in response to the Great Recession, three counterfactual experiments are conducted. First, throughout the ZLB period, the shadow rate is restricted to be greater than or equal to zero. This experiment essentially shuts off the implementation of unconventional monetary policies[11] Panel A of Figure 5 shows the implied path of GDP growth, inflation and the shadow rate in the absence of unconventional monetary policies, along with the model implied history of each time series. The counterfactual path of GDP growth implies the recovery would have been far more sluggish had there been no implementation of asset purchase facilities. In particular, in the quarter following QE1, the implied counterfactual for GDP is 1.14 percentage points lower than the actual simulated history. This exercise also suggests that inflation would have been substantially lower in the absence of QE. For instance during QE3 in 2014Q2, the implied rate of inflation is 1.29% lower than the actual simulated path.

The second additional counterfactual experiment assumes a different systematic component of the shadow rate Taylor rule throughout the ZLB period. Specifically, this counterfactual predicts outcomes for GDP growth, inflation, and the shadow rate if the Federal Reserve stopped considering inflation within their shadow rate Taylor rule following the 2008 recession. The final experiment shuts of the reaction of the shadow rate Taylor Rule to movements in GDP growth. Therefore the counterfactual reports the outcomes for US macroeconomic variables if the Federal Reserve reacted only to lagged interest rates and inflation during the ZLB period.

Panels B and C plot the model implied history of US macroeconomic data, along with the posterior median counterfactual paths under each respective scenario. From Panel B, it is clear that if the Federal Reserve stopped considering inflation in their monetary policy rules throughout the ZLB period, monetary policy would have been even more expansionary than

[10]To put these probabilities in the context of Cogley et al. (2010), these values imply that 96% and 98% of the joint distribution lie above the 45° line; assuming the impact coefficient associated to GDP growth from the Taylor rule is on the x-axis.

[11]In particular,the structural shocks of the model are manipulated such that the implied path of the shadow rate cannot surpass the zero threshold. Therefore, this exercise is free from the criticisms of structural VAR based policy counterfactuals of Sargent (1979). With regards the unconventional monetary policies, and following the theoretical results in Wu and Zhang (2019b), the influence of a negative shadow rate is equivalent to that of unconventional monetary policies. Therefore it can be perceived as a summary statistic for unconventional monetary policy that is mapped into the interest rate domain. Thus, constraining the shadow rate to be $\geq$0 can be thought of as an absence of QE and other asset purchase facilities.

realised. The differences between realised GDP growth and the implied path, as expected, are negligible. However, inflation would have been substantially lower during Operation Twist and QE3 entering negative territory prior to the Federal Reserve tightening monetary policy in 2015Q4. Turning to Panel C, t is clear that, had the Federal Reserve considered only inflation in their conduct of monetary policy, GDP growth volatility would have been notably larger during the ZLB period. Furthermore, the implied path for inflation indicates a much sharper decline than realised, followed by a stubborn recovery hitting the Fed's target of 2% during Operation Twist[12].

Taken together, these policy counterfactuals provide three key findings. First, had there been no asset purchase facilities the recovery of the US economy would have been substantially hindered throughout 2009–2015. Second if the Federal Reserve placed no weight on inflation, monetary policy would have been even more expansionary with little benefit in terms of GDP growth and the risk of entering a deflationary phase in 2015. Third, had the Federal Reserve placed no weight on output, the contraction in GDP growth would have been 1.5% larger in absolute magnitude in 2008 followed by sustained volatility over the ZLB period.

These simulations link well with those in: i) Kapetanios et al. (2012), who show that QE1 may have had a (peak) effect of 1.5% on the level of UK real GDP; ii) Baumeister and Benati (2013), who provide evidence that in the absence of QE1, US GDP growth reaches a trough of -10% in 2009Q1; and iii) Eberly et al. (2020), who use a structural VAR that uses high frequency jumps in asset prices around FOMC meetings as external instruments. Their results show that unconventional policies support recovery, with earlier unconventional policy action suggesting a faster recovery.

Overall, the comparison of conventional and shadow rate Taylor rules, and policy counterfactuals further justify the theoretical results in Wu and Zhang (2019b) that the shadow rate acts as a useful summary statistic to track unconventional monetary policies. These results go further in quantifying how unconventional monetary policy evolved over the ZLB period; and how economic fundamentals were influenced. Building on this, we can see that unconventional monetary policies contribute not only to hindering contractions in output during periods of recession, but also in stabilising price changes and GDP growth volatility during recovery. Ultimately, this highlights the empirical advantages of utilising shadow rates during an ultra-low interest rate environment.

[12]Of course the latter two counterfactuals suffer from the Lucas Critique. However unlike the counterfactual monetary policies presented in Sims and Zha (2006), the difference between actual (model implied in this case) and counterfactual paths for US macroeconomic variables are not large enough to warrant disbelief (Benati and Mumtaz, 2007). The only discernible difference is the implied and actual paths of the shadow rate in Panel B of Figure 5. However, noting that interpretation of the shadow rate is a summary statistic for monetary policy stance, it is entirely plausible that unconventional monetary policies could have been even more expansionary should economic activity have warranted such a response.

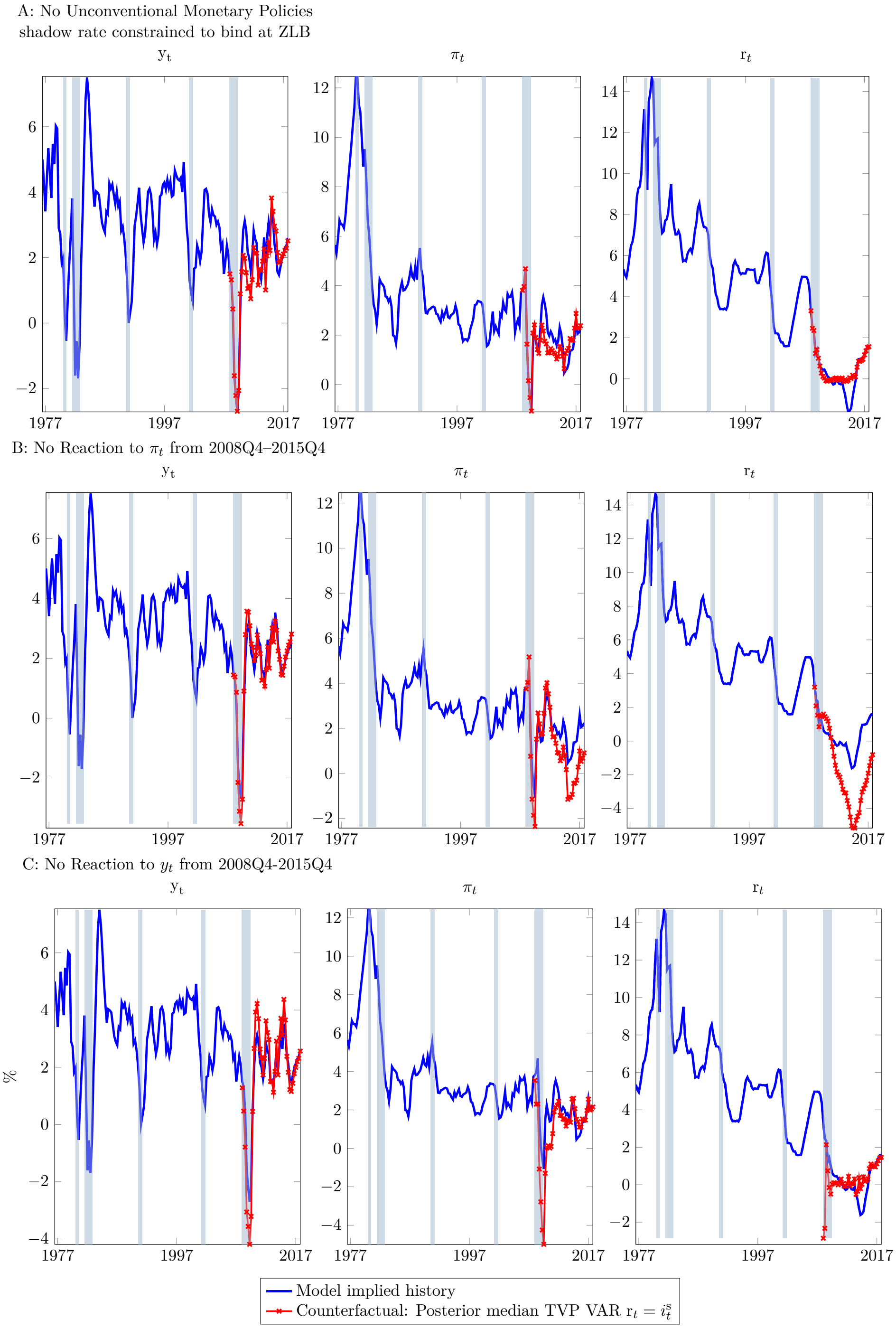

Figure 5: **Counterfactual Simulations of Shadow Rate Taylor Rules**

Notes: Panel A of this figure plots the model implied history for variable $x = \{y_t, \pi_t, r_t = i_t^s\}$ (blue lines) along with the counterfactual (implied) value (red lines) if there had been no monetary policy shocks from 2008Q4 to 2015Q4, and assuming that the ZLB constraint binds. Essentially implying no unconventional monetary policies were implemented. Panel B of this figure plots the model implied history for macroeconomic variables along with the counterfactual (implied) value if the Federal Reserve had stopped reacting to inflation in their structural monetary policy rule during the ZLB period. Panel C plots the implied history for macroeconomic variables along with the counterfactual (implied) values if the Federal Reserve stopped reacting to output in their structural monetary policy rule during the ZLB period. Grey bars indicate NBER recession dates.

3.5 Are there Differences in the Impact of Structural Shocks over the ZLB Period?

The empirical analysis in Debortoli et al. (2019) suggests that unconventional monetary policies have been highly effective at overcoming the issue of the ZLB. More specifically, their results from a 4 variable TVP VAR yield negligible differences in identified structural shocks when comparing average impulse response functions computed both prior to, and following, the 2008 recession[13]. The conjecture that an economy's performance is similar, empirically, under a binding and non-binding ZLB constraint is an emerging stream of literature.

Wu and Zhang (2019b) propose a New Keynesian model that includes a shadow rate to capture unconventional monetary policies that generates the data consistent result that a negative supply shock is always contractionary. Similarly, Wu and Zhang (2019a) extend on this by deducing an open-economy New Keynesian model with a shadow rate that overcomes the anomaly that output and terms of trade respond to supply shocks in opposite directions under a non-binding and binding ZLB constraint. Garín et al. (2019) provides empirical tests of the prediction from a New Keynesian model that positive supply shocks are less expansionary under a binding ZLB constraint. Using utilisation-adjusted total factor productivity data, they find that output and other measures relating to economic activity respond significantly more to positive productivity shocks under a binding ZLB constraint; while expected inflation remains similar across monetary regimes.

Furthermore, Sims and Wu (2020) show that QE and conventional monetary policy are substituable in a New Keynesian model. This analysis shows that the Federal Reserve's balance sheet expansion provides stimulus equivalent to cutting the Federal funds rate to -2%. This matches the empirical decline of the shadow rate (Wu and Xia, 2016) over the ZLB period. Meanwhile, Sims and Wu (2021) deduce a model that allows for the analysis of QE, forward guidance, and negative interest rate policies. They show such policies can all stimulate output as much as conventional monetary policy.

To explore this issue in the context of this study, Figure 6 plots the posterior median, along with the 90% highest posterior density intervals, of the difference in average impulse response functions for GDP growth, inflation and the policy rate, $\mathrm{r}_t = \{\mathrm{i}_t,\ \mathrm{i}^{\mathrm{s}}_t\}$, with respect to all identified structural shocks between the periods 2000Q1–2008Q4 and 2009Q1–2017Q4. Within each graph are results stemming from the TVP VAR using $\mathrm{r}_t = \mathrm{i}_t$ (red lines), and $\mathrm{r}_t = \mathrm{i}^{\mathrm{s}}_t$ (black lines). Panels A, B, and C, refer to supply, demand, and monetary policy shocks respectively. Statistically credible differences within these plots occur when the highest posterior density intervals do not include 0.

It is clear that the response of the Federal funds rate and shadow rate appear to be more responsive to both demand and supply shocks prior to the 2008 recession. At first glance, this looks to contradict the findings in Debortoli et al. (2019) and Garín et al. (2019). However, one should expect differences from the model using the Federal funds rate due to the fact that it contains no information regarding monetary policy stance. Therefore during the ZLB period the

[13] It should be noted here that Debortoli et al. (2019) do not include the Federal funds rate within their estimated TVP VAR models, instead they use the long-term government bond yield. Also note that they provide calibrations from theoretical models they reconcile with Wu and Zhang (2019b) as well as exercises analysing macroeconomic volatility.

response should be less sensitive to these shocks. Regarding the shadow rate, it is unsurprising to see that the shadow rate is less responsive to shocks under a binding ZLB constraint. Intuitively one would expect that any positive demand or supply shocks would have been welcomed by the Federal Reserve following the 2008 recession in order to facilitate expansion.

The difference in average impulse response functions of GDP growth and inflation with respect to both demand and supply shocks from the model using the shadow rate and Federal funds rate, on the whole, report no statistically credible or economically meaningful differences in the transmission of these shocks prior to and during the ZLB period. It is worth mentioning here that supply shocks in the shadow rate TVP VAR appear to be more persistent for inflation prior to the ZLB period. However, noting that structural analysis is carried out conditional on the current state of the economy and that the stochastic volatility of supply shocks is substantially larger prior to the ZLB period, this can be attributable to the fact that the size and magnitude of supply shocks is smaller during the ZLB period and that inflation was lower following the 2008 recession. Therefore, we cannot conclude that the persistence of supply shocks is solely a result of the ZLB.

Perhaps the most interesting result is that the average response of GDP growth and inflation to monetary policy shocks, when using the shadow rate, has not changed throughout the period 2000-2017; something that is completely contradicted from the model using the Federal funds rate. The stark increase in the relative sensitivity of GDP growth and inflation with respect to monetary policy shocks in the model using the Federal funds rate is a result of the high degree of posterior uncertainty in impulse response functions we observe in Figure 1. This suggests that, when accounting for unconventional monetary policy stance under a binding ZLB constraint using a shadow rate, the sensitivity of US macroeconomic fundamentals to monetary policy shocks has indeed remained similar over the last 17 years of the sample.

Table 5 assesses the statistical differences in the economic importance of supply and demand shocks throughout the binding ZLB constraint. Panel A reports the probability that the proportion of variance of variable $x = \{y_t,\ \pi_t,\ \mathrm{r}_t = \{i_t,\ i_t^{\mathrm{s}}\}\}$, during time $q = \{2006Q4,\ 2017Q4\}$, explained by supply shocks, $\mathrm{SVD}_{x,q}^{\mathrm{SUP}}$, is greater than the proportion of variance of variable x explained by a supply shocks in time $j = \{2009Q2,\ 2011Q1,\ 2012Q2,\ 2013Q2\}$, $\mathrm{SVD}_{x,j}^{\mathrm{SUP}}$. Panel B of this table reports analogous probabilities but for the proportion of variance explained by demand non-policy shocks. Note that statistical differences are observed when the probability is less (greater) than 0.1 (0.9). A value lower (greater) than 0.1 (0.9) implies that the proportion of variance explained, and therefore the overall economic importance, during the pre or post zero lower bound period is smaller (larger) than when the ZLB constraint is binding. As is clear there is no evidence in favour of statistical differences in the economic importance of these shocks throughout the ZLB period.

In general, these exercises are coherent with Debortoli et al. (2019) and Garín et al. (2019). These results go further by providing empirical confirmation of their New Keynesian model with a shadow rate not only in terms of the transmission mechanism, but also terms of the economic importance of supply and demand non policy shocks. The novel implication suggests that conventional and unconventional monetary policies may work in mutually exclusive harmony under a binding ZLB constraint. These results provide empirical support of the theoretical literature concerning unconventional monetary policies (see e.g. Wu and Zhang, 2019b,a; Sims and Wu, 2020, 2021). Consistent with conclusions drawn from Taylor rules and policy counterfactuals, this analysis provides further substance that the shadow rate is a useful summary statistic for unconventional monetary policies.

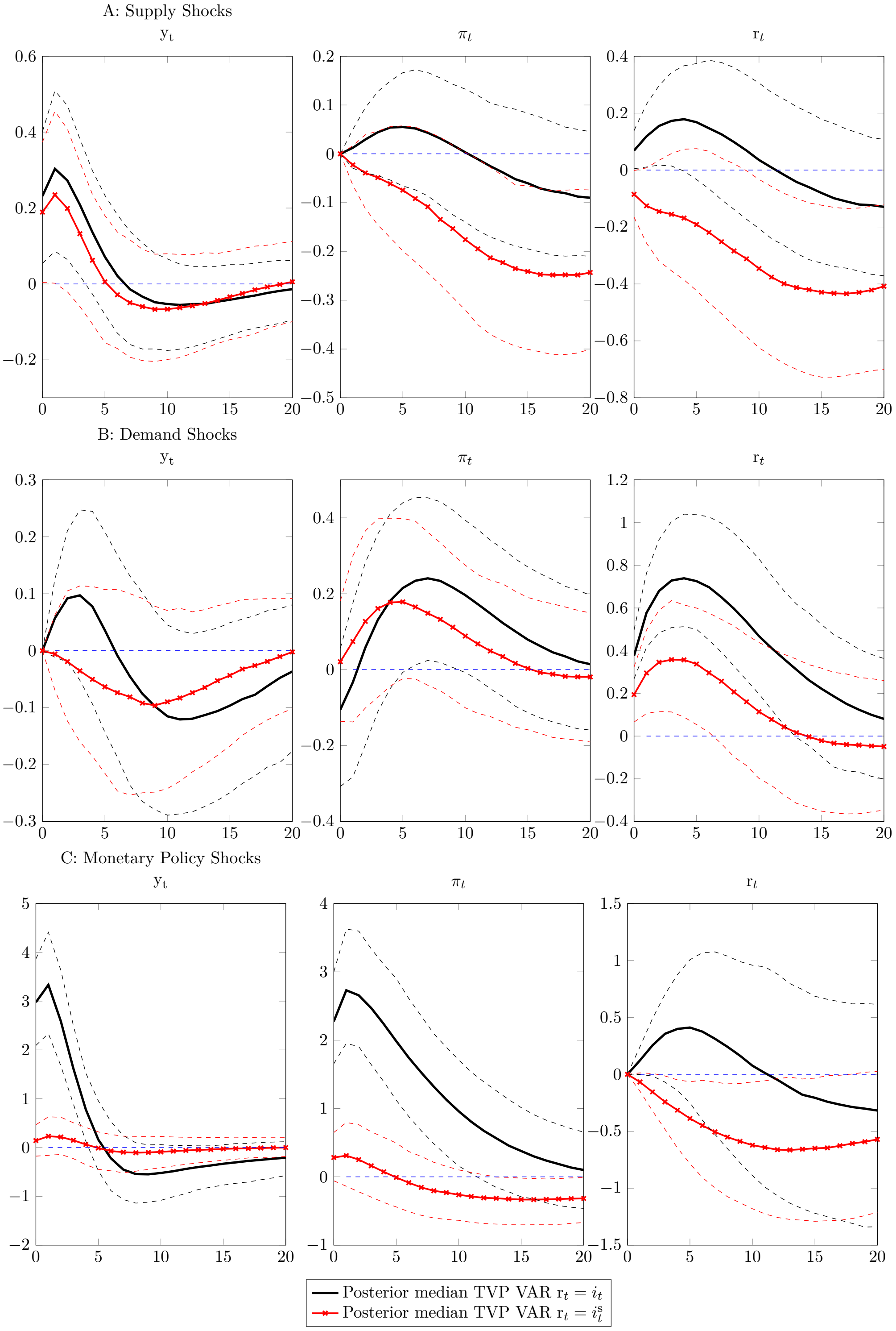

Figure 6: **Difference in Average Impulse Response Functions: Analysing the Impact of Shocks Pre and Post Zero Lower Bound**

Notes: Panel A of this Figure plots the posterior median and 90% highest posterior density intervals of the difference in average impulse response functions from 2000Q1–2008Q4 and 2009Q1–2017Q4 for GDP growth, y_t; inflation, π_t; and the short-term interest rate, $r_t = \{i_t, i_t^s\}$ with i_t denoting the Federal funds rate and i_t^s denoting the spliced Federal funds rate with the shadow rate (Wu and Xia, 2016) with respect to a supply shock. Panels B and C report the same plots, but with respect to a demand and monetary policy shock respectively.

Table 5: **Assessing the Statistical Differences in the Economic Importance of Supply and Demand Non-Policy Shocks over the Zero Lower Bound Period.**

Notes: Panel A of this table report the probability that percent of variance attributable to supply shocks of variable $x = \{y_t,\ \pi_t,\ \mathrm{r}_t = \{i_t,\ i_t^{\mathrm{s}}\}\}$, during time $q = \{2006Q4,\ 2017Q4\}$, $\mathrm{SVD}^{\mathrm{SUP}}_{x,q}$, is greater than the percent of variance explained by supply shocks for x in time $j = \{2009Q2,\ 2011Q1,\ 2012Q2,\ 2013Q2\}$, $\mathrm{SVD}^{\mathrm{SUP},4}_{x,j}$. Therefore, $\Pr\left(\mathrm{SVD}^{\mathrm{SUP}}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{SUP}}_{x,2009Q2}\right)$. Panels Al and A2 report these statistics from the TVP VAR using the Federal funds rate and shadow rate respectively. Panel B of this table reports analogous probabilities but for the percent of variance explained by demand non-policy shocks. Here Panels B1 and B2 refer to models using the Federal funds rate and shadow rate respectively. A statistical difference is observed when the probability is less (greater) than 0.1 (0.9). A value lower (greater) than 0.1 (0.9) implies that the percent variance explained by the shock during the pre or post zero lower bound period is smaller (larger) than when the constraint is binding.

A1: Supply Shocks, models using Federal funds rate				B1: Demand Shocks, models using Federal funds rate			
$x=$	y_t	π_t	r_t	$x=$	y_t	π_t	r_t
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2009Q2}\right)$	0.62	0.58	0.47	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2009Q2}\right)$	0.43	0.42	0.55
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2011Q1}\right)$	0.46	0.45	0.29	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2011Q1}\right)$	0.60	0.70	0.69
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2012Q2}\right)$	0.41	0.43	0.25	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2012Q2}\right)$	0.58	0.61	0.67
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2013Q2}\right)$	0.39	0.40	0.26	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2013Q2}\right)$	0.60	0.62	0.69
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2009Q2}\right)$	0.71	0.73	0.59	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2009Q2}\right)$	0.29	0.31	0.41
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2011Q1}\right)$	0.57	0.60	0.42	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2011Q1}\right)$	0.47	0.54	0.55
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2012Q2}\right)$	0.50	0.60	0.40	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2012Q2}\right)$	0.43	0.46	0.51
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t}_{x,2013Q2}\right)$	0.43	0.55	0.37	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t}_{x,2013Q2}\right)$	0.45	0.45	0.54
A2: Supply Shocks, models using shadow rate				B1: Demand Shocks, models using shadow rate			
$x=$	y_t	π_t	r_t	$x=$	y_t	π_t	r_t
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2009Q2}\right)$	0.59	0.55	0.53	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2009Q2}\right)$	0.43	0.39	0.47
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2011Q1}\right)$	0.51	0.43	0.34	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2011Q1}\right)$	0.53	0.58	0.66
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2012Q2}\right)$	0.48	0.33	0.21	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2012Q2}\right)$	0.61	0.77	0.79
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2013Q2}\right)$	0.41	0.27	0.25	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2006Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2013Q2}\right)$	0.72	0.78	0.80
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2009Q2}\right)$	0.63	0.65	0.65	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2009Q2}\right)$	0.35	0.33	0.37
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2011Q1}\right)$	0.57	0.60	0.45	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2011Q1}\right)$	0.41	0.45	0.51
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2012Q2}\right)$	0.53	0.45	0.33	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2012Q2}\right)$	0.46	0.64	0.68
$\Pr\left(\mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{SUP},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2013Q2}\right)$	0.50	0.41	0.36	$\Pr\left(\mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2017Q4} > \mathrm{SVD}^{\mathrm{DEM},\mathrm{r}_t=i_t^{\mathrm{s}}}_{x,2013Q2}\right)$	0.54	0.64	0.70

4 Robustness Analysis

It is necessary to assess how sensitive the preceding results to alternatives. GDP growth and CPI inflation in the baseline specification are exchanged for the output gap and GDP deflator inflation respectively. The output gap is the difference between the log of GDP and the log of potential GDP, and inflation is the annual growth in the GDP deflator; a specification similar to Belongia and Ireland (2016) and Wu and Zhang (2019b).

Figures 7 and 8 report the structural impact and long-run coefficients from the structural monetary policy rules from this alternative specification. Table 6 assesses statistical differences in the structural impact and long-run coefficients between these models during and following the ZLB period. Evidently these results are qualitatively similar to those in the baseline analysis. In particular, the structural impact coefficients associated to output in the shadow rate Taylor rule are statistically larger than the analogous coefficients from the conventional Taylor rule. Further, the long-run coefficient associated to GDP growth in the shadow rate Taylor rule is on average double that of the long-run coefficient stemming from the conventional Taylor rule.

Figure 9 reports the same policy counterfactuals from the TVP VAR using alternative measures of economic activity and inflation as in Figure 5. Notably, the counterfactual that reacts only to output during the ZLB period results in higher rates of inflation for no benefit in closing the output gap. Further, the counterfactual that imposes no reaction to output suggests the decline in the output gap would have been 2% larger (in absolute terms) in 2008 followed by a stubborn recovery and lower rates of inflation. These results are coherent with the baseline specification and conclusions remain the same.

In the Supplementary Appendix, two additional robustness checks that increase the information set are carried out. First, a macro-financial factor stemming from a subset of the FRED-MD database (McCracken and Ng, 2016) is included in the TVP VAR models. The second specification includes a measure of money growth resting on a transaction demand for money model (Benati, 2019). Conclusions hold after increasing the information set and the results are qualitatively similar to those presented in the main text.

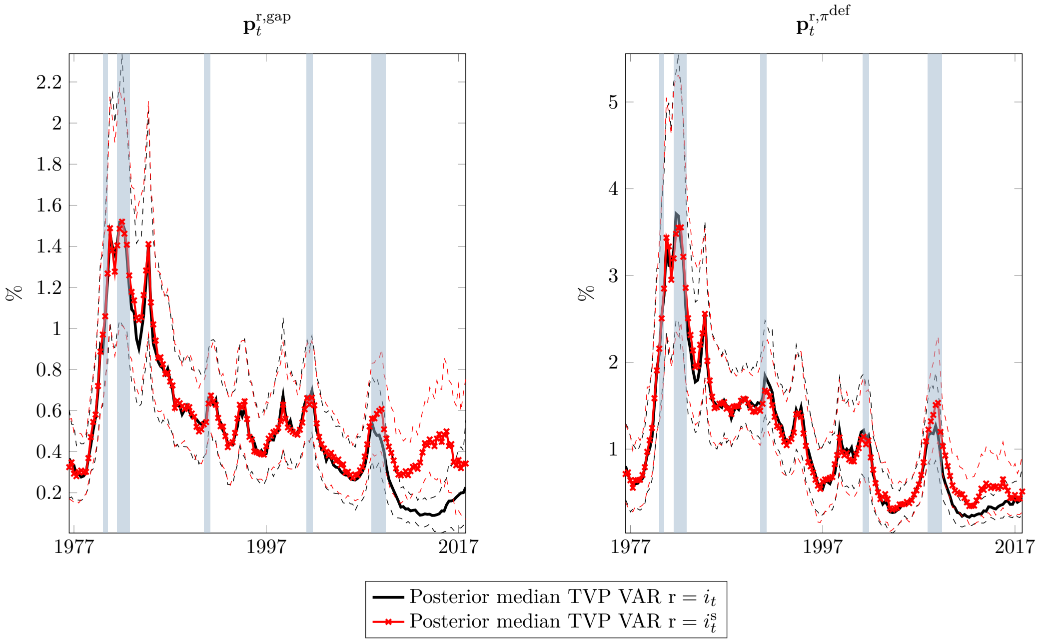

Figure 7: **Structural Impact Coefficients Implied by Structural Monetary Policy Rules from 1976Q3–2017Q4**

Notes: This figure plots the posterior median and 80% point-wise equal-tailed probability bands of the structural impact coefficients associated to real GDP growth, $\mathbf{p}_t^{\mathrm{r,gap}}$ and Consumer price inflation, $\mathbf{p}_t^{\mathrm{r},\pi^{\mathrm{def}}}$. In each model, the short-term interest rate is proxied by either the Federal funds rate, $\mathrm{r}_t = i_t$, or the shadow rate, $\mathrm{r}_t = i_t^{\mathrm{s}}$. Grey bars indicate NBER recession dates.

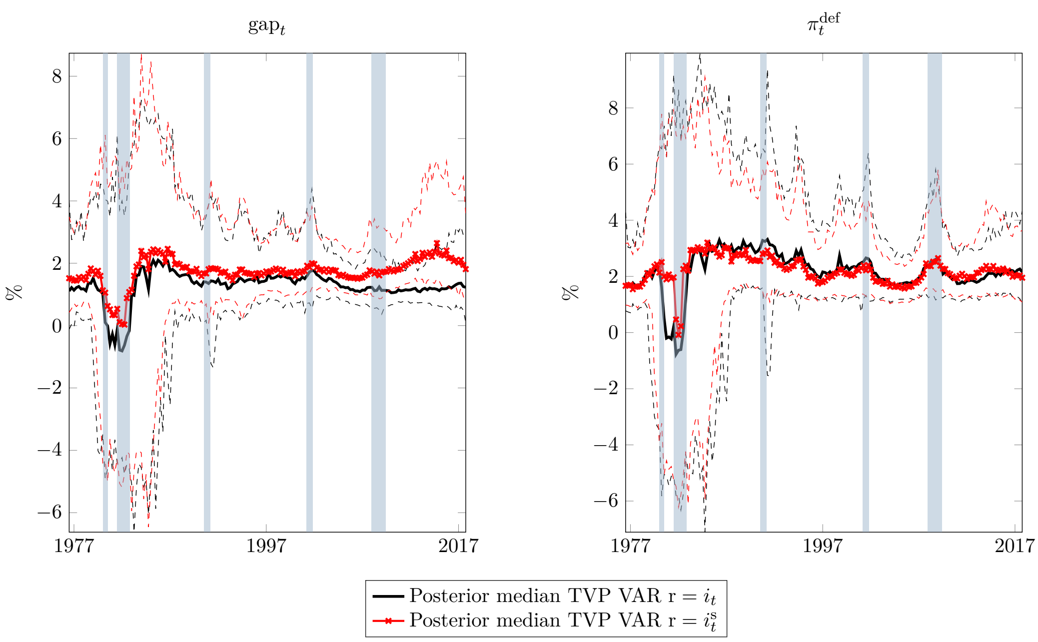

Figure 8: **Long-run Coefficients Implied by Structural Monetary Policy Rules from 1976Q3–2017Q4**

Notes: This figure plots the posterior median and 80% point-wise equal-tailed probability bands of the long-run coefficients associated to real GDP growth, gap_t and Consumer price inflation, π_t^{def} as implied by the structural monetary policy rule. In each model, the short-term interest rate is proxied by either the Federal funds rate, $\mathrm{r}_t = i_t$, or the shadow rate, $\mathrm{r}_t = i_t^{\mathrm{s}}$. Grey bars indicate NBER recession dates.

Table 6: **Assessing Statistical Differences in Taylor Rules throughout and Following the Zero Lower Bound Period. The Federal Funds Rate vs. the Shadow Rate**
Panel A of this table reports the probability that the structural impact coefficient associated to variable $x = \{\text{gap}_t,\ \pi_t^{\text{def}},\ \text{r}_t = \{i_t,\ i_t^{\text{S}}\}\}$, within the structural monetary policy rule from the TVP VAR model where the short-term interest rate is the Federal funds rate at period T, $\mathbf{p}_T^{\text{r}x,\text{r}=i_t}$, is greater than the structural impact coefficient associated to variable x, within the structural monetary policy rule from the TVP VAR model using the shadow rate of Wu and Xia (2016) as a proxy for the short-term interest rate at period T, $\mathbf{p}_T^{\text{r}x,\text{r}=i_t^{\text{S}}}$. Therefore: $\Pr\left(\mathbf{p}_T^{\text{r}x,\text{r}=i_t} > \mathbf{p}_T^{\text{r}x,\text{r}=i_t^{\text{S}}}\right)$. Panel B of this table reports analogous statistics, but for the long-run coefficients associated to variable x, $\text{LRC}_T^{\text{r}x}$, implied by the structural monetary policy rules from TVP VARs using the Federal funds rate and shadow rate respectively. Therefore: $\Pr\left(\text{LRC}_T^{\text{r}x,\text{r}=i_t} > \text{LRC}_T^{\text{r}x,\text{r}=i_t^{\text{S}}}\right)$. A statistical difference is observed when the probability is less (greater) than 0.1 (0.9). A value lower (greater) than 0.1 (0.9) implies that the structural impact/long-run coefficient implied by the TVP VAR using the Federal funds rate (shadow rate) in time period T is less than that implied by the TVP VAR using the shadow rate (Federal funds rate).

A: Structural Impact Coefficients		
$x=$	gap_t	π_t^{def}
$\Pr\left(\mathbf{p}_{2009Q2}^{\text{r}x,\text{r}=i_t} > \mathbf{p}_{2009Q2}^{\text{r}x,\text{r}=i_t^{\text{S}}}\right)$	0.37	0.34
$\Pr\left(\mathbf{p}_{2011Q1}^{\text{r}x,\text{r}=i_t} > \mathbf{p}_{2011Q1}^{\text{r}x,\text{r}=i_t^{\text{S}}}\right)$	0.18	0.21
$\Pr\left(\mathbf{p}_{2012Q2}^{\text{r}x,\text{r}=i_t} > \mathbf{p}_{2012Q2}^{\text{r}x,\text{r}=i_t^{\text{S}}}\right)$	0.09	0.33
$\Pr\left(\mathbf{p}_{2013Q2}^{\text{r}x,\text{r}=i_t} > \mathbf{p}_{2013Q2}^{\text{r}x,\text{r}=i_t^{\text{S}}}\right)$	0.03	0.25
$\Pr\left(\mathbf{p}_{2017Q4}^{\text{r}x,\text{r}=i_t} > \mathbf{p}_{2017Q4}^{\text{r}x,\text{r}=i_t^{\text{S}}}\right)$	0.40	0.42
B: Long Run Coefficients		
$x=$	y_t	π_t
$\Pr\left(\text{LRC}_{2009Q2}^{\text{r}x,\text{r}=i_t} > \text{LRC}_{2009Q2}^{\text{r}x,\text{r}=i_t^{\text{S}}}\right)$	0.31	0.49
$\Pr\left(\text{LRC}_{2011Q1}^{\text{r}x,\text{r}=i_t} > \text{LRC}_{2011Q1}^{\text{r}x,\text{r}=i_t^{\text{S}}}\right)$	0.22	0.42
$\Pr\left(\text{LRC}_{2012Q2}^{\text{r}x,\text{r}=i_t} > \text{LRC}_{2012Q2}^{\text{r}x,\text{r}=i_t^{\text{S}}}\right)$	0.21	0.47
$\Pr\left(\text{LRC}_{2013Q2}^{\text{r}x,\text{r}=i_t} > \text{LRC}_{2013Q2}^{\text{r}x,\text{r}=i_t^{\text{S}}}\right)$	0.16	0.41
$\Pr\left(\text{LRC}_{2017Q4}^{\text{r}x,\text{r}=i_t} > \text{LRC}_{2017Q4}^{\text{r}x,\text{r}=i_t^{\text{S}}}\right)$	0.38	0.55

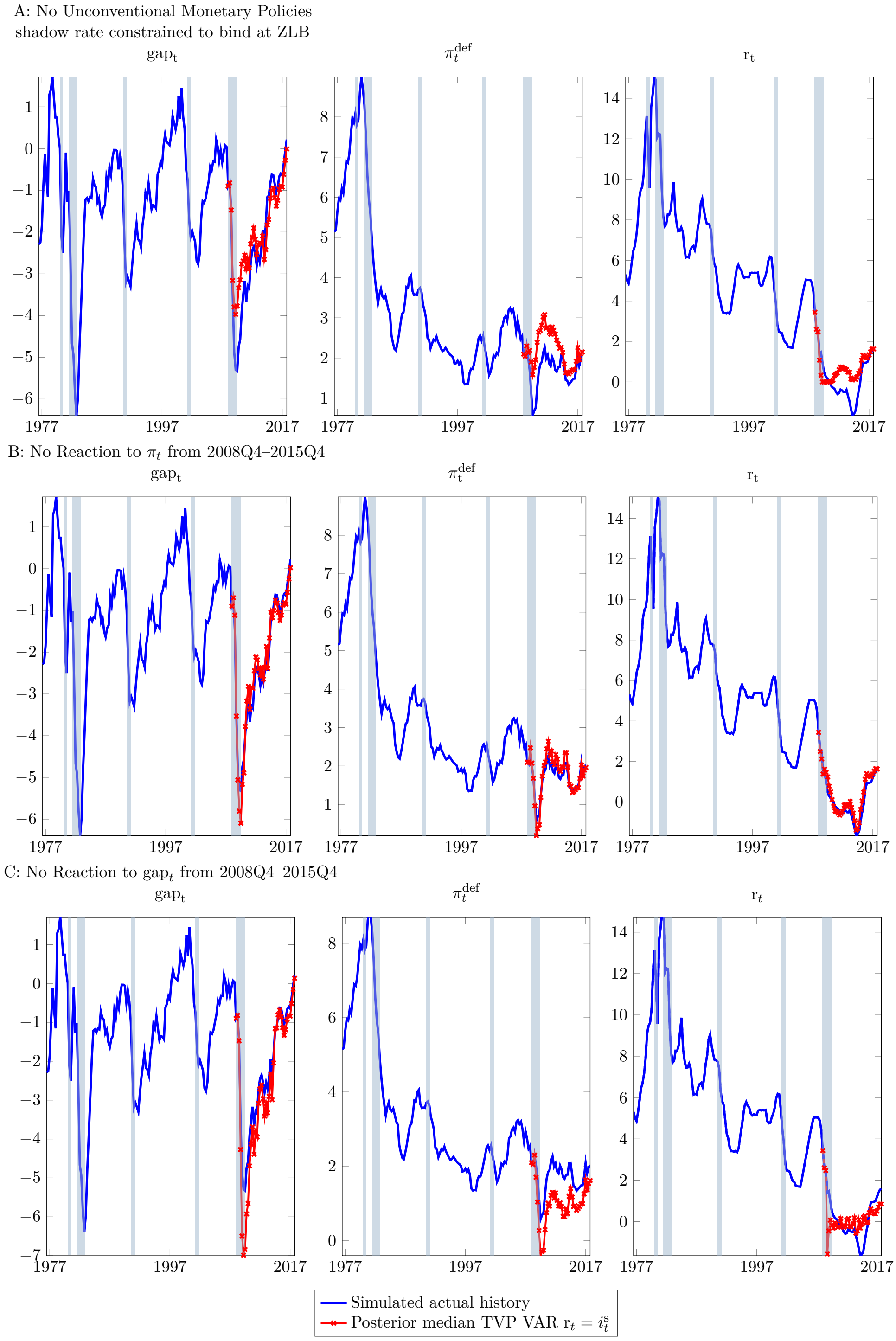

Figure 9: **Counterfactual Simulations of Shadow Rate Taylor Rules**
Notes: Panel A of this figure plots the model implied history for variable $x = \{y_t,\ \pi_t,\ r_t = i_t^s\}$ (blue lines) along with the counterfactual (implied) value (red lines) if there had been no monetary policy shocks from 2008Q4 to 2015Q4, and assuming that the ZLB constraint binds. Essentially implying no unconventional monetary policies were implemented. Panel B of this figure plots the model implied history for macroeconomic variables along with the counterfactual (implied) value if the Federal Reserve had stopped reacting to inflation in their structural monetary policy rule during the ZLB period. Panel C plots the implied history for macroeconomic variables along with the counterfactual (implied) values if the Federal Reserve stopped reacting to output in their structural monetary policy rule during the ZLB period. Grey bars indicate NBER recession dates.

5 Conclusion

The results in Wu and Zhang (2019b) suggest shadow rates are useful in overcoming issues in macroeconomic models and formalise the notion that shadow rates are useful summary statistics tracking monetary policy stance at the zero lower bound. This paper provides a comprehensive empirical assessment on the evolution of unconventional monetary policies under a binding ZLB constraint for the US economy. Following a demonstration that the Federal funds rate conveys no information regarding monetary policy stance under a binding ZLB constraint, analysis reveals that one can reconcile economically plausible results using shadow rates. Thus, the analysis here delivers further support for Wu and Xia (2016) who recommend researchers and policymakers use shadow rates in VAR models to account for monetary policy stance without loss of historical time-series.

These results go further by quantifying monetary policy stance under a binding ZLB constraint in the form of shadow rate Taylor rules permitted to evolve over time. In particular, they uncover a greater emphasis placed on output during QE2 and Operation Twist. Counterfactual simulations suggest that unconventional monetary policies are a viable response to deep recessions when nominal interest rates approach their ZLB. Specifically they indicate that had the Fed placed no emphasis on output during the Great Recession, then the contraction of GDP growth would have been 1.14% lower during QE1 followed by sustained increase in volatility; as well as higher risk of entering a deflationary period during QE2 and Operation Twist.

The importance and practical use of shadow rates is further justified by showing no statistical or economically meaningful differences in the transmission, or economic importance, of supply and demand non-policy shocks over the ZLB period both before the 2008 recession and after the Fed moves away from a binding ZLB constraint. These results provide strong support that conventional and unconventional monetary policies can work in mutually exclusive harmony in structural VAR models.

The immediate implication is that the Federal Reserve, and indeed other central banks, should consider shadow rates as an effective summary of monetary policy stance under a binding ZLB constraint. The ability of shadow rates under a binding zero lower bound constraint to reconcile economically plausible results consistent with those of conventional monetary policy highlight its practical use. A natural recommendation is that central banks use these results in combination with other methods to analyse the impact of unconventional monetary policies.

Supplementary Appendix (Not for Publication)

Plot of Economic Data

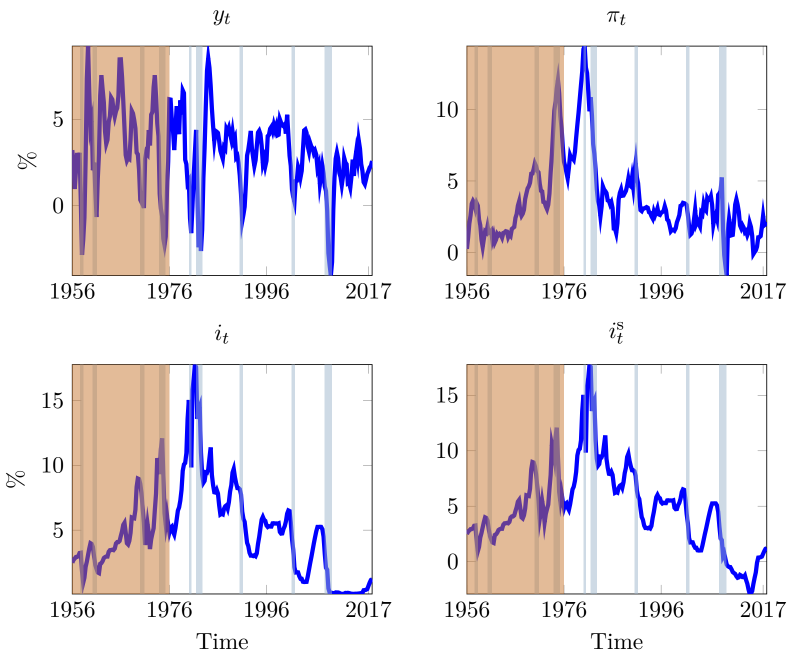

Figure 10: **US Macroeconomic data from 1956 to 2017**
Notes: This figure plots US macroeconomic data from 1956Q1–2017Q4. The top left plot reports the annual growth rate of GDP, y_t; the top right plot shows the annual CPI inflation rate, π_t; the bottom left plot is the Federal funds rate, i_t; and the bottom right plot reports the shadow rate as proposed in Wu and Xia (2016), i_t^s. The orange area indicates the time series used to calibrate our models. Grey bars indicate NBER recession dates.

Priors

Our prior specification involves estimating a Bayesian fixed coefficient VAR (BVAR) model over the training sample. The priors imposed on this BVAR model combine the traditional Minnesota prior of Doan et al. (1984) and Litterman (1986) on the coefficient matrices with an inverse-Wishart prior on the BVAR's covariance matrix. In our specification, the prior mean on the coefficient matrix sets all elements equal zero, except those corresponding to the own first lag of each dependent variable which are set to 0.9. This imposes the prior belief that our variables exhibit persistence whilst simultaneously ensuring shrinkage of the other VAR coefficients to zero. The prior variance of the coefficient matrix is set similar to Litterman (1986). Our prior for the BVAR's covariance matrix follows an inverse-Wishart distribution with the prior scale matrix and degrees of freedom set to an N-dimensional identity matrix and 1+N respectively.

We estimate the BVAR using a standard Gibbs sampler. For the sake of brevity, we do not explicitly outline our algorithm since it is well documented; see e.g. Koop and Korobilis (2010). Our alternative prior specification essentially replaces the conventional Cogley and Sargent (2005) prior with the posterior means from the draws of an estimated BVAR over the training sample

$$\bar{\mathbf{B}}_{\text{BVAR}} = \frac{1}{M}\sum_{i=1}^{M}\mathbf{B}_i, \tag{12}$$

$$\overline{\mathrm{V}(\mathbf{B})}_{\text{BVAR}} = \frac{1}{M}\sum_{i=1}^{M}\mathrm{V}(\mathbf{B}_i), \tag{13}$$

$$\bar{\Sigma}_{\text{BVAR}} = \frac{1}{M}\sum_{i=1}^{M}\Sigma_i \tag{14}$$

respectively. Here M denotes the number of saved draws from the estimated BVAR which we set to 20,000. $\mathbf{B}_i$ and $\mathrm{V}(\mathbf{B}_i)$ denote the ith draw of the coefficient matrix and the variance of the coefficient matrix respectively. Σ_i denotes the ith draw of the BVAR's covariance matrix. From these estimates, the initial conditions of the time-varying coefficient models, $\mathbf{B}_0$, $\mathbf{a}_0$, $\mathbf{h}_0$ are Normal and independent of one another, and the distributions of the hyperparameters. We set

$$\mathbf{B}_0 \backsim N\left[\bar{\mathbf{B}}_{\text{BVAR}},\ 4\cdot\overline{\mathrm{V}(\mathbf{B})}_{\text{BVAR}}\right] \tag{15}$$

for $\mathbf{a}_0$, $\mathbf{h}_0$, let $\bar{\Sigma}_{\text{BVAR}}$ be the posterior median of the estimated covariance matrix of the residuals from the time–invariant BVAR. Let C be the lower–triangular Choleski factor such that $CC' = \bar{\Sigma}_{\text{BVAR}}$. We then set

$$\ln \mathbf{h}_0 \backsim N(\ln \mu_0, 10 \times I_3) \tag{16}$$

where μ_0 collects the logarithms of the squared elements along the diagonal of C. We divide each column of C by the corresponding element on the diagonal; call this matrix $\tilde{C}$. We then set

$$\mathbf{a}_0 \backsim N\left[\tilde{\mathbf{a}}_0,\ \tilde{\mathrm{V}}(\tilde{\mathbf{a}}_0)\right] \tag{17}$$

with $\tilde{\mathbf{a}}_0 \equiv [\tilde{\mathbf{a}}_{0,11},\ \tilde{\mathbf{a}}_{0,21},\ \tilde{\mathbf{a}}_{0,31}]'$ which is a vector collecting all the elements below the diagonal of $\tilde{C}^{-1}$. We assume $\tilde{\mathrm{V}}(\tilde{\mathbf{a}}_0)$ is diagonal with each element equal to 10 times the absolute value of the corresponding element of $\tilde{\mathbf{a}}_0$. This is an arbitrary prior but correctly scales the variance of each element of $\mathbf{a}_0$ to account for their respective magnitudes.

For the time-varying coefficient model assuming $\mathbf{Q}_t = \mathbf{Q}$, $\mathbf{Q}$ is set to follow an inverse–Wishart distribution,

$$\mathbf{Q} \backsim IW(\underline{\mathbf{Q}}^{-1}, \mathrm{T}_0) \tag{18}$$

where $\underline{\mathbf{Q}} = (1+\dim(\mathbf{B}_t))\cdot\overline{\mathrm{V}}(\bar{\mathbf{B}}_{\text{BVAR}})\cdot 3.4\times10^{-4}$. The prior degrees of freedom, $(1+\dim(\mathbf{B}_t))$, are the minimum allowed for the prior to be proper. Our choice of scaling parameter of 3.4×10^{-4} is consistent with Cogley and Sargent (2005). We have also estimated our models using different priors, we allowed for a more restrictive scaling parameter of 1.0×10^{-4} and have also set the degrees of freedom to be the length of the training sample; in our case this is 80. The scaling parameter essentially sets the amount of drift within the $\mathbf{B}_t$ matrices. The results and conclusions presented within the main body are robust to changing the value of the scaling parameter, and the prior degrees of freedom imposed.

For the time-varying coefficient model assuming $\mathbf{Q}_t$ is diagonal where the elements follow a geometric random walk, let $C_{\overline{\mathrm{V}(\mathbf{B})}_{\mathrm{BVAR}}}$ be the lower-triangular Choleski factor such that $C_{\overline{\mathrm{V}(\mathbf{B})}_{\mathrm{BVAR}}} C'_{\overline{\mathrm{V}(\mathbf{B})}_{\mathrm{BVAR}}} = 3.4 \times 10^{-4} \overline{\mathrm{V}(\mathbf{B})}_{\mathrm{BVAR}}$. We then set

$$\ln \mathbf{q}_0 \backsim N \left[\ln \mu_{\mathbf{q}_0,0}, 10 \times I_3 \right] \tag{19}$$

with $\ln \mu_{\mathbf{q}_0,0}$ collecting the logarithmic squared diagonal elements of $3.4 \times 10^{-4} \overline{\mathrm{V}(\mathbf{B})}_{\mathrm{BVAR}}$. The variances of these stochastic volatility innovations follow an inverse-Gamma distribution for the elements of $\mathbf{Z}_{\mathbf{q}}$,

$$\mathbf{Z}_{\mathbf{q},i,i} \backsim IG\left(\frac{10^{-4}}{2}, \frac{1}{2}\right) \tag{20}$$

The blocks of $\mathbf{S}$ are also assumed to follow inverse–Wishart distributions with prior degrees of freedom equal to the minimum allowed (i.e. $1 + \dim(\mathbf{S}_i)$).

$$\mathbf{S}_1 \backsim IW(\underline{\mathbf{S}}_1^{-1}, 2) \tag{21}$$

$$\mathbf{S}_2 \backsim IW(\underline{\mathbf{S}}_2^{-1}, 3) \tag{22}$$

$$\tag{23}$$

we set S_1, S_2 in accordance with $\tilde{\mathbf{a}}_0$ such that $\underline{\mathbf{S}}_1 = 10^{-3} \times |\tilde{\mathbf{a}}_{0,11}|$, $\underline{S}_2 = 10^{-3} \times \mathrm{diag}([|\tilde{\mathbf{a}}_{0,21}|, |\tilde{\mathbf{a}}_{0,31}|]')$. This calibration is consistent with setting $\mathbf{S}_1, \mathbf{S}_2$ to 10^{-4} times the corresponding diagonal block of $\tilde{\mathrm{V}}(\tilde{\mathbf{a}}_0)$. The variances for the stochastic volatility innovations, as in Cogley and Sargent (2005), follow an inverse–Gamma distribution for the elements of $\mathbf{Z}_{\mathbf{h}}$,

$$\mathbf{Z}_{\mathbf{h},i,i} \backsim IG\left(\frac{10^{-4}}{2}, \frac{1}{2}\right) \tag{24}$$

Posterior Simulation

In order to simulate the posterior distribution of the hyperparameters and states, conditional on the data, we implement the following MCMC that combines elements from Primiceri (2005) and Cogley and Sargent (2005).

1) *Draw elements of* $\mathbf{B}_t$ Conditional on $\mathbf{Y}^T$, $\mathbf{a}^T$ and $\mathbf{H}^T$, the observation equation (1) is linear with Gaussian innovations with a known covariance matrix. Factoring the density of $\mathbf{B}_t$, $p(\mathbf{B}_t)$ in the following manner

$$p(\mathbf{B}^T | \mathbf{y}^T, \mathbf{A}^T, \mathbf{H}^T, V) = p(\mathbf{B}_T | \mathbf{Y}^T, \mathbf{A}^T, \mathbf{H}^T, \mathbf{V}) \prod_{t=1}^{T-1} p(\mathbf{B}_t | \mathbf{B}_{t+1}, \mathbf{Y}^t, \mathbf{A}^T, \mathbf{H}^T, \mathbf{V}) \tag{25}$$

the Kalman filter recursions pin down the first element on the right hand side of the above; $p(\mathbf{B}_T | Y^T, \mathbf{A}^T, \mathbf{H}^T, \mathbf{V}) \backsim N(\mathbf{B}_T, P_T)$, with P_T being the precision matrix of $\mathbf{B}_T$ from the Kalman filter. The remaining elements in the factorisation are obtained via backward recursions as in Cogley and Sargent (2005). Since $\mathbf{B}_t$ is conditionally Normal

$$\mathbf{B}_{t|t+1} = P_{t|t}P_{t+1|t}^{-1}(\mathbf{B}_{t+1} - \mathbf{B}_t) \tag{26}$$

$$P_{t|t+1} = P_{t|t} - P_{t|t}P_{t+1|t}^{-1}P_{t|t} \tag{27}$$

which yields, for every t from $T-1$ to 1, the remaining elements in the observation equation (1). More precisely, the backward recursion begins with a draw, $\tilde{\mathbf{B}}_T$ from $N(\mathbf{B}_T,\, P_T)$. Conditional on $\tilde{\mathbf{B}}_T$, the above produces $\mathbf{B}_{T-1|T}$ and $P_{T-1|T}$. This permits drawing $\tilde{\mathbf{B}}_{T-1}$ from $N(\mathbf{B}_{T-1|T},\, P_{T-1|T})$ until t=1.

2) *Drawing elements of* $\mathbf{a}_t$ Conditional on $\mathbf{Y}^T$, $\mathbf{B}^T$ and $\mathbf{H}^T$ we follow Primiceri (2005) and note that (1) can be written as

$$\mathbf{A}_t\tilde{\mathbf{Y}}_t \equiv \mathbf{A}_t(\mathbf{Y}_t - \mathbf{X}_t^{'}\mathbf{B}_t) = \mathbf{A}_t\mathbf{e}_t \equiv \mathbf{v}_t \tag{28}$$

$$\mathrm{Var}(\mathbf{v}_t) = \mathbf{H}_t \tag{29}$$

with $\tilde{\mathbf{Y}}_t \equiv [\tilde{\mathbf{Y}}_{1,t},\, \tilde{\mathbf{Y}}_{2,t},\, \tilde{\mathbf{Y}}_{3,t}]'$ and

$$\tilde{\mathbf{Y}}_{1,t} = \mathbf{v}_{1,t} \tag{30}$$

$$\tilde{\mathbf{Y}}_{2,t} = -\mathbf{a}_{21,t}\tilde{\mathbf{Y}}_{1,t} + \mathbf{v}_{2,t} \tag{31}$$

$$\tilde{\mathbf{Y}}_{3,t} = -\mathbf{a}_{31,t}\tilde{\mathbf{Y}}_{1,t} - \mathbf{a}_{32,t}\tilde{\mathbf{Y}}_{2,t} + \mathbf{v}_{3,t} \tag{32}$$

(33)

These observation equations and the state equation permit drawing the elements of $\mathbf{a}_t$ equation by equation using the same algorithm as above; assuming S is block diagonal.

3) *Drawing elements of* $\mathbf{H}_t$ Conditional on $\mathbf{Y}^T$, $\mathbf{B}^T$ and $\mathbf{a}^T$, the orthogonal innovations u_t, $\mathrm{Var}(\mathbf{e}_t) = \mathbf{H}_t$ are observable. Following Jacquier et al. (2002) the stochastic volatilities, $\mathbf{h}_{i,t}$'s, are sampled element by element; Cogley and Sargent (2005) provide details in Appendix B.2.5 of their paper.

4) *Drawing the hyperparameters* Conditional on Y^T, $\mathbf{B}^T$, $\mathbf{H}_t$ and $\mathbf{a}^T$, the innovations in $\mathbf{B}_t$, $\mathbf{a}_t$ and $\mathbf{h}_{i,t}$'s are observable, which allows one to draw the elements of $\mathbf{Q}_t = \mathbf{Q}$, $\mathbf{S}_1$, $\mathbf{S}_2$ and the $\mathbf{Z}_{\mathbf{h},i,i}$ from their respective distributions.

Note that for the model allowing for stochastic volatility in the innovation variances of the time-varying coefficients, $\mathbf{Q}_t$ being a diagonal matrix, we add an extra block into the MCMC algorithm.

3*a*) *Drawing the elements of* $\mathbf{Q}_t$ Conditional on $\mathbf{B}_t$, the innovations $u_t = \mathbf{B}_t - \mathbf{B}_{t-1}$, with $\mathrm{Var}(u_t) = \mathbf{Q}_t$ are observable. Therefore we sample the diagonal elements of $\mathbf{Q}_t$ applying the Jacquier et al. (2002) algorithm element by element. Following this, we can then sample the $\mathbf{Z}_{\mathbf{q},i,i}$ from the inverse-Gamma distribution in step 4 of the above algorithm.

Computational Issues

Impulse Response Computation

To compute generalised impulse responses, I follow an algorithm similar to Koop et al. (1996). The impulse response function is defined as the difference between two conditional expectations, with and without exogenous shocks

$$\mathbf{IRF}_{t+j} = \mathbb{E}\left[y_{t+j}|\mathbf{v}_t,\ \mathcal{F}_t\right] - \mathbb{E}\left[y_{t+j}|\mathcal{F}_t\right] \tag{34}$$

where y_{t+j} contains contains forecasts of the endogenous variables at horizon $j = 1, ..., 20$, $\mathcal{F}_t$ represents the current information set and $\mathbf{v}_t$ is a vector of current disturbance terms. The information set with which the forecasts are conditioned on, contains the actual values of the lagged variables and a random draw from the joint posterior distribution of the model parameters and hyperparameters. 500 random states of the economy are drawn and stochastically simulated the future paths of the coefficient vector and components of the covariance matrix based on the laws of motion 20 quarters into the future. In this manner, all potential sources of uncertainty are accounted for that may stem from the innovations, variations in lagged coefficients, and evolutions in the contemporaneous relations among the variables.

Following Arias et al. (2018) and Rubio-Ramirez et al. (2010), the time-varying structural impact matrix, $\mathbf{A}_{0,t}$ is calculated in the following manner. Given the current state of the economy, take the eigenvalue-eigenvector decomposition of the VAR's time-varying covariance matrix at time t, $\mathbf{\Omega}_t = \mathbf{P}_t\mathbf{D}_t\mathbf{P}_t'$. Draw an N $\times$ N matrix K from the $N(0, 1)$ distribution and compute the **QR** decomposition of K, normalising the elements of the diagonal matrix $\mathbf{R}$ to be positive; the matrix $\mathbf{Q}$ is a matrix whose columns are orthogonal to one another. The time-varying structural impact matrix is computed as $\mathbf{A}_{0,t} = \mathbf{P}_t\mathbf{D}_t^{\frac{1}{2}}\mathbf{Q}'$. Given $\mathbf{A}_{0,t}$, compute the reduced-form innovations using $\mathbf{e}_t = A_{0,t}\mathbf{v}_t$, where $\mathbf{v}_t$ contains the structural shocks obtained by drawing from a standard Normal distribution. The impulse response are computed by taking the difference between the evolution of the variables with and without a shock. In the former case, the shock is set to $\mathbf{v}_{i,t} + 1$ and in the latter the vector is left unchanged. From this set of impulse responses, only 50 of those that satisfy the whole set of sign restrictions are retained. Once 50 sets of responses meet the sign restrictions, the mean responses of the endogenous variables over the accepted simulations is taken.

Computing Frequency Domain Structural Variance Decompositions

Structural variance decompositions are computed following Benati and Mumtaz (2007), as the ratio of the conditional and unconditional spectral densities of each variable. The unconditional spectral density of variable $x = \{y_t,\ \pi_t,\ i_t,\ m_t\}$ at frequency ω is given by

$$f_{x,t|T}(\omega) = s_x(I_N - \tilde{\mathbf{B}}_{t|T}e^{-i\omega})^{-1}\frac{\mathbf{A}_{0,t|T}(\mathbf{A}_{0,t|T})'}{2\pi}\left[(I_N - \tilde{\mathbf{B}}_{t|T}e^{-i\omega})^{-1}\right]'s_x' \tag{35}$$

where $\mathbf{A}_{0,t|T}$ is the structural impact matrix of the fully identified model, $\tilde{\mathbf{B}}_{t|T}$ are the time-varying coefficient matrices excluding the constants, I_N is a N-dimensional identity matrix, and

s_x is a row vector selecting the variable of interest. The conditional spectral density of variable $x = \{y_t,\ \pi_t,\ i_t\}$ is

$$\bar{f}_{x,t|T}(\omega) = s_x(I_N - \tilde{\mathbf{B}}_{t|T}e^{-i\omega})^{-1}\frac{\tilde{\mathbf{A}}_{0,t|T}(\tilde{\mathbf{A}}_{0,t|T})'}{2\pi}\left[(I_N - \tilde{\mathbf{B}}_{t|T}e^{-i\omega})^{-1}\right]' s_x' \tag{36}$$

where $\mathbf{A}_{0,t|T}(\mathbf{A}_{0,t|T})'$ is replaced with $\tilde{\mathbf{A}}_{0,t|T}(\tilde{\mathbf{A}}_{0,t|T})'$ which shuts off all structural shocks except for the one of interest. It is not possible to uniquely identify the innovation variances of our structural shocks. However, it is plausible to compute the TVP-VAR covariance matrix at each point in time that results from setting one or more of the structural innovation variances to zero. Therefore the contribution of identified structural shocks to variable x at frequency ω is given by the ratio

$$\frac{\bar{f}_{x,t|T}(\omega)}{f_{x,t|T}(\omega)} \tag{37}$$

Additional Results

Increasing the Information Set

Results so far have relied on a small information set, thereby impacting the space spanned by the structural computation. It is necessary to investigate whether baseline results hold in light of increasing the information set[14]. To account for additional macro-financial variables, a subset of the FRED-MD database is used (McCracken and Ng, 2016). This includes 47 variables accounting for the housing market, stock market, money, and corporate and government bond yields reported in Table 7. Following the procedure outlined in McCracken and Ng (2016), and using the Expectations Maximisation algorithm of Stock and Watson (2002), the first principal component is extracted. Panels A and B of Figure 11 plot the macro-financial factor, F_t and the factor loadings respectively.

To identify the fourth structural shock, $\mathbf{v}_t^{\mathbf{f}}$, two sets of sign restrictions are applied and reported in Table 8. The first identification scheme postulates that the shock, $\mathbf{v}_t^{\mathbf{f}}$ results in a negative contemporaneous impact on GDP growth and inflation, and a positive impact on both the short-term interest rate and the macro-financial factor. The second identification scheme proposes that the shock yields a positive contemporaneous impact on F_t, whilst leaving all other variables unconstrained[15].

Tables 9 and 10 report tests for statistical significance of differences in the monetary policy transmission mechanism, and structural impact and long-run coefficients implied by monetary policy rules of the 4 variable TVP VARs respectively. It is clear from Table 8 that both identification schemes result in statistically significant differences in the four quarter accumulated response of real GDP growth, inflation and the interest rate in the very same periods as baseline

[14] Note also that results are robust to using alternative measures of economic activity. More specifically, loacted within the Supplementary Appendix is the baseline analysis reported for an output gap measure measured as the percent difference between actual and potential real GDP, and using Consumer Prices to proxy inflation.

[15] Analysis using a third identification scheme is also considered for $\mathbf{v}_t^{\mathbf{f}}$. Under this third identification scheme, the shock $\mathbf{v}_t^{\mathbf{f}}$ results in a negative contemporaneous impact of GDP growth, inflation, and the interest rate; and a positive impact on the macro-financial factor. Results are consistent with those reported in the main text and are available on request.

models[16]. Further echoing baseline analysis, Table 9 also shows that after accounting for a wider array of macroeconomic and financial variables, that there are negligible statistical differences in the structural impact, and long-run, coefficients when replacing the Federal funds rate with the shadow rate.

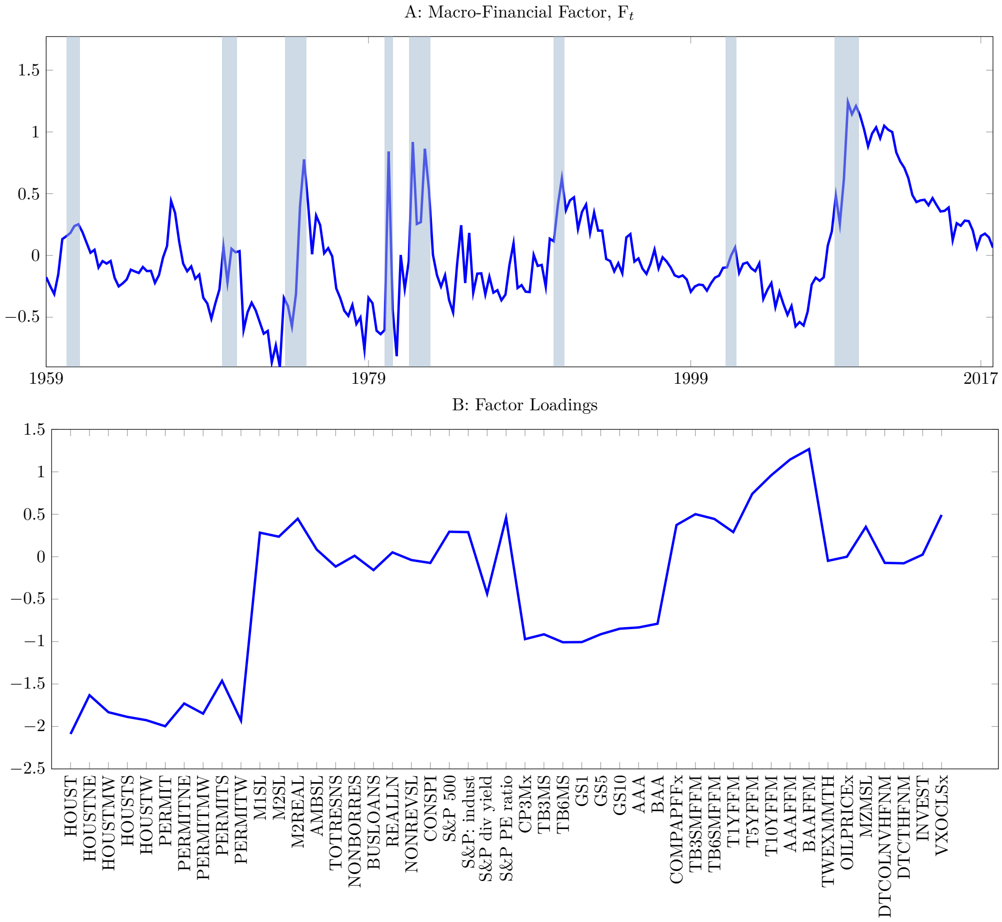

Figure 11: **Macro-Financial Factor from 1959–2017, and Loadings from Macro-Financial Dataset**
Notes: Panel A of this figure plots the extracted macro-financial factor, F_t, estimated from our US macro-financial dataset; a subset of the FRED-MD dataset in McCracken and Ng (2016). Grey bars indicate NBER recession dates. Panel B of this figure plots the factor loadings associated to each of the 47 variables in our dataset (labelled on the x-axis) for the extracted factor from our macro-financial dataset.

[16] Note also that imposing sign restrictions on the response of macroeconomic fundamentals to the response of a shock to $\mathbf{v}_t^{\mathbf{f}}$ also delivers statistically credible differences.

Table 7: **Variables Included in the Macro-Financial Dataset**
Notes: This table reports the variables included in our macro-financial dataset. This is a subset of the data used in McCracken and Ng (2016); the entire dataset is available from https://research.stlouisfed.org/econ/mccracken/fred-databases/. The column Variable Code and Description present the FRED Mnemonic and description of the time series respectively. The column T Code denotes the following data transformation for time series f_t: (1) No transformation; (2) Δf_t; (4) $\ln(f_t)$; (5) $\Delta \ln(f_t)$; (6) $\Delta^2 \ln(f_t)$; (7) $\Delta(f_t/f_{t-1} - 1)$.

Variable Code	Description	T Code
HOUST	Housing Starts: Total New Privately Owned	4
HOUSTNE	Housing Starts: North East	4
HOUSTMW	Housing Starts, Midwest	4
HOUSTS	Housing Starts, South	4
HOUSTW	Housing Starts, West	4
PERMIT	New Private Housing Permits (SAAR)	4
PERMITNE	New Private Housing Permits Northeast (SAAR)	4
PERMITMW	New Private Housing Permits Midwest (SAAR)	4
PERMITS	New Private Housing Permits South (SAAR)	4
PERMITW	New Private Housing Permits West (SAAR)	4
M1SL	M1 Money Stock	6
M2SL	M2 Money Stock	6
M2REAL	Real M2 Money Stock	5
AMBSL	St. Louis Adjusted Monetary Base	6
TOTRESNS	Total Reserves of Depository Institutions	6
NONBORRES	Reserves of Depository Institutions	7
BUSLOANS	Commercial and Industrial Loans	6
REALLN	Real Estate Loans at Commercial Banks	6
NONREVSL	Total Nonrevolving Credit	6
CONSPI	Nonrevolving consumer credit to Personal Income	2
S&P 500	S&P's Common Stock Price Index: Composite	5
S&P: indust	S&P's Common Stock Price Index: Industrials	5
S&P div yield	S&P's Common Stock Price Index: Dividend Yield	2
S&P PE ratio	S&P's Common Stock Price Index: Price-Earnings Ratio	5
CP3Mx	3-Month AA Financial Commercial Paper Rate	2
TB3MS	3-Month Treasury Bill	2
TB6MS	6-Month Treasury Bill	2
GS1	1-Year Treasury Rate	2
GS5	5-Year Treasury Rate	2
GS10	10-Year Treasury Rate	2
AAA	Moody's Seasoned Aaa Corporate Bond Yield	2
BAA	Moody's Seasoned Baa Corporate Bond Yield	2
COMPAPFFx	3-Month Commercial Paper minus FEDFUNDS	1
TB3SMFFM	3-Month Treasury C minus FEDFUNDS	1
TB6SMFFM	6-Month Treasury C minus FEDFUNDS	1
T1YFFM	1-Year Treasury C minus FEDFUNDS	1
T5YFFM	5-Year Treasury C minus FEDFUNDS	1
T10YFFM	10-Year Treasury C minus FEDFUNDS	1
AAAFFM	Moody's Aaa Corporate Bond minus FEDFUNDS	1
BAAFFM	Moody's Baa Corporate Bond minus FEDFUNDS	1
TWEXMMTH	Trade Weighted US Dollar Index: Major Currencies	5
OILPRICEx	Crude Oil, spliced WTI and Cushing	6
MZMSL	MZM Money Stock	6
DTCOLNVHFNM	Consumer Motor Vehicle Loans Outstanding	6
DTCTHFNM	Total Consumer Loans and Leases Outstanding	6
INVEST	Securities in Bank Credit at All Commercial Banks	6
VXOCLSx	VXO	1

Table 8: **Alternative Identification Schemes for Models using Macro-Financial Factor**
Notes: This table reports the contemporaneous response of real GDP growth, y_t; inflation, π_t; and the interest rate, i_t with respect to a supply shock, $\mathbf{v}_t^{\mathbf{s}}$; a demand non policy shock, $\mathbf{v}_t^{\mathbf{d}}$; a monetary policy shock, $\mathbf{v}_t^{\mathbf{mp}}$; and a factor shock, $\mathbf{v}_t^{\mathbf{f}}$, respectively. "x" denotes no restriction imposed.

Identification I	$\mathbf{v}_t^{\mathbf{s}}$	$\mathbf{v}_t^{\mathbf{d}}$	$\mathbf{v}_t^{\mathbf{mp}}$	$\mathbf{v}_t^{\mathbf{f}}$	Identification II	$\mathbf{v}_t^{\mathbf{s}}$	$\mathbf{v}_t^{\mathbf{d}}$	$\mathbf{v}_t^{\mathbf{mp}}$	$\mathbf{v}_t^{\mathbf{f}}$
y_t	$\geq$	$\geq$	$\leq$	$\leq$	y_t	$\geq$	$\geq$	$\leq$	x
π_t	$\leq$	$\geq$	$\leq$	$\leq$	π_t	$\leq$	$\geq$	$\leq$	x
r_t	x	$\geq$	$\geq$	$\geq$	r_t	x	$\geq$	$\geq$	x
F_t	x	$\geq$	$\leq$	$\geq$	F_t	x	$\geq$	$\leq$	$\geq$

Table 9: **Assessing Statistical Differences in the Transmission of Monetary Policy Shocks throughout and after the Zero Lower Bound Period using 4 Variable TVP VAR Models. The Federal Funds Rate vs. the Shadow Rate**
This table reports the probability that the four quarter accumulated response of variable $x = \{y_t, \pi_t, \mathrm{r}_t = \{i_t, i_t^{\mathrm{s}}\}\}$ with respect to a monetary policy shock, from the TVP VAR model where the short-term interest rate is the Federal funds rate, $\mathrm{IRF}_{x,T}^{\mathrm{MP},4,\mathrm{r}=i_t}$, is greater than the four quarter accumulated response of variable x, with respect to a monetary policy shock, from the TVP VAR model using the shadow rate of Wu and Xia (2016) as a proxy for the short-term interest rate for a given time period T, $\mathrm{IRF}_{x,T}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}$. Therefore: $\Pr\left(\mathrm{IRF}_{x,T}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,T}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$. Panel A and B report results from identification schemes I and II respectively. A statistical difference is observed when the probability is less (greater) than 0.1 (0.9). A value lower (greater) than 0.1 (0.9) implies that the IRF implied by the TVP VAR using the Federal funds rate (shadow rate) in time period T is smaller (larger) than that implied by the TVP VAR using the shadow rate (Federal funds rate).

Panel A: Identification I				
$x=$	y_t	π_t	r_t	F_t
$\Pr\left(\mathrm{IRF}_{x,2009Q2}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2009Q2}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.67	0.97	1.00	0.03
$\Pr\left(\mathrm{IRF}_{x,2011Q1}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2011Q1}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.75	0.95	1.00	0.06
$\Pr\left(\mathrm{IRF}_{x,2012Q2}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2012Q2}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.96	0.99	1.00	0.03
$\Pr\left(\mathrm{IRF}_{x,2013Q2}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2013Q2}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.95	0.98	1.00	0.00
$\Pr\left(\mathrm{IRF}_{x,2017Q4}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2017Q4}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.74	0.92	1.00	0.11
Panel B: Identification II				
$x=$	y_t	π_t	r_t	F_t
$\Pr\left(\mathrm{IRF}_{x,2009Q2}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2009Q2}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.59	0.98	1.00	0.50
$\Pr\left(\mathrm{IRF}_{x,2011Q1}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2011Q1}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.76	0.97	1.00	0.43
$\Pr\left(\mathrm{IRF}_{x,2012Q2}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2012Q2}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.94	1.00	0.99	0.47
$\Pr\left(\mathrm{IRF}_{x,2013Q2}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2013Q2}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.97	1.00	0.99	0.50
$\Pr\left(\mathrm{IRF}_{x,2017Q4}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2017Q4}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.78	0.97	0.99	0.32

Table 10: **Assessing Statistical Differences in Taylor Rules throughout and Following the Zero Lower Bound Period using 4 Variable TVP VAR Models. The Federal Funds Rate vs. the Shadow Rate**

The top half of this table reports the probability that the structural impact coefficient associated to variable $x = \{y_t,\ \pi_t,\ \mathrm{r}_t = \{i_t,\ i_t^{\mathrm{s}}\}\}$, within the structural monetary policy rule from the TVP VAR model where the short-term interest rate is the Federal funds rate at period T, $\mathbf{p}_T^{\mathrm{r}x,\mathrm{r}=i_t}$, is greater than the structural impact coefficient associated to variable x, within the structural monetary policy rule from the TVP VAR model using the shadow rate of Wu and Xia (2016) as a proxy for the short-term interest rate at period T, $\mathbf{p}_T^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}$. Therefore: $\Pr\left(\mathbf{p}_T^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_T^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$. The bottom half of this table reports analogous statistics, but for the long-run coefficients associated to variable x, $\mathrm{LRC}_T^{\mathrm{r}x}$, implied by the structural monetary policy rules from TVP VARs using the Federal funds rate and shadow rate respectively. Therefore: $\Pr\left(\mathrm{LRC}_T^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_T^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$. Panels A and B refer to results stemming from identification schemes I and II respectively. A statistical difference is observed when the probability is less (greater) than 0.1 (0.9). A value lower (greater) than 0.1 (0.9) implies that the structural impact/long-run coefficient implied by the TVP VAR using the Federal funds rate (shadow rate) in time period T is less than that implied by the TVP VAR using the shadow rate (Federal funds rate).

Panel A: Identification I				Panel B: Identification II			
Structural Impact Coefficients				Structural Impact Coefficients			
$x=$	y_t	π_t	F_t	$x=$	y_t	π_t	F_t
$\Pr\left(\mathbf{p}_{2009Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2009Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.24	0.57	0.31	$\Pr\left(\mathbf{p}_{2009Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2009Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.23	0.49	0.35
$\Pr\left(\mathbf{p}_{2011Q1}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2011Q1}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.37	0.56	0.38	$\Pr\left(\mathbf{p}_{2011Q1}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2011Q1}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.35	0.46	0.51
$\Pr\left(\mathbf{p}_{2012Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2012Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.26	0.70	0.23	$\Pr\left(\mathbf{p}_{2012Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2012Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.34	0.63	0.40
$\Pr\left(\mathbf{p}_{2013Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2013Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.40	0.75	0.15	$\Pr\left(\mathbf{p}_{2013Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2013Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.37	0.64	0.27
$\Pr\left(\mathbf{p}_{2017Q4}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2017Q4}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.33	0.48	0.38	$\Pr\left(\mathbf{p}_{2017Q4}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathbf{p}_{2017Q4}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.34	0.40	0.42
Long Run Coefficients				Long Run Coefficients			
$x=$	y_t	π_t	F_t	$x=$	y_t	π_t	F_t
$\Pr\left(\mathrm{LRC}_{2009Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2009Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.31	0.55	0.45	$\Pr\left(\mathrm{LRC}_{2009Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2009Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.24	0.43	0.44
$\Pr\left(\mathrm{LRC}_{2011Q1}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2011Q1}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.31	0.56	0.55	$\Pr\left(\mathrm{LRC}_{2011Q1}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2011Q1}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.31	0.56	0.59
$\Pr\left(\mathrm{LRC}_{2012Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2012Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.29	0.52	0.57	$\Pr\left(\mathrm{LRC}_{2012Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2012Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.24	0.57	0.61
$\Pr\left(\mathrm{LRC}_{2013Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2013Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.25	0.60	0.53	$\Pr\left(\mathrm{LRC}_{2013Q2}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2013Q2}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.28	0.58	0.58
$\Pr\left(\mathrm{LRC}_{2017Q4}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2017Q4}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.28	0.57	0.51	$\Pr\left(\mathrm{LRC}_{2017Q4}^{\mathrm{r}x,\mathrm{r}=i_t} > \mathrm{LRC}_{2017Q4}^{\mathrm{r}x,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.38	0.54	0.55

Models with Theoretical Underpinnings: The Inclusion of Money

Although informative and desirable, examining the impact by pooling data from a large number of variables lacks theoretical underpinning. Following the theoretical arguments in Benati (2019) and the empirical model of Ellington (2018), the Divisia M1 aggregate is added to the original data. This allows the fully identified structural model to include a money demand shock. Table 11 shows the contemporaneous sign restrictions imposed in this alternative framework.

Table 11: **Contemporaneous Sign Restrictions for a model with Money**
Notes: This table reports the contemporaneous response of real GDP growth, y_t; inflation, π_t; the interest rate, $\mathrm{r}_t = \{i_t,\ i_t^{\mathrm{s}}\}$; and Divisia M1 growth, m_t with respect to a supply shock, $\mathbf{v}_t^{\mathbf{s}}$; a demand non policy shock, $\mathbf{v}_t^{\mathbf{d}}$; a monetary policy shock, $\mathbf{v}_t^{\mathbf{mp}}$; and a money demand shock, $\mathbf{v}_t^{\mathbf{md}}$, respectively. "x" denotes no restriction imposed.

	$\mathbf{v}_t^{\mathbf{s}}$	$\mathbf{v}_t^{\mathbf{d}}$	$\mathbf{v}_t^{\mathbf{mp}}$	$\mathbf{v}_t^{\mathbf{md}}$
y_t	$\geq$	$\geq$	$\leq$	$\leq$
π_t	$\leq$	$\geq$	$\leq$	$\leq$
r_t	x	$\geq$	$\geq$	$\geq$
m_t	x	$\geq$	$\leq$	$\geq$

The underpinnings of these identifying assumptions can be motivated by a simple transaction demand for money model that allows for money in the utility function[17]. In particular, Benati (2019) shows that the velocity of the M1 monetary aggregate is a linear function of the short-term interest rate. In turn this suggests that the velocity of M1 is a close approximation to the permanent component of the short-term interest rate. Thus, any disequilibrium between M1 velocity, and therefore M1 balances, and the interest rate arise from the interest rate. Upon estimating a bi-variate VAR model, it is shown that the velocity of M1 increases in line the short-term interest rate with respect to transitory shocks. This justifies the positive sign associated to money growth and the interest rate.

Intuitively, the only ways in which velocity can increase are: i) money balances increase; ii) GDP falls; iii) money balances increase while GDP falls; and iv) money balances increase at a faster rate than GDP. Since any disequilibrium between M1 velocity and the interest rate implies movements of the interest rate, iv) can be disregarded. If interest rates rise, this implies GDP growth and inflation falls. Combining this with the fact that demand for money for transactions incorporates a forward looking component, indicates people may hold money for transactions in the future; thereby providing further substance to the identifying assumptions.

Results in this section add Divisia M1 annual growth to the baseline data. The models are estimated in the exact manner as those presented in the main results, with the exception of number of years used in the training sample to calibrate the initial conditions of the model. For these results the first 10 years of data are used because Divisia M1 data begins in 1967. Results

[17]Using the Divisia M1 monetary aggregate is consistent with this type of model as the component assets (currency and demand deposits) are used for transaction purposes. The Divisia M1 aggregate, along with broader measures of US money supply are available from the Center for Financial Stability see http://www.centerforfinancialstability.org/amfm˙data.php. The Divisia measure of M1 is used since its measurement is embedded within superlative index number theory (Barnett, 1980) and is more consistent with the methods used to calculate GDP. Divisia money also has a large historical literature, recent papers include Barnett and Chauvet (2011); Belongia and Ireland (2015); Barnett et al. (2016).

have also been calibrated using the first 20 years of data, and conclusions do not change from those presented here.

Table 12 reports tests for statistical differences in the four quarter accumulated impulse response functions of GDP growth, inflation, and Divisia M1 between models using the Federal funds rate and shadow rate; comparable to Table 3. It is even more clear from these results of the start differences in accounting for unconventional monetary policies using the shadow rate. In particular both throughout and following the zero lower bound period results in a larger (smaller) response of GDP and inflation (money growth) when using the Federal funds rate.

Table 12: **Assessing Statistical Differences in the Transmission of Monetary Policy Shocks throughout and after the Zero Lower Bound Period. The Federal Funds Rate vs. the Shadow Rate: Models including Divisia M1 Growth**

This table reports the probability that the four quarter accumulated response of variable $x = \{y_t, \pi_t, \mathrm{r}_t = \{i_t, i_t^{\mathrm{S}}\}\}$ with respect to a monetary policy shock, from the TVP VAR model where the short-term interest rate is the Federal funds rate, $\mathrm{IRF}_{x,T}^{\mathrm{MP},4,\mathrm{r}=i_t}$, is greater than the four quarter accumulated response of variable x, with respect to a monetary policy shock, from the TVP VAR model using the shadow rate of Wu and Xia (2016) as a proxy for the short-term interest rate for a given time period T, $\mathrm{IRF}_{x,T}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}$. Therefore: $\Pr\left(\mathrm{IRF}_{x,T}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,T}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$. A statistical difference is observed when the probability is less (greater) than 0.1 (0.9). A value lower (greater) than 0.1 (0.9) implies that the IRF implied by the TVP VAR using the Federal funds rate (shadow rate) in time period T is smaller (larger) than that implied by the TVP VAR using the shadow rate (Federal funds rate).

$x=$	y_t	π_t	r_t	m_t
$\Pr\left(\mathrm{IRF}_{x,2009Q2}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2009Q2}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.93	0.95	1.00	0.00
$\Pr\left(\mathrm{IRF}_{x,2011Q1}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2011Q1}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.93	0.99	0.98	0.00
$\Pr\left(\mathrm{IRF}_{x,2012Q2}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2012Q2}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.99	0.99	0.95	0.01
$\Pr\left(\mathrm{IRF}_{x,2013Q2}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2013Q2}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.99	0.99	0.95	0.01
$\Pr\left(\mathrm{IRF}_{x,2017Q4}^{\mathrm{MP},4,\mathrm{r}=i_t} > \mathrm{IRF}_{x,2017Q4}^{\mathrm{MP},4,\mathrm{r}=i_t^{\mathrm{S}}}\right)$	0.96	0.96	0.93	0.00

Figures 12 and 13 report the posterior median and 80% point-wise equal-tailed probability bands of the structural impact and long-run coefficients obtained from Taylor-rules implied by respective TVP VARs using the Federal funds rate and the shadow rate. The structural impact coefficients for real GDP and inflation from both models exhibit similar time profiles to the baseline results in Figure 3. Note that the impact coefficient associated to money follows a downward trend with surges during recessions; consistent with Ellington (2018).

Although the long-run coefficients for GDP growth are similar to those in Figure 4, inflation exhibits slightly different time profiles. In particular, and arguably more consistent with economic theory, the posterior median long-run coefficient fluctuates around unity. Then, following the 2008 recession, the long-run coefficient on inflation from these models diverge. The inflationary impact on the shadow rate is up to half a percentage point larger relative to the Taylor implied using the Federal funds rate; although economically meaningful, the differences are not statistically credible. This divergence also occurs for the long-run coefficient(s) associated to money[18]. Overall, model implied Taylor rules suggest a significant role for money within Taylor

[18]Although there are some economic differences in the impact and long-run coefficient associated to money,

rules which supports the results in Belongia and Ireland (2015).

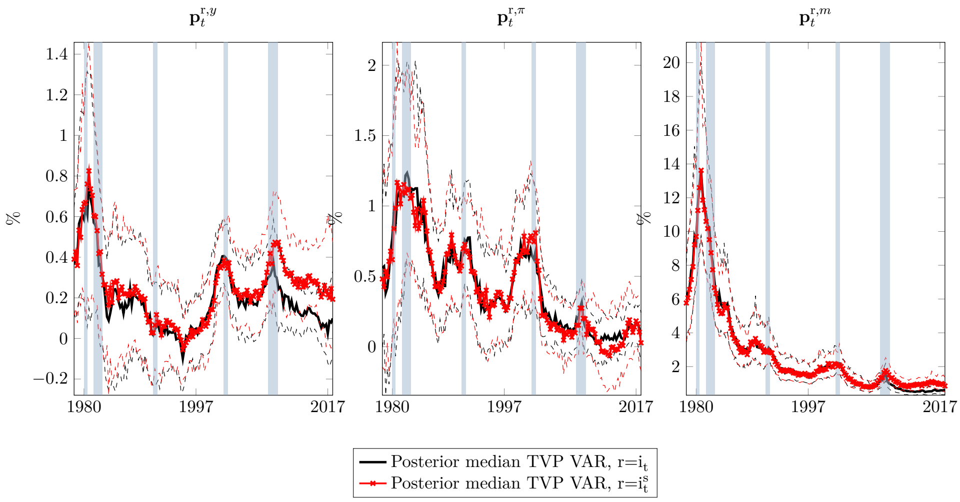

Figure 12: **Structural Impact Coefficients Implied by Structural Monetary Policy Rules from 1979Q2–2017Q4**

Notes: This figure plots the posterior median and 80% point-wise equal-tailed probability bands of the structural impact coefficients associated to real GDP growth, $\mathbf{p}_t^{r,y}$; Consumer price inflation, $\mathbf{p}_t^{r,\pi}$; and Divisia M1 growth, $\mathbf{p}_t^{r,m}$. In each model, the short-term interest rate is proxied by either the Federal funds rate, $r_t = i_t$, or the shadow rate, $r_t = i_t^s$. Grey bars indicate NBER recession dates.

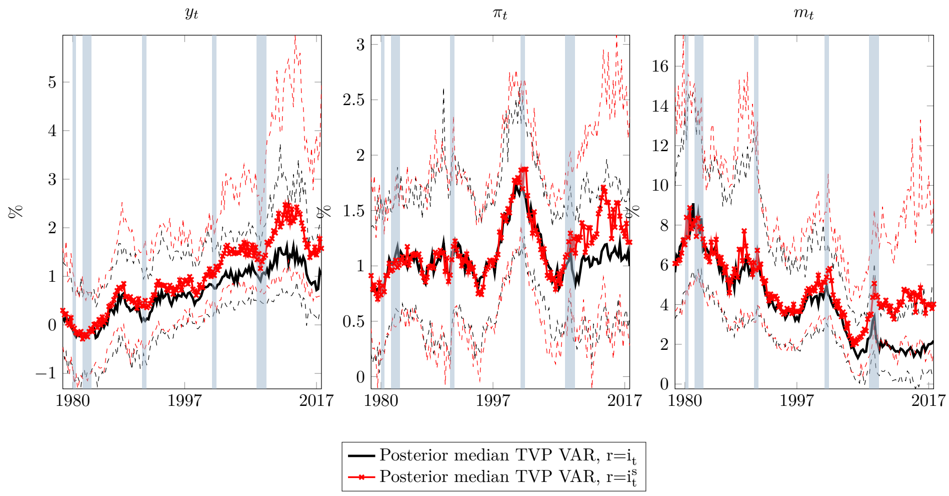

Figure 13: **Long-run Coefficients Implied by Structural Monetary Policy Rules from 1979Q2–2017Q4**

Notes: This figure plots the posterior median and 80% point-wise equal-tailed probability bands of the long-run coefficients associated to real GDP growth, y_t; Consumer price inflation, π_t; and Divisia M1 money growth, m_t, as implied by the structural monetary policy rule including money. In each model, the short-term interest rate is proxied by either the Federal funds rate, $r_t = i_t$, or the shadow rate, $r_t = i_t^s$. Grey bars indicate NBER recession dates.

they are not statistically significant.

In general from both sets of robustness analysis presented, it is clear that the results are robust to a both alternative data driven specifications, and theoretically founded set-up. This further highlights the importance of accounting for monetary policy stance, and indeed unconventional monetary policies; particularly over the last decade. These results add further substance to baseline analysis. In particular, the sensitivity of macroeconomic variables to monetary policy shocks when not accounting for unconventional monetary policies is over-stated. Furthermore, model implied Taylor rules show that using the shadow rate suggest that the Fed placed a larger weight on economic activity relative to inflation.

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