IASSNS-HEP-94/114
hep-ph/9501288

$Z_2 \times Z_2$ orbifold compactification – the origin of realistic free fermionic models

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ABSTRACT

All the realistic free fermionic models utilize a set of basis vectors, the NAHE set, that correspond to $Z_2 \times Z_2$ orbifold compactification with nontrivial background fields. I argue that the realistic features of free fermionic models, like the number of generations and the fermion mass spectrum are due to the underlying $Z_2 \times Z_2$ orbifold compactification.

As a unified theory of gravity and the gauge interactions, heterotic string theory[1] should reproduce the matter and symmetry content of the Standard Model and determine the fermion mass spectrum. Presently we do not know what is the dynamical mechanism that selects the unique string vacuum, and, a priori, there is a large number of potentially viable superstring models.

The notion, however, that there is a huge number of string models is somewhat misleading. By just imposing one or two phenomenological criteria, like three generations and a gauge group that can be reduced at low energies to the standard model gauge group, already one finds that the number of possibilities is substantially reduced. Imposing further phenomenological constraints may indeed single out a unique superstring model. If such a model is constructed, it will certainly be of use in trying to learn about the dynamical mechanism that chooses the string vacuum.

The task of constructing phenomenologically viable string models seems hopeless. While in ten dimensions the string vacuum is more or less unique, in four dimensions there is a huge number of equivalent candidates. The string consistency constraints impose a number of degrees of freedom and those degrees of freedom produce a symmetry that is larger than the observed symmetry at low energies. Furthermore the number of chiral generations is also determined in the four dimensional vacuum and is correlated with the gauge degrees of freedom. A bottom–up approach, in which different blocks of the standard model are assembled together piece by piece, is not adequate. Rather, what is required is a top–bottom approach in which the features of the standard model are carved out of the more symmetric string vacua.

To appear in Proceedings of the Beyond the Standard Model IV Conference, Lake Tahoe, California, December 13–18, 1994.

Is there a guiding principle that may distinguish among the equivalent string vacuum ? String vacua exhibits a new kind of symmetry, usually referred to as target–space duality[2], which is a generalization of the $R \to 1/R$ duality in the case of S^1. At the self–dual point, $R_j = 1/R_j$, space–time symmetries are enhanced. For appropriate choices of the background fields the space–time symmetries are maximally enhanced[3]. At the maximally symmetric point the internal degrees of freedom that are needed to cancel the conformal anomaly may be represented in terms of internal free fermions propagating on the string world–sheet. It is plausible that if string theory is realized in nature then the true string vacuum is in the vicinity of the highly symmetric self–dual point. It may turn out that near that point the free fermionic formulation[4] provides a good approximation to the true string vacuum. However, the number of consistent free fermionic models is still enormous.

As is well known in (2,2) string models that admit a geometrical interpretation the number of chiral generations is half the Euler number of the six dimensional compactified manifold. Following LEP data it is plausible to assume that only three complete generations exist in nature. How can three generations arise from a six dimensional compactified space. The answer may be simple. The six dimensional compactified space is divided into three factors of two. In the orbifold language[5], divide the six dimensional space, which is compactified on a flat torus, by a $Z_2 \times Z_2$ discrete symmetry. In that case the $Z_2 \times Z_2$ orbifold model produces exactly three twisted sectors. In the $Z_2 \times Z_2$ orbifold on a six dimensional space, the number three is deeply rooted in the structure of the models. Thus, the $Z_2 \times Z_2$ can very naturally lead to models with three generations. Namely, each light generation comes from a different twisted sector of the $Z_2 \times Z_2$ orbifold model.

It appears that $Z_2 \times Z_2$ orbifold on the flat torus of the six dimensional compactified space, can very naturally lead to three generations. However, in general, $Z_2 \times Z_2$ orbifold on generic lattices do not lead to three generation models. For example the $Z_2 \times Z_2$ orbifold on $SO(4)^3$ lattice did not yield three generation models. In contrast, the $Z_2 \times Z_2$ models at the free fermionic point in toroidal compactification space, realized by the NAHE set[6,7], do produce three generation models. The difference is seen by examining the number of fixed points in the two compactifications with (2,2) world–sheet supersymmetry. On the $SO(4)^3$ lattice the $Z_2 \times Z_2$ produces sixteen generations, from each twisted sector. On the $SO(12)$ lattice, which corresponds to the free fermionic point in the toroidal compactification space, it produces eight chiral generations, from each twisted sector. In the fermionic three generation constructions each one of the complex planes of the $Z_2 \times Z_2$ orbifold is modded out by additional Z_2^3 symmetries, thus reducing the number of generations to one generation from each twisted sector.

In the (2,2) fermionic constructions one starts from a set of boundary condition vectors that produces an $N = 4$ supersymmetric model with $SO(12) \times E_8 \times E_8$ gauge group[3]. One then adds two boundary condition vectors that correspond to the

$Z_2 \times Z_2$ twisting. The resulting gauge group is $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ with $N = 1$ space–time supersymmetry. In this model there are twenty four chiral generations from the boundary condition vectors that correspond to twisted sectors and three pairs of chiral and anti chiral generations from the untwisted sector. The number of twisted and untwisted moduli is equal to the number of generations. In addition the untwisted and twisted sectors produce $E_6 \times E_8$ singlets that are obtained by acting on the vacuum with oscillators that arise from the fermionic degrees of freedom that correspond to the six internal compactified dimensions.

In the orbifold formulation[5] the same model is obtained by applying a $Z_2 \times Z_2$ twist to a torodialy compactified $SO(12)$ lattice and $E_8 \times E_8$ gauge symmetry. The 36 free parameters of the six dimensional metric and the antisymmetric tensor field parameterize the six dimensional compactified space. For generic values of these parameters the gauge symmetry that arises from the six dimensional compactified torus is $U(1)^6$. For specific choices of the background parameters the $U(1)^6$ of the compactified torus is enlarged. To reproduce the $SO(12) \times E_8 \times E_8$ gauge group of the free fermionic model, the metric G_{ij} is the Cartan matrix of $SO(12)$ and the antisymmetric tensor field is given by, $B_{ij} = G_{ij}$ for $i > j$; $B_{ij} = 0$ for $i = j$ and $B_{ij} = -G_{ij}$ for $i < j$. For $R_I = \sqrt{2}$ and with the chosen background fields, the right–moving momenta produce the root vectors of $SO(12)$, thus reproducing the same gauge group as in the free fermionic model. The orbifold model is obtained by moding out the six dimensional torus by a discrete symmetry group. The massless spectrum contains states from the untwisted and twisted sectors. In the case of "standard embedding" the number of chiral families is given by one half of the Euler characteristic. To translate the fermionic boundary conditions to twists and shifts in the bosonic formulation the real fermionic degrees of freedom that correspond to the compactified dimensions are bosonized. The fermionic boundary condition vectors, b_1 and b_2, then translate to $Z_2 \times Z_2$ twist on the compactified coordinates and to shifts on the gauge degrees of freedom. It is then seen that symmetries and spectrum of the orbifold model coincide with those of the corresponding fermionic model[3].

The realistic free fermionic models correspond to models with (2,0), rather than (2,2), world–sheet supersymmetry. The transition from the (2,2) models to the (2,0) models can be regarded as choosing a GSO phase between the two boundary condition vectors that produce the spinorial of $SO(16)$. The GSO projection projects out the massless states from these sectors and the resulting gauge group is $SO(12) \times SO(16) \times SO(16)$, with $N = 4$ space–time supersymmetry. Alternatively, one of the spinorial vectors may be enlarged with additional four periodic complex fermions in the hidden sector. The $E_8 \times E_8$ gauge group is modified to $SO(16) \times SO(16)$, as in the first construction. The analysis with respect to the number of fixed points is identical to the case with (2,2) world–sheet supersymmetry. However, in this case the observable gauge group after applying the $Z_2 \times Z_2$ is $SO(10) \times U(1)$ rather than E_6, and the $U(1)$ is "anomalous". The twisted sectors produce spinorial and vectorial sixteen of the observable $SO(10)$ and hidden $SO(16)$ gauge groups, respectively.

The structure of the $Z_2 \times Z_2$ orbifold with (2,0) world–sheet supersymmetry and standard embedding, is common to all the realistic free fermionic models. Three generation models are obtained by adding three additional boundary condition basis vectors, beyond the NAHE set. The additional boundary condition vectors mod out each of the three complex planes of the $Z_2 \times Z_2$ orbifold by a Z_2^3 symmetry and break the observable $SO(10)$ symmetry to one of its maximal subgroups $SU(5) \times U(1)$, $SO(6) \times SO(4)$ or $SU(3) \times SU(2) \times U(1)^2$.

The fermion mass spectrum is also seen to originate from the $Z_2 \times Z_2$ orbifold structure, realized by the NAHE set. The untwisted sector produces three pairs of Higgs doublets and a combination of the vectors that break the $SO(10)$ symmetry produces one or two additional pairs. Due to the horizontal symmetries in the $Z_2 \times Z_2$ orbifold models, each pair of Higgs doublets couples only to states from one of the twisted sectors, producing couplings $16_j 16_j 10_j$ $j = 1, 2, 3$. The cancelation of the anomalous $U(1)$ D–term equation by singlet VEVs, gives Planck scale mass to several Higgs doublets. As a result, there exist models in which only one mass term, namely the top quark mass term, exist at the cubic level of the superpotential. The mass terms for the lighter quarks and leptons are obtained from nonrenormalizable terms. The nonrenormalizable terms contain $SO(10)$ singlets with nonvanishing VEVs, that are required to cancel the anomalous $U(1)$ D–term equation. Thus, the nonrenormalizable terms become effective renormalizable terms that are suppressed relative to the leading cubic level terms. Due to the horizontal symmetries and the singlet VEVs one generation is necessarily light[8]. Similarly, the mixing terms arise generically from nonrenormalizable terms of the form $16_i 16_j 10 16_i 16_j \phi^n$, where the first two 16 are in the spinorial representation of the observable $SO(10)$, the 10 is in the vector representation of the observable $SO(10)$, the last two 16 are in the vector representation of the hidden $SO(16)$ and ϕ^n is a combination of $SO(10) \times SO(16)$ scalar singlets[9,3]. The $Z_2 \times Z_2$ orbifold structure gives rise to the horizontal symmetries that may be needed to understand the matter mass spectrum. Requiring adequate generation mixing and the form of the mixing terms necessitates that we give nonvanishing VEVs to some of the hidden sector 16 representations. In Ref. [10] it was shown that this is possibly the source of supersymmetry breaking in these models.

Acknowledgements

This work is supported by DOE grant DE-FG02-90ER40542.

References

1. D.J. Gross *et. al.*, Nucl.Phys. B**256** (1985) 253.
2. A. Giveon *et. al.*, Phys.Rep. **244** (1994) 77, and references therein.
3. A.E. Faraggi, Phys.Lett. B**326** (1994) 62, hep-ph/9311312..
4. I. Antoniadis, C. Bachas, and C. Kounnas, Nucl.Phys. B**289** (1987) 87; H. Kawai, D.C. Lewellen, and S.H.-H. Tye, Nucl.Phys. B**288** (1987) 1.
5. L. Dixon, J.A. Harvey, C. Vafa and E. Witten, Nucl.Phys. B**274** (1986) 285.
6. I. Antoniadis *et. al.*, Phys.Lett. B**231** (1989) 65.
7. A.E. Faraggi, Nucl.Phys. B**387** (1992) 239, hep-th/9208024.

8. A.E. Faraggi, Nucl.Phys. B**403** (1993) 101, hep-th/9208023; Nucl.Phys. B**407** (1993) 57, hep-ph/9210256.
9. A.E. Faraggi and E.Halyo, Phys.Lett. B**307** (1993) 305, hep-ph/9301261; Nucl.Phys. B**416** (1994) 63, hep-ph/9306235.
10. A.E. Faraggi and E.Halyo, IASSNS–HEP–94/17, hep-ph/9405223.