Valuation Risk Revalued*

Oliver de Groot   Alexander W. Richter   Nathaniel A. Throckmorton

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ABSTRACT

This paper shows the success of valuation risk—time-preference shocks in Epstein-Zin utility—in resolving asset pricing puzzles rests sensitively on the way it is introduced. The specification used in the literature is at odds with several desirable properties of recursive preferences because the weights in the time-aggregator do not sum to one. When we revise the specification in a simple asset pricing model the puzzles resurface. However, when estimating a sequence of increasingly rich models, we find valuation risk under the revised specification consistently improves the ability of the models to match asset price and cash-flow dynamics.

Keywords: Recursive Utility; Asset Pricing; Equity Premium Puzzle; Risk-Free Rate Puzzle

JEL Classifications: C15; D81; G12

*de Groot, University of Liverpool Management School & CEPR, Chatham Street, Liverpool, L69 7ZH, UK (oliverdegroot@gmail.com); Richter, Research Department, Federal Reserve Bank of Dallas, 2200 N. Pearl Street, Dallas, TX 75201 (alex.richter@dal.frb.org); Throckmorton, Department of Economics, William & Mary, P.O. Box 8795, Williamsburg, VA 23187 (nat@wm.edu). We thank Victor Xi Luo for sharing the code to "Valuation Risk and Asset Pricing" and Winston Dou for discussing our paper at the 2019 NBER EFSF meeting. We also thank Martin Andreasen, Jaroslav Borovicka, Alex Chudik, Marc Giannoni, Ken Judd, Dana Kiku, Evan Koenig, Alex Kostakis, Holger Kraft, Wolfgang Lemke, Hanno Lustig, Walter Pohl, Karl Schmedders, Todd Walker, and Ole Wilms for comments that improved the paper. This research was supported in part through computational resources provided by the BigTex High Performance Computing Group at the Federal Reserve Bank of Dallas. The views in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

1 Introduction

In standard asset pricing models, uncertainty enters through the supply side of the economy, either through endowment shocks in a Lucas (1978) model or productivity shocks in a production economy model. Recently, several influential papers have included time preference shocks or "valuation risk" as a potential *demand side* driver of asset prices (Albuquerque et al., 2016, 2015; Basu and Bundick, 2017; Chen and Yang, 2019; Creal and Wu, 2020; Gomez-Cram and Yaron, 2020; Kliem and Meyer-Gohde, 2018; Maurer, 2012; Nakata and Tanaka, 2016; Schorfheide et al., 2018).[1]

The literature argues valuation risk is an important determinant of key asset pricing moments when embedded in Epstein and Zin (1989) recursive preferences. This paper contributes to the literature by theoretically and empirically re-examining the role of valuation risk. We first show the success of valuation risk rests sensitively on the way time preference shocks are introduced. In particular, we examine two specifications—Current (the specification used in the asset pricing literature) and Revised (our preferred alternative)—and show they lead to very different conclusions.

Given our theoretical findings, we use a rigorous simulated method of moments estimation approach to empirically re-evaluate the role of valuation risk in explaining asset pricing and cash-flow moments. After estimating a sequence of increasingly rich models, we find the role and contribution of valuation risk change dramatically relative to the literature. However, valuation risk under the revised specification consistently improves the ability to match moments in the data.

To evaluate the current and revised specifications, we identify four desirable properties of Epstein-Zin recursive preferences. This provides a practical guide for selecting valuation risk preferences in macro-finance.[2] The first property pertains to comparative risk aversion. It says, holding all else equal, an increase in the coefficient of relative risk aversion (RA, γ) equates to an increase in risk aversion. We show this property does not hold when the intertemporal elasticity of substitution (IES, ψ) is below unity under the current specification. An increase in γ equates to a decrease, rather than an increase, in aversion to valuation risk, flipping its standard interpretation.[3]

The second property is that preferences are well-defined with unitary IES. The IES measures the responsiveness of consumption growth to a change in the real interest rate. An IES of 1 is a focal point because this is when the substitution and wealth effects of an interest rate change exactly offset. We show this property does not hold under the current specification in the literature.

The third property is that recursive preferences nest time-separable log-preferences when $\gamma =$

[1] Time preference shocks are commonly referred to as discount factor shocks. The price of an asset is the present value of its future income stream. Valuation risk refers to the uncertainty households face about how to discount future income. These shocks have also been widely used in the macro literature (e.g., Christiano et al. (2011); Eggertsson and Woodford (2003); Justiniano and Primiceri (2008); Rotemberg and Woodford (1997); Smets and Wouters (2003)).

[2] Identifying a general set of admissible preference specifications that satisfy formal axioms is left to future work.

[3] The distinction between Epstein and Zin (1989) recursive preferences and constant relative risk aversion (CRRA) utility is that in the former, ψ and γ are distinct structural parameters, whereas in the latter the parameters are inverses.

$\psi = 1$. We show the current specification does not always nest log preferences in this case because it can even generate extreme curvature and aversion to valuation risk when γ and ψ are close to 1.

The final property is that equilibrium moments are continuous functions of the IES over its domain. We show there is a discontinuity (or vertical asymptote) under the current specification. When the IES is marginally above unity, households require an arbitrarily large equity premium and an arbitrarily small risk-free rate, while an IES marginally below unity predicts the opposite. This is because the utility function exhibits extreme concavity with respect to valuation risk when the IES is marginally above unity and extreme convexity when the IES is marginally below unity.[4]

The discontinuity is relevant because there is a tension between the finance and macroeconomics literatures as to whether the IES lies above or below unity. Setting the IES to 0.5, as is common in the macroeconomics literature, can inadvertently result in a sizable negative equity premium.[5] Imagine two researchers who want to estimate the IES set the domain to $[0, 1)$ and $(1, \infty)$, respectively. The estimates in the two settings would diverge due to the discontinuity. Therefore, awareness of these issues is important even if researchers continue to use the current preferences.

In a business cycle context, de Groot et al. (2018) propose a revised Epstein-Zin specification for valuation risk in which the time-varying weights in the CES time-aggregator sum to 1, a restriction the current specification does not impose. This specification satisfies all four desired properties. There is a well-defined equilibrium when the IES is 1 and asset prices are robust to small variations in the IES. Continuity is preserved because the weights in the time-aggregator always sum to unity. Another interpretation is that the time-aggregator maintains the well-known property that a CES aggregator tends to a Cobb-Douglas aggregator as the elasticity approaches 1.

The change in specification profoundly affects the equilibrium determination of asset prices. For example, the same RA and IES can lead to very different values for the equity premium and risk-free rate and comparative statics, such as the response of the equity premium to the IES, switch sign. Taken at face value, the current specification can resolve the equity premium (Mehra and Prescott, 1985) and risk-free rate (Weil, 1989) puzzles in an estimated model with *i.i.d.* cash-flow risk. Under the revised specification, valuation risk has a smaller role, RA is implausibly high, and the puzzles resurface. In light of these results, we estimate a sequence of increasingly rich models to empirically re-evaluate the role of valuation risk under the revised specification.

We begin by estimating the Bansal and Yaron (2004) long-run risk model (without time-varying uncertainty) without valuation risk and find it significantly under-predicts the standard deviation of the risk-free rate, even when these moments are targeted. When we introduce valuation risk, it accounts for roughly 40% of the equity premium, but at the expense of over-predicting the standard

[4] Kruger (2020) also shows aversion to valuation risk becomes infinite as $\psi \to 1$ under the current specification.

[5] Hall (1988) and Campbell (1999) provide empirical evidence for an IES close to zero. Basu and Kimball (2002) find an IES of 0.5 and Smets and Wouters (2007) estimate a value of roughly 0.7. In contrast, van Binsbergen et al. (2012) and Bansal et al. (2016) estimate models with Epstein-Zin preferences and report IES values of 1.73 and 2.18.

deviation of the risk-free rate. After targeting the risk-free rate dynamics, valuation risk only accounts for about 5% of the equity premium. Therefore, we find it is crucial to target these dynamics to accurately measure the contribution of valuation risk. Valuation risk is also able to generate the upward sloping term structure for real Treasury yields found in the data, whereas cash-flow risk alone predicts a counterfactually downward sloping term structure. While valuation risk (with or without the targeted risk-free rate moments) improves the fit of the long-run risk model, the model still fails a test of over-identifying restrictions. This is because the model fairs poorly in matching the low predictability of consumption growth from the price-dividend ratio, the high standard deviation of dividend growth, and the weak correlation between dividend growth and equity returns.

We consider two extensions that improve the model's fit: (1) an interaction term between valuation and cash-flow risk (a proxy for general equilibrium demand effects) following Albuquerque et al. (2016) (henceforth, "Demand" model) and (2) stochastic volatility on cash-flow risk as in Bansal and Yaron (2004) (henceforth, "SV" model). In a horse race between these extensions, we find the Demand model wins and passes the over-identifying restrictions test at the 5% level. However, the two extensions are complements and the combined model passes the test at the 10% level. This is because the demand extension lowers the correlation between dividend growth and equity returns, while the SV extension offsets the effect of higher valuation risk on risk-free rate dynamics. Targeting longer-term rates further increases the relative improvement of the combined model.

Related Literature This paper builds on the growing literature that examines the role of valuation risk in asset pricing models. Maurer (2012) and Albuquerque et al. (2016) were the first. They find valuation risk accounts for key asset pricing moments, such as the equity premium. Albuquerque et al. (2016) also focus on resolving the correlation puzzle (Campbell and Cochrane, 1999). Schorfheide et al. (2018) use a Bayesian mixed-frequency approach that targets entire time series and find valuation risk helps improve the empirical fit, particularly for the risk-free rate. Gomez-Cram and Yaron (2020) use a similar estimation strategy to show that preference shocks are important for explaining the nominal yield curve. Both papers use priors for the IES that encompass a unitary elasticity. Creal and Wu (2020) develop a term structure model where valuation risk is tied to consumption and inflation and does not have an independent stochastic element. Given an IES estimate close to unity (0.80), they find valuation risk is useful for matching time-variation in term premia. Nakata and Tanaka (2016) and Kliem and Meyer-Gohde (2018) study term premia in a New Keynesian model. The former calibrate the IES to 0.11 and generate a negative term premium, while the latter estimate the IES with a prior in the $[0, 1]$ range and obtain an IES of 0.09.

All of these papers use the current preferences and are potentially affected by the influence of the asymptote. For example, the papers with an IES estimate below unity find an estimate far below unity, since an IES close to but below one could generate large negative equity premia as a result of the asymptote. Furthermore, in studies where the IES is less than unity, an increase in the coefficient of relative risk aversion decreases aversion to valuation risk. In this paper, we first compare estimates based on the current and revised preferences in a simple endowment economy. We report significant differences in the estimates and contributions of valuation risk. We then use the revised preferences to estimate a sequence of increasingly rich models of long-run risk to re-evaluate the role of valuation risk in explaining asset prices. We find valuation risk has a smaller role in resolving the equity premium and risk-free rate puzzles, but it still plays an important role in matching particular moments. In related work, Rapach and Tan (2018) and Bianchi et al. (2018) use the revised specification to estimate real business cycle models. Both papers find valuation risk has an important role in explaining equity premia when it is interacted with other structural shocks. In all of these studies, including our own, time preference shocks are latent. Chen and Yang (2019) go a step further and proxy time preferences shocks using changes in life expectancy in the U.S.[6]

The paper proceeds as follows. Section 2 lays out desirable properties of recursive preferences and the consequences of the valuation risk specification. Section 3 discusses asset pricing implications. Section 4 describes our estimation method. Section 5 quantifies the effects of valuation risk in our baseline model with *i.i.d.* cash-flow risk. Section 6 estimates the basic long-run risk model with and without valuation risk, and Section 7 considers two key extensions. Section 8 concludes.

2 EPSTEIN-ZIN PREFERENCES WITH DISCOUNT FACTOR SHOCKS

2.1 BACKGROUND

Epstein and Zin (1989) preferences generalize standard expected utility time-separable preferences. Current-period utility is defined recursively over current-period consumption, c_t, and a certainty equivalent, $\mu_t(U_{t+1})$, of next period's random utility, U_{t+1}, as follows:

$$U_t = W(c_t, \mu_t(U_{t+1})), \tag{1}$$

where $\mu_t \equiv g^{-1}(E_t g(U_{t+1}))$, W is the *time-aggregator*, and g is the *risk-aggregator*. W and g are increasing and concave and W and μ_t are homogenous of degree 1. Note that $\mu_t(U_{t+1}) = U_{t+1}$ if there is no uncertainty, and $\mu_t(U_{t+1}) \leq E_t[U_{t+1}]$ if g is concave and future outcomes are uncertain. Most of the literature considers the following functional forms for W and g:

$$g(z) \equiv (z^{1-\gamma} - 1)/(1-\gamma), \qquad \text{for } 1 \neq \gamma > 0, \tag{2}$$

$$W(x, y) \equiv \left((1-\beta)x^{1-1/\psi} + \beta y^{1-1/\psi}\right)^{1/(1-1/\psi)}, \quad \text{for } 1 \neq \psi > 0. \tag{3}$$

When $\gamma = 1$, $g(z) = \log(z)$ and when $\psi = 1$, $W = x^{1-\beta}y^{\beta}$. Therefore, the time-aggregator is

[6] Two other strands of the literature have interesting connections to our work. One, disaster risk (see Barro, 2009 and Gourio, 2012) can generate variation in the stochastic discount factor analogous to valuation risk. Two, Bansal et al. (2014), identify "discount rate risk" as a component of risk premia distinct from cash-flow and volatility risks.

a CES function that converges to a Cobb-Douglas function as $\psi \to 1$.[7] It is also common in the literature to see the time-aggregator written without the $(1-\beta)$ coefficient on x as follows:

$$W(x, y) \equiv \left(x^{1-1/\psi} + \beta y^{1-1/\psi}\right)^{1/(1-1/\psi)}. \tag{3'}$$

In this case, (3') is undefined when $\psi = 1$. This is because the weights in the time-aggregator do not sum to 1. Nevertheless, the exact specification of W does not affect equilibrium behavior.[8]

Result 1. *Utility function* (1) *with time-aggregator* (3) *or* (3') *represents the same preferences.*

Result 1 holds because it is possible to switch between (3) and (3') with a positive monotonic transformation that multiplies the utility function by $(1-\beta)^{1/(1-1/\psi)}$.[9] To see this, note that the intertemporal marginal rate of substitution (or the stochastic discount factor, SDF) is given by

$$m_{t+1} \equiv \left(\frac{\partial U_t}{\partial c_{t+1}}\right) \Big/ \left(\frac{\partial U_t}{\partial c_t}\right) = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi} \left(\frac{U_{t+1}}{\mu_t\left(U_{t+1}\right)}\right)^{1/\psi-\gamma}. \tag{4}$$

Since μ_t is homogenous of degree 1, applying the positive monotonic transformation to U_{t+1} in the both numerator and denominator leaves the intertemporal marginal rate of substitution unchanged.[10]

The results thus far are standard, but they lay the groundwork for the discussion that follows. Valuation risk involves introducing discount factor shocks—exogenous stochastic time-variation in β. Whether one works with (3) and replaces both instances of β with $a_t\beta$ (where a_t is a log-normal mean zero stationary AR(1) stochastic process) or one works with (3') and replaces the only instance of β with $a_t\beta$ is *not* innocuous, even though one might conclude it is from Result 1. The specification matters and in what follows we will describe the consequences of these choices.

To determine a preferred specification of valuation risk, we first establish four desirable properties of standard Epstein-Zin preferences *without* discount factor shocks, and then assess whether the two specifications of Epstein-Zin preferences with discount factor shocks satisfy each of them.

Property 1. γ *is a measure of comparative risk aversion.*

Suppose there are two households, A and B, with Epstein-Zin preferences as defined above. The two households are identical in every way except in preference parameter γ. If γ measures risk aversion, then household A is more risk averse than household B if and only if $\gamma^A > \gamma^B$.

[7] The functional form for g implies $\mu_t = (E_t U_{t+1}^{1-\gamma})^{1/(1-\gamma)}$ when $\gamma \neq 1$ and $\mu_t = \exp(E_t \log(U_{t+1}))$ when $\gamma = 1$.

[8] Kraft and Seifried (2014) prove the continuous-time analog of recursive preferences (stochastic differential utility, Duffie and Epstein, 1992) is the continuous-time limit of recursive utility if the weights in the time-aggregator sum to 1.

[9] This is similar to the common practice of writing CRRA utility as $u(c) = c^\alpha/\alpha$ instead of $u(c) = (c^\alpha - 1)/\alpha$, even though the omitted constant term is necessary when proving the limit as $\alpha \to 0$ is given by $u(c) = \log(c)$.

[10] An equivalent observation is that time-preference is independent of the $(1-\beta)$ coefficient. In an environment without consumption growth and without risk, time-preference is captured by the discount factor (i.e., $m_{t+1} = \beta$).

Property 2. *ψ is a measure of the IES and preferences are well defined with unit IES.*

The IES is defined as the responsiveness of consumption growth to a change in the real interest rate. A rise in the real interest rate induces both a substitution effect (consumption today becomes relatively more expensive, decreasing current consumption) and an income effect (a saver feels wealthier, increasing current consumption). The substitution and income effects exactly offset when $\psi = 1$. Therefore, a unitary IES is an important focal point for any model of preferences.[11]

Property 3. *When $\gamma = \psi = 1$, Epstein-Zin preferences are equivalent to time-separable log-preferences given by $U_t = (1-\beta)\log(c_t) + \beta E_t U_{t+1}$ or, alternatively, $U_t = \log(c_t) + \beta E_t U_{t+1}$.*

Property 3 is a special case of the more general property that when $\gamma = 1/\psi$, Epstein-Zin preferences simplify to standard expected utility time-separable preferences. However, time-separable log preferences are a staple of economics textbooks, so this provides another useful benchmark.

Property 4. *Equilibrium moments are continuous functions of the IES, ψ, over its domain $\mathbb{R}^+$.*

This final property relates to the discussion of time-aggregator (3) versus (3'). Adopt (3') and suppose $x = 1$ and $y > 0$. In this case, $\lim_{\psi\to 1^-} W = 0$ and $\lim_{\psi\to 1^+} W = +\infty$. Therefore, (3') exhibits a discontinuity. However, as discussed, this discontinuity does not affect the intertemporal marginal rate of substitution, (4), and, as a result, does not materialize in equilibrium moments.

2.2 DISCOUNT FACTOR SHOCKS

There are two ways to introduce discount factor shocks into the Epstein-Zin time-aggregator. The first is denoted the "[C]urrent specification" and given by

$$W^C(x,y,a_t) \equiv \left((1-\beta)x^{1-1/\psi} + a_t\beta y^{1-1/\psi}\right)^{1/(1-1/\psi)}, \tag{3C}$$

where $a_t > 0$. The second is denoted the "[R]evised specification" and given by

$$W^R(x,y,a_t) \equiv \left((1-a_t\beta)x^{1-1/\psi} + a_t\beta y^{1-1/\psi}\right)^{1/(1-1/\psi)}, \tag{3R}$$

where $0 < a_t < 1/\beta$. The current specification is common. Its use is not surprising since, at face value, it is the natural extension of discount factor shocks to expected utility time-separable preferences given by $U_t = u(c_t) + a_t\beta E_t U_{t+1}$.[12] The revised specification extends Epstein and Zin (1991) to make the discount factor time-varying. Importantly, the two specifications are *not* equivalent.[13]

Result 2. *Utility function* (1) *given* (3C) *does not, in general, reflect the same preferences as* (3R).

[11] A unitary IES is also the basis of the "risk-sensitive" preferences in Hansen and Sargent (2008, Section 14.3).

[12] Kollmann (2016) introduces a time-varying discount factor into Epstein-Zin preferences in similar way as our revised specification. In that setup, however, the discount factor is a function of endogenously determined consumption.

[13] The presence of the $(1-\beta)$ coefficient in (3C) is irrelevant but we include it for symmetry. The domain of a_t is constrained to ensure the time-aggregator weights are always positive. With (3C), $a_t > 0$. With (3R), $0 < a_t < 1/\beta$.

To demonstrate this result, we show there is no positive monotonic transformation that maps the two specifications. Define $\tilde{U}_t^C = (\frac{1-a_t\beta}{1-\beta})^{1/(1-1/\psi)} U_t^C$, so the transformed preferences are given by

$$\tilde{U}_t^C = \left((1-a_t\beta) c_t^{1-1/\psi} + a_t\beta\mu_t \left(\tilde{a}_{t+1}^{1/(1-1/\psi)} \tilde{U}_{t+1}^C \right)^{1-1/\psi} \right)^{1/(1-1/\psi)}, \tag{5}$$

where $\tilde{a}_{t+1} \equiv (1-a_t\beta)/(1-a_{t+1}\beta)$. The revised preferences are given by

$$U_t^R = \left((1-a_t\beta) c_t^{1-1/\psi} + a_t\beta\mu_t \left(U_{t+1}^R \right)^{1-1/\psi} \right)^{1/(1-1/\psi)}. \tag{6}$$

Therefore, the equivalence only exists if $a_{t+1} = a_t$ for all t. Comparing (5) and (6), there are two striking features of the current specification. One, it has more risk since $\tilde{a}_{t+1}$ introduces additional variance. Two, it has more curvature in the certainty equivalent since $\tilde{a}_{t+1}$ is raised to $1/(1-1/\psi)$.

To gain further insight, we make a few simplifying assumptions. First, suppose $c_{t+1} = 1$ and $\Delta_{t+j} \equiv c_{t+j}/c_{t+j-1} = \Delta > 1$ for all $j \geq 2$. Second, suppose $a_{t+j} = 1$ for $j = 0$ and $j \geq 2$, but a_{t+1} is a random draw. The terms inside the expectations operators contained in μ_t are given by

$$\bar{U}_C(a_{t+1}) \equiv g(U_{t+1}^C) = g\left((1-\beta + a_{t+1}\beta\bar{x})^{1/(1-1/\psi)} \right), \tag{7}$$

$$\bar{U}_R(a_{t+1}) \equiv g(U_{t+1}^R) = g\left((1-a_{t+1}\beta + a_{t+1}\beta\bar{x})^{1/(1-1/\psi)} \right), \tag{8}$$

where $\bar{x} = \Delta^{1-1/\psi}(1-\beta)/(1-\beta\Delta^{1-1/\psi})$. One source of intuition is to examine the curvature of (7) and (8) with respect to a_{t+1} by defining an Arrow-Pratt type measure of risk aversion given by

$$\mathcal{A}^j \equiv -(\bar{U}_j''(a_{t+1})/\bar{U}_j'(a_{t+1}))|_{a_{t+1}=1},$$

where $j \in \{C, R\}$. The curvatures of the current and revised specifications are given by

$$\mathcal{A}^C = \left(\frac{\gamma - 1/\psi}{1-1/\psi} \right) \beta\Delta^{1-1/\psi} \quad \text{and} \quad \mathcal{A}^R = \left(\frac{\gamma - 1/\psi}{1-1/\psi} \right) \frac{\beta}{1-\beta} \left(\Delta^{1-1/\psi} - 1 \right). \tag{9}$$

To visualize this, Figure 1 plots state-space indifference curves following Backus et al. (2005). Suppose there are two equally likely states for $a_{t+1} \in \{a_1, a_2\}$. The 45-degree line represents certainty. We plot (a_1, a_2) pairs, derived in the Online Appendix, that deliver the same utility as the certainty equivalent. A convex indifference curve implies aversion with respect to valuation risk.

Result 3. *The current specification is at odds with Property 1 when $\psi < 1$ because increasing γ reduces aversion to valuation risk. Under the revised specification Property 1 is always satisfied.*

Result 3 states that under the current specification, a higher RA can lead to a fall in aversion to valuation risk ($\partial\mathcal{A}^C/\partial\gamma < 0$) for $\psi < 1$. This is shown in the top-row of Figure 1. Under the current specification with $\psi = 0.95$, an increase in γ from 0.1 to 3 causes the indifference curve to

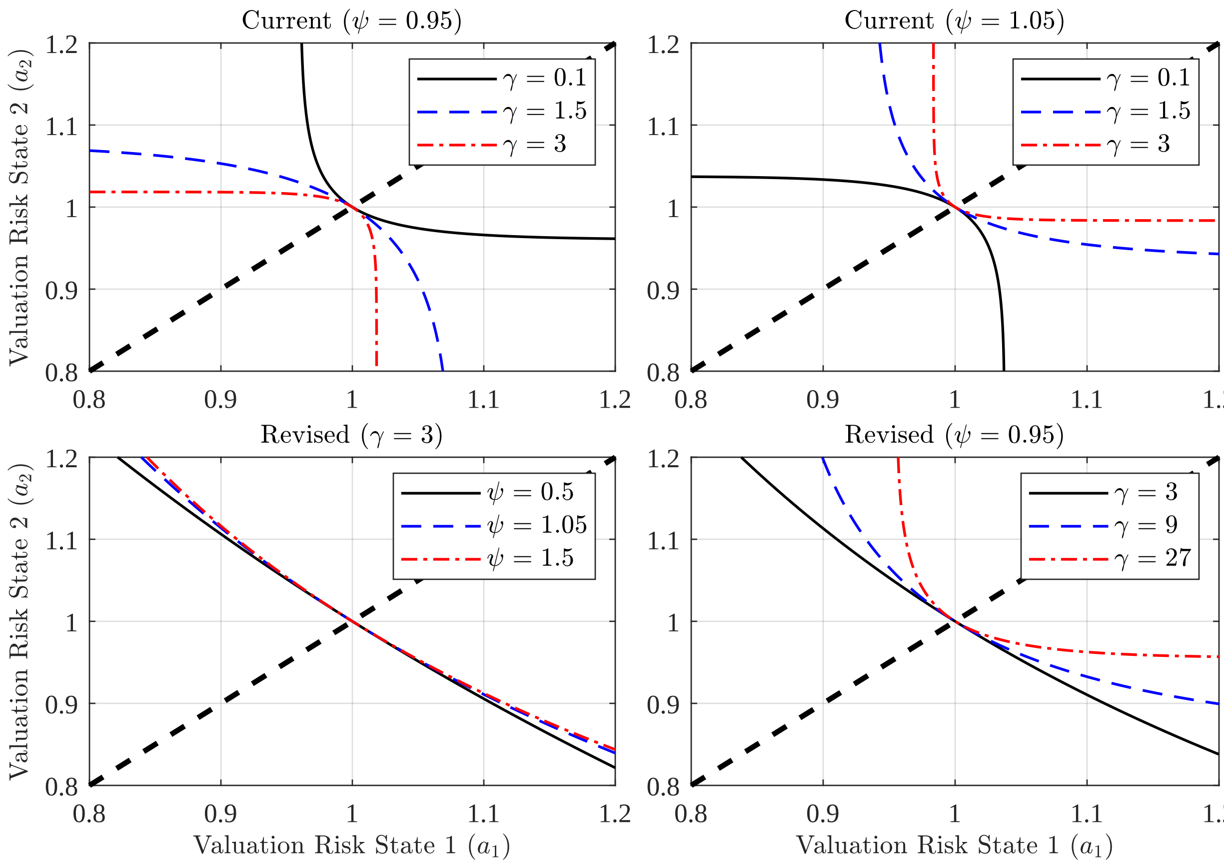

Figure 1: State-space indifference curves. We set $\beta = 0.9975$ and $\Delta = 1.0015$.

become less convex, indicating a decrease in aversion to valuation risk. When $\psi = 1.05$, the opposite occurs. Under the revised specification, $\partial \mathcal{A}^R/\partial\gamma > 0$ for all ψ, consistent with Property 1.

Result 4. *The current preferences are extremely concave with respect to valuation risk as $\psi \to 1^+$, extremely convex as $\psi \to 1^-$, and undefined when $\psi = 1$, which is at odds with Property 2. In contrast, the curvature of the revised preferences is continuous and increases only modestly in ψ.*

Result 4 states that under the current specification, risk aversion is very sensitive to the calibration of the IES. This is concerning since Epstein-Zin-type preferences are designed to separate risk attitudes from timing attitudes. Under the current specification, curvature and hence risk attitudes are primarily determined by the IES parameter. The revised specification resolves this problem.

One source of intuition is to examine an alternative version of the current specification given by

$$W^A(x, y, a_t) \equiv \left((1 - a_t\beta)x^{1-1/\psi} + \beta y^{1-1/\psi}\right)^{1/(1-1/\psi)}, \tag{3A}$$

where a_t only appears in the first position. A priori, if we accept the current specification, then (3A) should be an acceptable alternative. The curvature of the alternative specification is given by $\mathcal{A}^A = -\left(\frac{\gamma-1/\psi}{1-1/\psi}\right)\frac{\beta}{1-\beta}\left(1 - \beta\Delta^{1-1/\psi}\right)$, which has almost the exact opposite properties as $\mathcal{A}^C$

because the preferences become extremely convex with respect to valuation risk as $\psi \to 1^+$ and extremely concave as $\psi \to 1^-$. Since $\mathcal{A}^R = \mathcal{A}^C + \mathcal{A}^A$, the extreme curvature observed in both the current and alternative preference specifications broadly cancel out under the revised specification.[14]

A similar insight can be drawn from Maurer (2012), who introduces two separate shocks in the Epstein and Zin (1991) preference specification. The first is akin to the shock in (3A) and referred to as a taste shock, whereas the second is akin to (3C) and referred to as a time-preference shock. The Online Appendix shows that as the correlation between these two shocks tends to 1, we recover our revised preference specification. This gives a complementary interpretation of our revised preference specification as one in which taste and time-preference shocks are perfectly correlated.

Result 5. *Suppose $\gamma = 1 - \epsilon$ and $1 - 1/\psi = \epsilon^2$. As $\epsilon \to 0$, the current specification is at odds with Property 3, whereas the revised specification converges to $U_t = (1 - a_t\beta)\log c_t + a_t\beta E_t U_{t+1}$.*

Result 5 summarizes our investigation of Property 3 under valuation risk. If we begin with log-preferences and introduce discount factor shocks, then $U_t = (1 - a_t\beta)\log(c_t) + a_t\beta E_t U_{t+1}$ or $U_t = \log(c_t) + a_t\beta E_t U_{t+1}$ and there is no curvature with respect to valuation risk ($\mathcal{A} = 0$). Therefore, when $\gamma = \psi = 1$, Epstein-Zin preferences under valuation risk should always reduce to one of these utility functions and the SDF should reduce to $m_{t+1} \equiv a_t\beta(\frac{1-a_{t+1}\beta}{1-a_t\beta})\frac{c_t}{c_{t+1}}$ or $m_{t+1} \equiv a_t\beta\frac{c_t}{c_{t+1}}$. We show in the Online Appendix that this occurs under the revised specification, but *not* under the current specification when ψ approaches 1 at a faster rate than γ. Furthermore, suppose we calculate the limit as $\epsilon \to 0$, assuming $\gamma = 1 - \epsilon$ and $1 - 1/\psi = \epsilon^2$ to ensure ψ converges to 1 at a faster rate than γ. The current specification still exhibits extreme curvature with respect to valuation risk even though both γ and ψ become arbitrarily close to 1 as in the log-preference case.

3 CONSEQUENCES FOR ASSET PRICING

Thus far, we have described the alternative valuation risk specifications in terms of properties related to the curvature of the utility function. This section applies these ideas to asset pricing moments using our baseline asset pricing model and analyzes their consequences for Property 4.

3.1 BASELINE ASSET PRICING MODEL This section describes our baseline model with *i.i.d.* cash-flow risk. Later sections will introduce richer features, such as long-run cash-flow risk and stochastic volatility. We solve each model using a Campbell and Shiller (1988) approximation to facilitate estimation in the next section. Our theoretical results, however, do not rest on this choice. The Online Appendix shows the vertical asymptote identified in the current preference specification also appears when we derive an exact closed-form solution to the fully nonlinear model.[15]

[14] The Online Appendix shows (3A) is isomorphic to (3C) with a small change in the timing of the preference shock.

[15] Pohl et al. (2018) study the accuracy of Campbell-Shiller approximations for long-run risk asset pricing models.

Each period t denotes 1 month.[16] There are two assets: an endowment share, $s_{1,t}$, that pays income, y_t, and is in fixed unit supply, and an equity share, $s_{2,t}$, that pays dividends, d_t, and is in zero net supply. A representative household chooses $\{c_t, s_{1,t}, s_{2,t}\}_{t=0}^{\infty}$ to maximize utility (1) with time aggregator (3C) or (3R). The choices are constrained by the flow budget constraint given by

$$c_t + p_{y,t}s_{1,t} + p_{d,t}s_{2,t} = (p_{y,t} + y_t)s_{1,t-1} + (p_{d,t} + d_t)s_{2,t-1}, \tag{10}$$

where $p_{y,t}$ and $p_{d,t}$ are the endowment and dividend claim prices. The optimality conditions imply

$$E_t[m^j_{t+1} r_{y,t+1}] = 1, \quad r_{y,t+1} \equiv (p_{y,t+1} + y_{t+1})/p_{y,t}, \tag{11}$$

$$E_t[m^j_{t+1} r_{d,t+1}] = 1, \quad r_{d,t+1} \equiv (p_{d,t+1} + d_{t+1})/p_{d,t}, \tag{12}$$

where $r_{y,t+1}$ and $r_{d,t+1}$ are the gross returns on the endowment and dividend claims, and

$$m^C_{t+1} \equiv a^C_t \beta \left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi} \left(\frac{(U^C_{t+1})^{1-\gamma}}{\mu_t(U^C_{t+1})}\right)^{1/\psi-\gamma}, \tag{13}$$

$$m^R_{t+1} \equiv a^R_t \beta \left(\frac{1 - a^R_{t+1}\beta}{1 - a^R_t \beta}\right) \left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi} \left(\frac{(U^R_{t+1})^{1-\gamma}}{\mu_t(U^R_{t+1})}\right)^{1/\psi-\gamma}. \tag{14}$$

To permit an approximate analytical solution, we rewrite the optimality conditions as follows

$$E_t[\exp(\hat{m}^j_{t+1} + \hat{r}_{y,t+1})] = 1, \tag{15}$$

$$E_t[\exp(\hat{m}^j_{t+1} + \hat{r}_{d,t+1})] = 1, \tag{16}$$

where a hat denotes a log variable. A log-linear approximation of the SDF is given by

$$\hat{m}^j_{t+1} = \theta \log \beta + \theta(\hat{a}_t - \omega^j \hat{a}_{t+1}) - (\theta/\psi)\Delta\hat{c}_{t+1} + (\theta - 1)\hat{r}_{y,t+1}, \tag{17}$$

where $\theta \equiv (1-\gamma)/(1-1/\psi)$. The second term captures the direct effect of valuation risk on the stochastic discount factor, where $\omega^C = 0$, $\omega^R = \beta$, and $\hat{a}_t \equiv \hat{a}^C_t \approx \hat{a}^R_t/(1-\beta)$. Valuation risk also has an indirect effect through the return on the endowment. The log preference shock, $\hat{a}_{t+1}$, follows

$$\hat{a}_{t+1} = \rho_a \hat{a}_t + \sigma_a \varepsilon_{a,t+1},\ 0 \le \rho_a < 1,\ \varepsilon_{a,t+1} \sim \mathbb{N}(0,1). \tag{18}$$

The true time preference shocks in (13) and (14) can be recovered by mapping $\hat{a}_t$ into $\hat{a}^C_t$ and $\hat{a}^R_t$.

We apply a log-linear approximation to the asset returns to obtain

$$\hat{r}_{y,t+1} = \kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t} + \Delta\hat{y}_{t+1}, \tag{19}$$

$$\hat{r}_{d,t+1} = \kappa_{d0} + \kappa_{d1}\hat{z}_{d,t+1} - \hat{z}_{d,t} + \Delta\hat{d}_{t+1}, \tag{20}$$

[16] This frequency is supported by Bansal et al. (2016) who estimate a period is 33 days in a long-run risk model.

where $\hat{z}_{y,t+1}$ is the log price-endowment ratio, $\hat{z}_{d,t+1}$ is the log price-dividend ratio, and

$$\kappa_{y0} \equiv \log(1+\exp(\hat{z}_y)) - \kappa_{y1}\hat{z}_y, \quad \kappa_{y1} \equiv \exp(\hat{z}_y)/(1+\exp(\hat{z}_y)), \tag{21}$$

$$\kappa_{d0} \equiv \log(1+\exp(\hat{z}_d)) - \kappa_{d1}\hat{z}_d, \quad \kappa_{d1} \equiv \exp(\hat{z}_d)/(1+\exp(\hat{z}_d)), \tag{22}$$

are constants that are functions of the steady-state price-endowment and price-dividend ratios.

To close the model, the processes for log-endowment and log-dividend growth are given by

$$\Delta\hat{y}_{t+1} = \mu_y + \sigma_y\varepsilon_{y,t+1},\ \varepsilon_{y,t+1} \sim \mathbb{N}(0,1), \tag{23}$$

$$\Delta\hat{d}_{t+1} = \mu_d + \pi_{dy}\sigma_y\varepsilon_{y,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1},\ \varepsilon_{d,t+1} \sim \mathbb{N}(0,1), \tag{24}$$

where μ_y and μ_d are the steady-state growth rates, $\sigma_y \geq 0$ and $\psi_d\sigma_y \geq 0$ are the shock standard deviations, and π_{dy} determines the covariance between consumption and dividend growth. At this point, cash-flow growth is *i.i.d.* Later sections will introduce other empirically relevant features.

The asset market clearing conditions imply $s_{1,t} = 1$ and $s_{2,t} = 0$, so the resource constraint is $\hat{c}_t = \hat{y}_t$. Equilibrium includes sequences of prices $\{\hat{m}_{t+1}, \hat{z}_{y,t}, \hat{z}_{d,t}, \hat{r}_{y,t+1}, \hat{r}_{d,t+1}\}_{t=0}^{\infty}$, quantities $\{\hat{c}_t\}_{t=0}^{\infty}$, and exogenous variables $\{\Delta\hat{y}_{t+1}, \Delta\hat{d}_{t+1}, \hat{a}_{t+1}\}_{t=0}^{\infty}$ that satisfy (15)-(20), (23), (24), and the resource constraint, given the state of the economy, $\{\hat{a}_0\}$, and shock sequences, $\{\varepsilon_{y,t}, \varepsilon_{d,t}, \varepsilon_{a,t}\}_{t=1}^{\infty}$.

We posit the following solutions for the price-endowment and price-dividend ratios:

$$\hat{z}_{y,t} = \eta_{y0} + \eta_{y1}\hat{a}_t, \quad \hat{z}_{d,t} = \eta_{d0} + \eta_{d1}\hat{a}_t, \tag{25}$$

where $\hat{z}_y = \eta_{y0}$ and $\hat{z}_d = \eta_{d0}$. We apply the method of undetermined coefficients to solve the log-model. The Online Appendix provides derivations of the solution and equilibrium asset prices.

3.2 Asset Pricing Moments

We begin with a brief discussion of the asset pricing implications of the model without valuation risk. In particular, we review how Epstein-Zin preferences, by separating risk attitudes from timing attitudes, aid in matching the risk-free rate and equity premium. We then compare these moments under the current and revised valuation risk preferences.

3.2.1 Conventional Model

In the original Epstein-Zin preferences, there is no valuation risk ($\sigma_a = 0$). If, for simplicity, we further assume endowment and dividend risks are perfectly correlated ($\psi_d = 0$; $\pi_{dy} = 1$), then the average risk-free rate and average equity premium are given by

$$E[\hat{r}_f] = -\log\beta + \mu_y/\psi + ((1/\psi - \gamma)(1-\gamma) - \gamma^2)\sigma_y^2/2, \tag{26}$$

$$E[ep] = \gamma\sigma_y^2, \tag{27}$$

where the first term in (26) is the subjective discount factor, the second term accounts for endowment growth, and the third term accounts for precautionary savings. Endowment growth creates an incentive for households to borrow in order to smooth consumption. Since both assets are in fixed supply, the risk-free rate must be elevated to deter borrowing. When the IES, ψ, is high, households are willing to accept higher consumption growth so the interest rate required to dissuade borrowing is lower. Therefore, the model requires a fairly high IES to match the low risk-free rate in the data.

With CRRA preferences, higher RA lowers the IES and pushes up the risk-free rate. With Epstein-Zin preferences, these parameters are independent, so a high IES can lower the risk-free rate without lowering RA. The equity premium only depends on RA. Therefore, the model generates a low risk-free rate and modest equity premium with sufficiently high RA and IES parameter values. Of course, there is an upper bound on what constitute reasonable RA and IES values, which is the source of the risk-free rate and equity premium puzzles. Other prominent model features such as long-run risk and stochastic volatility à la Bansal and Yaron (2004) help resolve these puzzles.

3.2.2 VALUATION RISK MODEL COMPARISON We now turn to the model with valuation risk. Figure 2 plots the average risk-free rate, the average equity premium, and κ_1 (i.e., the marginal response of the price-dividend ratio on the equity return) under both preference specifications. For simplicity, we remove cash flow risk ($\sigma_y = 0$; $\mu_y = \mu_d$) and assume the time preference shocks are *i.i.d.* ($\rho_a = 0$). In this case, the standard deviation of the risk-free rate is common across the two models and matching the standard deviation of the risk-free rate in the data disciplines the parameter σ_a. Under these assumptions, the assets are identical so $(\kappa_{y0}, \kappa_{y1}, \eta_{y0}, \eta_{y1}) = (\kappa_{d0}, \kappa_{d1}, \eta_{d0}, \eta_{d1}) \equiv (\kappa_0, \kappa_1, \eta_0, \eta_1)$. We plot the results with and without cash-flow growth (μ_y).

In Figure 2, the current preferences are given by the solid-black (positive endowment growth) and red-diamond (no endowment growth) lines. In both cases, the average risk-free rate and average equity premium exhibit a vertical asymptote when the IES is 1. The risk-free rate approaches positive infinity as the IES approaches 1 from below and negative infinity as the IES approaches 1 from above. The equity premium has the same comparative statics with the opposite sign, except there is a horizontal asymptote as the IES approaches infinity (see the dashed-dotted black line).

Our results are consistent with Maurer (2012) and Kruger (2020). Maurer (2012) derives the equity premium and other conditional asset pricing moments in a continuous time version of our model without approximation. From equation 21 in his paper, it is possible to show that the equity premium tends to infinity as the IES approaches unity. Kruger (2020) shows the equity premium tends to infinity as the IES approaches unity because the variance of the SDF explodes. Kruger proposes alternative preferences where the shock is to current consumption rather than current utility, but notes that this completely eliminates valuation risk when the the IES is equal to unity.

Analytics provide further insights. Note that the risk premium is proportional to the covariance

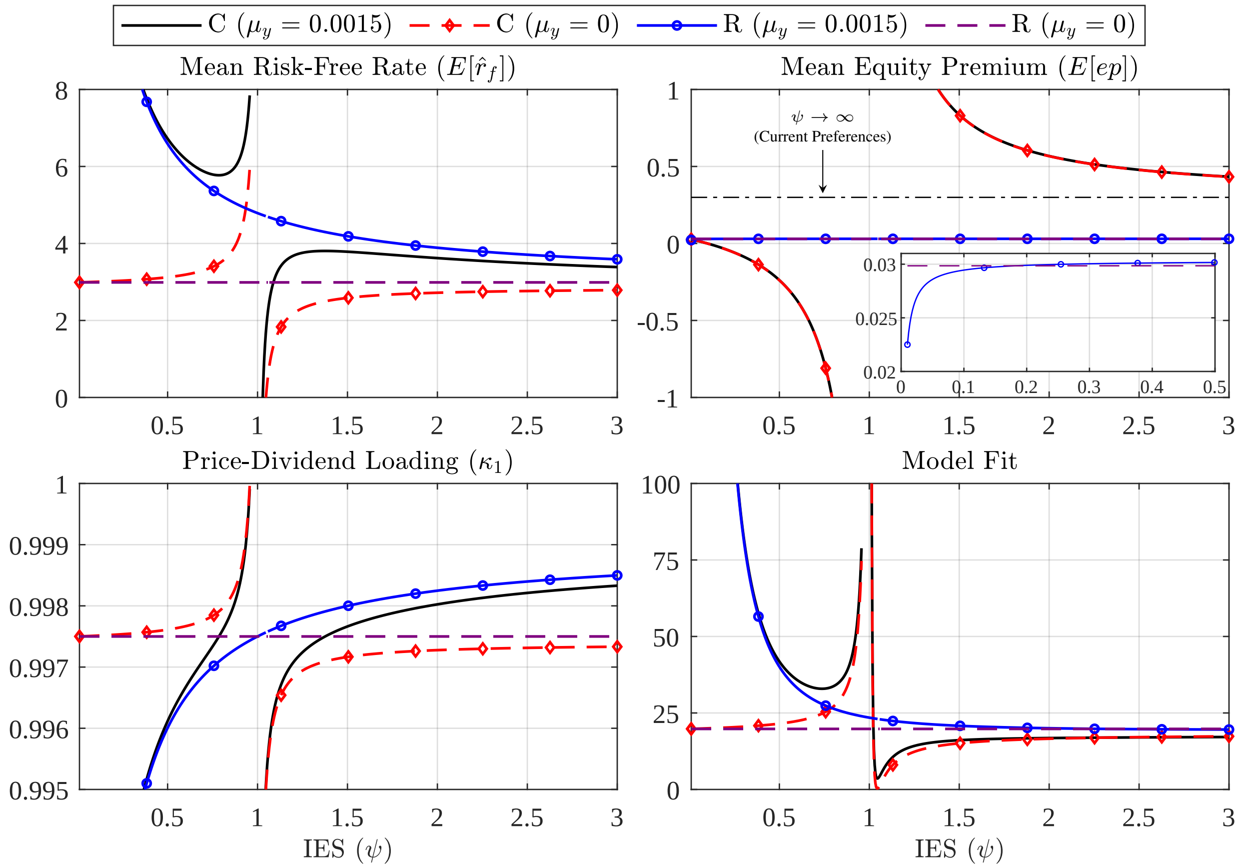

Figure 2: Equilibrium outcomes in the model without cash flow risk ($\sigma_y = 0$; $\mu_y = \mu_d$) and *i.i.d.* preference shocks ($\rho_a = 0$) under the current (C) and revised (R) preference specifications. We set $\beta = 0.9975$, $\gamma = 10$, and $\sigma_a = 0.005$. Model fit is the squared difference of the mean risk-free rate and mean equity return from their empirical counterparts. A value of zero is a perfect fit to the data. For both C specifications, the model fit is lowest when the IES equals 1.05.

between the marginal rate of substitution and the asset return. Innovations in the SDF are given by

$$m_{t+1} - E_t m_{t+1} = \lambda_a \sigma_a \varepsilon_{a,t+1}, \tag{28}$$

where $\lambda_a \equiv (\theta - 1)\kappa_1 \eta_1$ is the market price of valuation risk. Innovations in asset returns equal

$$\hat{r}_{d,t+1} - E_t \hat{r}_{d,t+1} = \kappa_1 \eta_1 \sigma_a \varepsilon_{a,t+1}. \tag{29}$$

The log-price-dividend ratio is given by $\hat{z}_t = \eta_0 + \hat{a}_t$, so the loading on the preference shock $\eta_1 = 1$.

These results show that a positive innovation in $\varepsilon_{a,t+1}$ raises the asset return. When the IES is close to but above 1, (28) shows that a positive $\varepsilon_{a,t+1}$ innovation causes an extreme fall in the marginal rate of substitution. This occurs because of the extreme curvature of the utility function when the IES is close to but above 1. Thus, the asset performs well in states where the marginal rate of substitution is low. The opposite holds (i.e., the asset is performing well in states where the marginal rate of substitution is high) when the IES is close to but below 1. In both cases, when the

IES is close to 1, the market price of valuation risk, λ_a, is large and the SDF is extremely volatile.

Making use of these insights, the average risk-free rate and equity premium are given by

$$E[\hat{r}_f] = -\log\beta + \mu_y/\psi + (\theta-1)\kappa_1^2\eta_1^2\sigma_a^2/2, \tag{30}$$

$$E[ep] = (1-\theta)\kappa_1^2\eta_1^2\sigma_a^2, \tag{31}$$

Since $\eta_1 = 1$, when the household becomes more patient and $\hat{a}_t$ rises, the price-dividend ratio rises one-for-one on impact and returns to the stationary equilibrium in the next period. Since η_1 is independent of the IES, there is no endogenous mechanism that prevents the asymptote in θ from influencing the risk-free rate or equity premium. θ dominates both of these moments when the IES is near 1 beause $0 < \kappa_1 < 1$. The following result describes the comparative statics with the IES.

Result 6. *Suppose $\gamma > 1$. The current preference specification is at odds with Property 4. As $\psi \to 1^+$, $\theta \to -\infty$, so $E[\hat{r}_f] \to -\infty$ and $E[ep] \to +\infty$. As $\psi \to 1^-$, $\theta \to +\infty$, so $E[\hat{r}_f] \to +\infty$ and $E[ep] \to -\infty$.*

Therefore, small and reasonable changes in the value of the IES (e.g., from 0.99 to 1.01) can result in dramatic changes in the predicted values of the average risk free rate and average equity premium. It also illustrates why valuation risk seems like such an attractive feature for resolving the risk-free rate and equity premium puzzles. As the IES tends to 1 from above, θ becomes increasingly negative, which dominates other determinants of the risk-free rate and equity premium. In particular, with an IES slightly above 1, the asymptote in θ causes the average risk-free rate to become arbitrarily small, while making the average equity premium arbitrarily large. The empirical implications are evident from the bottom right panel of Figure 2, which shows the model fit based on the squared difference of the mean risk-free rate and mean equity return from their counterparts in the data. The model is able to closely match the data with an IES just above 1 (1.05) because it exploits the amplification from the asymptote. Bizarrely, an IES marginally below 1—a popular value in the macro literature—generates the opposite predictions: the risk free rate approaches infinity and the equity premium approaches negative infinity, causing the model fit to deteriorate. As the IES approaches infinity, $1-\theta$ tends to γ. This shows that even when the IES is far above 1, the last term in (30) and (31) is scaled by γ and can still have a meaningful effect on asset prices.

In Figure 2, the revised preferences are given by circle-blue (positive endowment growth) and dashed-black (no endowment growth) lines. In both cases, the average risk-free rate and average equity premium are continuous in the IES, regardless of μ_y. When $\mu_y = 0$, the endowment stream is constant. This means the household is indifferent about the timing of when the preference uncertainty is resolved, so both κ_1 and the average equity premium are independent of the IES. When $\mu_y > 0$, the household's incentive to smooth consumption interacts with uncertainty about how it will value the higher future endowment stream.[17] When the IES is large, the household has a stronger preference for an early resolution of uncertainty, so the equity premium rises as a result of the valuation risk (see the Figure 2 inset). Therefore, the qualitative relationship between the IES and the equity premium has different signs under the current and revised specifications. Moreover, the increase in the equity premium is quantitatively small and converges to a level well below the value with the current preferences. It is this difference in the sign and magnitude of the relationship between the IES and the average equity premium that will explain many of our empirical results.

In this case, the market price of valuation risk is given by $\lambda_a \equiv (\theta-1)\kappa_1\eta_1 - \theta\beta$, and

$$E[\hat{r}_f] = -\log\beta + \mu_y/\psi + ((\theta-1)\kappa_1^2\eta_1^2 - \theta\beta^2)\sigma_a^2/2, \tag{32}$$

$$E[ep] = ((1-\theta)\kappa_1\eta_1 + \theta\beta)\kappa_1\eta_1\sigma_a^2. \tag{33}$$

Relative to the current specification, η_1, is unchanged.[18] However, the market price of valuation risk and both asset prices include a new term that captures the effect of valuation risk on current utility, so a rise in a_t that makes the household more patient raises the value of future certainty equivalent consumption and lowers the value of present consumption. The asymptote occurs under the current specification because it does not account for the effect of valuation risk on current-period consumption. With the revised preferences, $\kappa_1 = \beta$ when $\psi = 1$, so the terms involving θ cancel out and the asymptote disappears. As a result, the market price of valuation risk, λ_a, is continuous in ψ, and the volatility of the SDF remains modest relative to the current preferences.

Result 7. *The revised preferences satisfy Property 4, as λ_a, $E[\hat{r}_f]$ and $E[ep]$ are continuous in ψ.*

When $\psi = 1$, valuation risk lowers the average risk-free rate by $\beta^2\sigma_a^2/2$ and raises the average equity return by the same amount. Therefore, the average equity premium equals $\beta^2\sigma_a^2$, which is invariant to the RA parameter. When $\psi > 1$, $\kappa_1 > \beta$, so an increase in RA lowers the risk-free rate and raises the equity return. As $\psi \to \infty$, the equity premium with the revised specification relative to the current specification equals $1 + \beta(1-\gamma)/(\gamma\kappa_1)$. This means the disparity between the predictions of the two models grows as RA increases. As a consequence, the revised preferences would require much larger RA to generate the same equity premium as the current preferences.

Finally, it is worth emphasizing that Result 7 is not a consequence of any simplifying parameter restrictions. Under the revised preferences, an equilibrium solution at $\psi = 1$ is well defined, regardless of whether time-preference shocks are persistent ($\rho_a > 0$) or the consumption and dividend processes are perfectly correlated. The Online Appendix provides a proof of this result.

[17] Andreasen and Jørgensen (2020) show how to decouple the household's timing attitude from the RA and IES.

[18] Notice κ_1 is a function of the steady-state price-dividend ratio, z_d. When the IES is 1, $z_d = \beta/(1-\beta)$, which is equivalent to its value absent any risk. Therefore, when the IES is 1, valuation risk has no effect on the price-dividend ratio. This result points to a connection with income and substitution effects, which usually cancel when the IES is 1.

3.3 FURTHER DISCUSSION The previous section shows the current and revised preferences generate different predictions. This section covers three miscellaneous questions readers may have.

Question 1: Is the valuation risk specification under CRRA preferences important?

Since we have demonstrated that the valuation risk specification is important under Epstein-Zin preferences, it is worth addressing whether the same is true under CRRA preferences. In particular, is the choice between $U_t = u(c_t) + a_t\beta E_t U_{t+1}$ and $U_t = (1 - a_t\beta)u(c_t) + a_t\beta E_t U_{t+1}$ important? In terms of first-order dynamics, both specifications generate the same impulse response functions with an appropriate rescaling of σ. The rescaling is by the factor $1 - \rho_a\beta$, where ρ_a is unchanged across the specifications. There is a numerically small difference in $E[\hat{r}_f]$ and $E[ep]$, which is easy to see by setting $\theta = 1$ in equations (30)-(33). This stems from the conditional expectation of a_{t+1}.

Question 2: Are the revised preferences the only alternative?

A potential alternative to the revised specification is the following:

$$V_t = W(c_t, a_t\mu_t) = [c_t^{1-1/\psi} + \beta(a_t\mu_t)^{1-1/\psi}]^{1/(1-1/\psi)}. \tag{34}$$

We refer to this specification as "disaster risk" preferences following Gourio (2012). That paper shows how a term like a_t can arise endogenously in a production economy asset pricing model.

Technically, since the disaster risk shock affects the certainty equivalent of future utility and does not alter the time-aggregator, these preferences are consistent with the four desirable properties described in Section 2. However, they do not represent a household's intrinsic time preference uncertainty. To appreciate why, once again set $\gamma = 1/\psi = 1$, giving $V_t = \log c_t + \log(a_t) + E_t V_{t+1}$. The model reduces to time-separable log-preferences with an additive shock term. As a result a_t disappears from any equilibrium condition, so the disaster risk preferences are not able to capture an exogenous change in the household's impatience, even though there is no plausible reason why a household with time-separable log-preferences cannot become more or less patient over time. This means valuation risk must be linked to time-variation in the discount factor, as in (5) and (6).

Question 3: Is the current specification reasonable if the IES is set far from unity?

Figure 2 shows the current preferences generate counterfactual comparative statics with an IES near 1. For example, the risk-free rate is increasing in the IES as the influence of the asymptote wanes. Far from the asymptote, this effect disappears, so the comparative statics of the two specifications coincide. This could provide a heuristic for deciding whether the current preferences are reasonable. However, even when the IES is large, the two specifications have different quantitative predictions, and the difference is increasing in γ. As such, it is not possible to define an IES threshold for which the asymptote will no longer matter. When the IES is high in Figure 2, the equity premium is around 0.3 under the current preferences and near 0 under the revised preferences.

Beyond these differences, the properties in Section 2 do not provide any further guidance on the reasonableness of the current specification when the IES is large. Epstein et al. (2014) call for more experimental evidence to discipline preferences in asset pricing models. This is a similar situation. Theory suggests that the current preferences produce counterintuitive results, but future experimental work is needed to provide additional evidence that helps discipline models with valuation risk.

4 DATA AND ESTIMATION METHODS

We construct our data using the procedure in Bansal and Yaron (2004), Beeler and Campbell (2012), Bansal et al. (2016), and Schorfheide et al. (2018). The moments are based on seven time series from 1929 to 2017: real per capita consumption expenditures on nondurables and services, the real equity return, real dividends, the real risk-free rate, the price-dividend ratio, and the real 5- and 20-year U.S. Treasury yields. Nominal equity returns are calculated with the CRSP value-weighted return on stocks. We obtain data with and without dividends to back out a time series for nominal dividends. Both series are converted to real series using the consumer price index (CPI).

The nominal risk-free rate is based on the CRSP yield-to-maturity on 90-day Treasury bills. We first convert the nominal time series to a real series using the CPI. Then we construct an *ex-ante* real rate by regressing the *ex-post* real rate on the nominal rate and annual inflation rate three month ahead. The data on personal consumption expenditures is annual. To match this frequency, the monthly asset pricing data are converted to annual time series using the last month of each year.

The estimation procedure has two stages. The first stage estimates our mean target moments, $\hat{\Psi}_T^D$, using a two-step Generalized Method of Moments (GMM) estimator, where $T = 87$ is the sample size.[19] Conditional on $\hat{\Psi}_T^D$, the second stage estimates the parameters of our structural model with a Simulated Method of Moments (SMM) procedure. For parameterization θ and shocks $\mathcal{E}_T = [\varepsilon_{y,t}, \varepsilon_{d,t}, \varepsilon_{a,t}]_{t=1}^T$, we solve the model and simulate it $R = 1{,}000$ times for T periods. This allows us to compute the mean moments across the R simulations, $\bar{\Psi}_{R,T}^M(\theta, \mathcal{E}) = \frac{1}{R}\sum_{r=1}^R \Psi_T^M(\theta, \mathcal{E}_r)$.

The parameter estimates, $\hat{\theta}$, are obtained by minimizing the following loss function:

$$J(\theta, \mathcal{E}) = [\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\theta, \mathcal{E})]'[\hat{\Sigma}_T^D(1 + 1/R)]^{-1}[\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\theta, \mathcal{E})],$$

where $\hat{\Sigma}_T^D$ is the diagonal of the GMM estimate of the variance-covariance matrix.[20] A bootstrap procedure is used to calculate the standard errors on the parameters.[21] Specifically, we run our SMM algorithm $N_s = 500$ times, each time conditional on a particular sequence of shocks $\mathcal{E}^s$ but

[19] In total, there are 89 periods in our sample, but we lose one period for growth rates and one for serial correlations.

[20] For the revised preferences, we impose the restriction $\beta \exp(4(1-\beta)\sqrt{\sigma_a^2/(1-\rho_a^2)}) < 1$ when estimating the model parameters. This ensures the time-aggregator weights are positive in 99.997% of the simulated observations.

[21] Ruge-Murcia (2012) applies SMM to several nonlinear business cycle models and finds that asymptotic standard errors tend to overstate the variability of the estimates. In the conclusion, he acknowledges that "[a] possible way to address the limitations of asymptotic theory would be to use the bootstrap to construct accurate confidence intervals."

holding fixed the empirical targets, $\hat{\Psi}_T^D$, and weighting matrix, $\hat{\Sigma}_T^D$, in the loss function. Given the set of parameter estimates $\{\hat{\theta}^s\}_{s=1}^{N_s}$, we report the mean, $\bar{\theta} = \sum_{s=1}^{N_s} \hat{\theta}^s / N_s$, and $(5, 95)$ percentiles.[22] This method has two benefits. First, it provides more reliable estimates of the standard errors than using the asymptotic variance of the estimator, which is commonly used in the literature. Second, it makes it possible to determine whether the estimation method has settled on a global optimum. Appendix A describes our data sources and the Online Appendix outlines our estimation algorithm.

The baseline model targets 15 moments: the means and standard deviations of consumption growth, dividend growth, equity returns, the risk-free rate, and the price-dividend ratio, the correlation between dividend growth and consumption growth, the autocorrelations of the price-dividend ratio and risk-free rate, and the cross-correlations of consumption growth, dividend growth, and equity returns. These targets are common in the literature and the same as Albuquerque et al. (2016), except we exclude 5- and 10-year correlations between equity returns and cash-flow growth. We omit the long-run correlations to allow a longer sample that includes the Great Depression period.

Real Yields		TIPS-Implied Yields	
$E[r_{f,5}]$	$E[r_{f,20}]$	$E[r_{f,5}]$	$E[r_{f,20}]$
0.30 (0.23)	1.62 (0.19)	0.43 (0.29)	1.35 (0.20)

Table 1: Comparison of long-term yields using data from 2004-2017. Standard errors are shown in parentheses.

We also show the empirical performance of our model when we target the real return on 5- and 20-year Treasury bonds. Since longer-term assets are more sensitive to time preference shocks, the real yield curve could help identify the time preference shock parameters. In the literature, there is no widely accepted method for removing inflation risk from nominal yields. To facilitate comparison, we follow the procedure in Albuquerque et al. (2016). We obtain the intermediate and long-term nominal Treasury yields from Morningstar Direct (formerly Ibbotson Associates). We then convert to real yields by regressing the *ex-post* real long-term rate on the nominal rate and 12-month ahead inflation rate. A common alternative approach is to use treasury inflation protected securities (TIPS). Table 1 shows both methods produce a similar upward sloping yield curve. We decided to use the regression-based approach because TIPS data is only available since 2004.[23]

[22] The practice of re-estimating with different sequences of shocks follows the recommendation of Fabio Canova (see `http://apps.eui.eu/Personal/Canova/Teachingmaterial/Smm_eui2014.pdf`, slide 16).

[23] A third option used in parts of the literature (see, for example, Creal and Wu (2020) and Gomez-Cram and Yaron (2020)) is to estimate an exogenous equation for inflation dynamics and fit the model to nominal yield curve data.

5 Estimated Baseline Model

This section takes the baseline model from Section 3.1 and compares the estimates from the current and revised preference specifications. We fix the IES to 2.5, which is near the upper end of the plausible range of values in the literature.[24] This restriction helps us compare the estimates from the two preference specifications because the model fit, as measured by the J value, is insensitive to the value of the IES in the revised specification, but the unconstrained global minimum prefers an implausibly high IES. For example, the J value is only one decimal point lower with an IES equal to 10. Therefore, we are left with estimating nine parameters to match 17 empirical targets.

	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Ptr	Current	Revised	Max RA	Current	Revised	Max RA
γ	1.58 (1.55, 1.60)	75.11 (73.68, 76.56)	10.00 (10.00, 10.00)	1.41 (1.38, 1.43)	98.44 (97.09, 99.68)	10.00 (10.00, 10.00)
β	0.9977 (0.9976, 0.9978)	0.9956 (0.9956, 0.9957)	0.9973 (0.9972, 0.9973)	0.9979 (0.9978, 0.9980)	0.9963 (0.9963, 0.9964)	0.9978 (0.9977, 0.9978)
ρ_a	0.9969 (0.9968, 0.9970)	0.9903 (0.9902, 0.9904)	0.9882 (0.9881, 0.9884)	0.9974 (0.9973, 0.9975)	0.9897 (0.9896, 0.9898)	0.9882 (0.9880, 0.9883)
σ_a	0.00030 (0.00029, 0.00030)	0.03482 (0.03461, 0.03502)	0.03828 (0.03807, 0.03846)	0.00027 (0.00026, 0.00027)	0.03592 (0.03573, 0.03609)	0.03841 (0.03822, 0.03861)
μ_y	0.0016 (0.0016, 0.0016)	0.0016 (0.0016, 0.0016)	0.0017 (0.0017, 0.0017)	0.0016 (0.0016, 0.0016)	0.0017 (0.0016, 0.0017)	0.0016 (0.0016, 0.0016)
μ_d	0.0015 (0.0015, 0.0015)	0.0021 (0.0021, 0.0021)	0.0010 (0.0009, 0.0010)	0.0010 (0.0010, 0.0010)	0.0017 (0.0016, 0.0017)	0.0005 (0.0005, 0.0005)
σ_y	0.0058 (0.0057, 0.0058)	0.0057 (0.0057, 0.0058)	0.0058 (0.0058, 0.0059)	0.0058 (0.0058, 0.0058)	0.0055 (0.0055, 0.0056)	0.0060 (0.0059, 0.0060)
ψ_d	1.51 (1.47, 1.55)	0.96 (0.92, 1.00)	1.07 (1.04, 1.11)	1.49 (1.45, 1.53)	1.12 (1.09, 1.14)	1.02 (0.99, 1.05)
π_{dy}	0.811 (0.785, 0.840)	0.431 (0.415, 0.446)	0.616 (0.594, 0.639)	0.809 (0.783, 0.838)	0.606 (0.595, 0.617)	0.604 (0.583, 0.629)
J	28.63 (28.03, 29.30)	47.63 (47.37, 47.91)	55.47 (55.04, 55.89)	30.81 (30.22, 31.46)	49.67 (49.37, 49.99)	59.22 (58.89, 59.57)
pval	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)
df	6	6	6	8	8	8

Table 2: Baseline model. Average and $(5, 95)$ percentiles of the parameter estimates. The IES is 2.5.

Table 2 shows the parameter estimates and Table 3 reports the data and model-implied moments for six variants of our baseline model: with and without targeting the yield curve (5- and 20-year average risk-free bond yields); with the current preferences; and with the revised preferences, with and without an upper bound on RA. For each parameter, we report the average and $(5, 95)$ percentiles across 500 estimations of the model. For each moment, we provide the mean and t-statistic for the null hypothesis that a model-implied moment equals its empirical counterpart.

We begin with the model that excludes the yield curve moments. In both specifications, the data prefers a very persistent valuation risk process with $\rho_a > 0.98$. Given the estimates for ρ_a and

[24] Estimation results with $\psi = 1.5$ and $\psi = 2.0$ for each specification considered below are in the Online Appendix. In total, we estimate 54 variants of our model. Since each variant is estimated 500 times, there are 27,000 estimations. The estimations are run in Fortran and the time per estimation ranges from 1-24 hours depending on model complexity.

		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Moment	Data	Current	Revised	Max RA	Current	Revised	Max RA
$E[\Delta c]$	1.89	1.89 (0.00)	1.94 (0.20)	2.01 (0.49)	1.89 (0.00)	1.98 (0.37)	1.96 (0.27)
$E[\Delta d]$	1.47	1.79 (0.33)	2.50 (1.07)	1.17 (−0.31)	1.22 (−0.26)	1.99 (0.54)	0.59 (−0.92)
$E[z_d]$	3.42	3.45 (0.24)	3.49 (0.49)	3.56 (1.04)	3.49 (0.49)	3.53 (0.76)	3.60 (1.29)
$E[r_d]$	6.51	5.60 (−0.57)	5.62 (−0.55)	4.06 (−1.53)	5.05 (−0.91)	5.00 (−0.94)	3.38 (−1.96)
$E[r_f]$	0.25	0.25 (0.00)	0.37 (0.19)	1.09 (1.37)	0.12 (−0.22)	0.26 (0.01)	0.45 (0.32)
$E[r_{f,5}]$	1.19	1.21 (0.03)	1.74 (0.81)	2.18 (1.45)	0.91 (−0.41)	1.22 (0.04)	1.51 (0.46)
$E[r_{f,20}]$	1.88	3.10 (2.04)	3.50 (2.70)	3.32 (2.40)	2.53 (1.08)	2.30 (0.71)	2.63 (1.25)
$SD[\Delta c]$	1.99	1.99 (0.00)	1.99 (−0.02)	2.00 (0.01)	2.00 (0.01)	1.91 (−0.16)	2.07 (0.16)
$SD[\Delta d]$	11.09	3.42 (−2.80)	2.10 (−3.29)	2.48 (−3.15)	3.39 (−2.82)	2.43 (−3.16)	2.45 (−3.16)
$SD[r_d]$	19.15	17.96 (−0.63)	13.49 (−2.98)	13.29 (−3.09)	17.99 (−0.61)	13.31 (−3.08)	12.97 (−3.26)
$SD[r_f]$	2.72	3.25 (1.04)	3.68 (1.88)	3.85 (2.22)	3.04 (0.62)	3.68 (1.88)	3.74 (2.01)
$SD[z_d]$	0.45	0.48 (0.44)	0.26 (−3.07)	0.23 (−3.43)	0.50 (0.73)	0.25 (−3.24)	0.23 (−3.53)
$AC[r_f]$	0.68	0.94 (4.00)	0.89 (3.28)	0.88 (3.06)	0.94 (4.05)	0.89 (3.21)	0.88 (3.05)
$AC[z_d]$	0.89	0.91 (0.42)	0.84 (−0.99)	0.82 (−1.44)	0.91 (0.52)	0.84 (−1.14)	0.82 (−1.46)
$Corr[\Delta c, \Delta d]$	0.54	0.47 (−0.32)	0.41 (−0.62)	0.49 (−0.20)	0.48 (−0.30)	0.48 (−0.30)	0.51 (−0.14)
$Corr[\Delta c, r_d]$	0.05	0.09 (0.58)	0.06 (0.22)	0.09 (0.62)	0.09 (0.58)	0.09 (0.54)	0.09 (0.67)
$Corr[\Delta d, r_d]$	0.07	0.19 (1.42)	0.15 (1.01)	0.18 (1.38)	0.18 (1.39)	0.18 (1.33)	0.19 (1.41)

Table 3: Baseline model. Data and average model-implied moments. t-statistics are in parentheses.

σ_a, the volatility of the actual time-preference shocks is the same order of magnitude across the two specifications ($SD(\hat{a}_t^C) = SD(\hat{a}_t) = 0.38\%$ and $SD(\hat{a}_t^R) \approx (1-\beta)SD(\hat{a}_t) = 0.12\%$). In the current specification, the risk aversion parameter, γ, is 1.58. In the revised specification $\gamma = 75.11$, which is well outside what is considered acceptable in the asset pricing literature.[25] Both specifications generate a sizable equity premium (the estimates are about 1% lower than the empirical equity premium) and a near zero risk-free rate. However, they significantly under-predict the standard deviation of dividend growth and over-predict the autocorrelation of the risk-free rate in the data.[26]

[25]Mehra and Prescott (1985, p. 154) say "Any of the above cited studies…constitute an *a priori* justification for restricting the value of [RA] to be a maximum of ten, as we do in this study." Weil (1989, p. 411) describes $\gamma = 40$ as "implausibly" high. Swanson (2012) shows γ does not equate to risk aversion when households have a labor margin. Therefore, only in production economies can γ be reasonably above 10, where it is common to see values around 100.

[26]The estimate of the valuation risk shock standard deviation, σ_a, is two orders of magnitude larger in the revised specification than the current specification. Recall that the valuation risk term in the SDF is given by $\hat{a}_t - \omega\hat{a}_{t+1}$. When the valuation risk shock is *i.i.d.*, the estimates of the shock standard deviation are very similar. However, as the persistence increases with the revised preferences, $SD_t[\hat{a}_t - \omega\hat{a}_{t+1}]$ shrinks, so σ_a rises to compensate for the extra term.

Using the analytical expressions for the average risk-free rate and average equity premium (see E.15 and E.16 in the Online Appendix), it is possible to break down the fraction of each moment explained by cash-flow and valuation risk.[27] With the current specification valuation risk explains 98.9% and 99.2% of the risk-free rate and the equity premium, whereas with the revised preferences it explains only 63.2% and 79.2%. Since the estimate of the cash-flow shock standard deviation is unchanged, cash-flow risk has a bigger role in explaining the equity premium due to higher RA.

The revised specification has a significantly poorer fit than the current specification ($J = 47.6$ vs. $J = 28.6$), although both specifications fail the over-identifying restrictions test. The poorer fit is mostly due to the model significantly over-predicting the volatility of the risk-free rate and under-predicting the volatilities of the price-dividend ratio and equity return. The intuition is as follows. In the revised specification, risk-free rate volatility is relatively more sensitive to valuation risk than equity return volatility. Since the volatility of equity returns is higher than the volatility of the risk-free rate in the data, valuation risk alone does not allow the model to match these moments. Dividend growth volatility, however, cannot rise to compensate for the lack of the equity return volatility because the target correlation between equity returns and dividend growth is near zero.

The revised preferences not only have a worse fit, but the risk aversion parameter is implausibly large. When we restrict γ to a maximum of 10—the upper end of the values used in the asset pricing literature—the fit deteriorates further ($J = 55.5$ vs. 47.6). The primary source of the poorer fit is the larger estimate of the risk-free rate (1.1% vs. 0.4%) and lower equity return (4.1% vs. 5.6%).

Intuition suggests that valuation risk should also be informative about the long-term risk-free interest rates, not just the short-term rate. When longer-term moments are omitted from the estimation routine, both preferences over-predict the slope of the yield curve ($E[r_{f,20}] - E[r_f]$ is 2.8% and 3.1% for the current and revised preferences, relative to the 1.6% in the data). Once the yield curve moments are included, however, the slopes fall to 2.4% and 2.0%, respectively. For the revised preferences, this flattening of the yield curve is generated by a rise in RA. Overall, the inclusion of these moments worsens the fit of the model but does not materially change the results.

The results in Tables 2 and 3 demonstrate that the valuation risk specification matters empirically. Figure 3 provides a broader comparison of the two specifications to highlight the properties of the current specification around the asymptote at $\psi = 1$. Conditional on different degrees of risk aversion (γ), we report the model fit and selected parameters from re-estimating the model for a range of IES values.[28] The influence of the asymptote under the current preferences is immediately apparent. As the IES approaches 1 from either direction, the model fit rapidly deteriorates.[29] The

[27] The mean risk-free rate is given by $E[\hat{r}_{f,t}] = \alpha_1 + \alpha_2\sigma_a^2 + \alpha_3\sigma_y^2$ and the mean equity premium is given by $E[ep_t] = \alpha_4\sigma_a^2 + \alpha_5\sigma_y^2$ for some function of model parameters α_i, $i \in \{1,\ldots,5\}$. Therefore, the contribution of valuation risk to the risk-free rate and equity premium is given by $\alpha_2\sigma_a^2/(\alpha_2\sigma_a^2 + \alpha_3\sigma_y^2)$ and $\alpha_4\sigma_a^2/(\alpha_4\sigma_a^2 + \alpha_5\sigma_y^2)$.

[28] We also conducted this exercise without fixing γ. In this case, the data prefers a γ extremely close to 1 to eliminate the influence of the asymptote when ψ is near 1. Fixing γ allows us to highlight the implications of the asymptote.

[29] As shown in Figure 2, the model fit could improve as the IES approaches 1 because the algorithm exploits the

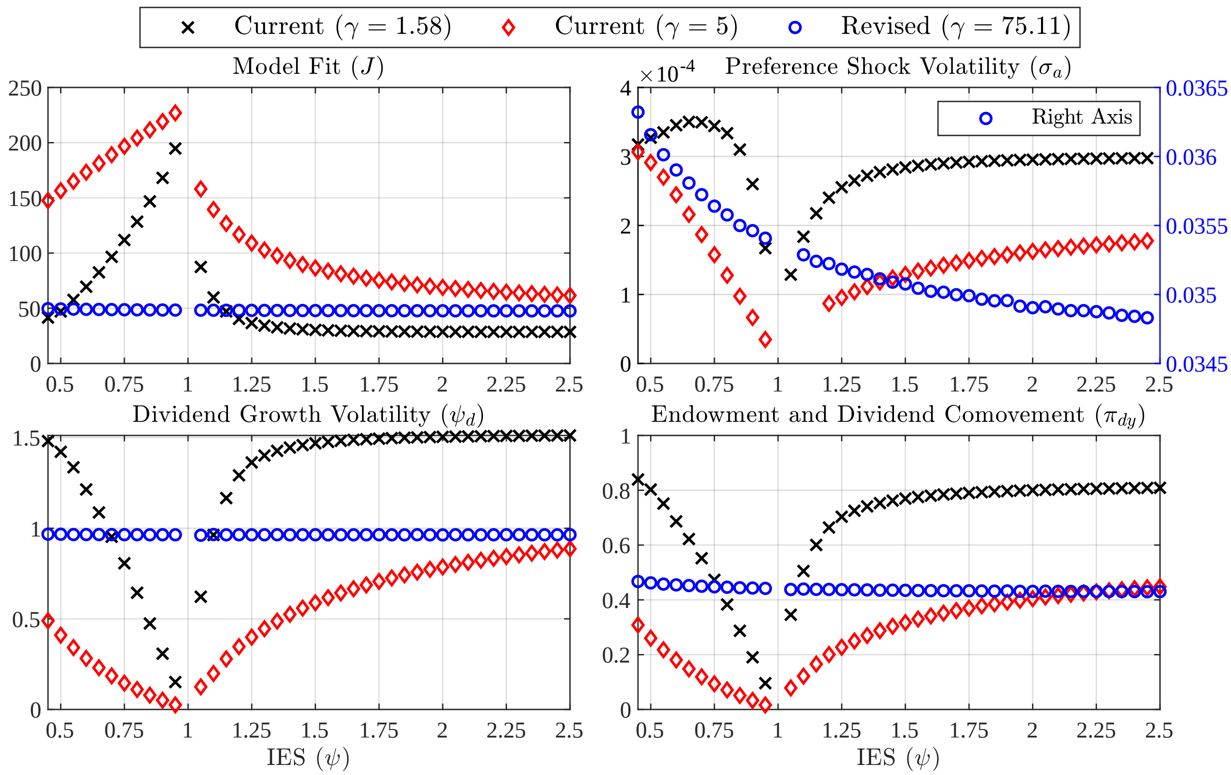

Figure 3: Baseline model estimates as a function of the IES.

estimation compensates for the influence of the asymptote by reducing the standard deviations of the time-preference shocks (σ_a) and cash flow shocks (ψ_d and π_{dy}). In contrast, both the model fit and equilibrium outcomes are continuous with respect to the IES under the revised preferences.

Importantly, a modest increase in risk aversion under the current preferences causes the asymptote to have a wider influence: the model fit is uniformly worse for a given IES and there is an effect on model outcomes for IES values further away from 1. This shows the influence of the asymptote even when the IES is well above 1. A priori, the researcher does not know the sensitivity of the asymptote to the degree of risk aversion. Conditional on a given set of parameter estimates, it is difficult to determine whether equilibrium outcomes are an artifact of the data or driven by the asymptote. The revised preferences eliminate this problem by removing the asymptote from the model.

The current preferences also exhibit comparative statics that are counterintuitive relative to the Epstein-Zin asset pricing literature. The Epstein-Zin literature tells us that as the IES increases, agents demand a larger equity premium. In order to match the equity premium in the data, we should therefore expect an estimation with a higher IES to compensate with less exogenous volatility (i.e., a lower σ_a, σ_y, and π_{dy}). The revised preferences generate this intuitive comparative static result. Under the current preferences, the exogenous volatility is increasing in the IES for $\psi > 1$,

asymptote. Whether the asymptote causes the model fit to improve or deteriorate is model and estimation dependent.

because the estimation needs to compensate for the waning influence of the asymptote. Given the potential influence of the asymptote on equilibrium outcomes, the rest of the paper concentrates on the revised specification when examining the role of valuation risk in richer asset pricing models.

6 ESTIMATED LONG-RUN RISK MODEL

Long-run risk provides a well-known resolution to many asset pricing puzzles. This section introduces this feature into our baseline model and re-examines the marginal contribution of valuation risk with the revised preferences. To introduce long-run risk, we modify (23) and (24) as follows:

$$\Delta\hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_y\varepsilon_{y,t+1},\ \varepsilon_{y,t+1} \sim \mathbb{N}(0,1), \tag{35}$$

$$\Delta\hat{d}_{t+1} = \mu_d + \phi_d\hat{x}_t + \pi_{dy}\sigma_y\varepsilon_{y,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1},\ \varepsilon_{d,t+1} \sim \mathbb{N}(0,1), \tag{36}$$

$$\hat{x}_{t+1} = \rho_x\hat{x}_t + \psi_x\sigma_y\varepsilon_{x,t+1},\ \varepsilon_{x,t+1} \sim \mathbb{N}(0,1), \tag{37}$$

where the specification of the persistent component, $\hat{x}_t$, follows Bansal and Yaron (2004). We apply the same estimation procedure as the baseline model, except there are three additional parameters, ϕ_d, ρ_x, and ψ_x. We also match up to five additional moments: the autocorrelations of consumption growth, dividend growth, and the equity return and two predictability moments—the correlations of consumption growth and the equity premium with the lagged price-dividend ratio.

The long-run risk model also prefers a high IES even though it does not significantly lower the J value. As a result, we continue to set the IES to 2.5 and estimate the remaining parameters. The parameter estimates are shown in Table 4 and the data and model-implied moments are reported in Table 5. The tables show the results for six variants of the model: with and without targeting both the yield curve and higher-order risk-free rate moments; with and without targeting the yield curve but always including higher-order risk-free rate moments; and with and without valuation risk.

We begin with the model without valuation risk and without the yield curve and risk-free rate moments (column 1). This is a typical model estimated in the literature. The model fails to pass the over-identifying restrictions test at the 5% level, signalling that the standard long-run risk model is insufficient to adequately describe the behavior of asset prices and cash flows. The parameter estimates are similar to the estimates in the literature. In particular, the data requires a small but very persistent shock that generates risk in long-run cash-flow growth ($\rho_x = 0.9986$; $\psi_x = 0.0265$).

The literature typically excludes the standard deviation and autocorrelation of the risk-free rate when estimating the long-run risk model because the model does not generate sufficient volatility (a standard deviation of 0.53 vs. 2.72 in the data) and over-predicts the autocorrelation (0.95 vs. 0.68 in the data). Even when these two moments are targeted, as shown in column 3, long-run cash-flow risk is unable to significantly improve on these moments (the standard deviation rises to 0.70 and the autocorrelation falls to 0.94). The standard long-run risk model also fairs poorly on

	Omits $SD[r_f]$, $AC[r_f]$, $E[r_{f,5}]$, & $E[r_{f,20}]$		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$		All Moments	
Parameter	No VR	Revised	No VR	Revised	No VR	Revised
γ	2.74 $(2.64, 2.84)$	2.52 $(2.41, 2.67)$	2.84 $(2.69, 2.95)$	2.63 $(2.53, 2.76)$	2.59 $(2.38, 2.76)$	2.37 $(2.27, 2.50)$
β	0.9991 $(0.9990, 0.9991)$	0.9980 $(0.9979, 0.9981)$	0.9990 $(0.9990, 0.9991)$	0.9989 $(0.9989, 0.9990)$	0.9985 $(0.9985, 0.9986)$	0.9985 $(0.9985, 0.9985)$
ρ_a	–	0.9813 $(0.9806, 0.9820)$	–	0.9563 $(0.9554, 0.9570)$	–	0.9584 $(0.9575, 0.9592)$
σ_a	–	0.0481 $(0.0473, 0.0491)$	–	0.0169 $(0.0167, 0.0172)$	–	0.0178 $(0.0175, 0.0181)$
μ_y	0.0016 $(0.0015, 0.0017)$	0.0016 $(0.0014, 0.0017)$	0.0016 $(0.0015, 0.0016)$	0.0016 $(0.0015, 0.0017)$	0.0016 $(0.0015, 0.0016)$	0.0016 $(0.0015, 0.0017)$
μ_d	0.0012 $(0.0010, 0.0014)$	0.0011 $(0.0009, 0.0014)$	0.0014 $(0.0012, 0.0016)$	0.0012 $(0.0010, 0.0014)$	0.0012 $(0.0010, 0.0014)$	0.0010 $(0.0008, 0.0013)$
σ_y	0.0039 $(0.0039, 0.0040)$	0.0039 $(0.0038, 0.0040)$	0.0047 $(0.0047, 0.0047)$	0.0039 $(0.0039, 0.0040)$	0.0045 $(0.0044, 0.0045)$	0.0037 $(0.0037, 0.0037)$
ψ_d	3.39 $(3.31, 3.46)$	2.73 $(2.63, 2.84)$	3.17 $(3.09, 3.22)$	3.27 $(3.19, 3.35)$	3.30 $(3.18, 3.40)$	3.40 $(3.32, 3.49)$
π_{dy}	0.595 $(0.496, 0.676)$	0.926 $(0.854, 0.999)$	0.029 $(-0.098, 0.146)$	0.710 $(0.623, 0.781)$	0.122 $(-0.033, 0.267)$	0.832 $(0.752, 0.903)$
ϕ_d	2.39 $(2.30, 2.48)$	1.43 $(1.38, 1.50)$	2.24 $(2.14, 2.33)$	2.22 $(2.14, 2.32)$	2.39 $(2.24, 2.53)$	2.33 $(2.24, 2.42)$
ρ_x	0.9986 $(0.9985, 0.9987)$	0.9995 $(0.9995, 0.9995)$	0.9974 $(0.9971, 0.9977)$	0.9988 $(0.9987, 0.9990)$	0.9974 $(0.9970, 0.9979)$	0.9989 $(0.9988, 0.9991)$
ψ_x	0.0267 $(0.0261, 0.0273)$	0.0265 $(0.0258, 0.0273)$	0.0327 $(0.0318, 0.0335)$	0.0261 $(0.0255, 0.0268)$	0.0314 $(0.0304, 0.0323)$	0.0253 $(0.0247, 0.0259)$
J	20.72 $(20.10, 21.38)$	13.34 $(13.12, 13.56)$	54.99 $(54.24, 55.84)$	19.60 $(19.06, 20.14)$	61.74 $(61.03, 62.53)$	24.42 $(23.88, 24.93)$
pval	0.008 $(0.006, 0.010)$	0.038 $(0.035, 0.041)$	0.000 $(0.000, 0.000)$	0.012 $(0.010, 0.015)$	0.000 $(0.000, 0.000)$	0.007 $(0.005, 0.008)$
df	8	6	10	8	12	10

Table 4: Long-run risk model. Average and $(5, 95)$ percentiles of the parameter estimates. The IES is 2.5.

three additional moments: (1) the standard deviation of dividend growth (too low), (2) the correlation between dividend growth and the return on equity (too high), and (3) the predictability of consumption growth (too high). All of them are significantly different from their empirical targets.

Adding valuation risk (columns 2 and 4) significantly improves the fit of the model. With the restricted set of moments, the J value declines from 20.7 to 13.3. More importantly, the p-value from the over-identifying restrictions test rises from 0.01 to 0.04, even though the valuation risk model contains two more parameters than the standard model (6 degrees of freedom instead of 8).

Unlike cash-flow risk, valuation risk directly affects the time-series properties of the risk-free rate, which makes it important to target these moments in the estimation. In column 2, the model includes valuation risk but targets neither the standard deviation nor the autocorrelation of the risk-free rate. As a result, the estimated model significantly over-predicts both moments (the standard deviation is 5.64 vs. 2.72 in the data and the autocorrelation is 0.83 vs. 0.68 in the data). However, once these moments are targeted in the estimation (column 4), the standard deviation of the risk-free rate is 2.83 and the autocorrelation of the risk-free rate is 0.69, consistent with the data.

In both columns 2 and 4, the model closely matches the mean risk-free rate and equity return.

		Omits $SD[r_f]$, $AC[r_f]$, $E[r_{f,5}]$, & $E[r_{f,20}]$		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$		All Moments	
Moment	Data	No VR	Revised	No VR	Revised	No VR	Revised
$E[\Delta c]$	1.89	1.88 (−0.01)	1.89 (0.01)	1.88 (−0.02)	1.89 (0.00)	1.89 (0.00)	1.89 (0.01)
$E[\Delta d]$	1.47	1.50 (0.03)	1.36 (−0.12)	1.68 (0.22)	1.43 (−0.04)	1.48 (0.00)	1.25 (−0.23)
$E[z_d]$	3.42	3.42 (−0.03)	3.42 (−0.05)	3.41 (−0.08)	3.42 (−0.01)	3.42 (0.01)	3.43 (0.06)
$E[r_d]$	6.51	6.44 (−0.05)	6.92 (0.26)	5.91 (−0.37)	6.61 (0.06)	5.69 (−0.51)	6.55 (0.02)
$E[r_f]$	0.25	0.27 (0.03)	0.27 (0.03)	0.27 (0.03)	0.25 (0.00)	1.39 (1.87)	1.20 (1.55)
$E[r_{f,5}]$	1.19	0.12 (−1.58)	1.03 (−0.24)	0.06 (−1.68)	0.25 (−1.39)	1.24 (0.07)	1.25 (0.09)
$E[r_{f,20}]$	1.88	−0.29 (−3.61)	1.01 (−1.44)	−0.50 (−3.95)	−0.13 (−3.34)	0.83 (−1.74)	0.98 (−1.49)
$SD[\Delta c]$	1.99	1.93 (−0.14)	2.02 (0.05)	2.41 (0.87)	1.92 (−0.14)	2.24 (0.52)	1.79 (−0.43)
$SD[\Delta d]$	11.09	5.72 (−1.96)	4.49 (−2.41)	6.54 (−1.66)	5.47 (−2.05)	6.43 (−1.70)	5.35 (−2.10)
$SD[r_d]$	19.15	17.74 (−0.74)	19.34 (0.10)	18.69 (−0.24)	17.71 (−0.76)	18.72 (−0.23)	17.71 (−0.76)
$SD[r_f]$	2.72	0.53 (−4.32)	5.64 (5.76)	0.70 (−3.99)	2.83 (0.22)	0.64 (−4.11)	2.93 (0.41)
$SD[z_d]$	0.45	0.55 (1.54)	0.46 (0.17)	0.52 (1.14)	0.54 (1.40)	0.53 (1.19)	0.54 (1.43)
$AC[\Delta c]$	0.53	0.43 (−1.06)	0.47 (−0.67)	0.48 (−0.60)	0.43 (−1.06)	0.46 (−0.77)	0.42 (−1.17)
$AC[\Delta d]$	0.19	0.28 (0.87)	0.20 (0.07)	0.33 (1.28)	0.27 (0.72)	0.32 (1.24)	0.26 (0.65)
$AC[r_d]$	−0.01	0.00 (0.15)	−0.05 (−0.46)	0.00 (0.07)	−0.01 (0.00)	0.00 (0.07)	−0.01 (0.01)
$AC[r_f]$	0.68	0.95 (4.17)	0.83 (2.35)	0.94 (4.05)	0.69 (0.16)	0.94 (4.06)	0.70 (0.30)
$AC[z_d]$	0.89	0.93 (0.76)	0.88 (−0.11)	0.91 (0.51)	0.92 (0.70)	0.91 (0.52)	0.92 (0.71)
$Corr[\Delta c, \Delta d]$	0.54	0.48 (−0.27)	0.53 (−0.05)	0.43 (−0.52)	0.49 (−0.20)	0.44 (−0.49)	0.50 (−0.16)
$Corr[\Delta c, r_d]$	0.05	0.07 (0.31)	0.05 (0.08)	0.08 (0.53)	0.06 (0.24)	0.08 (0.53)	0.06 (0.21)
$Corr[\Delta d, r_d]$	0.07	0.24 (2.09)	0.18 (1.33)	0.28 (2.60)	0.23 (1.93)	0.28 (2.53)	0.22 (1.85)
$Corr[ep, z_{d,-1}]$	−0.16	−0.16 (0.00)	−0.13 (0.32)	−0.13 (0.37)	−0.17 (−0.03)	−0.13 (0.36)	−0.17 (−0.07)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.65 (2.62)	0.58 (2.20)	0.68 (2.81)	0.65 (2.60)	0.67 (2.74)	0.64 (2.55)

Table 5: Long-run risk model. Data and average model-implied moments. t-statistics are in parentheses.

However, the contribution of valuation risk is quite different across the various sets of moments. Recall that in the baseline model, valuation risk explains a sizable majority of the risk-free rate and equity premium.[30] In column 2, valuation risk has a smaller but still meaningful contribution (49.4% of the risk-free rate and 42.3% of the equity premium). In column 4, however, it explains

[30]The contribution of valuation risk under the current preferences is larger than under the revised preferences. In the model without the higher-order risk-free rate or term structure moments, valuation risk under the current preferences explains 91.5% of the risk-free rate and 91.1% of the equity premium. If only the term structure moments are excluded, valuation risk explains a smaller percentage but it is still bigger than with the revised preferences (31.2% and 18.7%).

very little of these moments (9.4% and 5.4%) because the model requires smaller and less persistent valuation risk shocks ($\rho_a = 0.9563$ and $\sigma_a = 0.0169$) to match the dynamics of the risk-free rate.

Finally, we turn to the yield curve. In columns 1 and 3, which exclude valuation risk and do not target longer-term risk-free rates, the presence of cash-flow risk generates a (counterfactual) downward sloping yield curve. This is because households in the model dislike long-run risks to cash-flow growth and longer-term risk-free bonds provide additional insurance against these risks. Valuation risk, however, generates a positive term premium for longer-term risk-free bonds because it creates the possibility that households will revalue future cash flows. A longer-term asset increases exposure to this risk. This results in a lower price and higher return for risk-free assets with a longer maturity, leading to an upward sloping yield curve. In columns 2 and 4, which add valuation risk, the yield curve is humped shaped due to the competing effects of the two risks.

The failure of the long-run risk model to predict an upward sloping yield curve is not resolved by targeting the yield curve moments. In column 5, which excludes valuation risk but targets the yield curve moments, the yield curve remains downward sloping. However, the entire curve is raised, resulting in a short-term risk free rate of 1.4%. The addition of valuation risk (column 6) improves the slope of the yield curve, lowering $E[r_f]$ by 19 basis points and raising $E[r_{f,20}]$ by 15 basis points. However, the constraints imposed by also targeting the standard deviation and autocorrelation of the risk-free rate limit the role of valuation risk in fully matching the yield curve.

These results show that valuation risk does not unilaterally resolve the risk-free rate and equity premium puzzles, but the improvements in fit show that it helps match the data. Despite these improvements, the long-run risk model with valuation risk still performs poorly on the three moments listed above as well as the yield curve. Furthermore, all six specifications fail to pass the over-identifying restrictions test at the 5% level. The next section addresses these shortcomings.

7 ESTIMATED EXTENDED LONG-RUN RISK MODEL

We consider two extensions to the long-run risk model. First, we allow valuation risk shocks to directly affect cash-flow growth, in addition to their effect on asset prices through the SDF (henceforth, the "Demand" shock model). This feature is similar to a discount factor shock in a production economy model. For example, in the workhorse New Keynesian model, an increase in the discount factor looks like a negative demand shock that lowers interest rates, inflation, and consumption.[31] Therefore, it provides another mechanism for valuation risk to help fit the data, especially the

[31] See, for example, Smets and Wouters (2003). However, without a carefully microfounded model, it is not clear whether $\varepsilon_{a,t+1}$ should be correlated with $\Delta\hat{y}_{t+1}$ or $\hat{x}_t$ (or both) and what restrictions should be placed on the shock coefficients. While there are limitations to using this reduced-form specification, it is very useful for informing what description of the shock processes best explain the data and for developing models with deeper microfoundations.

correlation moments. Following Albuquerque et al. (2016), we modify (35) and (36) as follows:

$$\Delta\hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_y\varepsilon_{y,t+1} + \pi_{ya}\sigma_a\varepsilon_{a,t+1}, \tag{38}$$

$$\Delta\hat{d}_{t+1} = \mu_d + \phi_d\hat{x}_t + \pi_{da}\sigma_a\varepsilon_{a,t+1}, \tag{39}$$

where π_{ya} and π_{da} control the covariances between valuation risk shocks and cash-flow growth.[32]

Second, we add stochastic volatility to cash-flow risk following Bansal and Yaron (2004) (henceforth, the "SV" model). SV introduces time-varying uncertainty. Bansal et al. (2016) show SV leads to a significant improvement in fit. An important question is therefore whether the presence of SV will affect the role of valuation risk. To introduce SV, we modify (35)-(37) as follows:

$$\Delta\hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_{y,t}\varepsilon_{y,t+1}, \tag{40}$$

$$\Delta\hat{d}_{t+1} = \mu_d + \phi_d\hat{x}_t + \pi_{dy}\sigma_{y,t}\varepsilon_{y,t+1} + \psi_d\sigma_{y,t}\varepsilon_{d,t+1}, \tag{41}$$

$$\hat{x}_{t+1} = \rho_x\hat{x}_t + \psi_x\sigma_{y,t}\varepsilon_{x,t+1}, \tag{42}$$

$$\sigma^2_{y,t+1} = \sigma^2_y + \rho_{\sigma_y}(\sigma^2_{y,t} - \sigma^2_y) + \nu_y\varepsilon_{\sigma_y,t+1}, \tag{43}$$

where ρ_{σ_y} is the persistence of the SV process and ν_y is the standard deviation of the SV shock.

Table 6 and Table 7 present estimates from three versions of the extended long-run risk model: (1) the SV model without valuation risk (columns 1 and 4), (2) the demand shock model (columns 2 and 5), and (3) the combination of the demand shock and SV models (columns 3 and 6). In each case, we report the results from including and excluding longer-term rates as targeted moments.

We begin with the models that exclude longer-term returns as targeted moments.[33] A key finding is that all three extensions improve on the p-values from the simpler long-run risk models in the previous section. Adding SV to the model without valuation risk increases the p-value from near zero (Table 4, column 3) to 0.02 (Table 6, column 1). The estimated SV process is very persistent ($\rho_{\sigma_y} = 0.9646$) and the shock is statistically significant, consistent with the literature. The improved fit largely occurs because SV helps match the higher-order risk-free rate moments (the standard deviation is 2.44 vs. 2.72 in the data and the autocorrelation is 0.69 vs. 0.68 in the data).

The Demand model increases the p-value from 0.012 (Table 4, column 4) to 0.098 (Table 6, column 2). Thus, the Demand model easily passes the over-identifying restrictions test at the 5% level. Consistent with the predictions of a production economy model, π_{ya} and π_{da} are negative in the estimation. More specifically, a positive valuation risk shock, which makes households more patient, reduces consumption and dividend growth. In a direct horse race between the SV model

[32] With the inclusion of π_{ya} and π_{da}, π_{dy} and ψ_d are redundant so we exclude them from the Demand specifications.

[33] The No VR+SV model is the same model BKY estimate. In that paper, the model passes the over-identifying restrictions test at the 5% level, while in our case it does not. The key difference is that BKY do not target the correlations between cash-flows and the equity return. When we exclude these moments, our p-value jumps to 0.15.

	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Ptr	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
γ	2.67 (2.61, 2.73)	3.39 (3.28, 3.49)	6.31 (5.84, 6.84)	1.25 (1.09, 1.46)	3.59 (3.46, 3.74)	8.51 (8.17, 8.87)
β	0.9983 (0.9982, 0.9983)	0.9991 (0.9990, 0.9991)	0.9981 (0.9980, 0.9982)	0.9982 (0.9981, 0.9983)	0.9987 (0.9987, 0.9988)	0.9976 (0.9976, 0.9977)
ρ_a	–	0.9608 (0.9600, 0.9615)	0.9933 (0.9931, 0.9935)	–	0.9630 (0.9622, 0.9637)	0.9934 (0.9933, 0.9936)
σ_a	–	0.0188 (0.0185, 0.0191)	0.0289 (0.0285, 0.0293)	–	0.0197 (0.0193, 0.0200)	0.0286 (0.0283, 0.0289)
μ_y	0.0016 (0.0015, 0.0017)	0.0015 (0.0015, 0.0016)	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0018)	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0016)
μ_d	0.0013 (0.0011, 0.0015)	0.0015 (0.0013, 0.0016)	0.0015 (0.0014, 0.0017)	0.0000 (0.0000, 0.0001)	0.0013 (0.0012, 0.0015)	0.0015 (0.0013, 0.0016)
σ_y	0.0003 (0.0002, 0.0003)	0.0040 (0.0039, 0.0040)	0.0000 (0.0000, 0.0001)	0.0002 (0.0000, 0.0004)	0.0036 (0.0036, 0.0036)	0.0000 (0.0000, 0.0000)
ψ_d	3.04 (2.97, 3.10)	–	–	2.79 (2.73, 2.86)	–	–
π_{dy}	0.788 (0.691, 0.881)	–	–	0.961 (0.858, 1.051)	–	–
ϕ_d	1.91 (1.86, 1.95)	2.76 (2.68, 2.83)	2.86 (2.78, 2.95)	1.69 (1.65, 1.72)	3.30 (3.21, 3.38)	2.97 (2.93, 3.01)
ρ_x	0.9989 (0.9988, 0.9990)	0.9972 (0.9970, 0.9974)	0.9955 (0.9953, 0.9958)	0.9995 (0.9995, 0.9995)	0.9967 (0.9965, 0.9968)	0.9953 (0.9951, 0.9955)
ψ_x	0.0270 (0.0263, 0.0277)	0.0309 (0.0303, 0.0315)	0.0376 (0.0367, 0.0386)	0.0258 (0.0250, 0.0265)	0.0309 (0.0303, 0.0315)	0.0375 (0.0366, 0.0384)
π_{ya}	–	−0.049 (−0.053, −0.046)	−0.049 (−0.051, −0.047)	–	−0.033 (−0.036, −0.030)	−0.044 (−0.046, −0.042)
π_{da}	–	−1.021 (−1.036, −1.007)	−0.864 (−0.876, −0.853)	–	−0.997 (−1.010, −0.983)	−0.884 (−0.896, −0.873)
ρ_{σ_y}	0.9646 (0.9633, 0.9658)	–	0.8324 (0.7939, 0.8606)	0.9609 (0.9577, 0.9638)	–	0.5861 (0.5455, 0.6234)
ν_y	1.2e−5 (1.1e−5, 1.2e−5)	–	2.2e−5 (2.0e−5, 2.4e−5)	1.3e−5 (1.2e−5, 1.4e−5)	–	3.3e−5 (3.2e−5, 3.4e−5)
J	18.25 (17.76, 18.75)	13.43 (13.03, 13.85)	8.88 (8.64, 9.13)	25.30 (24.61, 26.08)	18.27 (17.90, 18.66)	9.72 (9.41, 10.03)
pval	0.020 (0.016, 0.023)	0.098 (0.086, 0.111)	0.180 (0.166, 0.195)	0.005 (0.004, 0.006)	0.051 (0.045, 0.057)	0.285 (0.263, 0.309)
df	8	8	6	10	10	8

Table 6: Extended long-run risk models. Average and $(5, 95)$ percentiles of the parameter estimates. The IES is 2.5.

and the Demand model, which have the same number of parameters, the Demand model wins. The superior fit of the Demand model comes from the fact that it better matches the high volatility of dividend growth and the low correlation between dividend growth and equity returns. The model is better able to match these moments because the volatility of dividend growth increases with π_{da} while partially offsetting the positive relationship between valuation risk and the return on equity.

The Demand+SV model (column 3) raises the p-value to 0.18, passing the over-identifying restrictions test at the 10% level. This result reveals that the two extensions to the long-run risk model are complements, rather than substitutes, which is not obvious *a priori* because both features help match risk-free rate dynamics. It also occurs even though the two additional parameters in the model reduce the degrees of freedom and the critical value for the over-identifying restrictions test.

		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Moment	Data	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
$E[\Delta c]$	1.89	1.91 (0.07)	1.85 (−0.14)	1.89 (0.01)	1.96 (0.27)	1.89 (0.02)	1.91 (0.10)
$E[\Delta d]$	1.47	1.54 (0.07)	1.75 (0.28)	1.83 (0.38)	0.02 (−1.51)	1.59 (0.12)	1.78 (0.32)
$E[z_d]$	3.42	3.42 (−0.05)	3.40 (−0.14)	3.40 (−0.19)	3.52 (0.69)	3.41 (−0.06)	3.40 (−0.17)
$E[r_d]$	6.51	6.80 (0.18)	5.92 (−0.37)	5.75 (−0.48)	6.64 (0.08)	5.60 (−0.57)	5.70 (−0.51)
$E[r_f]$	0.25	0.08 (−0.28)	0.43 (0.29)	0.20 (−0.08)	0.80 (0.90)	1.21 (1.57)	0.31 (0.08)
$E[r_{f,5}]$	1.19	−0.81 (−2.96)	0.39 (−1.19)	0.55 (−0.95)	1.46 (0.39)	1.25 (0.08)	1.42 (0.34)
$E[r_{f,20}]$	1.88	−2.24 (−6.84)	−0.02 (−3.16)	0.39 (−2.46)	1.40 (−0.78)	0.97 (−1.51)	1.58 (−0.49)
$SD[\Delta c]$	1.99	2.07 (0.16)	1.99 (0.00)	2.08 (0.18)	2.12 (0.27)	1.76 (−0.49)	2.09 (0.20)
$SD[\Delta d]$	11.09	5.35 (−2.10)	7.71 (−1.24)	9.65 (−0.53)	5.07 (−2.20)	7.90 (−1.17)	9.83 (−0.46)
$SD[r_d]$	19.15	18.24 (−0.48)	18.03 (−0.59)	18.52 (−0.33)	17.17 (−1.04)	18.39 (−0.40)	18.39 (−0.40)
$SD[r_f]$	2.72	2.44 (−0.55)	2.99 (0.53)	2.70 (−0.04)	2.72 (−0.01)	3.06 (0.66)	2.64 (−0.17)
$SD[z_d]$	0.45	0.52 (1.16)	0.51 (0.94)	0.48 (0.52)	0.55 (1.66)	0.50 (0.80)	0.49 (0.66)
$AC[\Delta c]$	0.53	0.45 (−0.92)	0.43 (−1.07)	0.45 (−0.89)	0.45 (−0.93)	0.42 (−1.22)	0.45 (−0.90)
$AC[\Delta d]$	0.19	0.25 (0.56)	0.22 (0.28)	0.17 (−0.20)	0.23 (0.37)	0.23 (0.32)	0.18 (−0.14)
$AC[r_d]$	−0.01	−0.04 (−0.31)	0.02 (0.32)	−0.04 (−0.33)	0.03 (0.48)	0.01 (0.30)	−0.01 (0.02)
$AC[r_f]$	0.68	0.69 (0.09)	0.72 (0.51)	0.70 (0.24)	0.67 (−0.25)	0.72 (0.66)	0.71 (0.37)
$AC[z_d]$	0.89	0.91 (0.44)	0.91 (0.51)	0.89 (0.08)	0.93 (0.82)	0.91 (0.40)	0.90 (0.18)
$Corr[\Delta c, \Delta d]$	0.54	0.51 (−0.13)	0.48 (−0.27)	0.52 (−0.08)	0.54 (0.03)	0.45 (−0.43)	0.50 (−0.16)
$Corr[\Delta c, r_d]$	0.05	0.06 (0.20)	0.10 (0.67)	0.11 (0.84)	0.05 (−0.01)	0.10 (0.71)	0.11 (0.91)
$Corr[\Delta d, r_d]$	0.07	0.22 (1.77)	0.14 (0.80)	0.06 (−0.08)	0.20 (1.61)	0.13 (0.76)	0.06 (−0.14)
$Corr[ep, z_{d,-1}]$	−0.16	−0.23 (−0.64)	−0.13 (0.37)	−0.12 (0.44)	−0.23 (−0.64)	−0.12 (0.47)	−0.11 (0.61)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.65 (2.60)	0.65 (2.62)	0.62 (2.42)	0.66 (2.65)	0.64 (2.56)	0.61 (2.42)

Table 7: Extended long-run risk models. Data and average model-implied moments. t-statistics are in parentheses.

The model continues to fail on one key moment: the predictability of consumption growth given the price dividend ratio (i.e., $Corr[\Delta c, z_{d,-1}]$) remains too high (0.62 vs. 0.19 in the data). The overall improvement in fit occurs because the Demand+SV model does a much better job matching dividend growth dynamics. Specifically, it better matches the standard deviation of dividend growth (9.65 vs. 11.09 in the data) and the weak correlation between dividend growth and equity returns (0.06 vs. 0.07 in the data). In this model, valuation risk has a bigger role than in the Demand model ($\rho_a = 0.993$ vs. $\rho_a = 0.961$; $\sigma_a = 0.0289$ vs. $\sigma_a = 0.0188$), while the SV process is not as persistent ($\rho_{\sigma_y} = 0.832$ vs. $\rho_{\sigma_y} = 0.965$) as in the No VR+SV model. Also, σ_y is significantly smaller, so the contribution of consumption growth volatility from pure endowment risk is smaller when compared to the Demand model. The Demand model has trouble matching dividend growth dynamics while simultaneously matching risk-free rate dynamics. An expanded role of valuation risk is crucial for matching dividend growth dynamics. Without SV, this is not possible because it would cause the model to miss on the risk-free rate dynamics. Introducing SV, however, permits a lower σ_y, which helps offset the effect of valuation risk on the risk-free rate dynamics.

In terms of the yield curve, the No VR+SV and Demand models both improve on this dimension. Once the long-term rates are targeted, the yield curve slope (i.e., $E[r_{f,20}] - E[r_f]$) rises from -2.3% to 0.6% with the No VR+SV model (column 4) and from -0.4% to -0.2% with the Demand model (column 5). However, in both cases, the yield curve is hump-shaped and the addition of the yield curve moments decreases the p-values. The Demand+SV model performs the best. The p-value rises from 0.18% (column 3) to 0.29% (column 6) and the yield curve is no longer hump-shaped. All three yield curve moments are insignificantly different from their data counterparts.

8 CONCLUSION

Although valuation risk has become the subject of a substantial body of research to address asset pricing puzzles, the literature has ignored the full implications of the current preference specification. This paper first documents four desirable properties of Epstein-Zin recursive preferences without valuation risk. It then shows the current valuation risk specification is at odds with these properties because the distributional weights in the time-aggregator of the utility function do not sum to 1. In contrast, our revised preferences, which restrict the distributional weights, satisfy all four properties. These results caution against using the current specification.

Under our revised preferences, valuation risk has a much smaller role in resolving the equity premium and risk-free rate puzzles. However, we find valuation risk still plays an important role in matching the standard deviation and autocorrelation of the risk-free rate as well as the yield curve. Furthermore, allowing valuation risk to directly affect cash-flow growth, similar to a production economy model, adds a source of volatility that significantly improves the empirical fit of the model and helps match the standard deviation of dividend growth and its correlation with equity returns.

Despite the importance of valuation risk, our paper and the literature is silent on its structural foundations. As a consequence, there are several open research questions. For example, what does it mean for a representative household to have a time-varying time-preference? Is there an economy with multiple (heterogenous) households that supports these preferences? Is there a decision-theoretic explanation and is it possible to back out the dynamics of a time-varying time-preference from experiments or data? We believe these questions are important avenues for future research.

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A DATA SOURCES

We drew from the following data sources to estimate our models:

1. [$RCONS$] **Per Capita Real PCE (excluding durables)**: Annual, chained 2012 dollars. Source: Bureau of Economic Analysis, National Income and Product Accounts, Table 7.1.

2. [$RETD$] **Value-Weighted Return (including dividends)**: Monthly. Source: Wharton Research Data Services, CRSP Stock Market Indexes (CRSP ID: VWRETD).
3. [$RETX$] **Value-Weighted Return (excluding dividends)**: Monthly. Source: Wharton Research Data Services, CRSP Stock Market Indexes (CRSP ID: VWRETX).
4. [CPI] **Consumer Price Index for All Urban Consumers**: Monthly, not seasonally adjusted, index 1982-1984=100. Source: Bureau of Labor Statistics (FRED ID: CPIAUCNS).
5. [RFR] **Risk-free Rate**: Monthly, annualized yield calculated from nominal price. Source: Wharton Research Data Services, CRSP Treasuries, Risk-free Series (CRSP ID: TMYTM).
6. [$RFR5$] **5-year U.S. Treasury Yield**: Monthly, intermediate-term, annualized. Source: Ibbotson Associates via Morningstar Direct, IA SBBI US IT (ID: FOUSA05XQC).
7. [$RFR20$] **20-year U.S. Treasury Yield**: Monthly, long-term, annualized. Source: Ibbotson Associates via Morningstar Direct, IA SBBI US LT (ID: FOUSA05XQ8).

We applied the following transformations to the above data sources:

1. **Annual Per Capita Real Consumption Growth (annual frequency)**:

$$\Delta\hat{c}_t = 100\log(RCONS_t/RCONS_{t-1})$$

2. **Annual Real Dividend Growth (monthly frequency)**:

$$P_{1928M1} = 100, \quad P_t = P_{t-1}(1 + RETX_t), \quad D_t = (RETD_t - RETX_t)P_{t-1},$$
$$d_t = \textstyle\sum_{i=t-11}^{t} D_i/CPI_t, \quad \Delta\hat{d}_t = 100\log(d_t/d_{t-12})$$

3. **Annual Real Equity Return (monthly frequency)**:

$$\pi_t^m = \log(CPI_t/CPI_{t-1}), \quad \hat{r}_{d,t} = 100\textstyle\sum_{i=t-11}^{t}(\log(1 + RETD_i) - \pi_i^m)$$

4. **Annual Real Risk-free Rate (monthly frequency)**:

$$rfr_t = RFR_t - \log(CPI_{t+3}/CPI_t), \quad \pi_t^q = \log(CPI_t/CPI_{t-12})/4,$$
$$\hat{r}_{f,t} = 400(\hat{\beta}_0 + \hat{\beta}_1 RFR_t + \hat{\beta}_2\pi_t^q),$$

where $\hat{\beta}_j$ are OLS estimates from regressing the quarterly *ex-post* real rate, rfr, on the quarterly nominal rate, RFR, and inflation, π^q. The fitted values estimate the *ex-ante* real rate.

5. **5- and 20-year Real Risk-free Rate (monthly frequency)**:

$$rfrX_t = RFRX_t - \log(CPI_{t+12}/CPI_t), \quad \pi_t^a = \log(CPI_t/CPI_{t-12}),$$
$$\hat{r}_{f,X,t} = 100(\hat{\beta}_0 + \hat{\beta}_1 RFRX_t + \hat{\beta}_2 \pi_t^a),$$

where $\hat{\beta}_j$ are the OLS estimates from regressing the annual *ex-post* real long-term rate, $rfr5$ or $rfr20$, on the annual nominal rate, $RFR5$ or $RFR20$, and inflation, π^a. The fitted values estimate the *ex-ante* real long-term rate. The *ex-ante* real rates are similar if they are constructed by regressing on average annual inflation over the maturity of the bond. We opted to use 12-month ahead inflation rates to maintain our balanced sample that extends to 2017.

6. **Price-Dividend Ratio (monthly frequency)**:

$$\hat{z}_{d,t} = \log(P_t / \textstyle\sum_{i=t-11}^{t} D_i)$$

We use December of each year to convert each of the monthly time series to an annual frequency.